Structural Operational Semantics for Stochastic Systems

V. Sassone

(joint work with B. Klin, Cambridge)

What we do...

We deal in models....

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Hmm, not this kind...

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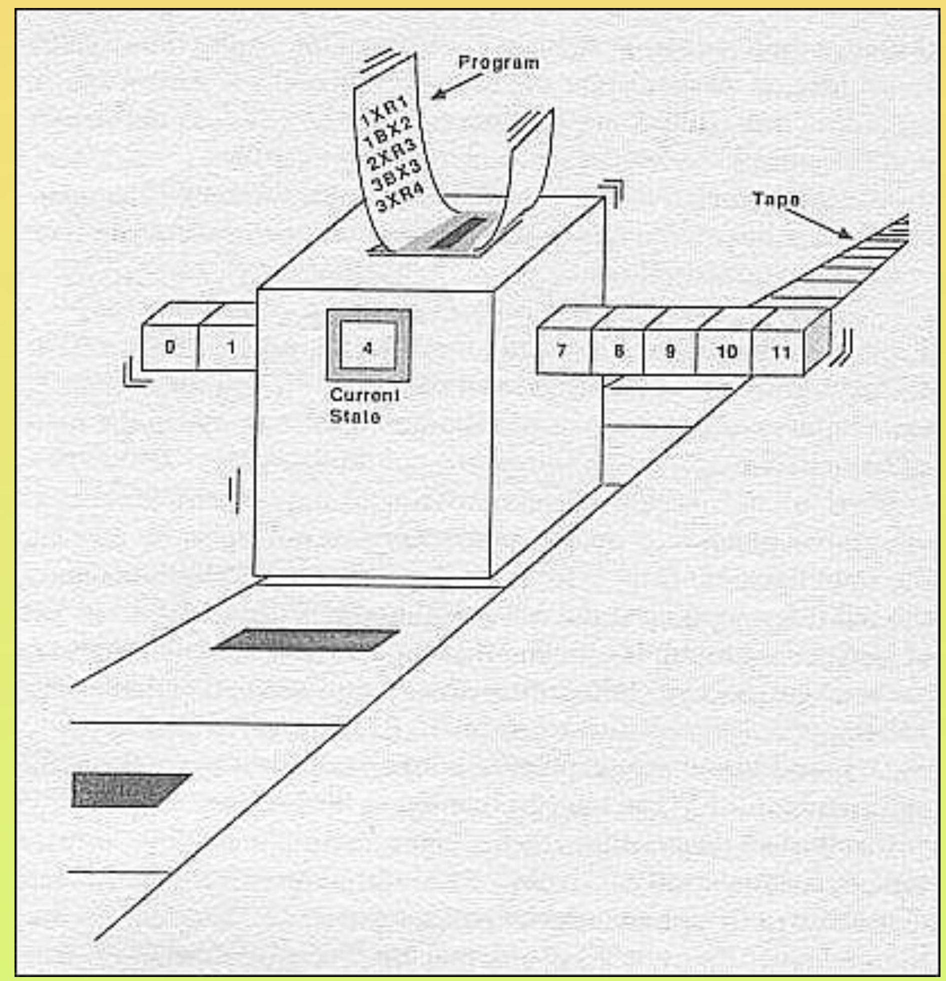

Yeah, more like this kind...

Why we do it...

We want to engineer systems fit for purpose

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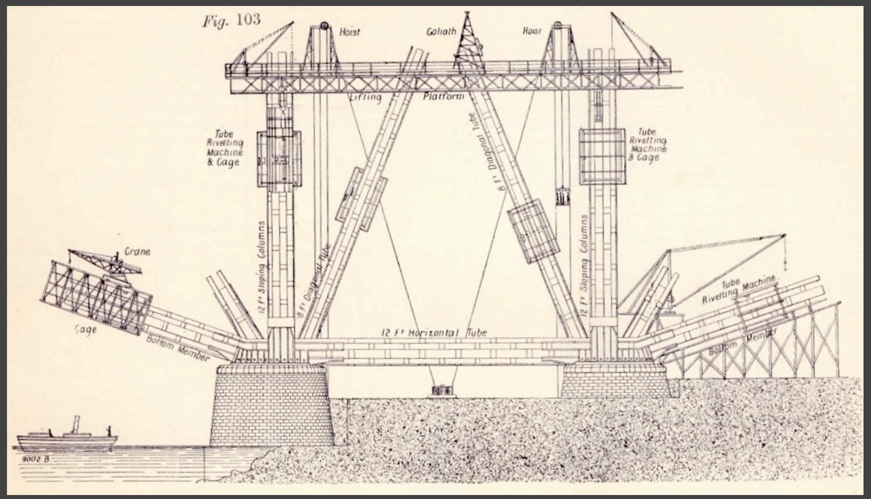

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uptime: 45K+ days & running

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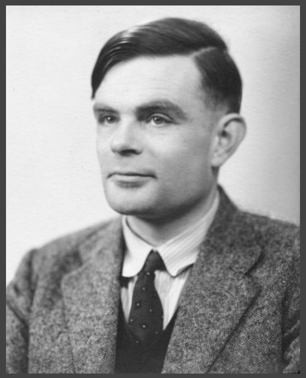

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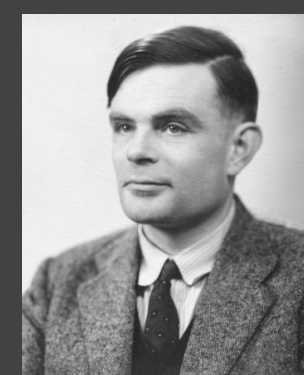

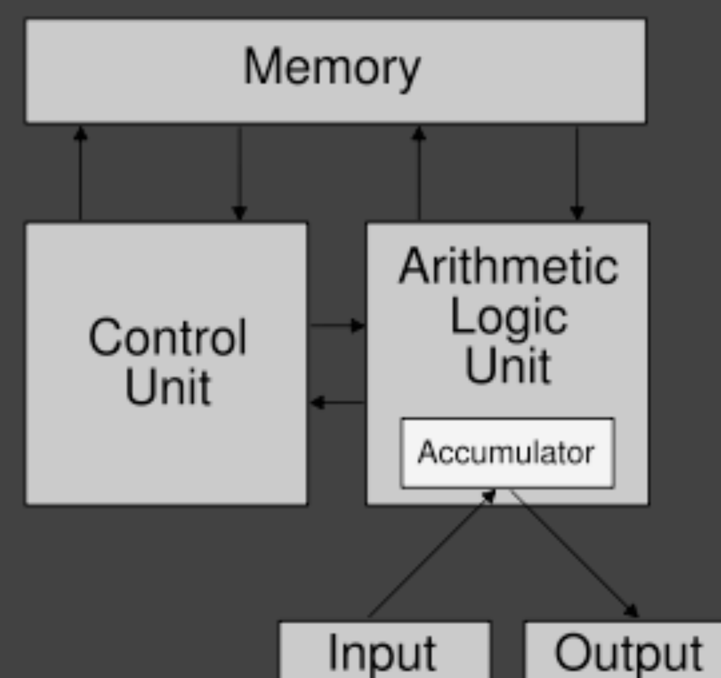

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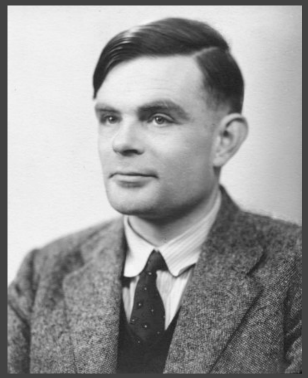

How we do it...

Don’t just build model of specific systems:

we want laws, principles and methodologies;
in fact, engineering techniques and tools.

specify, design, program, transform, validate

we call these collectively Theory, or less ambitiously Software Science.

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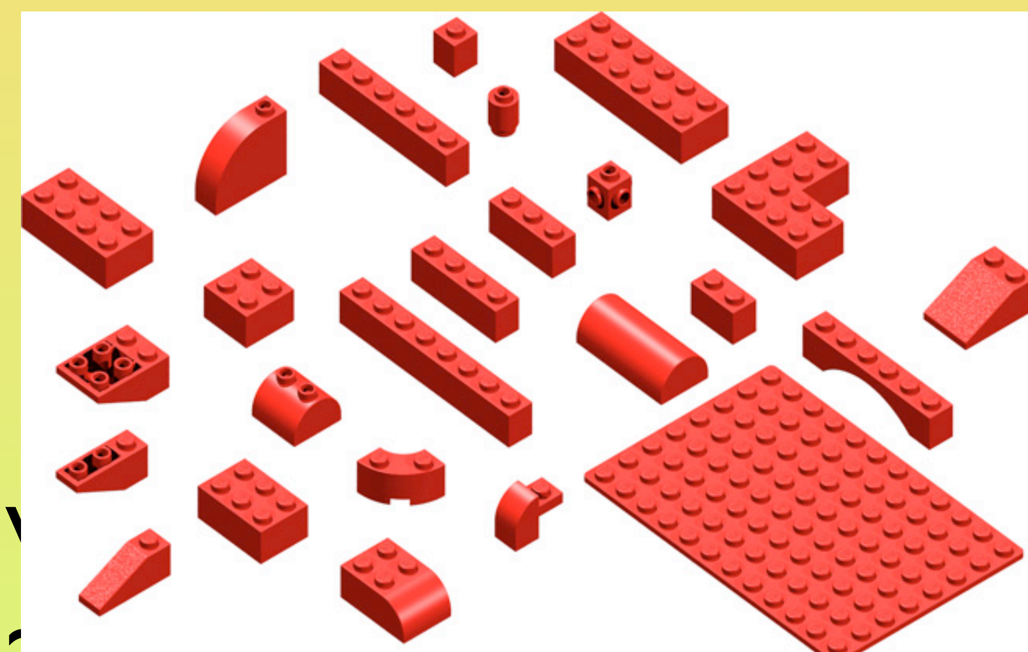

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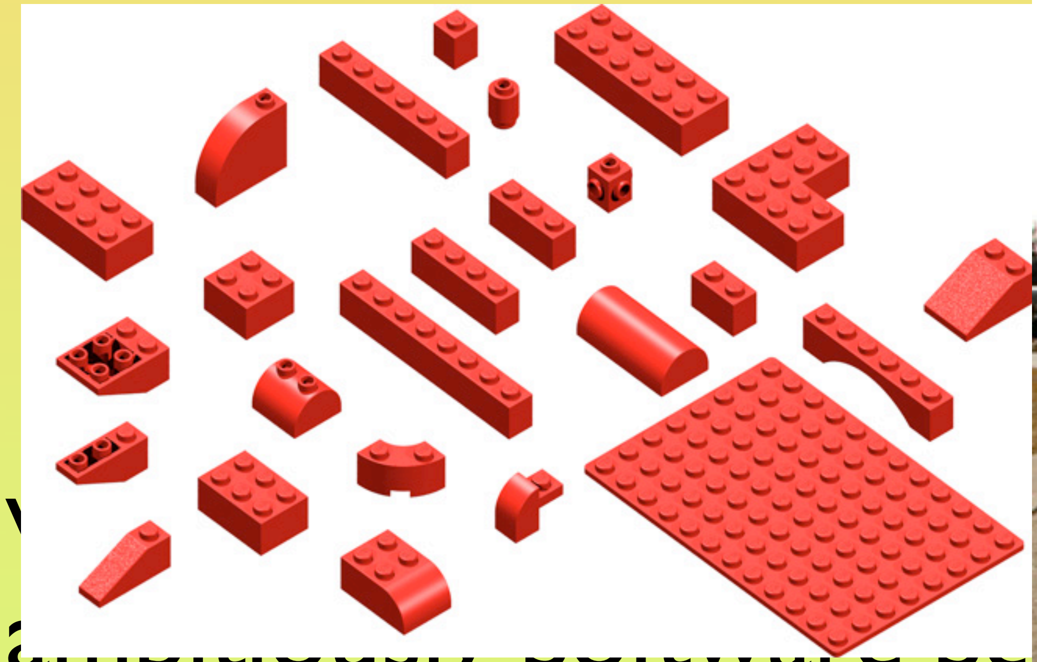

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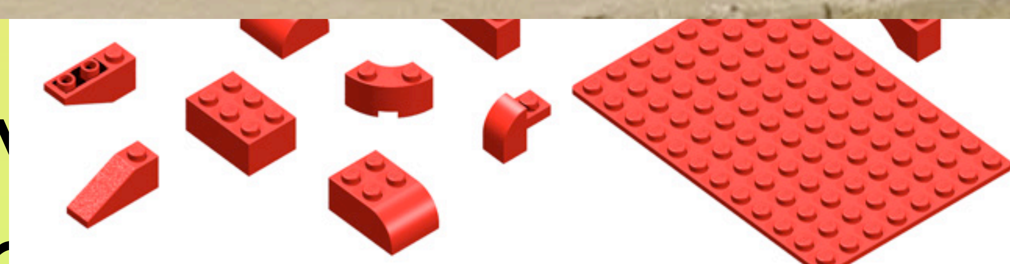

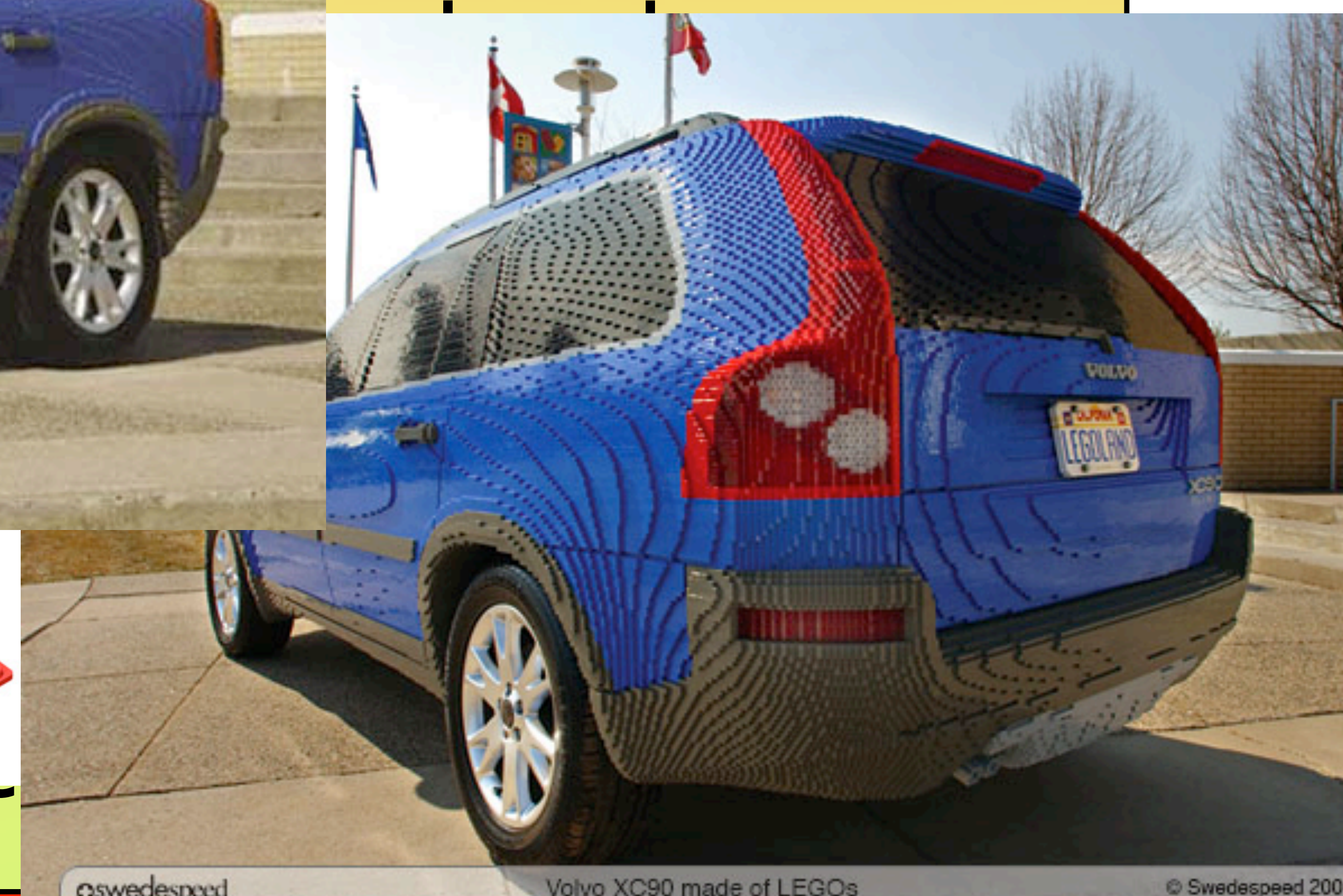

overarching notion: compositionality

Fixation with SOS rules & congruences

SOS = Structural Operational Semantics

Syntax-driven framework to specify the systems' behaviours in a "principled" way

For $\approx$ a notion of systems' equivalence, $\approx$ is a congruence if $\approx$-equivalent systems can be replaced for each other indistinguishably.

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a fundamental engineering & compositionality principle

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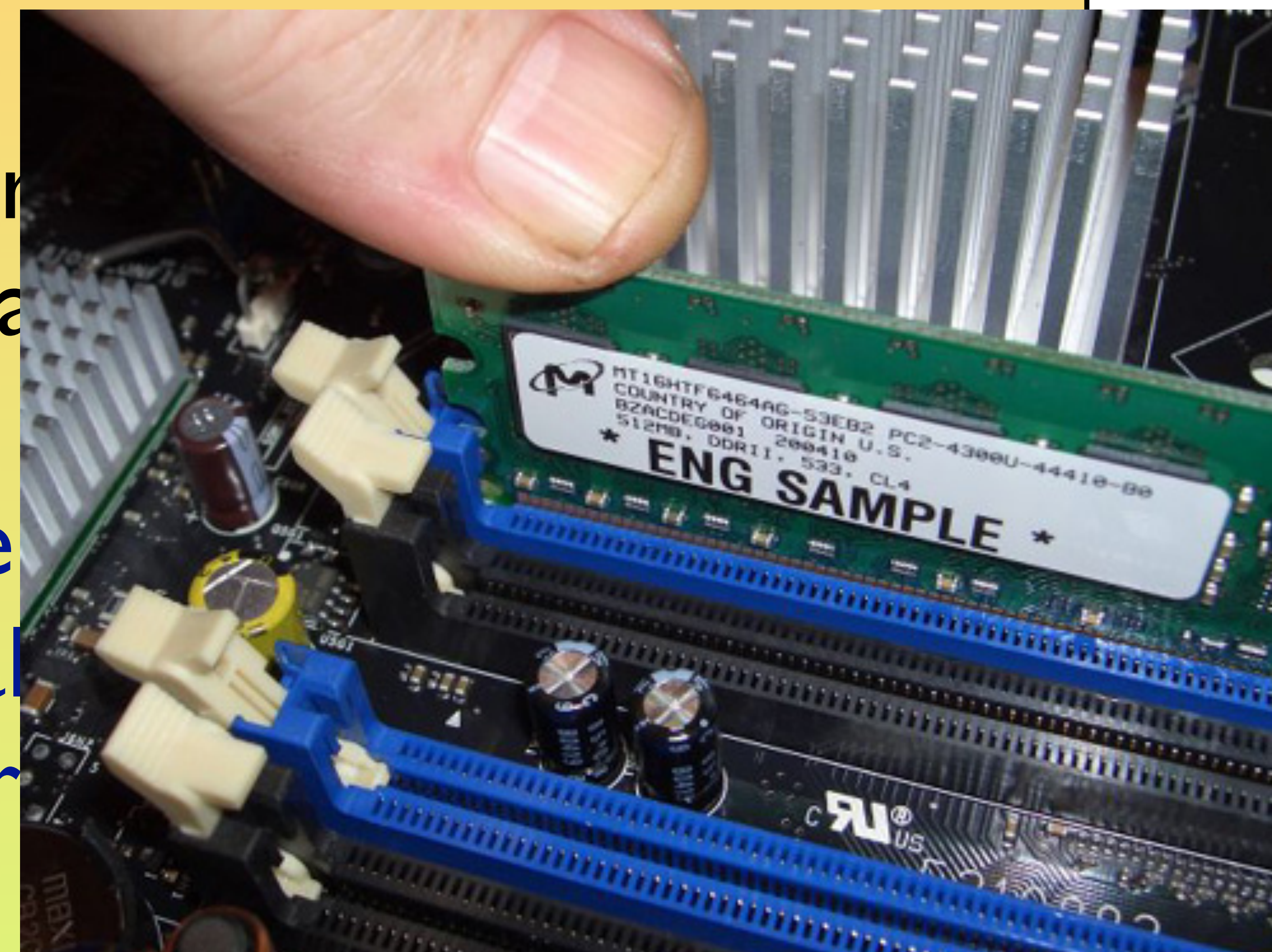

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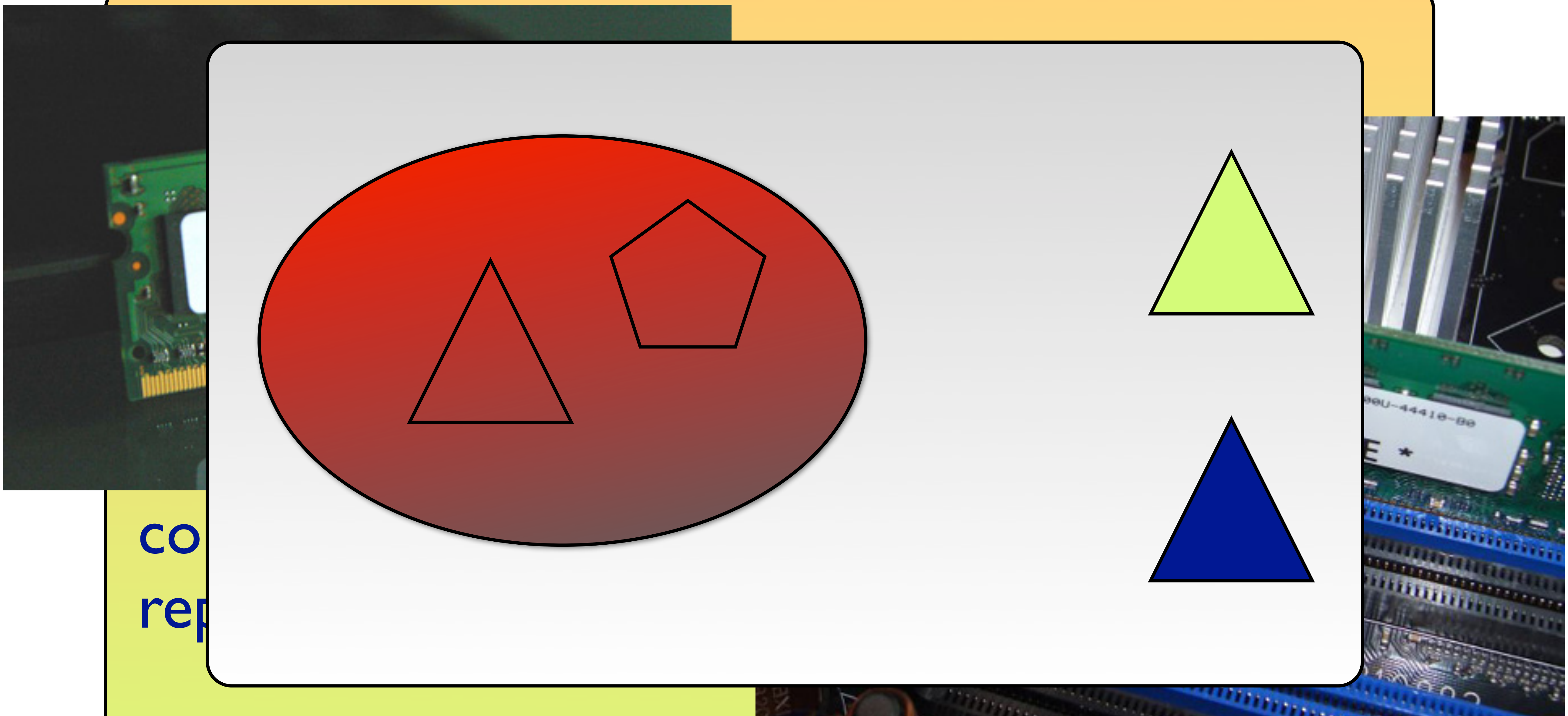

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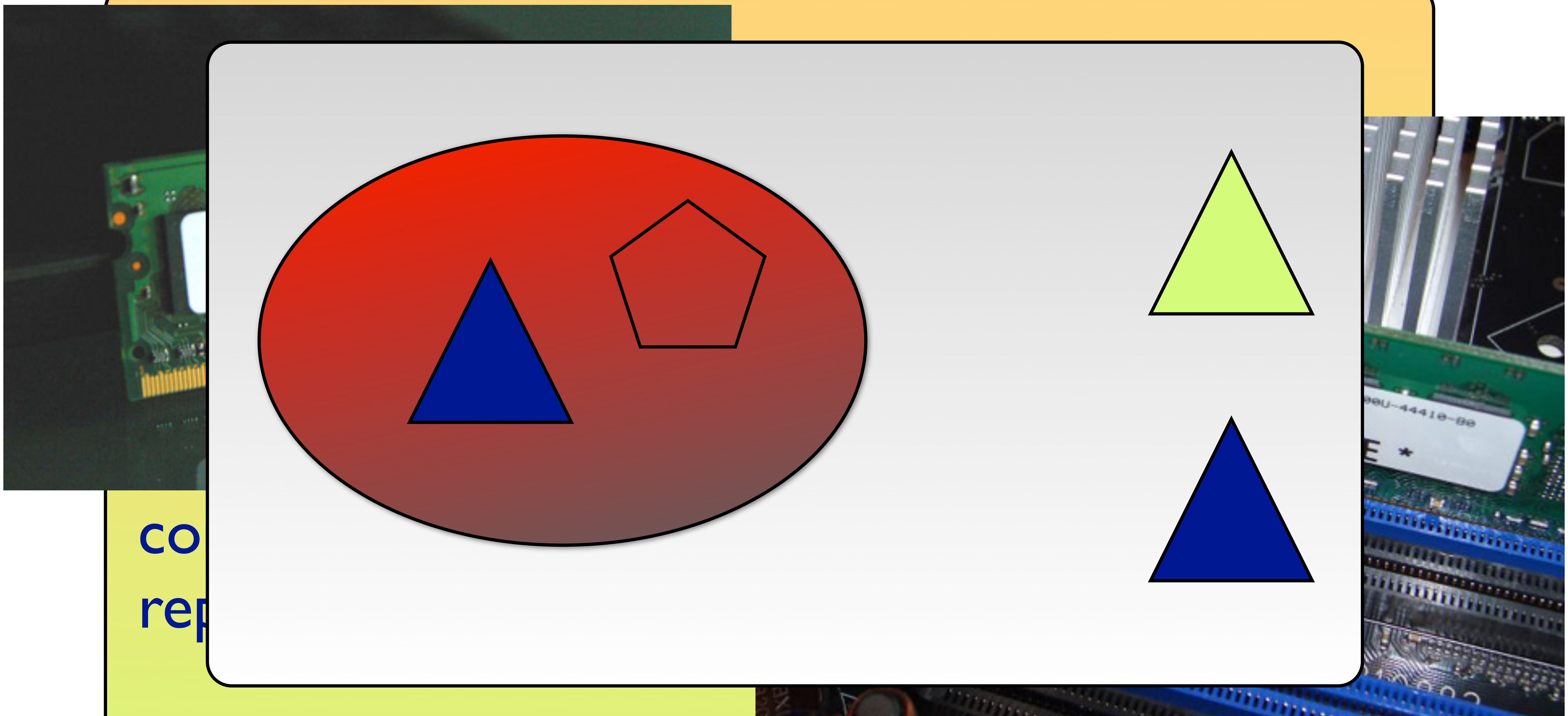

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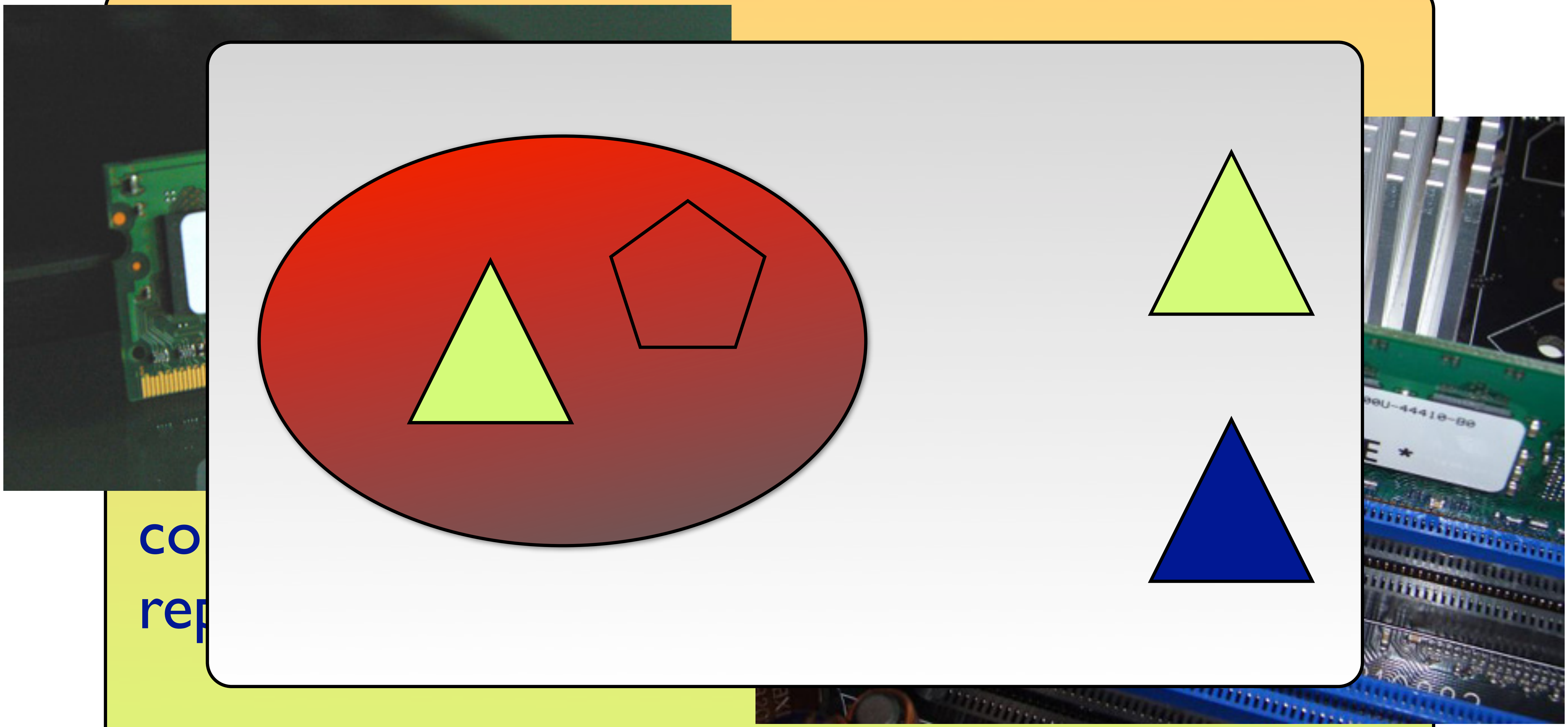

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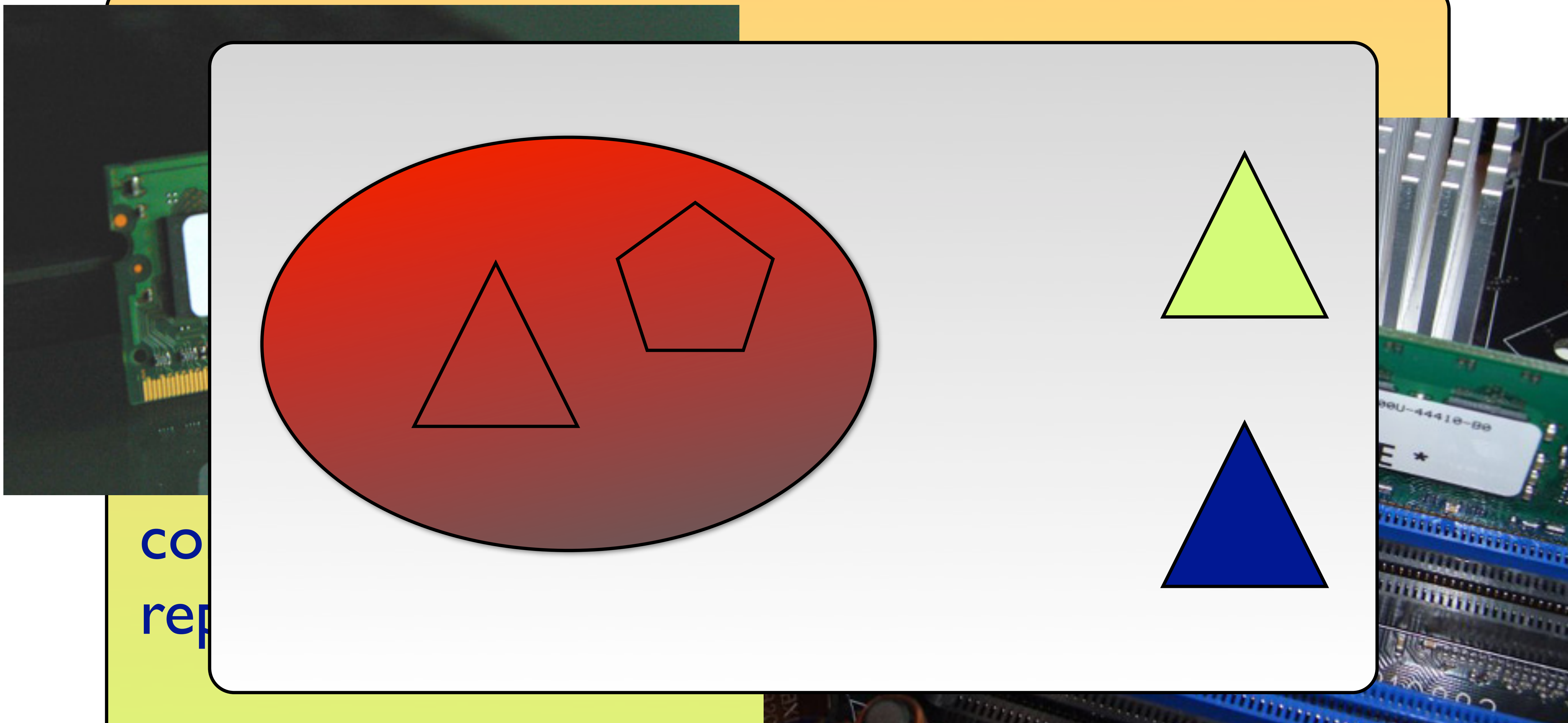

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Executive summary

A syntactic format for SOS
of stochastic systems

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1. Rated transition systems (RTSs)

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2. Some approaches to their structural description
3. SGSOS: a new approach

Abstract

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Two worlds of SOS:

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Nondeterministic systems

$$\frac{\mathtt{x}_1 \xrightarrow{a} \mathtt{y}_1 \quad \mathtt{x}_2 \xrightarrow{\bar{a}} \mathtt{y}_2}{\mathtt{x}_1 \| \mathtt{x}_2 \xrightarrow{\tau} \mathtt{y}_1 \| \mathtt{y}_2}$$

Stochastic systems

$$\frac{\mathtt{x}_1 \xrightarrow{a, r_1} \mathtt{y}_1 \quad \mathtt{x}_2 \xrightarrow{a, r_2} \mathtt{y}_2}{\mathtt{x}_1 \bowtie_L \mathtt{x}_2 \xrightarrow{a, R} \mathtt{y}_1 \bowtie_L \mathtt{y}_2}$$

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Rich SOS theory

- GSOS: a rule format
- ...

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LTSs and inference rules

Labelled transition system:

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if this is in this,
then this is too

$$\frac{}{a.x \xrightarrow{a} x} \qquad \frac{x_1 \xrightarrow{a} y}{x_1 + x_2 \xrightarrow{a} y} \qquad \frac{x_2 \xrightarrow{a} y}{x_1 + x_2 \xrightarrow{a} y}$$

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Fact: Bisimilarity is a congruence.

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A GSOS rule is of the form:

$$\frac{\{\mathtt{x}_{i_j} \xrightarrow{a_j} \mathtt{y}_j\}_{1\leq j\leq m} \quad \{\mathtt{x}_{i_k} \not\xrightarrow{b_k}\}_{1\leq k\leq l}}{\mathtt{f}(\mathtt{x}_1,\ldots,\mathtt{x}_n) \xrightarrow{c} \mathtt{t}}$$

s.t. all $\mathtt{x}_i, \mathtt{y}_j$ distinct and $\mathtt{t}$ has no other variables.

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A GSOS spec: a set of rules where for each $\mathtt{f}$ and c, finitely many rules are triggered by each $A_1,\ldots,A_n$.

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This yields a CTMC for each label

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In an i.f.RTS, apparent rates exist: $r_a(x) = \rho(x \xrightarrow{a} X)$

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1) A naive approach:

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2) Multitransition semantics (PEPA)

3) Proved semantics (Stochastic Pi)

Serious approaches

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$$(a,2).P + (a,2).P \begin{array}{l} \xrightarrow{+_1(a,2)}\!\!\triangleright\ P \\ \xrightarrow[+_2(a,2)]{}\!\!\triangleright\ P \end{array}$$

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$$\frac{}{(a,r).\mathtt{x} \xrightarrow{a,r}\!\!\rhd\ \mathtt{x}} \qquad \frac{\mathtt{x}_1 \xrightarrow{a,r}\!\!\rhd\ \mathtt{y}}{\mathtt{x}_1{+}\mathtt{x}_2 \xrightarrow{a,r}\!\!\rhd\ \mathtt{y}} \qquad \frac{\mathtt{x}_2 \xrightarrow{a,r}\!\!\rhd\ \mathtt{y}}{\mathtt{x}_1{+}\mathtt{x}_2 \xrightarrow{a,r}\!\!\rhd\ \mathtt{y}}$$

$$(a,2).P + (a,2).P \xrightarrow{a,4} P$$

Pepa

Proved semantics:

$$\frac{}{(a,r).\mathtt{x} \xrightarrow{(a,r)}\!\!\rhd\ \mathtt{x}} \qquad \frac{\mathtt{x}_1 \xrightarrow{\theta}\!\!\rhd\ \mathtt{y}}{\mathtt{x}_1{+}\mathtt{x}_2 \xrightarrow{+_1\theta}\!\!\rhd\ \mathtt{y}} \qquad \frac{\mathtt{x}_2 \xrightarrow{\theta}\!\!\rhd\ \mathtt{y}}{\mathtt{x}_1{+}\mathtt{x}_2 \xrightarrow{+_2\theta}\!\!\rhd\ \mathtt{y}}$$

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StocPi

Serious approaches

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$$\frac{}{(a,r).\mathrm{x} \xrightarrow{a,r} \mathrm{x}} \qquad \frac{\mathrm{x}_1 \xrightarrow{a,r} \mathrm{y}}{\mathrm{x}_1 + \mathrm{x}_2 \xrightarrow{a,r} \mathrm{y}} \qquad \frac{\mathrm{x}_2 \xrightarrow{a,r} \mathrm{y}}{\mathrm{x}_1 + \mathrm{x}_2 \xrightarrow{a,r} \mathrm{y}}$$

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A two-step procedure: first a rich LTS, then drop information to get a RTS.

$$(a,2).P + (a,2).P \xrightarrow{a,4} P$$

StocPi

Compositionality issues

When is stoch. bisim. a congruence on the RTS?

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vs.

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Multitransition semantics:

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vs.

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vs.
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Problem: too much information in the labels

Mu

$$\frac{\mathtt{x} \xrightarrow{a,r} \mathtt{y}}{\mathtt{f(x)} \xrightarrow{a,\max(r,5)} \mathtt{y}}$$

$(a, 3).P + (a, 4).P$
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The abstract approach

Transition systems are coalgebras

Distributive laws are formats for SOS

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Transition systems are coalgebras

Distributive laws are formats for SOS

LTS	
Prob. TS	
RTS	

The abstract approach

Transition systems are coalgebras

Distributive laws are formats for SOS

LTS	$$\frac{\{\mathtt{x}_{i_j} \xrightarrow{a_j} \mathtt{y}_j\}_{1\leq j\leq m} \quad \{\mathtt{x}_{i_k} \stackrel{b_k}{\not\rightarrow}\}_{1\leq k\leq l}}{\mathtt{f}(\mathtt{x}_1,\ldots,\mathtt{x}_n) \xrightarrow{c} \mathtt{t}}$$
Prob. TS	
RTS	

[TP97]

The abstract approach

Transition systems are coalgebras

Distributive laws are formats for SOS

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Prob. TS	$$\frac{\left\{\mathtt{x}_i \xrightarrow{a}\right\}_{a\in R_i, 1\leq i\leq n} \quad \left\{\mathtt{x}_i \stackrel{a}{\not\to}\right\}_{a\in P_i, 1\leq i\leq n} \quad \left\{\mathtt{x}_{i_j} \xrightarrow{b_j[u_j]} y_j\right\}_{1\leq j\leq m}}{\mathtt{f}(\mathtt{x}_1,\ldots,\mathtt{x}_n) \xrightarrow{c[w\cdot u_1\cdot\ldots\cdot u_m]} \mathtt{t}}$$	[Bar04]
RTS		

The abstract approach

Transition systems are coalgebras

Distributive laws are formats for SOS

LTS	$$\frac{\{\mathtt{x}_{i_j} \xrightarrow{a_j} \mathtt{y}_j\}_{1\le j\le m} \quad \{\mathtt{x}_{i_k} \stackrel{b_k}{\not\longrightarrow}\}_{1\le k\le l}}{\mathtt{f}(\mathtt{x}_1,\ldots,\mathtt{x}_n) \xrightarrow{c} \mathtt{t}}$$	[TP97]
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RTS	$$\frac{\left\{\mathtt{x}_i \xrightarrow{a@r_{ai}}\right\}_{a\in D_i,1\le i\le n} \quad \left\{\mathtt{x}_{i_j} \xrightarrow{b_j} \mathtt{y}_j\right\}_{1\le j\le k}}{\mathtt{f}(\mathtt{x}_1,\ldots,\mathtt{x}_n) \xrightarrow{c@R} \mathtt{t}}$$	[this08]

SGSOS rules

$$\frac{\left\{r_a(x_i) = r_{a,i}\right\}_{a_i \in D_i,\ 1 \leq i \leq n} \qquad \left\{x_{i_j} \xrightarrow{bj} y_j\right\}_{b_j \in D_{i_j},\ 1 \leq j \leq k}}{\mathtt{f}(\mathtt{x}_1, \ldots, \mathtt{x}_n) \xrightarrow{c@W} \mathtt{t}}$$

- $D_i \subseteq A$ and $W, r_{a,i} \in \mathbb{R}^+$
- all $\mathtt{x}_i, \mathtt{y}_{a,i}$ distinct and $\mathtt{t}$ has no other variables
- all $\mathtt{y}_{a,i}$ appear in $\mathtt{t}$ exactly if

SGSOS rules

apparent rate of *a* in x*i*

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SGSOS spec: set of rules subject to a size condition.

How SGSOS rules induce RTSs

$$P = \mathtt{f}(P_1, \ldots, P_n)$$

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Conditional probability of $P_i \xrightarrow{a} Q_{a,i}$.

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4. Add $W \cdot \prod_{a,i} p_{a,i}$ to $\rho(P \xrightarrow{c} \mathtt{t}[^{P_i}/_{\mathtt{x_i}}, {}^{Q_{a,i}}/_{\mathtt{y_{a,i}}}])$

Theorem

Stochastic bisimilarity on the RTS induced by an SGSOS specification is a congruence.

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Proof: SGSOS is to RTSs what GSOS is to LTSs.

Examples

Atomic actions

$$P ::= \texttt{nil} \mid (a, r).P$$

$$\frac{}{(a, r).\texttt{X} \xrightarrow{a@r} \texttt{X}}$$

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Choice

$$P ::= \cdots \mid P + P$$

$$\frac{\texttt{x}_1 \xrightarrow{a@r} \texttt{y}}{\texttt{x}_1 + \texttt{x}_2 \xrightarrow{a@r} \texttt{y}} \qquad \frac{\texttt{x}_2 \xrightarrow{a@r} \texttt{y}}{\texttt{x}_1 + \texttt{x}_2 \xrightarrow{a@r} \texttt{y}}$$

Examples II

Synchronisation

$$P ::= \ldots \mid P \underset{L}{\bowtie} P$$

$$\frac{\mathtt{x}_1 \xrightarrow{a@r} \mathtt{y}}{\mathtt{x}_1 \underset{L}{\bowtie} \mathtt{x}_2 \xrightarrow{a@r} \mathtt{y} \underset{L}{\bowtie} \mathtt{x}_2} \qquad \frac{\mathtt{x}_2 \xrightarrow{a@r} \mathtt{y}}{\mathtt{x}_1 \underset{L}{\bowtie} \mathtt{x}_2 \xrightarrow{a@r} \mathtt{x}_1 \underset{L}{\bowtie} \mathtt{y}}$$

$$\frac{\mathtt{x}_1 \xrightarrow{b@r_1} \mathtt{y}_1 \quad \mathtt{x}_2 \xrightarrow{b@r_2} \mathtt{y}_2}{\mathtt{x}_1 \underset{L}{\bowtie} \mathtt{x}_2 \xrightarrow{b@R} \mathtt{y}_1 \underset{L}{\bowtie} \mathtt{y}_2}$$

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minimal rate law: $R = \min(r_1, r_2)$

mass action law: $R = r_1 \cdot r_2$

Examples III

Communication

$$P ::= \ldots \mid P \parallel P$$

$$\frac{\mathtt{x}_1 \xrightarrow{a@r} \mathtt{y}}{\mathtt{x}_1 \parallel \mathtt{x}_2 \xrightarrow{a@r} \mathtt{y} \parallel \mathtt{x}_2} \qquad \frac{\mathtt{x}_2 \xrightarrow{a@r} \mathtt{y}}{\mathtt{x}_1 \parallel \mathtt{x}_2 \xrightarrow{a@r} \mathtt{x}_1 \parallel \mathtt{y}}$$

$$\frac{\mathtt{x}_1 \xrightarrow{a@r_1} \mathtt{y}_1 \quad \mathtt{x}_2 \xrightarrow{\bar{a}@r_2} \mathtt{y}_2}{\mathtt{x}_1 \parallel \mathtt{x}_2 \xrightarrow{\tau@R} \mathtt{y}_1 \parallel \mathtt{y}_2}$$

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Examples IV

Catalysts / Inhibitors

$$\frac{\mathtt{x} \xrightarrow{a@r} \mathtt{y}}{\mathtt{cat}_a(\mathtt{x}) \xrightarrow{a@2r} \mathtt{cat}_a(\mathtt{y})} \qquad \frac{\mathtt{x} \xrightarrow{a@r} \mathtt{y}}{\mathtt{inh}_a(\mathtt{x}) \xrightarrow{a@r/2} \mathtt{inh}_a(\mathtt{y})}$$

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Other things

$$\frac{\mathtt{x}_1 \xrightarrow{a@r_1} \mathtt{y} \quad \mathtt{x}_2 \xrightarrow{a@r_2}}{\mathtt{x}_1 !! \mathtt{x}_2 \xrightarrow{a@r_1} \mathtt{y} !! \mathtt{x}_2} \qquad \frac{\mathtt{x}_1 \xrightarrow{a@r_2} \quad \mathtt{x}_2 \xrightarrow{a@r_1} \mathtt{y}}{\mathtt{x}_1 !! \mathtt{x}_2 \xrightarrow{a@r_1} \mathtt{x}_1 !! \mathtt{y}}$$

(for $r_1 > r_2$)

Associativity of parallel composition

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CCS-style (e.g. Stoch-Pi)

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Thm: $||$ associative iff $R = c \cdot r_1 \cdot r_2$ for c constant.

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Thm: $\underset{L}{\bowtie}$ associative iff R associative:

$$R(R(r_1, r_2), r_3) = R(r_1, R(r_2, r_3))$$

Conclusions

- A rule format for RTS specifications
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- Missing: recursive definitions, name passing in Pi