Unitary Linear Dispersion Code Design and Optimisation for MIMO Communication Systems

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Abstract

Linear Dispersion Codes (LDCs) have recently attracted numerous research interests. Thanks to their efficient spreading of data across both the time and spatial domains, LDCs are capable of achieving a desired Diversity-Multiplexing Trade-off (DMT) in Multiple Input Multiple Output (MIMO) broadband wireless access systems. This paper proposes a novel LDC design method, which relies on the unitary matrix theory combined with a Genetic Algorithm (GA) aided optimisation procedure. The proposed design provides a flexible framework, where new LDCs attaining higher data rates and better error resilience than a number of classic MIMO schemes can be generated.

Index Terms

Diversity-multiplexing trade-off, genetic algorithm, multiple-input multiple-output, linear dispersion code.

I. Introduction

THE Diversity-Multiplexing Trade-off (DMT) represents a compromise between the achievable data rate (link throughput) and diversity gain (error resilience) in Broadband Wireless Access (BWA) systems. Among other techniques, the family of Linear Dispersion Codes (LDCs) [1], [2] was found to be capable of achieving a flexible DMT, thanks to their capability of exploiting both spatial and time (or frequency) diversity.

The concept of LDCs [1] introduces a generalised Multiple-Input Multiple-Output (MIMO) framework, which subsumes the Vertical Bell Labs Layered Space-Time (V-BLAST) [3] and Spatial Multiplexing (SM) type techniques, as well as a wide range of Space-Time Codes (STC), such as Alamouti's [4] and Tarokh's codes [5]. At the receiver side, the different LDCs can be decoded by the same set of decoders as those designed for the conventional STCs. These include the Minimum Mean Square Error Decoder (MMSED), the Maximum Likelihood Decoder (MLD), the Sphere Decoder (SD) and other BLAST-type decoders.

Conventional LDC design methods include the classic constructions [4], [5], gradient-based search algorithms [1], [6], [7], the frame theory [2] and the algebraic theory [8]–[10]. Design criteria such as maximising the ergodic

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channel capacity or Mutual Information (MI) [1], minimising the Pair-wise Error Probability (PEP) [5] and the Block Error Probability (BLEP) [7], etc. are often used for optimising LDCs [11]. Since analytical expressions of the above-mentioned criteria are not always available, numerical methods, many of which employ gradient-based search algorithms [1], [6], [7] have been found to be more flexible and effective for a wide range of scenarios. However, most of these methods may converge to a local minimum only [7].

In contrast to the above design options, the novelty of this paper is that we devise a new design approach using Genetic Algorithms (GA), where a specific unitary matrix transform module is employed. Unitary LDCs have been shown to be asymptotically optimal [6]. *To the best of our knowledge, the proposed novel LDC design framework appears to be the first GA-aided technique, which is capable of exploring the entire space of unitary matrices according to the unique Haar measure, where optimised unitary LDCs exhibiting a better performance than many of their existing counterparts can be found.*

GAs constitute powerful global optimisation methods that are efficient in solving complex non-linear optimisation problems [12]. We have demonstrated that with the aid of appropriate operators, GAs are capable of conducting a 'guided', rather than a purely random search [13], thus distinguishing them from many random optimisation methods. Constrained by the LDC optimisation problem, the proposed GA exploiting a non-binary matrix encoding method increases the search granularity, thus providing a higher degree of freedom in our design. We also design a new hybrid non-binary mutation operator constituted by the joint modulo and polarity-flipping controller. Thanks to the flexibility of the new design framework, it is also possible to produce LDCs with various dimensions for both square and non-square encoding matrices.

The organisation of this paper is as follows. A brief review of LDCs is provided in Section II-A, followed by our elaboration on the unitary matrix transform module in Section II-B. The proposed GA-aided optimisation procedure is described in Section II-C. Finally, we present our simulation results and conclusions in Sections III and IV, respectively.

II. Proposed LDC Design Framework

A. Linear Dispersion Codes

An LDC is fully specified, given a set of so-called dispersion matrices or encoding matrices, which can have real or complex valued entries [1]. Let M and N denote the number of transmit and receive antennas, respectively. The $(T \times M)$ LDC codeword matrix $\mathbf{S}$, where T is the number of time slots in each codeword, is defined by:

$$\mathbf{S} = \sum_{q=1}^{Q} (\alpha_q \mathbf{A}_q + j\beta_q \mathbf{B}_q), \tag{1}$$

where $\mathbf{A}_q$ and $\mathbf{B}_q$ are the set of dispersion matrices. The set of $\{\alpha_q, \beta_q\}$ is defined by $s_q = \alpha_q + j\beta_q$, where s_q is a complex symbol from a given m-PSK or m-QAM constellation. In the quasi-static $(M \times N)$ MIMO system model, we have:

$$\mathbf{X} = \sqrt{\frac{\rho}{M}}\mathbf{SH} + \mathbf{V}, \tag{2}$$

where $\mathbf{X}$ is the $(T \times N)$ received signal matrix, $\mathbf{H}$ is the $(M \times N)$ channel matrix which is assumed to be time-invariant during the duration T, $\mathbf{V}$ is the $(T \times N)$ Additive White Gaussian Noise (AWGN) matrix whose entries are zero-mean, unit-variance, complex-Gaussian variables, and ρ is the Signal-to-Noise Ratio (SNR) at each of the N receive antennas, which is independent of M. The effective data rate is defined as $R = R_c \cdot \log_2(m)$, where $R_c = \frac{Q}{T}$ is the LDC's code rate.

B. Matrix Unitarisation

Our objective is to devise a specific design framework that produces the set of unitary dispersion matrices $\mathbf{A}_q$ and $\mathbf{B}_q$ based on a given criterion. In order to find good unitary LDCs, it is beneficial to explore the entire unbiased unitary space under specific LDC design constraints. The LDC matrix unitarisation method employed in our design framework of Fig. 1 is based on the random unitary matrix theory [14]–[16]. Note that the unitary matrices' elements are not independent random variables, making the numerical generation of such random matrices complicated [16]. However, Hurwitz [15] proposed a convenient method to parameterise the space of unitary matrices by K^2 independent parameters, where K is the dimension of the matrices concerned. Provided random Euler angles as input, the method forms elementary unitary transformations in two-dimensional subspaces to create an arbitrary unitary transformation. Combined with the GA-based search method of Section II-C, optimised unitary LDCs may be found spanning the entire unbiased space of unitary matrices, rather than spanning only a fraction of it. The proposed procedure is highlighted below.

Let $K = \max(T, M)$. We carry out the following steps to construct a unitary LDC:

Step 1: Initialise the MIMO and LDC parameters, namely M, N, T, Q, ρ, etc. Generate an external control matrix $\mathbf{\Lambda}$:

$$\mathbf{\Lambda} = [\mathbf{\Lambda}_1, \mathbf{\Lambda}_2, \cdots, \mathbf{\Lambda}_{2Q}], \tag{3}$$

where the q-th column vector $\mathbf{\Lambda}_q$ is defined as $\mathbf{\Lambda}_q = [\psi_q^{(r,s)}, \chi_q^{(s)}, \xi_q^{(r,s)}, \alpha_q]^T$, and $\psi_q^{(r,s)}, \chi_q^{(s)}, \alpha_q, \xi_q^{(r,s)}$ $(q = 1, \cdots, 2Q, 0 \leq r < s \leq K-1)$ are real numbers within the predefined ranges:

$$\psi_q^{(r,s)} \in [0, 2\pi), \chi_q^{(s)} \in [0, 2\pi), \xi_q^{(r,s)} \in [0, 1), \alpha_q \in [0, 2\pi). \tag{4}$$

It is worth pointing out that for all legitimate combinations of $\psi_q^{(r,s)}, \chi_q^{(s)}, \xi_q^{(r,s)}, \alpha_q$ in (4), the ensemble of the

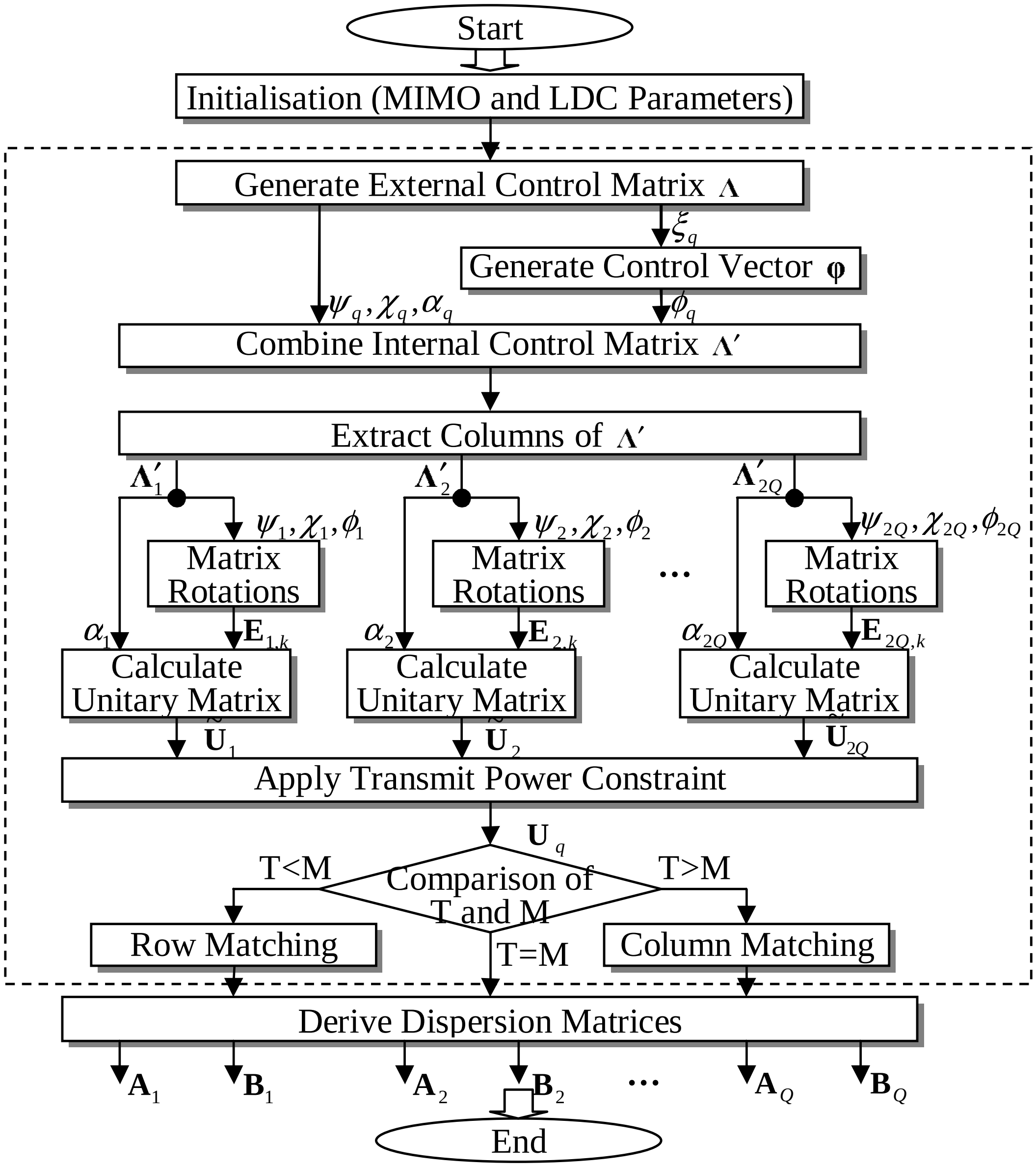

Fig. 1. The flowchart of the LDC matrix unitarisation process.

resultant unitary dispersion matrices will constitute the full unitary matrix space according to the unique Haar measure [16]. We will discuss how to initialise $\mathbf{\Lambda}$ in Section II-C.

Step 2: Calculate a control vector $\mathbf{\Phi} = [\phi_1^{(r,s)}, \phi_2^{(r,s)}, \cdots, \phi_{2Q}^{(r,s)}],\ \phi_q^{(r,s)} = \arcsin[(\xi_q^{(r,s)})^{1/(2r+2)}]$, where $\xi_q^{(r,s)}$ is defined by (4). Then, from $\mathbf{\Lambda}$ and $\mathbf{\Phi}$, create an internal control matrix $\mathbf{\Lambda}' = [\mathbf{\Lambda}'_1, \mathbf{\Lambda}'_2, \cdots, \mathbf{\Lambda}'_{2Q}]$, where $\mathbf{\Lambda}'_q = [\psi_q^{(r,s)}, \chi_q^{(s)}, \phi_q^{(r,s)}, \alpha_q]^T,\ q = 1, \cdots, 2Q,\ 0 \leq r < s \leq K-1$.

Step 3: Conduct unitary transformation according to the random unitary matrix theory [14], [16]. First, for each column vector $\mathbf{\Lambda'}_q$, construct $(K-1)$ number of rotated unitary matrices $\mathbf{E}_{q,k}, k = 1, \cdots, K-1$:

$$\begin{aligned} &\mathbf{E}_{q,1} = \mathbf{E}_q^{(K-1,K)}(\phi_q^{(0,1)}, \psi_q^{(0,1)}, \chi_q^{(1)}) \\ &\mathbf{E}_{q,2} = \mathbf{E}_q^{(K-2,K-1)}(\phi_q^{(1,2)}, \psi_q^{(1,2)}, 0) \cdot \\ &\quad \mathbf{E}_q^{(K-1,K)}(\phi_q^{(0,2)}, \psi_q^{(0,2)}, \chi_q^{(2)}) \\ &\quad \vdots \\ &\mathbf{E}_{q,K-1} = \mathbf{E}_q^{(1,2)}(\phi_q^{(K-2,K-1)}, \psi_q^{(K-2,K-1)}, 0) \cdot \\ &\quad \mathbf{E}_q^{(2,3)}(\phi_q^{(K-3,K-1)}, \psi_q^{(K-3,K-1)}, 0) \cdot \\ &\quad \cdots \mathbf{E}_q^{(K-1,K)}(\phi_q^{(0,K-1)}, \psi_q^{(0,K-1)}, \chi_q^{(K-1)}) \end{aligned} \tag{5}$$

where the set of so-called elementary unitary matrices $\mathbf{E}_q^{(r,s)}(\phi_q^{(r,s)}, \psi_q^{(r,s)}, \chi_q^{(s)})$ are given by [16]:

$$\mathbf{E}_q^{(r,s)} = \begin{bmatrix} E_{q,11}^{(r,s)} & E_{q,12}^{(r,s)} & \cdots & E_{q,1K}^{(r,s)} \\ E_{q,21}^{(r,s)} & E_{q,22}^{(r,s)} & \cdots & E_{q,2K}^{(r,s)} \\ \vdots & \vdots & \ddots & \vdots \\ E_{q,K1}^{(r,s)} & E_{q,K2}^{(r,s)} & \cdots & E_{q,KK}^{(r,s)} \end{bmatrix}, \tag{6}$$

where the non-zero elements are defined by: $E_{q,kk}^{(r,s)} = 1 \ (k = 1, \cdots, K; k \neq r, s)$, $E_{q,rr}^{(r,s)} = \cos\phi_q^{(r,s)} \cdot \exp(j\psi_q^{(r,s)})$, $E_{q,rs}^{(r,s)} = \sin\phi_q^{(r,s)} \cdot \exp(j\chi_q^{(s)})$, $E_{q,sr}^{(r,s)} = -\sin\phi_q^{(r,s)} \cdot \exp(-j\chi_q^{(s)})$, $E_{q,ss}^{(r,s)} = \cos\phi_q^{(r,s)} \cdot \exp(-j\psi_q^{(r,s)})$. Then, the $2Q$ unitary matrices $\tilde{\mathbf{U}}_q$ can be calculated as [16]:

$$\tilde{\mathbf{U}}_q = \exp(j\alpha_q) \cdot \mathbf{E}_{q,1} \cdot \mathbf{E}_{q,2} \cdots \mathbf{E}_{q,K-1}, q = 1, \cdots, 2Q. \tag{7}$$

Apply the transmit power constraint such that $\mathbf{U}_q = \sqrt{\frac{K}{Q}} \cdot \tilde{\mathbf{U}}_q$.

Step 4: Derive the LDC dispersion matrices based on the associated dimension requirement. If $T = M$, we have:

$$\begin{cases} \mathbf{A}_q = \mathbf{U}_q, q = 1, \cdots, Q \\ \mathbf{B}_q = \mathbf{U}_q, q = Q+1, \cdots, 2Q \end{cases}. \tag{8}$$

If $T < M$, the dispersion matrices are created by selecting T out of the K rows in the matrix $\mathbf{U}_q$ to form a new $(T \times M)$ matrix. The process for the case of $T > M$ is similar.

C. Genetic Algorithm aided LDC Optimisation

The algorithm described in Section II-B provides a convenient way of generating the ensemble of unitary matrices, but it does not impose any constraint on the characteristics of these unitary LDC matrices. Therefore, the GA-aided LDC optimisation framework of Fig. 2 is proposed for optimising the candidate unitary LDCs. The genetic operators were carefully selected to ensure that their combination constitutes an efficient heuristic mechanism. Due to space limitations, in the sequel we only describe the proposed framework without elaborating on the GA's operations. Interested readers are referred to [12] for more detailed information on GAs.

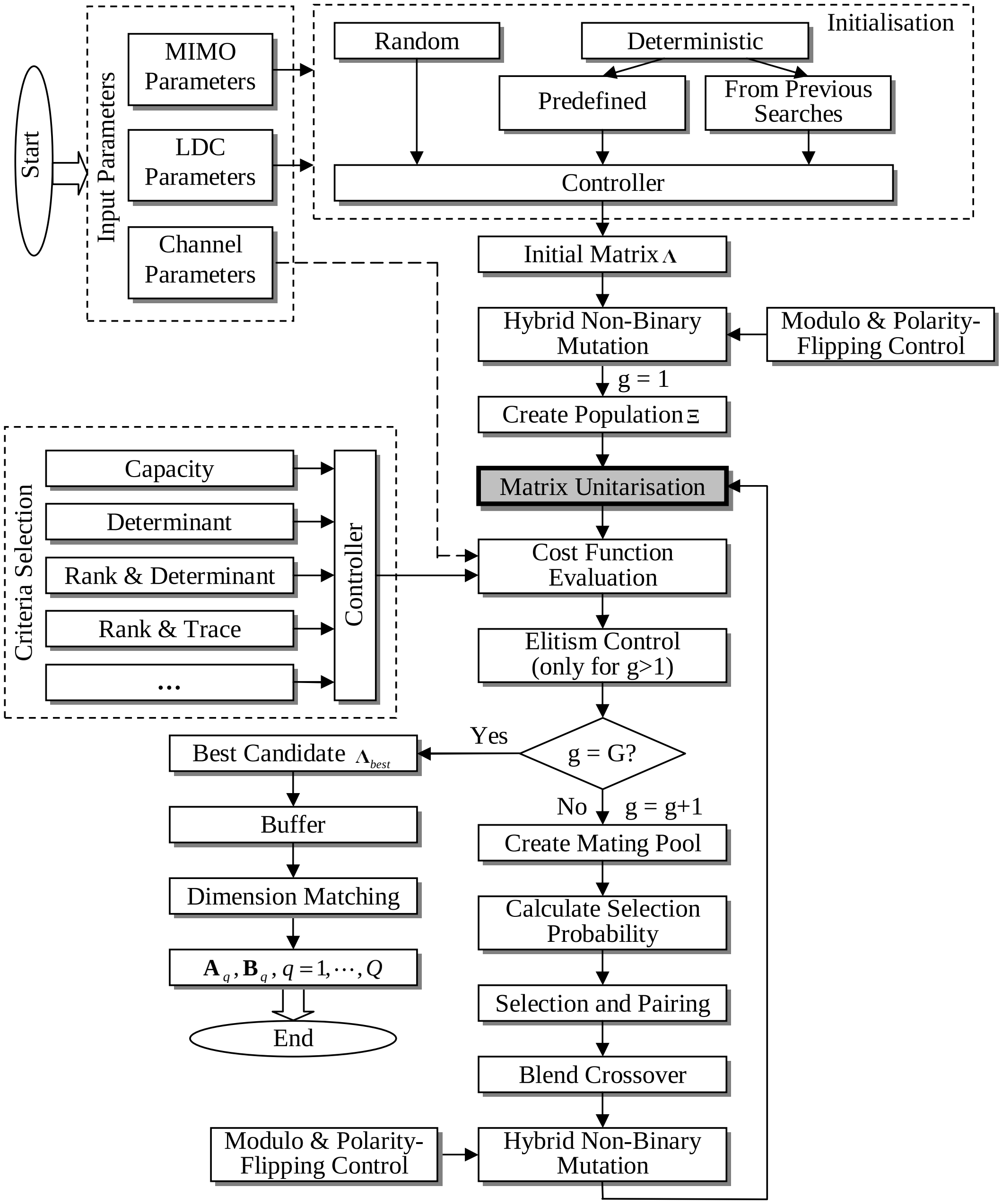

Fig. 2. The flowchart of the process for generating unitary LDCs.

First, the control matrix $\mathbf{\Lambda}$ defined in (3) has to be initialised. Subject to (4), its elements' values can be generated randomly, or according to deterministic rules, for example, by deriving the corresponding $\mathbf{\Lambda}$ from a known unitary LDC (such as the Alamouti code [1]), or by using the values optimised in a previous search. According to our experience, a combination of the above three options can help to provide good results, while at the same time mitigating premature convergence to local minima. Based on $\mathbf{\Lambda}$, we create a so-called *Population* set, $\Xi = \{\mathbf{\Lambda}[i]\}$ $(i = 1, \cdots, P)$, with the aid of a hybrid non-binary mutation operator, which jointly perturbs its elements by constrained random quantities to prevent early convergence to a local optimum, and hence facilitates finding the global optimum. More specifically, a modulo-(2π) operator is applied to ψ_q, χ_q, α_q and a polarity-flipping operator is applied to ξ_q, respectively, ensuring that their mutated values fall within the ranges specified in (4):

$$\begin{aligned} \psi_{q,new} &= (\psi_{q,old} + \lambda \cdot \Delta) \bmod (2\pi) \\ \chi_{q,new} &= (\chi_{q,old} + \lambda \cdot \Delta) \bmod (2\pi) \\ \alpha_{q,new} &= (\alpha_{q,old} + \lambda \cdot \Delta) \bmod (2\pi) \qquad\qquad , \\ \xi_{q,new} &= \begin{cases} -\xi, & \text{if } \xi < 0 \\ 2-\xi, & \text{if } \xi > 1 \end{cases} , \quad \xi = \xi_{q,old} + \lambda \cdot \Delta \end{aligned} \tag{9}$$

where we have $\Delta = 1/(2\pi)$ and $\lambda \in \{1, -1\}$. The entire Population Ξ is referred to as a *Generation*. Each element in Ξ has the same dimension as $\mathbf{\Lambda}$ and is called an *Individual*.

The matrix unitarisation process of Section II-B is invoked for each GA Individual, in order to transform it to $2Q$ number of unitary dispersion matrices $\mathbf{A}_q$ and $\mathbf{B}_q$ $(q = 1, \cdots, Q)$. These dispersion matrices, which together fully specify an LDC, will be evaluated according to a cost function, namely the determinant criterion of [6]. However, diverse criteria can be applied or combined, as indicated by the "Criteria Selection" module shown in Fig. 2. The evaluation process will assign a score to each Individual, which quantifies its fitness with respect to the specific criteria employed. The Individuals having high fitness values will be selected to form a *Mating Pool*, where each of them is assigned a specific selection probability [12]. According to their selection probabilities, two Individuals are then selected for forming a pair of *Parents*. The total number of Parent pairs is $P/2$, where P is the Population size. Each pair is subject to a blend crossover and the aforementioned hybrid mutation operations, resulting in two *Offspring*, namely in two new Individuals. All Offspring jointly will form a new Generation and their fitness will be evaluated accordingly. In order to retain the high-fitness Individuals from one Generation to the next, an *Elitism* control function is applied to replace the worst Individuals of the new Generation with the best ones of the previous Generation [12].

The above-mentioned optimisation process iterates, until the Generation index reaches a maximum predefined value of G. In the last generation, the Individual that yields the highest score will be considered as the solution, namely the optimised LDC. In addition, the "Dimension Matching" function seen in Fig. 2 and detailed in Section II-B is invoked to provide non-square LDCs, if required.

The complexity of the optimisation procedure depends on a number of design aspects, such as for example the dimensions of the LDCs and the Population size P as well as the number of Generations G used. When these are large, the complexity can be high. However, we point out that the proposed framework is employed offline, making the search for high-dimension LDCs possible for open-loop transmissions.

III. Simulation Results

In this section, we provide link-level performance results for LDC examples produced by our design framework and by classic MIMO benchmark schemes, including Alamouti code [1], SM [3], Hassibi's code [1], Gohary's code [6] and Diagonal Algebraic Space-Time (DAST) codes [8]. For simplicity, M, N and T were set to 2. We have $Q = 2$ for the Alamouti/DAST code and $Q = 4$ for the other codes, resulting in the code rate of 4 and 8 Bits per Symbol (BPS), respectively. The standard MLD receiver and an i.i.d. Rayleigh fading channel model with perfect channel estimation were used. The Orthogonal Frequency Division Multiplexing (OFDM) system employed a frame size of 12 OFDM symbols, each consisting of 1024 subcarriers.

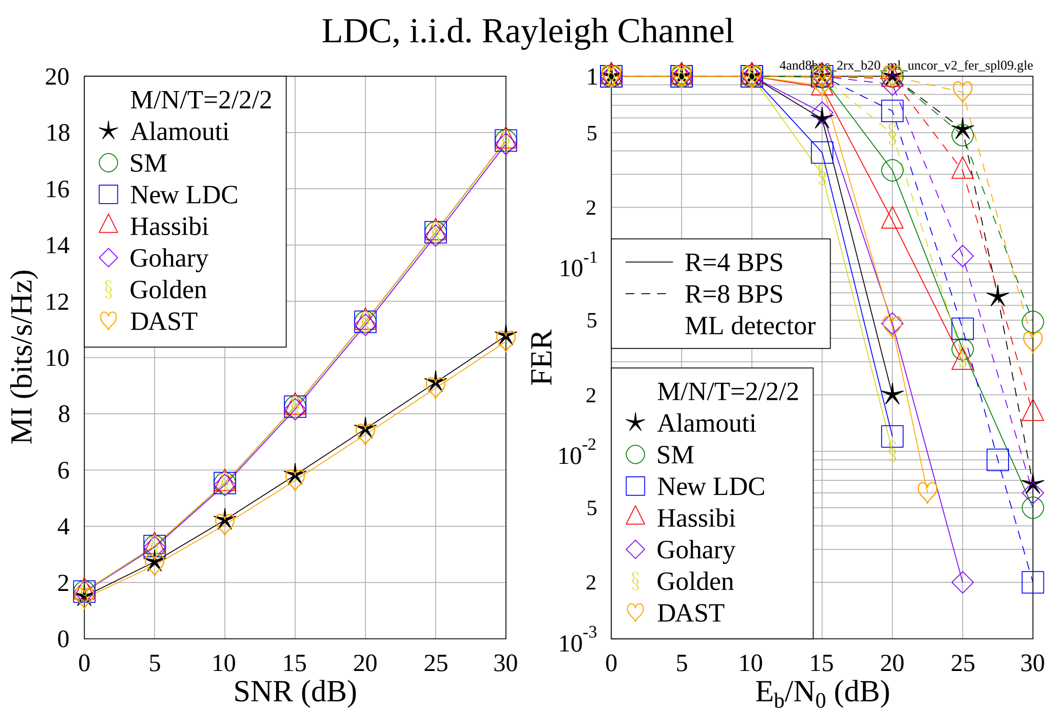

Fig. 3. MI and FER performances.

In Fig. 3, the achievable Mutual Information (MI) and the corresponding Frame Error Rate (FER) performance of the different schemes are plotted, where the effective system data rates defined in Section II-A, namely $R = 4$ and $R = 8$ BPS were considered. As expected, SM is the scheme that yields the maximal achievable MI. Furthermore, Fig. 3 shows that the new LDC, among a few others, is also capable of attaining the highest MI, while the Alamouti and DAST codes only achieve a significantly lower MI value. This result indicates that the LDC optimised by our design method is capable of reaching the best possible performance in terms of MI, and thus a high effective data throughput.

Moreover, Fig. 3 shows that our optimised LDC provides a similar performance as that of the optimum Golden code [9] for both cases, demonstrating its high robustness. This proves that a near-optimum DMT is practically achievable by LDCs generated from our optimisation framework. Specifically, we would like to point out that the proposed LDC design framework can offer more than the Golden code does. Firstly, our design method has the flexibility to produce LDCs with arbitrary dimensions for both square and non-square cases, which can not be achieved by the Golden code or its higher-order counterparts, namely the Perfect codes [10], which are square codes with fixed dimensions. Secondly, our design framework allows us to easily adapt the optimisation criteria. This is beneficial for finding optimum codes against different design targets. A combination of several criteria within a single optimisation process is also possible.

IV. Conclusions

Finally, we briefly conclude that our new LDC design framework, which exploits the unitary matrix theory, is capable of exploring the entire space of unitary matrices according to the unique Haar measure. Further, the specific GA equipped with a new hybrid mutation operator provides a flexible and efficient framework for searching for new codes. We demonstrated that unitary LDCs with better performance than many known codes can be found with the aid of the proposed design method. The optimised LDCs offer both a high data rate and a high link robustness, thus achieving a near-optimum DMT, as verified by our simulation results.

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