

# Exponential Decay in Probabilistic Trust Models

**Mogens Nielsen's 60th Fest**

**Aarhus, 4/10/09**



*To your sharpness of mind and kindness of heart*

**Vladimiro Sassone**

I dag er det  
Mogens' fødselsdag

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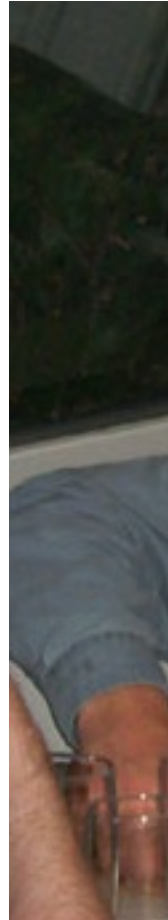


# I dag er det Mogens' fødselsdag





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- Features of Ubiquitous Computing like *scalability*, *mobility*, and *incomplete information* deeply affect security requirements.
- One of the proposed approaches is to use a notion of *computational trust*, resembling the concept of trust among human beings.

- *Probabilistic models* are used to evaluate trust.
- A probabilistic model assigns a degree of confidence to a principal's ability to predict another principal's behaviour.
- Eg, the behaviour of a principal  $A$  may be defined as the probability that interaction with  $A$  yields a certain outcome (eg, success or failure).

# Beta Trust Model

- The outcome of an interaction between a principal  $a$  and a partner  $b$  is either *successful* or *unsuccessful*:

$$o \in \{Succ, Fail\}$$

- The probability that a partner  $b$  interacts successfully with  $a$  is governed by the parameter  $\theta$  where

$$\theta = \Pr(o = Succ)$$



# Beta Trust Model

- The behaviour of the partner  $b$  represented by  $\theta$  is assumed to be fixed over time.
- The estimated probability of success,  $B(Succ | o)$ , at time  $t$  is the expected value of  $\theta$  *given the sequence of outcomes*  $o = \{o_0, o_1, \dots, o_t\}$

$$B(Succ | o) = E[\theta | o]$$

# Beta Trust Model

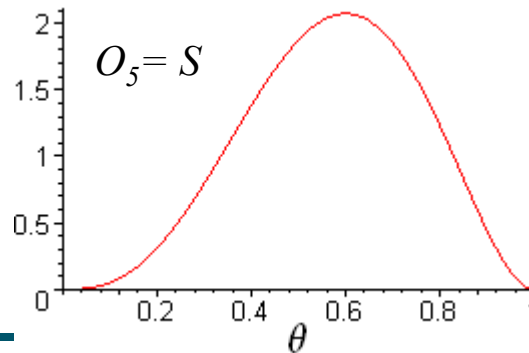
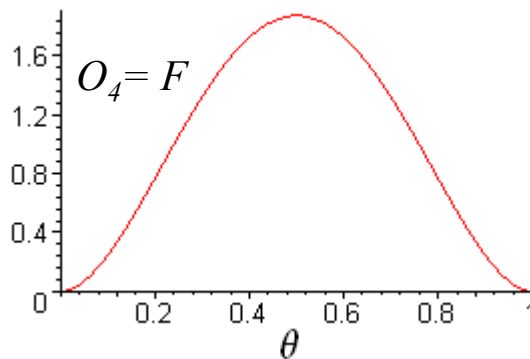
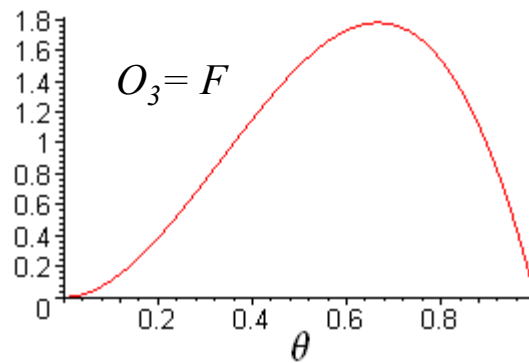
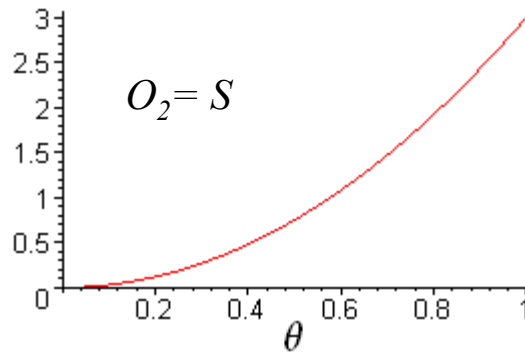
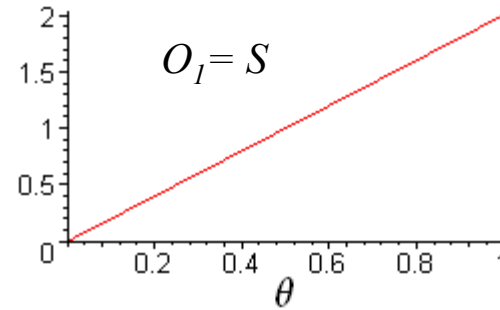
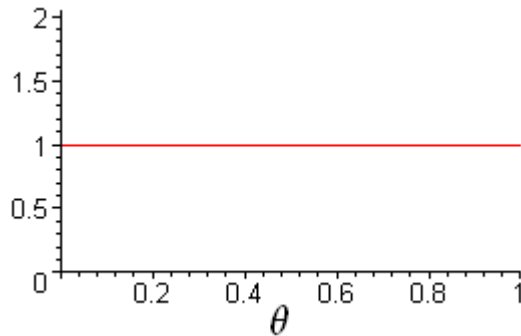
Using Bayesian inference to learn the parameter  $\theta$  from observations  $o$ .

The random variable  $\theta$  follows Beta distribution function, and therefore

$$B(Succ \mid o) = E[\theta \mid o] = \frac{m(o) + 1}{m(o) + n(o) + 2}$$

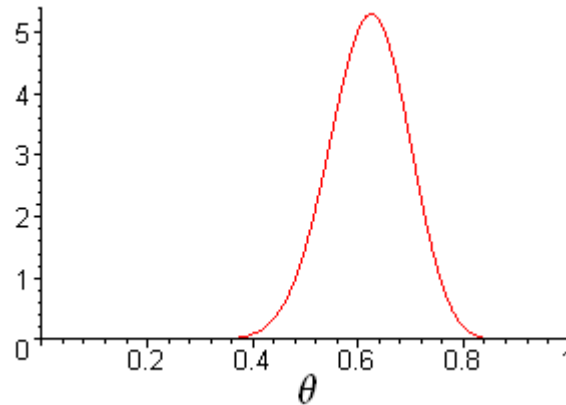
where  $m(o)$  is the number of successful interactions in  $o$  and  $n(o)$  that of unsuccessful ones in  $o$ .

# Trust Inference Process



# Trust Inference Process

The distribution of  $\theta$  after 40 interactions  
25 Successful and 15 Failed





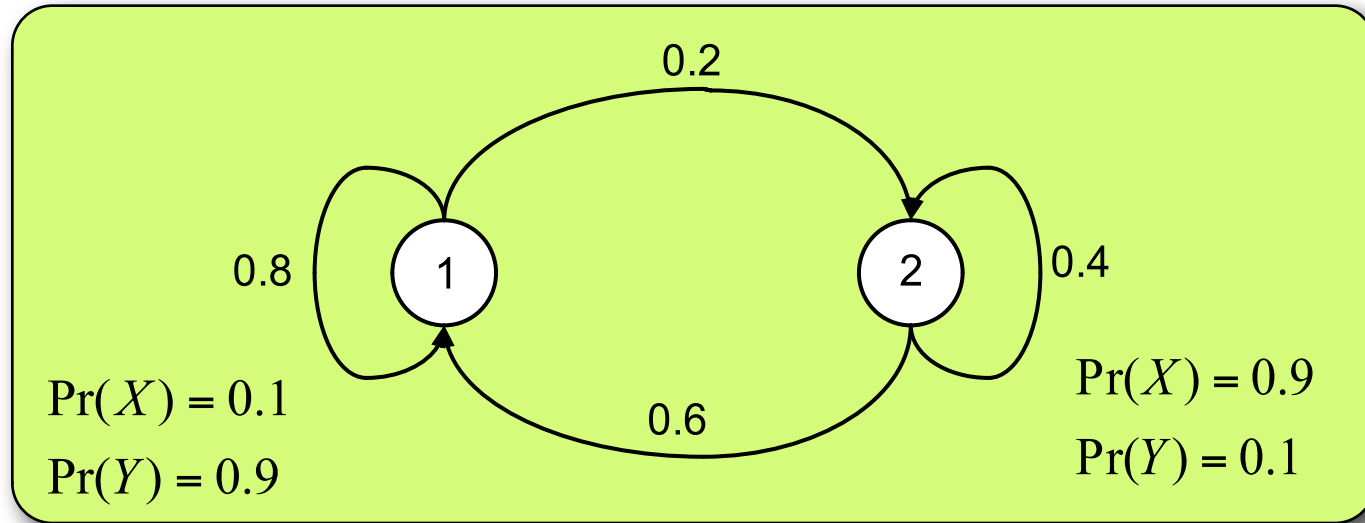
# Limitation of the Beta model

- The assumption that a principal behaviour is fixed is not always realistic:
- The behaviour of a principal may depend on its internal state which may change over time.

# Modelling Dynamic Behaviour

- Modelling static behaviour as a probability distribution over outcomes leads to modelling the dynamic behaviour by a *Hidden Markov Model (HMM)*.
- A single state in an HMM models the system behaviour at a particular time.

# Hidden Markov Model:



$$S = \{1, 2\}$$

$$V = \{X, Y\}$$

$$A = \begin{bmatrix} 0.8 & 0.2 \\ 0.6 & 0.4 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

# Beta Trust Model with Decay

- The probability distribution over outcomes changes over time.
- Old observations are given less weight (decayed) than more recent observations.
- Weights of observations are controlled by the decay factor  $r$ .



# Beta Trust Model with Decay

Given a decay factor  $0 \leq r < 1$  and an observation sequence  $O = \{o_L, \dots, o_1, o_0\}$ , where  $o_0$  is the last outcome,  $o_1$  is the previous outcome, and so on, then

$$B_r(Succ|O) = \frac{m_r(O) + 1}{m_r(O) + n_r(O) + 2} \quad , \quad B_r(Fail|O) = \frac{n_r(O) + 1}{m_r(O) + n_r(O) + 2}$$

Where

$$m_r(O) = \sum_{i=0}^L r^i \delta_i(Succ) \quad , \quad n_r(O) = \sum_{i=0}^L r^i \delta_i(Fail)$$

And

$$\delta_i(Succ) = \begin{cases} 1 & \text{if } o_i = Succ \\ 0 & \text{otherwise} \end{cases} \quad , \quad \delta_i(Fail) = \begin{cases} 1 & \text{if } o_i = Fail \\ 0 & \text{otherwise} \end{cases}$$

# How good is the model ?

- Given a dynamic system modelled by an HMM  $\lambda$  we define Beta estimation error as follows

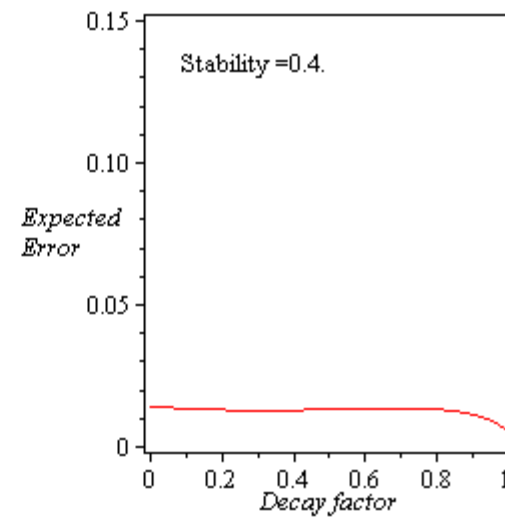
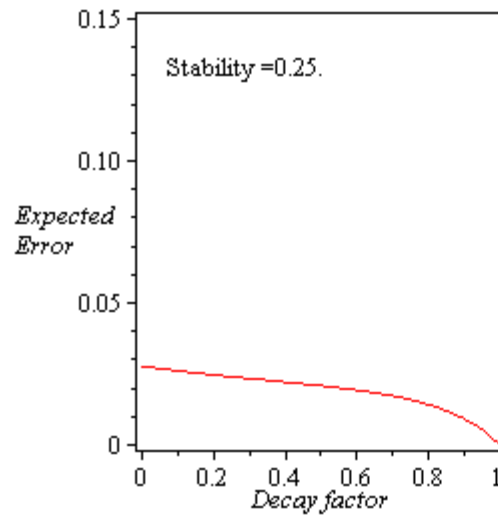
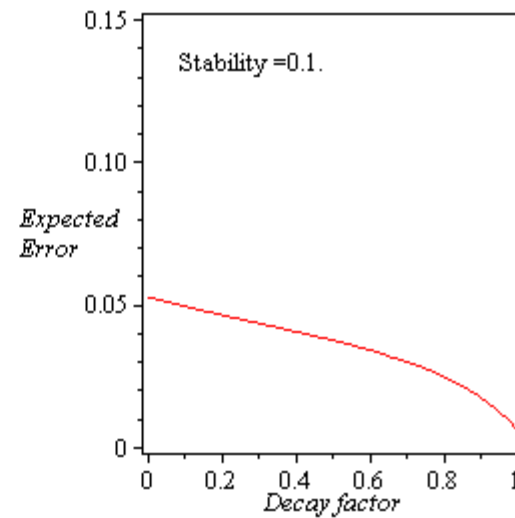
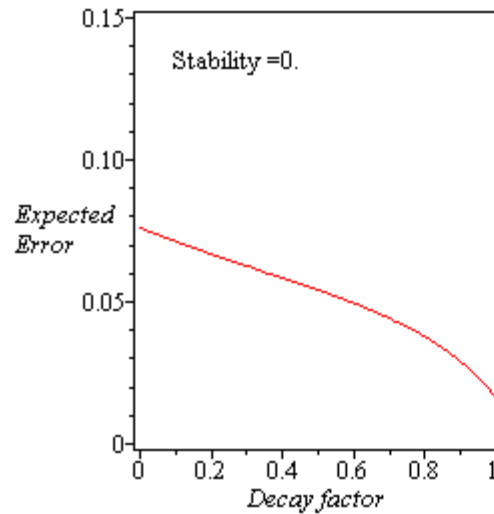
$$\text{Error}(\lambda, r) = E \left[ (B(\text{Succ} \mid o) - \alpha)^2 \right]$$

where  $r$  is the decay factor, and  $\alpha$  is the real probability that next outcome is Success

- *System stability* is the expected probability of the HMM remaining in the same state.
- Consider the system modelled by HMM:

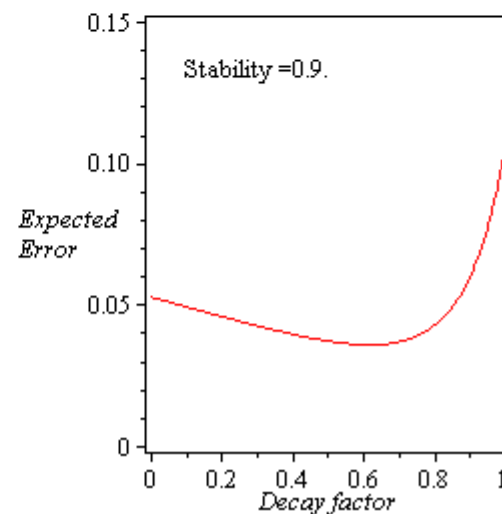
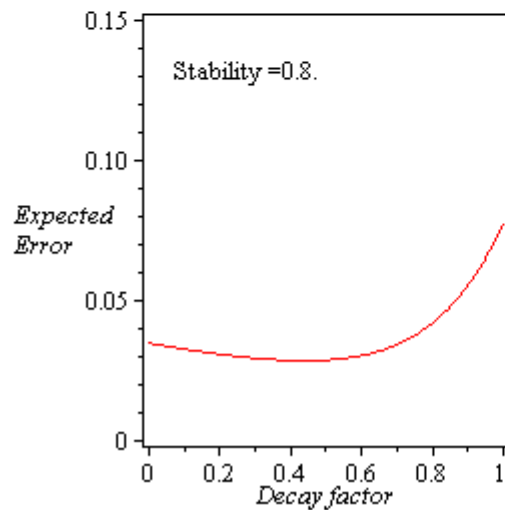
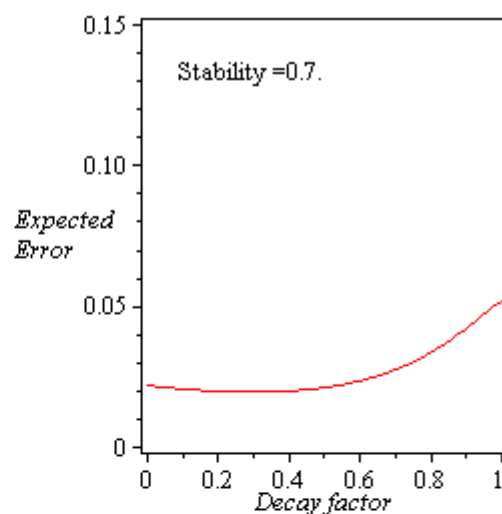
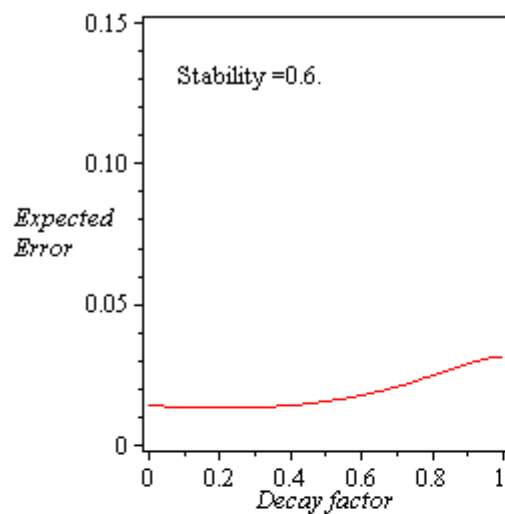
$$A_{\lambda} = \begin{bmatrix} s & \frac{1-s}{3} & \frac{1-s}{3} & \frac{1-s}{3} \\ \frac{1-s}{3} & s & \frac{1-s}{3} & \frac{1-s}{3} \\ \frac{1-s}{3} & \frac{1-s}{3} & s & \frac{1-s}{3} \\ \frac{1-s}{3} & \frac{1-s}{3} & \frac{1-s}{3} & s \end{bmatrix} \quad \Theta_{\lambda} = \begin{bmatrix} 1.0 \\ 0.7 \\ 0.3 \\ 0.0 \end{bmatrix}$$

# Unstable System

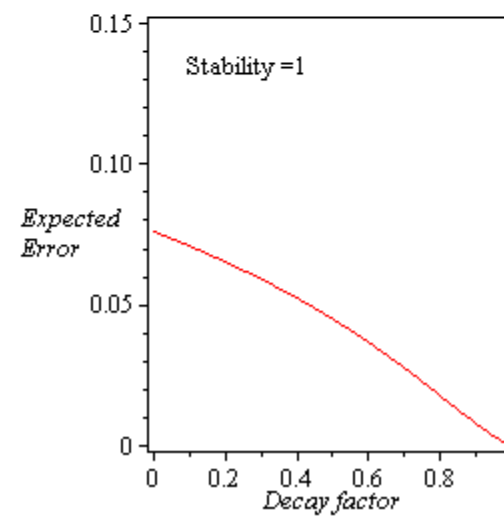
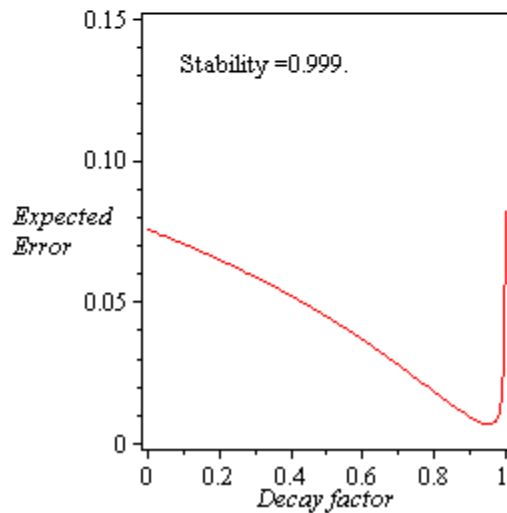
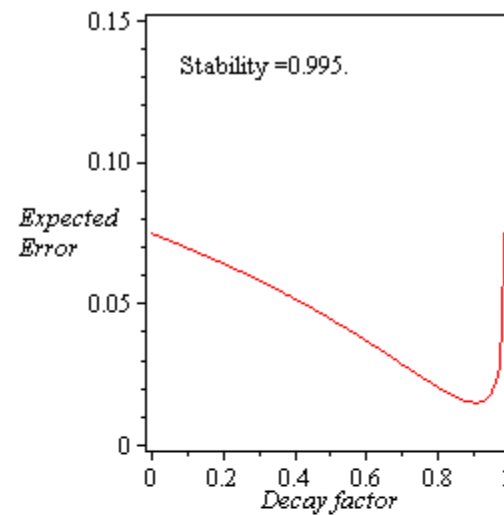
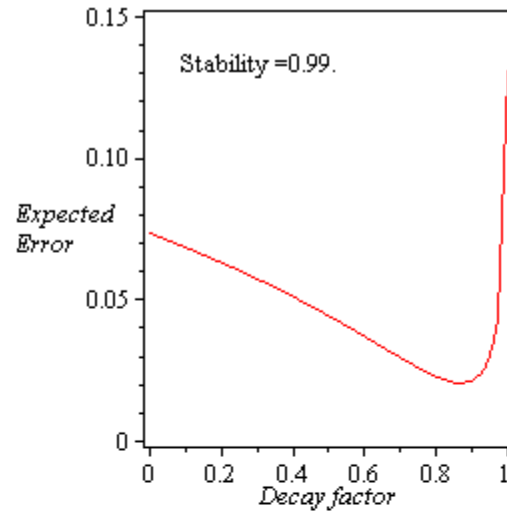




# Stable System



# Very Stable System



- Traditional Beta trust models are unable of coping with dynamic behaviour systems.
- Using a decay scheme enhances Beta estimation in cases where the system is very stable.
- Beta estimation error is subject to choosing the optimal value of decay which depends on the system parameters.

- Investigate using HMM as a trust model instead of using existing Beta models [ FAST 2009 with MN ]
  
- Handling Reputation problems
  - Modelling reputation reports
  - Combining reputation reports to update trust
  - Evaluating confidence in the evaluated trust