A Game-Theoretic Analysis of Market Selection Strategies for Competing Double Auction Marketplaces

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ABSTRACT

In this paper, we propose a novel general framework for analysing competing double auction markets that vie for traders, who then need to choose which market to go to. Based on this framework, we analyse the competition between two markets in detail. Specifically, we game-theoretically analyse the equilibrium behaviour of traders' market selection strategies and adopt evolutionary game theory to investigate how traders dynamically change their strategies, and thus, which equilibrium, if any, can be reached. In so doing, we show that it is unlikely for these competing markets to coexist. Eventually, all traders will always converge to locating themselves at one of the markets. Somewhat surprisingly, we find that sometimes all traders converge to the market that charges higher fees. Thus we further analyse this phenomenon, and specifically determine the factors that affect such migration.

Categories and Subject Descriptors

J.4 [**Social and Behavioural Sciences**]: Economics

General Terms

Economics

Keywords

Competing Markets, Nash equilibrium, Evolutionary Game Theory

1. INTRODUCTION

Exchanges, in which securities, futures, stocks and commodities can be traded, are becoming ever more prevalent. Now, many of these adopt the double auction market mechanism which is a particular type of two-sided markets with multiple buyers (one side) and multiple sellers (the other side). Specifically, in such a mechanism, traders can submit offers at any time in a specified trading round, and can be matched by the market at a specified time. The advantages of this mechanism are that traders can enter the market at any time and they can trade multiple homogeneous or heterogeneous items in one place without travelling around several markets. In addition, this mechanism provides high allocative efficiency [3]. These benefits have led many electronic marketplaces to also use this format. For example, Google offers a DoubleClick Ad Exchange, which is a real-time double auction market enabling large online ad publishers, on one side, and ad networks and agencies, on the other side, to buy and sell advertising space. However, because of the globalised economy, these markets do not exist in isolation. Thus they compete against each other to attract traders and make profits. For example, stock exchanges compete to attract companies to list their stocks in their marketplaces and Google competes against Yahoo! in order to obtain a large share of the display advertising market. Thus such competition is becoming an increasingly important area of research. Given this, and in order to promote further research on this issue, an annual Market Design Competition (CAT) was introduced as part of the Trading Agent Competition (TAC) [4]. In this, entrants need to design effective market policies and set appropriate fees to attract traders and so make profits. Against this background, this paper adopts a game-theoretic approach to analyse the equilibrium behaviour of the strategies that traders should use to select which marketplace to enter and to determine how competing markets should set appropriate fees to obtain market share and make profits.

In more detail, when markets compete against each other, traders usually choose the market which maximises their profits. Specifically, in an environment with multiple competing two-sided markets, buyers will prefer markets with more sellers and sellers will prefer markets with more buyers. This is referred to as a *positive* network externality [2], by which, on average, buyers(sellers) receives higher profits in the market which has the larger number of sellers(buyers) since a large number of sellers(buyers) gives the buyers(sellers) access to more choices. Thus when two-sided markets compete with each other, they need to try to attract traders on both sides. However, this gives rise to a "chicken and egg" problem: to attract buyers, a market should have a large base of the sellers, but these will be willing to participate in the market only if they expect many buyers to show up.

In order to analyse this problem, a number of theoretical models have been proposed [1, 2, 8]. These models analyse how the two-sided markets set appropriate fees to compete effectively to attract traders. However, in these works, they usually assume that buyers(sellers) are homogeneous and they adopt the same strategy to choose a market. They also assume that transaction prices are the same for all transactions. However, such assumptions are unrealistic. In real-world markets, traders are usually heterogeneous as

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they are likely to have different private values[1]. In addition, transaction prices are usually not fixed, as they are usually determined by current demand and supply. Both of these factors typically result in different profits for the traders and so cause different market selection strategies. Furthermore, when transaction prices are affected by demand and supply, in addition to the positive network externality of buyers(sellers) preferring the market with more sellers(buyers), there also exists a *negative* network externality. For example, buyers will prefer market with fewer buyers since this decreased demand decreases transaction prices, and thus brings more profits for them. In our work, we will consider all these factors and thus we are the first to do so from a theoretical perspective. The other strand of work that explores this area is primarily related to CAT, but is largely empirical in nature. In [9], a simple game-theoretic model was proposed to analyse market selection, but this model assumes a game with complete information about the traders' preferences. In [5], researchers theoretically analyse how competing sellers set their reserve prices to attract traders. However, in their analysis, there is only one seller in each market (i.e. a single-sided market). Thus this work is also not relevant for our setting.

Against this background, in this paper, we analyse competition between double auction markets in a game-theoretic way. Specifically, we consider heterogeneous traders with different private values and a discriminatory pricing policy where transaction prices will be affected by current demand and supply. This setting results in both negative and positive externalities, and we will analyse how these externalities affect traders' market selections.

In particular, this paper advances the state of the art in the following ways. First, we propose a novel general framework for analysing competing double auction markets, and in doing so, introduce a general equation to calculate a trader's expected utility. Second, we formulate the trader's expected utility equation given a specific market setting, and then game-theoretically analyse the equilibrium behaviour of traders' market selection strategies in the two competing markets environment with incomplete information. Third, we further analyse how traders dynamically change their market selection strategies and which of the equilibria can be reached using evolutionary game theory (EGT) [10]. We show that it is unlikely for these competing markets to co-exist, and all traders will always converge to one of the markets. Counter-intuitively, we find that sometimes all traders converge to the market charging higher fees. This means that the market can maintain both high number of traders and high profits. We then analyse this interesting phenomenon in detail. Specifically, we analyse in what situations traders migrate to the market charging higher fees and what factors can affect this migration.

The structure of the paper is as follows. In Section 2, we describe our general framework. In Section 3, we analyse the equilibrium behaviour of traders' market selection strategies. In Section 4, we use EGT to analyse the dynamics of traders' selection strategies and investigate the relationship between traders' migration and market fees. Finally, we conclude in Section 5.

[1]The private value of a buyer is its *limit price* which is the highest price that it is willing to buy the item for, and the private value of a seller is its *cost price* which is the lowest price that it is willing to sell the item for.

2. GENERAL FRAMEWORK

In this section, we start by introducing basic notations of our framework. Then we introduce the markets and their policies. Finally, we describe the market selection strategies in detail and give a general equation for a trader's expected utility.

2.1 Preliminaries

In this framework we assume that there are a set of buyers, $\mathcal{B} = \{1, 2, ...B\}$, and a set of sellers, $\mathcal{S} = \{1, 2, ...S\}$. Each buyer and seller has a type[2], which is denoted as θ^b and θ^s respectively. We assume that types of all buyers are independently drawn from the same cumulative distribution function F^b, with support $[\underline{l}, \bar{l}]$, and the types of all sellers are independently drawn from the cumulative distribution function F^s, with support $[\underline{c}, \bar{c}]$. The distributions F^b and F^s are assumed to be common knowledge and differentiable. The probability density functions are f^b and f^s respectively. In our framework, the type of each specific trader is not known to the other traders and markets, and only the type distribution functions are public. In addition, we assume that there is a set of competing markets $\mathcal{M} = \{1, 2, ...M\}$, that offer places for trade and provide a matching service between the buyers and sellers.

2.2 Markets and Fees

Since we consider marketplaces to be commercial enterprises that seek to make a profit, we assume they charge fees for their service as match makers. The fee structure of a market m is defined, as per CAT, as $\mathcal{P}_m = (p_m^b, p_m^s, q_m^b, q_m^s)$, $p_m^b, p_m^s \geq 0$ and $q_m^b, q_m^s \in [0, 1]$, where p_m^b, p_m^s are fixed flat fees (such as registration fees) charged to buyers and sellers respectively, and q_m^b, q_m^s are percentage fees charged on profits made by buyers and sellers respectively (in the following, we refer to such fees as profit fees). Then the fees of all competing markets constitute the fee system $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, ...\mathcal{P}_M)$. Furthermore, the transaction price of a successful transaction in market m is determined by a parameter $k_m \in [0, 1]$, i.e. a *discriminatory k-pricing* policy, which sets the transaction price of a matched buyer and seller at the point determined by k_m in the interval between their offers. The pricing parameters of all markets constitute the pricing system $\mathcal{K} = (k_1, k_2, ..., k_M)$.

2.3 Trader Market Selection

We assume that traders can only choose a single market at a time (called *single-homing*), but they can freely migrate to a different market in the next trading round. A trading round proceeds as follows. First, all markets publish their fees and pricing parameters. Second, based on the observed fees and pricing parameters, each trader selects a market according to its market selection strategy. Third, traders submit their offers according to their bidding strategies. Finally, after all traders have submitted their offers, the market matches buyers and sellers and then executes transactions. For simplicity, we assume that only one unit of commodity can be traded by each trader in a giving trading round. Intuitively, we can see that traders' choice of markets is important since this significantly affects the markets' positions in the competition. Given this, in the following, we present the traders' market selection strategies in more detail.

[2]The types of buyers and sellers represent the buyers' limit prices and the sellers' cost prices respectively.

We consider a mixed market selection strategy, where each market is selected with some probability. A pure strategy can be regarded as a degenerate case of a mixed strategy, where the particular pure strategy is selected with probability 1 and every other strategy with probability 0. Now, a mixed market selection strategy of buyer i is defined as $\omega_i^b : [\underline{l}, \bar{l}] \times \mathcal{M} \to [0,1]$, which means the probability that buyer i with type θ^b chooses the market m is $\omega_i^b(\theta^b, m)$, where $\sum_{m \in \mathcal{M}} \omega_i^b(\theta^b, m) \leq 1$. Here, $1 - \sum_{m \in \mathcal{M}} \omega_i^b(\theta^b, m)$ is the probability that buyer i with type θ^b chooses no market. The complete mixed market selection strategy of buyer i with type θ^b is given by:

$$\delta_i^b(\theta^b) = \langle \omega_i^b(\theta^b, 1), \omega_i^b(\theta^b, 2), ...\omega_i^b(\theta^b, M) \rangle, \quad \delta_i^b(\theta^b) \in \Delta,$$

where Δ is the set of all possible mixed strategies of a trader:

$$\Delta = \left\{ \langle x_1, ..., x_M \rangle \in [0,1]^M : \sum_{m=1}^{m=M} x_m \leq 1 \right\}$$

Similarly, we use $\omega_j^s : [\underline{c}, \bar{c}] \times \mathcal{M} \to [0,1]$ to define the probability of selecting a market of seller j, and write the complete strategy as $\delta_j^s(\theta^s) = \langle \omega_j^s(\theta^s, 1), \omega_j^s(\theta^s, 2), ...\omega_j^s(\theta^s, M) \rangle$, $\delta_j^s(\theta^s) \in \Delta$.

Now we use $\delta^b = \langle \delta_1^b(\cdot), \delta_2^b(\cdot), ...\delta_B^b(\cdot) \rangle$ to denote the strategy profile of buyers, and $\delta^s = \langle \delta_1^s(\cdot), \delta_2^s(\cdot), ...\delta_S^s(\cdot) \rangle$ that of the sellers. Furthermore, we use δ_{-i}^b to represent the strategy profile of all buyers except for buyer i, and use δ_{-j}^s to represent the strategy profile of all sellers except for seller j. Then δ^b and δ^s can be rewritten as $\delta^b = \langle \delta_i^b, \delta_{-i}^b \rangle$ and $\delta^s = \langle \delta_j^s, \delta_{-j}^s \rangle$. Given a buyers' strategy profile δ^b and a sellers' strategy profile δ^s, the expected utility of a buyer i with type θ^b is defined by:

$$\tilde{U}_i^b(\mathcal{P}, \mathcal{K}, \langle \delta_i^b, \delta_{-i}^b \rangle, \delta^s, \theta^b) = \sum_{m=1}^{m=M} \omega_i^b(\theta^b, m) \times \tilde{U}_{i,m}^b(\mathcal{P}, \mathcal{K}, \langle \delta_i^b, \delta_{-i}^b \rangle, \delta^s, \theta^b) \quad (1)$$

where $\tilde{U}_{i,m}^b(\mathcal{P}, \mathcal{K}, \langle \delta_i^b, \delta_{-i}^b \rangle, \delta^s, \theta^b)$ is buyer i's expected utility if it chooses to trade in the market m, which depends on the specific matching policy adopted by market m. We will detail this equation in the next section where we consider a particular market setting. The expected utility of the sellers is defined analogously.

3. GAME-THEORETIC ANALYSIS

In the above we have specified a general framework for analysing competing double auction markets. Before we can theoretically analyse the competition between these markets, we first need to specify the bidding strategies adopted by traders and the matching policies adopted by the markets. Since our main focus is to analyse how competing markets with different fees and pricing parameters affect traders' market selections, and thus we consider a simple bidding strategy for traders. Specifically, we assume that traders use a truthtelling bidding strategy, which means they will submit their types as their offers during the trading process. For the matching policy, we consider *equilibrium matching* since this aims to maximise traders' profits and thus maximises the allocative efficiency for the market. In detail, this policy will match the buyer with v-th highest limit price with the seller with v-th lowest cost price if the seller's cost price is not greater than the buyer's limit price.

In the following, we will first derive the traders' expected utilities for the above setting and then analyse the equilibrium behaviour of the traders' market selections. Since the trader's market selection strategy is affected by other traders' selection strategies and furthermore, there is incomplete information about other traders' types, the Bayes-Nash equilibrium (BNE) solution concept, in which each player's strategy maximises its expected utility given other players' strategies, is appropriate to define this equilibrium behaviour.

3.1 A Trader's Expected Utility

In what follows, we assume a trader's expected utility only depends on its type and whether it is a buyer or a seller. Furthermore, we are interested in calculating *symmetric* BNEs, as is common in game theory, and so we can assume that (in equilibrium) traders with the same type will employ the same strategy. Thus in the following equations, we omit the index i when it is intuitively clear. We now calculate the trader's expected utility given the fee system $\mathcal{P}$, pricing system $\mathcal{K}$ and the market selection strategies of the other traders. In the following, we give the equations for a *buyer*, but the seller's expected utility is calculated analogously.

According to Equation 1, we need to calculate the trader's expected utility in each market, which depends on the type distribution function of traders in this market. To this end, we derive the type distribution function in market m as follows. Firstly, the probability that the type of a buyer is less than θ^b in the market m is:

$$H_m^b(\theta^b) = \int_{\underline{l}}^{\theta^b} f^b(x) * \omega^b(x, m) dx \quad (2)$$

Then, to obtain a proper type distribution function of buyers in market m, we need to normalise the above equation:

$$G_m^b(\theta^b) = \frac{H_m^b(\theta^b)}{H_m^b(\bar{l})} \quad (3)$$

Furthermore, the probability density function of buyer types is:

$$g_m^b(\theta^b) = \frac{f^b(\theta^b) * \omega^b(\theta^b, m)}{H_m^b(\bar{l})} \quad (4)$$

In addition, we can see that the prior probability that a buyer will choose market m is given by $H_m^b(\bar{l})$. The equations of the sellers can be derived in the same way.

Now, we calculate the probabilities that there are *exactly* τ^b buyers (excluding the buyer itself) and τ^s sellers choosing market m, which are given by the binomial distributions:

$$\rho_m^b(\tau^b) = \binom{B-1}{\tau^b} * \left(H_m^b(\bar{l})\right)^{\tau^b} * \left(1 - H_m^b(\bar{l})\right)^{B-1-\tau^b} \quad (5)$$

$$\rho_m^s(\tau^s) = \binom{S}{\tau^s} * \left(H_m^s(\bar{c})\right)^{\tau^s} * \left(1 - H_m^s(\bar{c})\right)^{S-\tau^s} \quad (6)$$

In the *equilibrium matching* policy, the market matches the buyer with the v-th highest limit price with the seller with the v-th lowest cost price. Therefore, we need to calculate the probability that a certain trader is at a certain position. When $\tau^b + 1$ buyers choose market m, the probability that the buyer with type θ^b is the v-th ($v = 1, ..., \tau^b + 1$) highest is given by:

$$Pr_m^b(v|\theta^b) = \begin{pmatrix} \tau^b \\ v-1 \end{pmatrix} * \left(1 - G_m^b(\theta^b)\right)^{v-1} * \left(G_m^b(\theta^b)\right)^{\tau^b+1-v} \quad (7)$$

Similarly, the probability that the seller's type θ^s is v-th ($v = 1, ..., \tau^s$) lowest among τ^s sellers in market m is given by:

$$Pr_m^s(v|\theta^s) = \begin{pmatrix} \tau^s - 1 \\ v-1 \end{pmatrix} * \left(G_m^s(\theta^s)\right)^{v-1} * \left(1 - G_m^s(\theta^s)\right)^{\tau^s-v} \quad (8)$$

Furthermore, the prior probability that a seller is the v-th lowest is given by:

$$Pr^s(v) = \int_{\underline{c}}^{\bar{c}} Pr_m^s(v|\theta^s) * g_m^s(\theta^s) d\theta^s \quad (9)$$

Now using Bayes' theorem, we can calculate the probability density function of a seller at position v:

$$g_m^s(\theta^s|v) = \frac{Pr_m^s(v|\theta^s) * g_m^s(\theta^s)}{Pr^s(v)} \quad (10)$$

At this moment, we can get the buyer's expected gross profit (without subtracting fees) in market m:

$$\tilde{\Lambda}_m^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, \theta^b) = \left[\sum_{\tau^b=0}^{B-1} \rho_m^b(\tau^b) * \sum_{v=1}^{\tau^b+1} Pr_m^b(v|\theta^b) * \left(\sum_{\tau^s=v}^{S} \rho_m^s(\tau^s) * \int_{\underline{c}}^{\theta^b} k_m * (\theta^b - \theta^s) * g_m^s(\theta^s|v) d\theta^s \right) \right] \quad (11)$$

where $\theta^b - \theta^s$ is called the trading surplus, and $k_m * (\theta^b - \theta^s)$ is the share of the buyer's surplus. By adding the fixed flat fee and profit fee from market m, a buyer's expected utility in this market becomes:

$$\tilde{U}_m^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, \theta^b) = \tilde{\Lambda}_m^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, \theta^b) * (1 - q_m^b) - p_m^b \quad (12)$$

The above equations give the expected utility in a particular market. Therefore, given the market selection strategy, a buyer's expected utility is:

$$\tilde{U}^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, \theta^b) = \sum_{m=1}^{M} \omega^b(\theta^b, m) * \tilde{U}_m^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, \theta^b) \quad (13)$$

3.2 Market Selection Strategy Equilibria

With the equations for the traders' expected utilities established, we are now ready to analyse the market selection equilibrium behaviour of traders. To do so, we use the game-theoretic concept of mixed Bayes-Nash equilibrium (BNE). We consider the equilibrium behaviour of both buyers and sellers simultaneously. Formally, the mixed Bayes-Nash equilibrium in our setting is defined as:

Definition *Given the fee system $\mathcal{P}$ and pricing system $\mathcal{K}$, market selection strategy profiles δ^{b*} and δ^{s*} constitute a mixed Bayes-Nash equilibrium, if*

$\forall i \in \mathcal{B}, \forall \theta^b \in [\underline{l}, \bar{l}], \forall \delta_i^b(\theta^b) \in \Delta$:

$$\tilde{U}_i^b(\mathcal{P}, \mathcal{K}, \langle \delta_i^{b*}, \delta_{-i}^{b*} \rangle, \delta^{s*}, \theta^b) \geq \tilde{U}_i^b(\mathcal{P}, \mathcal{K}, \langle \delta_i^{b}, \delta_{-i}^{b*} \rangle, \delta^{s*}, \theta^b)$$

and $\forall j \in \mathcal{S}, \forall \theta^s \in [\underline{c}, \bar{c}], \forall \delta_j^s(\theta^s) \in \Delta$:

$$\tilde{U}_j^s(\mathcal{P}, \mathcal{K}, \delta^{b*}, \langle \delta_j^{s*}, \delta_{-j}^{s*} \rangle, \theta^s) \geq \tilde{U}_j^s(\mathcal{P}, \mathcal{K}, \delta^{b*}, \langle \delta_j^{s}, \delta_{-j}^{s*} \rangle, \theta^s)$$

As we said before, in this paper, we focus on the symmetric BNE which means that traders with the same type will adopt the same strategy in equilibrium. Furthermore, in order to get insights from this complicated game with more traders and more types, we make several simplifying assumptions. First of all, we initially restrict our analysis to two buyers and two sellers, i.e. $B = S = 2$, (although we will relax this in Section 4.2), and only consider the competition between two markets, i.e. $M = 2$. In addition, to allow for tractable results, at this stage, we restrict our analysis to discrete trader types (we intend to analyse continuous types in future work). In particular, we assume that there are two types of buyers and two types of sellers: rich and poor, which are denoted by t_2^b and t_1^b respectively for buyers, and t_1^s and t_2^s for sellers. A rich buyer is defined as having a higher limit price than a poor buyer, i.e. $t_2^b > t_1^b$, and a rich seller is defined as having a lower cost price than a poor seller, i.e. $t_1^s < t_2^s$. Trader types are independently drawn from the discrete uniform distribution (i.e. both types are equally likely). In addition, for simplicity, we only consider profit fee at this stage[3], i.e. $p_1^b = p_2^b = p_1^s = p_2^s = 0$. Given this, traders will always choose one of the markets since no fixed flat fee is charged and they have non-negative profits. Thus $\omega^b(\theta^b, 1) = 1 - \omega^b(\theta^b, 2)$ and similar for sellers.

Given these assumptions, we now investigate the traders' market selection equilibrium behaviour. Intuitively, we can see that all traders selecting one market constitutes a pure strategy BNE, since given all other traders selecting one market, the best response of a trader is also to select this market (otherwise they will have nobody to trade with). In addition to the pure strategy BNEs, we are also interested in the mixed symmetric BNE of traders' market selection strategies since we would like to know whether two competing markets can coexist. To do so, we first need to adapt Equation 12 to a discrete probability distribution. This is straightforward and because of space limitations, we will not show this in detail. As we know, in the mixed Nash equilibrium, a player should be indifferent to choosing each pure strategy, i.e. its expected utility for each pure strategy should be the same [6]. Thus we get the following equations to calculate the mixed BNE:

$$\tilde{U}_1^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_2^b) = \tilde{U}_2^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_2^b) \quad (14)$$

$$\tilde{U}_1^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_1^b) = \tilde{U}_2^b(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_1^b) \quad (15)$$

$$\tilde{U}_1^s(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_1^s) = \tilde{U}_2^s(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_1^s) \quad (16)$$

$$\tilde{U}_1^s(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_2^s) = \tilde{U}_2^s(\mathcal{P}, \mathcal{K}, \delta^b, \delta^s, t_2^s) \quad (17)$$

We can use a mathematical tool, such as Matlab or Mathematica, to solve the above quadratic equations with four unknown variables: $\omega^b(t_1^b, 1)$, $\omega^b(t_2^b, 1)$, $\omega^s(t_1^s, 1)$ and $\omega^s(t_2^s, 1)$. If the solution is in the range $[0, 1]$, then this constitutes a mixed symmetric BNE.

By analysing the above equations, we find that the solution such that

$$\omega^b(t_1^b, 1) = \omega^b(t_2^b, 1) \quad (18)$$

and

$$\omega^s(t_1^s, 1) = \omega^s(t_2^s, 1) \quad (19)$$

always exists. This means that there always exists a mixed BNE whereby buyers adopt the same mixed strategy no matter whether they are rich or poor, and the same for sellers. This appears somewhat counter-intuitive and so we analyse

[3] This is similar to recent years' CAT competition, where most competing markets only charge profit fees, and charge no fixed flat fees in order to avoid negative profits for traders.

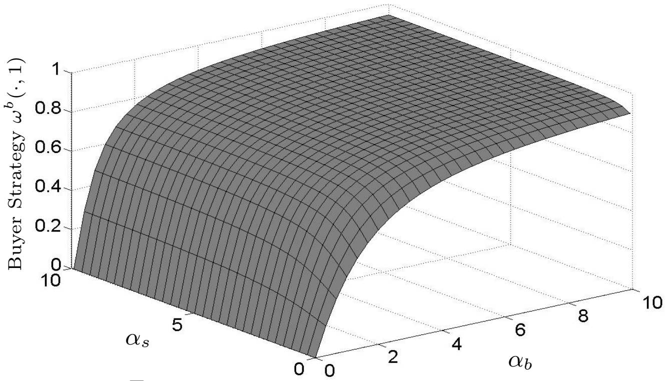

Figure 1: The mixed BNE strategy of buyers.

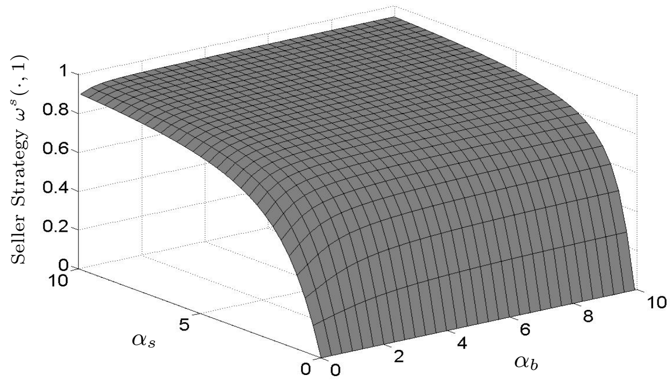

Figure 2: The mixed BNE strategy of sellers.

this solution in more detail to understand more about the traders' equilibrium behaviour.

To this end, we introduce two new notations, α_b and α_s, where $\alpha_b = \frac{k_2*(1-q_2^b)}{k_1*(1-q_1^b)}$ and $\alpha_s = \frac{(1-k_2)*(1-q_2^s)}{(1-k_1)*(1-q_1^s)}$, to represent the buyers' profit ratio and the sellers' profit ratio in market 2 compared to market 1 respectively. We can see that $\alpha_b > 0$, $\alpha_s > 0$, and they can be chosen independently. Using these notations, Equations 14-17 can be rewritten to the following two equations:

$$2*\omega^s(t_1^s,1) - \left(\omega^s(t_1^s,1)\right)^2 - \omega^b(t_1^b,1)*\omega^s(t_1^s,1) + \omega^b(t_1^b,1)*\left(\omega^s(t_1^s,1)\right)^2 = \left[1 - \omega^s(t_1^s,1) + \omega^b(t_1^b,1)*\omega^s(t_1^s,1) - \omega^b(t_1^b,1)*\left(\omega^s(t_1^s,1)\right)^2\right]*\alpha_b \quad (20)$$

$$2*\omega^b(t_1^b,1) - \left(\omega^b(t_1^b,1)\right)^2 - \omega^b(t_1^b,1)*\omega^s(t_1^s,1) + \omega^s(t_1^s,1)*\left(\omega^b(t_1^b,1)\right)^2 = \left[1 - \omega^b(t_1^b,1) + \omega^b(t_1^b,1)*\omega^s(t_1^s,1) - \omega^s(t_1^s,1)*\left(\omega^b(t_1^b,1)\right)^2\right]*\alpha_s \quad (21)$$

Note that the solution of the above two equations depends on the profit ratio parameters α_b and α_s. Given this, the resulting mixed BNEs are shown in Figures 1 and 2. Specifically, we find that there exist equilibria in which traders have a higher probability of choosing market 1 even when it allocates less profit to them. This is because when initially more sellers stay in market 1, then buyers prefer to choose market 1 even though it allocates less profit to them, and they still have same expected utilities in markets 1 and 2. Vice versa for sellers. This implies that it is possible for traders to be maintained in the market charging higher fees. In Section 4.2, we will analyse this phenomenon in detail.

4. EVOLUTIONARY ANALYSIS

In the above, we analysed the traders' equilibrium behaviour with regards to market selection strategies and showed that there exist at least three BNEs: all traders choosing market 1 or 2 and the mixed BNE. However, such equilibria only provide a static explanation for why populations playing BNE strategies remain in that state since each population makes a best response to the other populations' strategies. Therefore, this solution concept fails to indicate whether the BNE can be reached and which of these equilibria is most likely to occur in practice. To overcome this, in what follows, we use evolutionary game theory (EGT) to analyse this game. EGT is different from traditional game theory because it focuses on the dynamic change of strategies rather than the static properties of Nash equilibria [10]. In EGT, players gradually adjust their strategies over time in response to the repeated observation of their opponents' strategies. In particular, the *replicator dynamics* equation is often used to specify the dynamic adjustment of the probability of which pure strategy should be played. For example, in [7], replicator dynamics are used to show how traders bid in a double auction market. However, existing literature only provides single or 2-population replicator dynamics. In our work, we have 4 populations (rich buyers, poor buyers, rich sellers and poor sellers). Thus first we introduce the 4-population replicator dynamics equations which show the dynamic changes of traders' selection strategies with respect to time t:

$$\begin{aligned}\dot{\omega}^b(t_1^b,1) &= \frac{d\omega^b(t_1^b,1)}{dt} \\ &= \left(\tilde{U}_1^b(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_1^b) - \tilde{U}^b(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_1^b)\right)*\omega^b(t_1^b,1)\end{aligned} \quad (22)$$

$$\begin{aligned}\dot{\omega}^b(t_2^b,1) &= \frac{d\omega^b(t_2^b,1)}{dt} \\ &= \left(\tilde{U}_1^b(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_2^b) - \tilde{U}^b(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_2^b)\right)*\omega^b(t_2^b,1)\end{aligned} \quad (23)$$

$$\begin{aligned}\dot{\omega}^s(t_1^s,1) &= \frac{d\omega^s(t_1^s,1)}{dt} \\ &= \left(\tilde{U}_1^s(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_1^s) - \tilde{U}^s(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_1^s)\right)*\omega^s(t_1^s,1)\end{aligned} \quad (24)$$

$$\begin{aligned}\dot{\omega}^s(t_2^s,1) &= \frac{d\omega^s(t_2^s,1)}{dt} \\ &= \left(\tilde{U}_1^s(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_2^s) - \tilde{U}^s(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_2^s)\right)*\omega^s(t_2^s,1)\end{aligned} \quad (25)$$

As an example, $\dot{\omega}^b(t_1^b,1)$ describes how the poor buyer with type t_1^b changes its probability of choosing market 1. Here, $\tilde{U}_1^b(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_1^b)$ is the poor buyer's expected utility when choosing market 1 given other traders' strategies, and $\tilde{U}^b(\mathcal{P},\mathcal{K},\delta^b,\delta^s,t_1^b)$ is the poor buyer's overall expected utility (see Section 3.1). In order to get the dynamics of the strategies, we need to calculate *trajectories*, which indicate how the mixed strategies evolve. In more detail, initially, a mixed strategy is chosen as a starting point. For convenience, we use $(\omega^b(t_2^b,1),\omega^b(t_1^b,1),\omega^s(t_1^s,1),\omega^s(t_2^s,1))$ to represent this starting point. The dynamics are then calculated according to the above replicator equations. According to the dynamic changes of traders' strategies, their current mixed strategy can be calculated. Such calculations are repeated until $\dot{\omega}^b(\cdot,1)$ and $\dot{\omega}^s(\cdot,1)$ become zero, at which point, the equilibrium is reached. The replicator dynamics show the trajectories and how they converge to an equilibrium. We call an equilibrium to which trajectories converge an *attractor*, and call the equilibrium to which no trajectories converge a *saddle point*. The region where all trajectories converge to a particular equilibrium is called the *basin of attraction* of this equilibrium. The basin is very useful since its size indicates how likely the population is to converge to that equilibrium.

4.1 Dynamics of Traders' Market Selection

In order to visualise how trajectories converge to the equilibrium and which equilibrium is the most likely to hap-

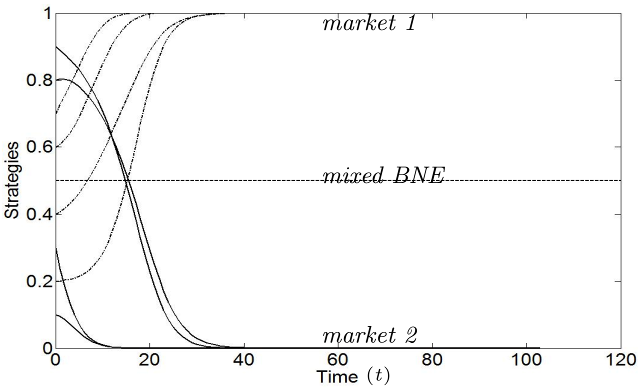

Figure 4: Equilibrium behaviour of 2 buyers and 2 sellers with three chosen starting points.

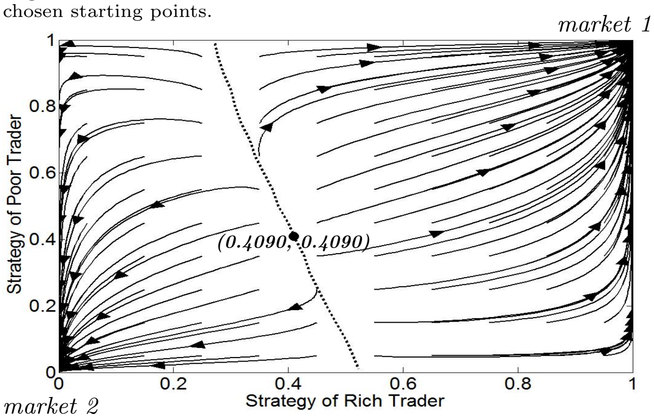

Figure 3: Market selection of rich(poor) traders having the same behaviour. The dotted line denotes the boundary between the basins of attraction.

pen, firstly, we consider a simple case with rich(poor) buyers and rich(poor) sellers having the same behaviour, which will reduce the 4-populations replicator dynamics to the 2-populations replicator dynamics. In this case, the market should charge the same profit fee to buyers and sellers, i.e. $q_1^b = q_1^s$, $q_2^b = q_2^s$, and set transaction prices fairly to buyers and sellers, i.e. $k_1 = k_2 = 0.5$. Furthermore, the surplus should be symmetric. To this end, we let $t_2^b - t_1^s = 8$, $t_2^b - t_2^s = 5$, $t_1^b - t_1^s = 5$ and $t_1^b - t_2^s = 2$, which means all traders can make profits and rich traders make more profits than poor ones. Specifically, we consider 2 buyers and 2 sellers and assume $\alpha_b = \alpha_s = 0.8$. Then the mixed BNE satisfying Equations 18 and 19 is (0.4090, 0.4090, 0.4090, 0.4090). The evolutionary results are shown in Figure 3, where the x-axis is the the rich buyer(seller)'s probability of choosing market 1, and the y-axis is the poor buyer(seller)'s probability of choosing market 1. We find that all traders finally converge to market 1 or 2. We can see that the basin of attraction to market 1 is bigger, which means that traders have a higher probability of converging to market 1 since it allocates more profits to them. We find that no trajectory converges to (0.4090, 0.4090) (the solid circle in Figure 3), i.e. the mixed BNE is a saddle point. This indicates that this equilibrium is hard to reach in practice.

We now consider the general cases where rich(poor) traders may not have the same behaviour. Thus we let traders have asymmetric surpluses. In the following, we let $t_2^b - t_1^s = 8$, $t_2^b - t_2^s = 5$, $t_1^b - t_1^s = 4$ and $t_1^b - t_2^s = 1$ unless otherwise specified. We firstly consider the case that two competing markets are identical, i.e. $k_1 = k_2$, $q_1^b = q_2^b$ and $q_1^s = q_2^s$. According to Equations 20 and 21, we know that (0.5, 0.5 0.5

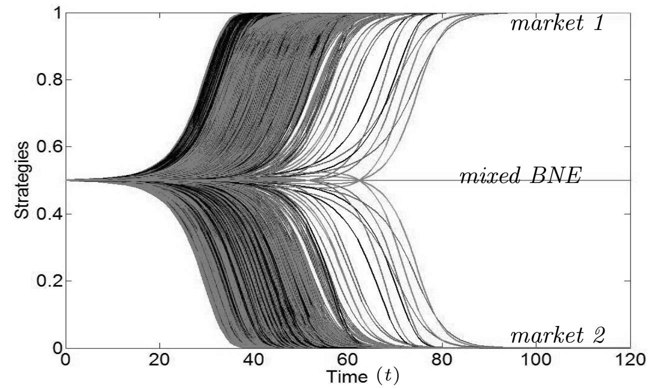

Figure 5: Equilibrium behaviour of 2 buyers and 2 sellers with starting points around (0.5, 0.5, 0.5 0.5).

0.5) is a mixed BNE. We now show the evolutionary results of two representative starting points (0.6, 0.4, 0.7, 0.2), (0.1, 0.8, 0.3, 0.9) and a specific starting point (0.5, 0.5 0.5 0.5) in Figure 4. The x-axis is the time at which the mixed strategies evolve, the points at $t = 0$ correspond to the starting points, from which traders evolve their strategies. As can be seen, traders eventually converge to market 1 or 2 except when starting at the mixed BNE. Furthermore, we analyse the area of starting points around the equilibrium (0.5, 0.5, 0.5, 0.5) and find that these do not converge to (0.5, 0.5, 0.5, 0.5). This is described in Figure 5, which shows the evolutionary results where starting points are chosen from 0.499 to 0.501 with step size 0.0005. Therefore, even though (0.5, 0.5, 0.5, 0.5) is a mixed BNE, it is a saddle point and unlikely to be reached in practice.

Now we consider the traders' evolved strategies when fees and pricing parameters are different across markets. As an example, we let $\alpha_b = 0.8$ and $\alpha_s = 0.7$, which means compared to market 1, market 2 allocates less profits to traders. By solving Equations 20 and 21 with $\alpha_b = 0.8$ and $\alpha_s = 0.7$, we find one mixed BNE at (0.3679, 0.3679, 0.3994, 0.3994). Then we show the evolutionary results of two representative starting points (0.2, 0.4, 0.3, 0.1), (0.7, 0.9, 0.8, 0.6), and the specific starting point (0.3679, 0.3679, 0.3994, 0.3994) in Figure 6 and the evolutionary results with starting points around the equilibrium (0.3679, 0.3679, 0.3994, 0.3994) in Figure 7. We still find that traders finally converge to either market 1 or 2, and the mixed BNE is unlikely to occur in practice. Furthermore, we find the same result for a wide range of settings, including more traders, different starting points and different fees. This indicates that although there exists a negative network externality (where a trader prefers less traders on the same side), the positive network externality (where a trader prefers more traders on the other side) is stronger, and then pushes all traders into one market. In addition, we find that from the starting point (0.2, 0.4, 0.3, 0.1), traders finally converge to market 2, which is the market allocating less profits to traders. This is counter-intuitive and we will analyse it in detail in the following.

4.2 Trader Migration and Market Fees

When analysing the dynamics of traders' market selection strategies, we found that traders evolving from some starting points converged to the market allocating less profits to them. When $k_1 = k_2$, this phenomenon directly means that traders may converge to the market charging a higher profit fee (we call this an expensive market, and call the market

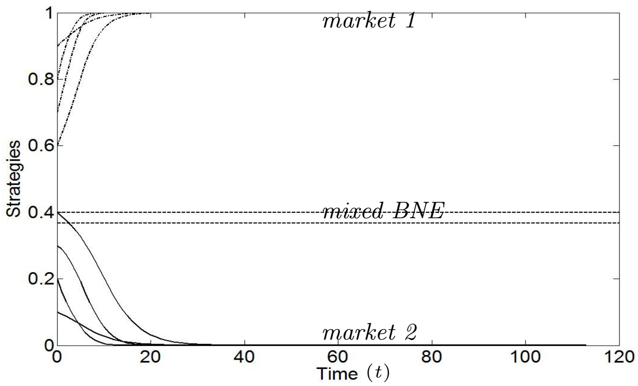

Figure 6: Equilibrium behaviour of 2 buyers and 2 sellers with three chosen starting points.

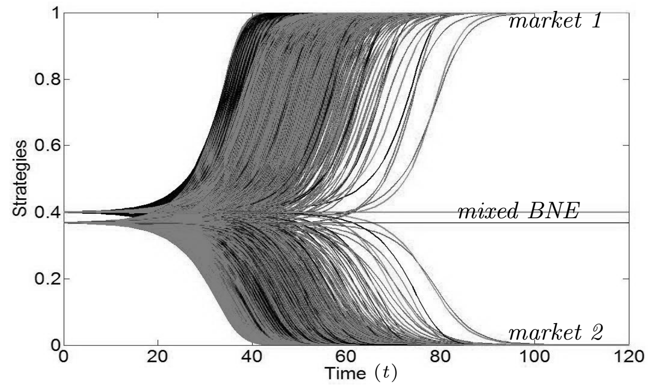

Figure 7: Equilibrium behaviour of 2 buyers and 2 sellers with starting points around (0.3679, 0.3679, 0.3994, 0.3994).

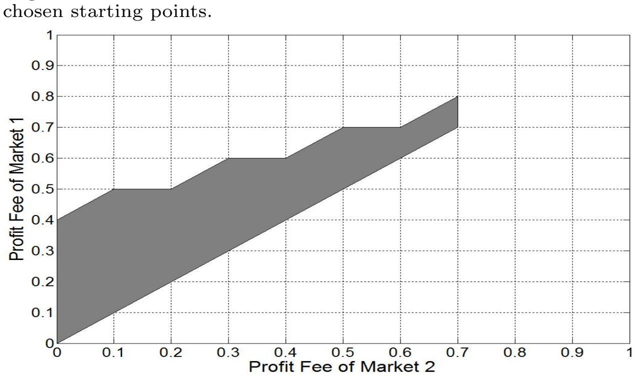

Figure 8: Lock-in region with 2 buyers and 2 sellers.

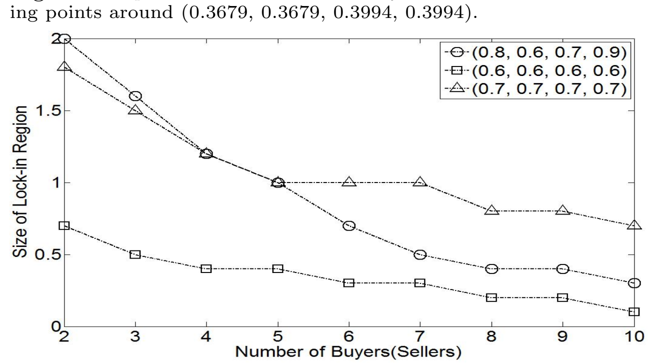

Figure 9: The size of lock-in region with respect to the number of traders.

charging a lower profit fee a cheap market in the following). This somewhat counter-intuitive result is interesting since it suggests that a competing market can maintain traders and make profits at a good level. To this end, we will hence investigate the relationship between the traders' migration and market fees in detail. Such an analysis is insightful to guide the design of charging policies in competing markets.

We first consider the case with two buyers and two sellers with rich and poor types respectively, but we will consider more buyers and sellers below. For convenience of showing results, we assume that the market charges the same profit fees to buyers and sellers, i.e. $q_1^b = q_1^s$ and $q_2^b = q_2^s$. We discretize the continuous profit fee from 0 and 1 with a step size of 0.1. The traders' evolution of their market selection strategies depends on two factors: starting point and fees charged to them. We now choose a starting point (0.8, 0.6, 0.7, 0.9), where the traders have a higher initial probability of choosing market 1. Then we evolve traders' market selection strategies in the competing markets with different profit fees. The results are given in Figure 8. The gray area is what we call the "lock-in region". This area is very interesting since when the profit fees charged by the two markets are within this area, then even though market 1 charges a higher profit fee than market 2, traders still converge to market 1. Note that when the profit fee of market 2 is higher than 70%, market 1 can no longer maintain traders if its profit fee is higher than market 2, i.e. the lock-in region disappears. Now we can see that it is possible for traders to converge to the expensive market if currently traders have higher probabilities of choosing this market. Thus, in the initial stage of the competition, a market has to lower its fees to attract or maintain traders. After obtaining an advantageous position, the market can then increase its fees slightly higher than its opponents, but still can keep traders since traders still have higher expected utilities in the expensive market.

After obtaining the preliminary conclusion that it is possible for traders to stay in the expensive market, we investigate what factors can affect the size of the lock-in region. In particular, we investigate how the number of traders can affect the size of lock-in region. In the following, we calculate the size of the lock-in region as the sum of the differences of two markets' discretized profit fees in the lock-in region. For example, the size of the lock-in region in Figure 8 is two[4]. From Figure 9, we find that as the number of traders in the competing market environment increases, the size of the lock-in region decreases, which means traders will increasingly select cheap markets. The reason for this is as follows. The traders' choice of markets is determined by their expected utilities, which, in turn, depend on two parts: the gross profit and fees charged to them (see Equation 12). From Figure 10, we can see that as the number of traders in the multiple competing markets environment increases, the difference of the traders' gross profits in two markets (i.e. $\Lambda_1^b(\cdot) - \Lambda_2^b(\cdot)$, $\Lambda_1^s(\cdot) - \Lambda_2^s(\cdot)$, see Equation 11) gradually decreases. This means that the gross profits of traders in two markets gradually become closer to each other. Then the traders' choice of market is mainly determined by the market fees. Thus they will increasingly choose the cheap market. This indicates that, in a multiple competing markets context with a large number of traders, it is difficult for the market to maintain both a high number of traders and high profits. Intuitively, markets want to attract traders of

[4]This is the sum of discretized profit fee difference of two markets: $(0.4-0.0)+(0.5-0.1)+(0.5-0.2)+(0.6-0.3)+(0.6-0.4)+(0.7-0.5)+(0.7-0.6)+(0.8-0.7)$.

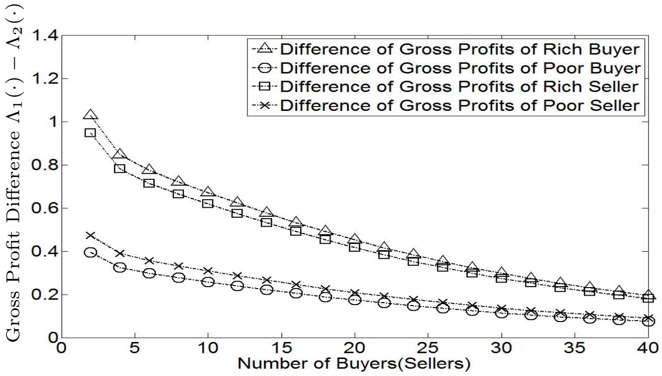

Figure 10: Difference of gross profits in two markets with mixed strategy (0.7, 0.7, 0.7, 0.7).

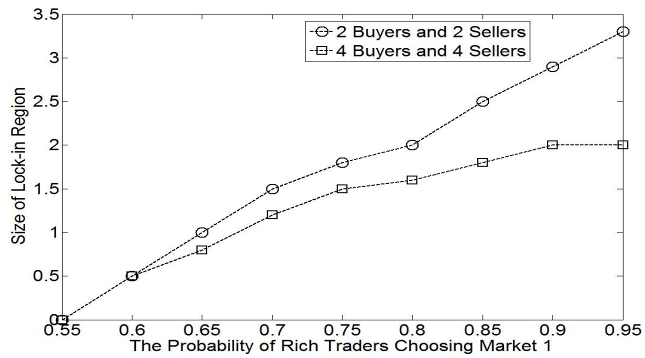

Figure 11: The size of lock-in region with respect to rich traders' strategy.

a rich type since they are more likely to make transactions. Given this, we now analyse what happens when a certain type of trader initially has a bias towards selecting a particular market. First, we consider the rich type's effect on the lock-in region, where we fix the poor traders' probabilities of choosing market 1 to be 0.5, and then change the strategies of the rich traders from 0.55 to 0.95 with step size 0.05. The results are shown in Figure 11. From this, we can see that, when rich traders have a higher initial probability of selecting market 1, the size of the lock-in region increases. This means that rich traders have a positive effect on the lock-in region. In contrast, if we fix the rich traders' probabilities of choosing market 1 to be 0.5, and then increase the poor traders' probabilities of choosing market 1 starting from 0.05, we find no lock-in regions exist. This is because the surpluses were chosen such that poor traders can make relatively good profits, which means they are not poor enough. Thus we reduce the poor traders' surpluses to enhance their effect on the lock-in region. If we let $t_2^b - t_1^s = 8$, $t_2^b - t_2^s = 2$, $t_1^b - t_1^s = 1$ and $t_1^b - t_2^s = 0$, we get the following result. When the poor traders' probability of choosing market 1 is 0.1, the lock-in region exists and its size is 0.2. When the probability increases to 0.2, the lock-in region disappears. Thus we can see that poor traders have a negative effect on the size of the lock-in region. Furthermore, considering again Figure 9, we can see that the lines with circles and triangles cross. This suggests that, if the proportion of poor traders is high, and as the number of traders increases, poor traders have an even larger negative effect.

5. CONCLUSIONS

We have developed a novel general framework for theoretically analysing competing markets. Based on this framework, we game-theoretically analysed the static equilibrium behaviour of traders' market selection strategies. We then adopted EGT to analyse the dynamics of traders' market selection strategies considering a wide range of settings. From this, we found that traders will always converge to one market unless they start off from the mixed BNE. This indicates that the competing markets cannot coexist. We then analysed how market fees affect traders' migration. In so doing, we found that it is possible for the competing market to keep traders even when charging higher fees if it already has a larger market share. We also found that, as the number of traders increases, this becomes more difficult and traders prefer the cheaper market. We further showed that rich traders have a positive effect on a market in terms of helping to attract traders and poor traders have a negative effect. Such insights are particularly useful to guide the competing market to set its fees. Specifically, when the competing market obtains a larger market share with many rich traders than its opponents, then according to the lock-in region, it can charge slightly higher fees to earn high profit, but still keep traders. In contrast, for the market with smaller market share and poor traders, it should lower its fees to "escape" from the lock-in region.

In the future, we would like to generalise our analysis of the market selection equilibrium behaviour as well as the bidding strategies by considering traders with continuous types. Furthermore, at present, we have assumed that the competing markets use the same matching technology. However, in practice, they may well use different technologies, as we see in the CAT competition. In addition, we intend to analyse the equilibrium behaviour of the markets themselves in terms of setting fees and pricing parameters taking into account the equilibrium behaviour of traders' market selection strategies. We also plan to use insights from the theoretical analysis to guide the design of a competing double auction market for the CAT competition.

6. REFERENCES

[1] M. Armstrong. Competition in two-sided markets. *The RAND Journal of Economics*, 37(3):668–691, 2006.
[2] B. Caillaud and B. Jullien. Chicken & egg: Competition among intermediation service providers. *The RAND Journal of Economics*, 34(2):309–328, 2003.
[3] D. Friedman and J. Rust. *The double auction market: institutions, theories and evidence*, volume XIV of Santa Fe Institute Studies in the Science of Complexity. Perseus Publishing, 1993.
[4] E. H. Gerding, P. McBurney, J. Niu, S. Parsons, and S. Phelps. Overview of CAT: A market design competition. Technical report, Dept. of Computer Science, University of Liverpool, 2007.
[5] E. H. Gerding, A. Rogers, R. K. Dash, and N. R. Jennings. Sellers competing for buyers in online markets: reserve prices, shill bids, and auction fees. In *Proceedings of the 20th International Joint Conference on Artificial Intelligence*, pages 1287–1293, 2007.
[6] M. J. Osborne and A. Rubinstein. *A Course in Game Theory*. The MIT Press, 1994.
[7] S. Phelps, P. McBurney, and S. Parsons. Evolutionary mechanism design: a review. *Journal of Autonomous Agents and Multi-Agent Systems*, 2010.
[8] J. Rochet and J. Tirole. Platform competition in two-sided markets. *Journal of European Economic Association*, 1(4):990–1029, 2003.
[9] J. Sohn, S. Lee, and T. Mullen. Attracting intra-marginal traders across multiple markets. In *Workshop on Trading Agent Design and Analysis (TADA 2009)*, 2009.
[10] J. W. Weibull. *Evolutionary Game Theory*. The MIT Press, 1996.