On Optimal Agendas for Multi-Issue Negotiation

Shaheen Fatima1   Michael Wooldridge2   Nicholas R. Jennings3

1 Department of Computer Science
Loughborough University, Loghborough, LE11 3TU, U.K.
S.S.Fatima@lboro.ac.uk

2 Department of Computer Science
University of Liverpool, Liverpool, L69 7ZF, U.K.

mjw@liv.ac.uk

3 School of Electronics and Computer Science
University of Southampton, Southampton SO17 1BJ, U.K.
nrj@ecs.soton.ac.uk

Abstract. This paper analyzes bilateral multi-issue negotiation where the issues are *divisible*, there are time constraints in the form of *deadlines* and *discount factors*, and the agents have different preferences over the issues. The issues are negotiated using the *package deal* procedure. The set of issues to be negotiated is a choice variable in that the agents can decide what issues to negotiate. This set is called the negotiation *agenda*. Since the outcome of negotiation depends on the agenda, it is important to determine what agenda maximizes an agent's utility and is therefore its *optimal agenda*. To this end, this paper presents polynomial time methods for finding an agent's optimal agenda.

1 Introduction

Negotiation has long been studied by economists and game theorists but is now receiving increasing attention from researchers in multi-agent systems [10, 13]. In the existing work, the analysis of negotiation typically begins with a given set of issues and the parties' preferences (in the form of their utilities) for different possible settlements of the issues. Within this framework, theorists have investigated a range of negotiation procedures such as the *package deal* procedure (PDP), the *simultaneous* procedure (SP), and the *sequential* procedure (SQP) [7, 8, 2].

It is well known that different procedures result in different outcomes and, therefore, give different utilities to the agents [6]. So it is important that agents choose the right procedure. Moreover, for a given procedure, it is also important that the agents choose an appropriate *agenda*. The term agenda refers to the set of issues included for negotiation [11]. The agenda is important because, irrespective of the procedure, the outcome of negotiation depends on the agenda [7, 14]. Thus, given the utility maximizing feature of agents, it is important to find what agenda maximizes their utility and is therefore *optimal* for them. For example, consider a car dealer who has five second hand cars to sell. A potential buyer may be interested in buying two of these. So he must first choose which two cars to negotiate the price for (i.e., from all possible subsets of size

two, he must choose the one that maximizes his utility and so is his *optimal agenda*). Note that here, the buyer has choice over the agenda but not the seller.

Although the importance of agendas has been recognized [7, 11], most existing work has taken the agenda as given and then analyzed the outcome for different procedures [8, 5, 6]. But as the above example illustrates, the set of issues to be negotiated themselves are often choice variables (i.e., what issues to negotiate can be chosen by a negotiator) whose ultimate configuration can have decisive effects on the negotiation outcome [14]. Thus, in addition to knowing what procedure is best for an agent, it is important to know what agenda maximizes an agent's utility and is therefore its optimal agenda. To this end, this paper analyzes the problem of finding optimal agendas in the context of the PDP (future work will deal with SP and SQP). The key contribution of this paper lies in presenting, for the first time, polynomial time methods for determining optimal agendas for the PDP.

The rest of the paper is structured as follows. Section 2 provides a discussion of related literature. Section 3 describes the setting and defines 'agenda' and 'optimal agenda'. Section 4 shows how to find optimal agendas for the complete information setting. Section 5 builds on Section 4 to show how to find optimal agendas in an incomplete information setting. Section 6 concludes.

2   Related Literature

There are different procedures such as the PDP, the SP, and the SQP for multi-issue negotiation, and the term agenda has different meanings in these different contexts. For the PDP (which is the focus of this paper) and the SP, the term *agenda* refers to the set of issues to be included for negotiation. But for the SQP, the term refers to not just the set of issues to be included for negotiation but also the order in which they will be negotiated.

Although the importance of agendas has been recognized [7, 11], most existing work has taken the set of issues as given and then analyzed the equilibrium outcome for different procedures [9, 8, 6]. In the context of the SQP, [1] takes the set of issues to be negotiated as given and provides an analysis of the role of information and time preferences on the equilibrium, while [4, 5] has dealt with taking the set of issues to be negotiated as given and finding an optimal ordering for the given set. But as the example outlined in the Introduction illustrates, the set of issues to be negotiated themselves are often choice variables (i.e., what issues to negotiate can be chosen by a negotiator) whose ultimate configuration can have decisive effects on the negotiation outcome [14]. Thus, a negotiator must make three key choices: he/she must decide the following:

1. what negotiation procedure to use,
2. what set of issues to negotiate, and
3. for the SQP, what ordering to use for negotiating a given set of issues.

To our knowledge, there is no existing work that deals with finding what set of issues to negotiate. Hence, the novelty of this paper lies in showing how to find the set of issues to negotiate. Specifically, it analyzes the problem of finding optimal agendas in the context of the PDP. The key contribution of this paper lies in presenting, for the first time, polynomial time methods for determining optimal agendas for the PDP.

3 The Negotiation Setting

An agent's optimal agenda depends on its equilibrium utility from all possible agendas. So we first give an overview of the equilibrium for single and multi-issue negotiation. On the basis of this equilibrium, we will show how to find optimal agendas in Sections 4 and 5. A formal definition of optimal agenda is given toward the end of this section.

3.1 Equilibrium for Single Issue

We use the single issue model of [6] in which two agents, a and b, negotiate over a single divisible issue which is a 'pie' of size 1. The agents want to determine how to split it between themselves. Let $n \in N^+$ be the deadline and $0 < \delta \leq 1$ the discount factor for both agents. The agents use an alternating offers protocol [12], which proceeds through a series of time periods. One of the agents, say a, starts in the first time period (i.e., $t = 1$) by making an offer ($0 \leq x^a \leq 1$) to b. Agent b can accept/reject the offer. If it accepts, negotiation ends in an agreement with a getting x^a and b getting $x^b = 1 - x^a$. Otherwise, negotiation goes to the next time period, in which b makes a counter-offer. This process continues until one of the agents either accepts an offer or quits negotiation (resulting in a conflict).

Agent a's utility at time t from a share x^a is $u^a(x^a, t) = x^a \delta^{t-1}$ if $t \leq n$, otherwise it is zero. For b, u^b is analogous. For this setting, the equilibrium offers are obtained using backward induction (BI) as follows. Let $n = 1$ and let a be the first mover. If b accepts a's proposal at $t = 1$, the division occurs as agreed; if not, neither agent gets anything (since the deadline is $n = 1$). Here, a is in a powerful position and is able to keep 100% of the pie and give nothing to b[1]. Agent b accepts and agreement takes place at $t = 1$.

Now, let $n = 2$ and $\delta = 1/2$. The first mover (say a) decides what to offer at $t = 1$, by looking ahead to $t = 2$ and reasoning backwards. Agent a reasons that if negotiation goes to $t = 2$, b will take 100% of the shrunken pie by offering $[0, 1/2]$. So, at $t = 1$, if a offers b anything less than 1/2, b will reject the offer. So, at $t = 1$, a offers $[1/2, 1/2]$. Agent b accepts and an agreement occurs at $t = 1$. In this way, BI was used to obtain the equilibrium offer for $t > 2$.

3.2 Equilibrium for Multiple Issues for the PDP

Let $I = \{1, 2, \ldots, m\}$ be the set of issues. Agent a's (b's) preference for issue i is represented with a weight $w_i^a \in R_+$ ($w_i^b \in R_+$). Each issue is a pie of size 1 and can be split between the agents such that if x_i^a and x_i^b are a's and b's shares for issue i, then $x_i^a + x_i^b = 1$. Here n is the deadline for all the issues, and δ the discount factor for all of them. Agent a's cumulative utility at time $t \leq n$ is given by $U^a(I, x^a, t) = \delta^{t-1} \sum_{i=1}^m w_i^a x_i^a$. For b, $U^b(I, x^b, t)$ is analogous. As before, an agent's utility for $t > n$ is zero.

---

[1] It is possible that b may reject such a proposal. But, irrespective of whether b accepts or rejects, it gets zero utility (since the deadline is $n = 1$). So, b accepts a's offer.

For the above model, the equilibrium was obtained, using BI, as follows [6]. We give an overview of this first for complete information setting, and then explain its extension to an incomplete information setting.

The complete information setting. Let $\text{SA}(t)$ ($\text{SB}(t)$) denote a's (b's) equilibrium strategy for time t. For $t = n$, the offering agent gets a 100% of all the shrunken pies. For all previous time periods, $t < n$, the offering agent (say a) proposes a package ($[x^a, x^b]$) such that b's cumulative utility from it is what b would get from its own offer for $t + 1$. If there is more than one such package, then a must choose the one that maximizes its own cumulative utility. Thus, a must solve the following trade-off problem (called TA): maximize $\sum_{i=1}^{m} w_i^a x_i^a$, subject to $\sum_{i=1}^{m} w_i^b (1 - x_i^a) = Y$ where $0 \leq x_i^a \leq 1$. Here, Y is b's cumulative utility from its own offer SB(t+1). On the other hand, if a receives an offer $[x^a, x^b]$ at time t, then it accepts if $U^a(I, x^a, t) = Z$ where Z is a's cumulative utility from its own offer $\text{SA}(t + 1)$. The equilibrium strategy for b (in terms of TB) is analogous. Thus we have:

$$\text{SA}(n) = \begin{cases} \text{OFFER } [\mathbf{1}, \mathbf{0}] & \text{If } a\text{'s turn to offer} \\ \text{ACCEPT} & \text{If } a\text{'s turn to receive} \end{cases}$$

where $\mathbf{0}$ ($\mathbf{1}$) denotes a vector of m zeros (ones). For all preceding time periods $t < n$, the strategies are as follows:

$$\text{SA}(t) = \begin{cases} \text{OFFER } \text{TA} & \text{If } a\text{'s turn to offer} \\ \text{If } (U^a(I, x^a, t) \geq Z) \text{ ACCEPT} & \text{If } a \text{ receives } x^a \\ \text{else REJECT} \end{cases}$$

For b, $\text{SB}(t)$ is analogous. Both TA and TB are standard *fractional knapsack* problems [3]. The solution to TA is for a to consider issues in ascending order of w_i^a / w_i^b and allocate to b maximum possible share for the individual issues until b's cumulative utility equals Y. If the issues $1, \ldots, m$ are in ascending order of w_i^a / w_i^b, the equilibrium solution is $x^a = \{0, .., 0, x_c^a, 1, .., 1\}$, $x^b = \{1, .., 1, 1 - x_c^a, 0, .., 0\}$. The solution to TB is analogous.

An incomplete information setting. Here, the agents are uncertain about their utilities. Let T be the number of possible utility function pairs. The jth possible pair (U_j^a, U_j^b) occurs with probability γ_j. For a, the jth function is: $U_j^a(I, x^a, t) = \delta^{t-1} \sum_{i=1}^{m} w_{ij}^a x_i^a$, and its expected utility is $EU^a(I, x^a, t) = \delta^{t-1} \sum_{j=1}^{T} \gamma_j \times U_j^a(I, x^a, t)$. Agent a's expected weight for issue i is $ew_i^a = \delta^{t-1} \sum_{j=1}^{T} \gamma_j w_{ij}^a$. For b, U_j^b, $EU^b(I, x^b, t)$, and ew_i^b are analogous.

Given this, agent a's tradeoff problem at time t is to find a package $[x^a, x^b]$ that solves the following problem:

$$\text{TA-I} \quad \text{maximize} \quad EU^a(I, x^a, t)$$
$$\text{subject to} \quad EU^b(I, x^b, t) = EY \qquad 0 \leq x_i^a \leq 1$$

Here I is fixed and EY is b's equilibrium utility for $t + 1$. Given this, a's equilibrium strategy for time t is the same as SA(t) (defined earlier) with TA replaced with TA-I, and

U^a replaced with EU^a. Likewise for agent b. The problem TA-I is also the standard fractional knapsack problem, and is solvable using a greedy approach.

On the basis of the above equilibrium, we will show how to find an optimal agenda, for the PDP, for each agent.

3.3 The Negotiation Agenda

As noted in Section 1, in many cases, the set of issues to be negotiated are choice variables. So, before negotiation begins, the agents must decide upon an *agenda* which we define as follows:

Definition 1. *In the context of the PDP, an agenda A^g of size $g \leq m$ is a set of g issues, i.e., $A^g \subseteq I$ such that $|A^g| = g$. Let AG^g denote the set of all possible agendas of size g.*

Definition 2. *For the PDP, an agenda (AA^g) of size $g \leq m$ is agent a's optimal agenda if*

$$AA^g = arg\ max_{X \in AG^g} EU^a(X, x^a, 1)$$

where x^a denotes a's equilibrium allocation (for agenda X, for $t = 1$). For b, AB^g is analogous. For the complete information setting, EU^a and EU^b are replaced with U^a and U^b respectively.

For the agenda I containing m issues, Section 3.2 showed how to find equilibrium outcomes (i.e., U^a, EU^a, U^b, and EU^b). Given this equilibrium, our problem now is to find each agent's optimal agenda: AA^g and AB^g for $1 \leq g \leq m$. We show how to find these agendas in Section 4.

Below, we focus on the case where one of the agents, say a, prefers different issues differently but b prefers all the issues equally[2]. So, for a, different issues have different weights but b has the same[3] weight for all the issues. We let the issues $\{1, \ldots, m\}$ be such that $w_i^a \leq w_{i+1}^a$ for $1 \leq i \leq n - 1$. Also, for agenda $X \in AG^g$, we let $X = \{1, \ldots, g\}$ be such that $w_{X_i}^a \leq w_{X_{i+1}}^a$ for $1 \leq i < g$. Note that, although we are viewing X as a list, as per its definition, an agenda is a set because the equilibrium for the PDP is independent of the ordering of issues. However, we view X as a list because it makes it easier to refer to the individual elements of X.

4 Optimal Agendas: Complete Information

We first show how to find optimal agenda for the complete information setting and then for the incomplete information setting described in Section 3.2. For the complete information setting, Theorem 1 (Theorem 2) shows how to find a's (b's) optimal agenda.

---

[2] This is situation occurs often: for the previous car dealer example, a buyer may have different preferences over the seat color and the car color, but the dealer may be indifferent between colors.

[3] Future work will deal with those situations where, for both a and b, the weights are different for different issues.

Then, for the incomplete information setting described in Section 3.2, Theorem 3 (Theorem 4) gives a's (b's) optimal agenda. Theorem 5 gives the time complexity of computing the optimal agendas.

Theorem 1. *In a setting where agent a has different weights for different issues and b has the same weight for all m issues, a's optimal agenda of size g is a set of g issues that are associated with the g highest weights in w^a (i.e., $AA^g = \{m-g+1, \ldots, m\}$).*

Proof. Here AA^g is obtained using BI. If an agenda is optimal for a for the last time period $t = n$, then by BI, it will be optimal for all previous ones. Consider the last time period $(t = n)$ for which two possibilities can arise: either a or b could be the last mover. Consider first the case where a is the last mover. As per the equilibrium for the PDP, irrespective of the agenda, a gets a 100% of all the shrunken pies. So, from among all possible agendas, its cumulative utility is maximized for the agenda $AA^g = \{m-g+1, \ldots, m\}$.

Now, consider the case where b is the last mover. Here, as per the equilibrium for PDP, b gets a 100% of all the shrunken pies and a gets nothing (i.e., a's utility for $t = n$ is zero). Also, since b has equal weights for all the issues, its cumulative utility for $t = n$ is independent of the agenda. We therefore consider the previous time period $(t = n-1)$ for which it is a's turn to offer. As per the equilibrium for $t = n-1$, b's utility must be equal to its equilibrium utility for $t = n$. If U^b denotes b's equilibrium utility for $t = n$, then in the equilibrium for $t = n-1$, b must get U^b. Note that since b has equal weights, U^b is independent of the agenda. But a's utility depends on the agenda. Given U^b, a's equilibrium utility for $t = n-1$ (for an agenda $X \in AG^g$) will be $U^a_X = (\sum_{i=1}^{g} w^a_{X_i}) - U^b$. Then, a's optimal agenda is the one that maximizes U^a_X. Since all weights are positive, and the issues in $X = \{1, \ldots, g\}$ are in ascending order of w^a_i, U^a_X is maximized when $X = \{m-g+1, \ldots, m\}$. Thus, $AA^g = \{m-g+1, \ldots, m\}$.
□

Regarding b's optimal agenda, one might think that AB^g will be the set of g issues that correspond to the g lowest weights for a. But this is not so, because an agent's cumulative utility from an agenda depends not just on its weights but also on its equilibrium shares for the g issues. The following example clarifies this point.

Example 1. Let $m = 4$, $I = \{1, 2, 3, 4\}$, $g = 3$, $\delta = 0.5$, $n = 2$, $w^a = \{1, 2, 3, 4\}$, $w^b = \{1, 1, 1, 1\}$, and b be first mover. There are four possible agendas of size $g = 3$: $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, and $\{2, 3, 4\}$. For each of them, the agents' equilibrium utilities for $t = 1$ (i.e., U^a_1 and U^b_1) are as given in Table 1. Agent b's utility U^b_1 is highest for the agenda $\{1, 2, 4\}$, and so $AB^g = \{1, 2, 4\}$ is b's optimal agenda. Likewise, $AA^g = \{2, 3, 4\}$.

As Example 1 shows, the optimal agenda may be different for different agents. But in many practical cases, only one of the two agents has a choice over the agenda. This is also the case in the car dealer example (outlined in Section 1) where the buyer can choose an agenda but not the seller.

We will now show how to find b's optimal agenda AB^g using the following method. Initially, the optimal agenda is empty. Then, we add issues to it one by one using a greedy approach. For $1 \leq k \leq g$, let AB^g_k denote the issue that is b's optimal choice at step k. Then, b's optimal agenda AB^k of size k is $AB^k = \cup_{i=1}^{k} AB^g_i$.

Agenda	U_1^a	U_1^b	b's Optimal Agenda?
$\{2,3,4\}$	4.5	1.833	No
$\{1,2,3\}$	3	2	No
$\{\mathbf{1,2,4}\}$	**3.5**	**2.125**	**Yes**
$\{1,3,4\}$	4	2	No

Table 1. Agents' utilities (for $t = 1$) for possible agendas.

Theorem 2. *In a setting where agent a has different weights for different issues and b has the same weight for all of them, b's optimal agenda AB^g (for $g \geq 2$) is obtained as follows. Agent b's optimal choice for the first two issues is:*

$$AB_1^g = 1 \quad \text{and} \quad AB_2^g = m.$$

Then AB_k^g ($3 \leq k \leq g$) is given by the following rule. If we let $\cup_{i=1}^{k-1} AB_i^g = \{1, \ldots, p, q, \ldots, m\}$ where ($1 \leq p < q \leq m$) and ($p + m - q + 1 = k - 1$), then if

$$\sum_{i=q}^{m} w_i^a \geq \delta \sum_{i=1}^{k-1} w_{AB_i^g}^a + \delta w_{p+1}^a \tag{1}$$

then $AB_k^g = p + 1$. Otherwise, if $\exists Z \in I - \cup_{i=1}^{k-1} AB_i^g$ such that

$$w_Z^a + \sum_{i=q}^{m} w_i^a \geq \delta \sum_{i=1}^{k-1} w_{AB_i^g}^a + \delta w_Z^a \tag{2}$$

then if

$$\delta \sum_{i=1}^{k-1} w_{AB_i^g}^a \geq \sum_{r=q}^{m} w_{AB_r^g}^a \tag{3}$$

then $AB_k^g = p + 1$. Otherwise (i.e., Eq. 2 or 3 is false), $AB_k^g = q - 1$.

Proof. An agent's optimal agenda problem exhibits the 'greedy choice property' and the 'optimal sub-structure property'. So a globally optimal solution can be found by making locally optimal choices. A problem has 'optimal substructure' if an optimal solution can be constructed from optimal solutions to its subproblems. An agent's optimal agenda problem has both these properties since all weights are positive and so an agent's cumulative utility is maximized if its utility from the individual issues is maximized. Given that b has equal weights for all the issues, b's cumulative utility is maximized if the issues are chosen such that its cumulative equilibrium share for them is the maximum over all possible agendas. So if x_X^b denotes b's equilibrium share for agenda X, then $AB^g = \arg\max_{X \in AG^g} \sum_{i=1}^{g} x_{X_i}^b$ or $AB^g = \arg\min_{X \in AG^g} \sum_{i=1}^{g} x_{X_i}^a$ (since, $x_i^a = 1 - x_i^b$). To find AB^g, we will choose one issue at a time. At step k ($1 \leq k \leq g$) we will choose AB_k^g such that a's shares for the issues chosen thus far are minimized (relative to all possible agendas of size k). In more detail, this is done as follows.

As before, if an agenda is optimal for b for $t = n$, then by BI, it will be optimal for all previous time periods. Now, either a or b could be the last mover. If it is a, it gets a 100% of all the shrunken pies and b gets zero utility. But if b is last mover, then it gets a 100% of all shrunken pies and a gets nothing. So irrespective of who the last mover is, b's utility for $t = n$ is independent of the agenda. Hence, we look at the previous time period $t = n - 1$ to find AB^g. Consider the case where b is the offering agent for $t = n - 1$. If x^a_X denotes a's equilibrium share for $t = n - 1$, then b's optimal agenda for this time period is $AB^g = \arg\min_{X \in AG^g} \sum_{i=1}^{g} x^a_{X_i}$.

To begin, we show how to find the first and second issues (i.e., AB_1^g and AB_2^g) that will be included in AB^g. Then, we will show how to find issues one by one (using a greedy approach) to include in AB^g until the number of issues in it is g. Let the m issues $\{1, \ldots, m\}$ be such that $w_i^a \leq w_{i+1}^a$ for $1 \leq i < m - 1$. We decide what AB_1^g and AB_2^g should be on the basis of the relation between $w^a_{AB_1^g}$ and $w^a_{AB_2^g}$. Irrespective of what these two issues are, there are two[4] possible relations between their weights: $w^a_{AB_1^g} < w^a_{AB_2^g}$ or else $w^a_{AB_1^g} > w^a_{AB_2^g}$. We consider each of these below.

For the case $w^a_{AB_1^g} < w^a_{AB_2^g}$: At this stage, we know the relation between $w^a_{AB_1^g}$ and $w^a_{AB_2^g}$ but not the actual weights. Given this relation, as per the equilibrium for $t = n - 1$ (see Section 3.2), b will first allocate a share to a for the issue AB_2^g and then for AB_1^g. This will be done such that a's cumulative utility (say Y) from AB_1^g and AB_2^g is equal to its cumulative utility for $t = n$ (i.e., $Y = \delta^{n-1} \sum_{i=1}^{2} w^a_{AB_i^g}$). When doing this allocation for $t = n - 1$, one of two possible cases (C1.1 or C1.2) can arise:

C1.1 The entire utility Y can be given to a from the issue AB_2^g alone so a's share for AB_1^g is zero, so we have:

$$w^a_{AB_2^g} \delta^{n-2} \geq (w^a_{AB_1^g} + w^a_{AB_2^g})\delta^{n-1} \tag{4}$$

If $x^a_{AB_2^g}$ denotes a's equilibrium share for AB_2^g for $t = n - 1$, then as per Section 3.2, we have:

$$x^a_{AB_2^g} = (w^a_{AB_1^g} + w^a_{AB_2^g})\delta^{n-1} / w^a_{AB_2^g} \delta^{n-2} \tag{5}$$

Clearly, it is optimal for b to choose AB_1^g and AB_2^g such that $x^a_{AB_2^g}$ is minimized. In Equation 5, since δ is a constant, $x^a_{AB_2^g}$ is minimized when $w^a_{AB_1^g}$ is minimized and $w^a_{AB_2^g}$ is maximized. Thus, AB_1^g is the issue with lowest weight in w^a, and AB_2^g, is the one with highest weight in w^a. So $AB_1^g = 1$ and $AB_2^g = m$.

C1.2 The utility Y cannot be given just from the issue AB_2^g so a's share for AB_2^g is a 100% of it and its share for AB_1^g is non-zero, so Equation 4 is false. If $x^a_{AB_1^g}$ denotes a's equilibrium share for AB_1^g for $t = n - 1$, then as per Section 3.2, we have:

$$(w^a_{AB_1^g} x^a_{AB_1^g} + w^a_{AB_2^g})\delta^{n-2} = (w^a_{AB_1^g} + w^a_{AB_2^g})\delta^{n-1}$$

---

[4] For the case $w^a_{B1} = w^a_{B2}$, the agents' equilibrium shares are independent of $w^a_{AB_1^g}$ and $w^a_{AB_1^g}$. This can be verified by substituting $w^a_{AB_1^g} = w^a_{AB_1^g}$ in Equations 5 and 6.

and

$$x^a_{AB^g_1} = \delta - \frac{w^a_{AB^g_2}}{w^a_{AB^g_1}}(1 - \delta) \tag{6}$$

Here, $x^a_{AB^g_1}$ is minimized when $w^a_{AB^g_2}$ is maximized and $w^a_{AB^g_1}$ is minimized. So $AB^g_1 = 1$ and $AB^g_2 = m$.

Thus, if $w^a_{AB^g_1} < w^a_{AB^g_2}$, then $AB^g_1 = 1$ and $AB^g_2 = m$ for both cases (C1.1 and C1.2).

For the case $w^a_{AB^g_1} > w^a_{AB^g_2}$: This case is the same as the previous one, with $w^a_{AB^g_1}$ and $w^a_{AB^g_2}$ swapped.

Now, we will choose AB^g_3. Since $AB^g_1 = 1$ and $AB^g_2 = m$, we have $w^a_1 < w^a_{AB^g_3} < w^a_m$. So, in the equilibrium for $t = n - 1$, b will first allocate to a, a share for the issue AB^g_2, then for AB^g_3, and finally for AB^g_1. In the equilibrium allocation for $t = n - 1$, a's utility is $Y = \delta^{n-1}\sum_{i=1}^{3} w^a_{AB^g_i}$. We find a's equilibrium shares in this allocation by considering the following three possible cases (C3.1 that corresponds to Equation 1 being true, C3.2 that corresponds to Equation 1 being false and Equation 2 being true, or C3.3 that corresponds to Equations 1 and 2 being false) that can arise:

C3.1 The entire utility Y can be given to a from the issue AB^g_2 alone so a's share for AB^g_1 and AB^g_3 is each zero. So we have:

$$w^a_{AB^g_2}\delta^{n-2} \geq \sum_{i=1}^{3} w^a_{AB^g_i}\delta^{n-1} \tag{7}$$

If $x^a_{AB^g_2}$ denotes a's equilibrium share for AB^g_2 for $t = n - 1$, then as per Section 3.2, we have:

$$x^a_{AB^g_2} = (\sum_{i=1}^{3} w^a_{AB^g_i})\delta^{n-1}/w^a_{AB^g_2}\delta^{n-2} \tag{8}$$

Here, $x^a_{AB^g_2}$ is minimized by choosing as AB^g_3 the issue with lowest weight from the remaining issues in $I - \{AB^g_1 \cup AB^g_2\}$. This gives $AB^g_3 = 2$. Moreover, Equation 7 is true iff $AB^g_3 = 2$ because, in $I - \{AB^g_1 \cup AB^g_2\}$, 2 is the issue with least weight.

C3.2 The utility Y cannot be given just from AB^g_2 but can be given from AB^g_2 and AB^g_3. So a's share for AB^g_2 is a hundred percent of it and its share for AB^g_3 is non-zero, i.e., Equation 7 is false $\forall AB^g_3 \in I - \{AB^g_1 \cup AB^g_2\}$ and $\exists AB^g_3 \in I - \{AB^g_1 \cup AB^g_2\}$ s.t.:

$$(w^a_{AB^g_2} + w^a_{AB^g_3})\delta^{n-2} \geq \sum_{i=1}^{3} w^a_{AB^g_i}\delta^{n-1} \tag{9}$$

If $x^a_{AB^g_3}$ denotes a's equilibrium share for AB^g_3 for $t = n - 1$, then as per Section 3.2, we have:

$$(w^a_{AB^g_3}x^a_{AB^g_3} + w^a_{AB^g_2})\delta^{n-2} = \sum_{i=1}^{3} w^a_{AB^g_i}\delta^{n-1} \tag{10}$$

Solving the above equation for $x^a_{AB^g_3}$, we get:

$$x^a_{AB^g_3} = \frac{\sum_{i=1}^{2} \delta w^a_{AB^g_i} - w^a_{AB^g_2}}{w^a_{AB^g_3}} + \delta$$

Here, if Equation 3 is true, then $x^a_{AB^g_3}$ is minimized if $w^a_{AB^g_3}$ is maximized. So AB^g_3 must be the issue with lowest weight from the remaining issues, or $AB^g_3 = 2$. But if Equation 3 is false, $x^a_{AB^g_3}$ is minimized when $AB^g_3 = m - 1$.

C3.3 *The entire utility Y cannot be given from AB^g_2 and AB^g_3 so a's share for each of AB^g_2 and AB^g_3 is a hundred percent and its share for AB^g_1 is non-zero, i.e., Equations 7 and 9 are false $\forall AB^g_3 \in I - \{AB^g_1 \cup AB^g_2\}$, so:*

$$\left(\sum_{i=1}^{3} w^a_{AB^g_i}\right)\delta^{n-2} \geq \sum_{i=1}^{3} w^a_{AB^g_i}\delta^{n-1}$$

If $x^a_{AB^g_1}$ denotes a's equilibrium share for AB^g_1 for $t = n - 1$, then as per Section 3.2, we have:

$$\left(w^a_{AB^g_1}x^a_{AB^g_1} + w^a_{AB^g_2} + w^a_{AB^g_3}\right)\delta^{n-2} = \sum_{i=1}^{3} w^a_{AB^g_i}\delta^{n-1}$$

Solving the above equation for $x^a_{AB^g_1}$, we get:

$$x^a_{AB^g_1} = \left(\delta w^a_{AB^g_1} + \delta w^a_{AB^g_2} - w^a_{AB^g_2} - w^a_{AB^g_3}(1 - \delta)\right)/w^a_{AB^g_1}$$

Here, $x^a_{AB^g_1}$ is minimized when $AB^g_3 = m - 1$.

Thus, for the case C3.1, $AB^g_3 = 2$. For C3.2, $AB^g_3 = 2$ or $AB^g_3 = m - 1$ (depending on Equation 3). And for C3.3 $AB^g_3 = m - 1$.

In general, to decide what to choose as AB^g_k ($4 \leq k \leq g$), we find a's equilibrium allocation for each of the k issues (chosen thus far) for $t = n - 1$. At stage k, we already have the optimal agenda of size $k - 1$, i.e., $\cup_{i=1}^{k-1} AB^g_i$. Also, we know that for stage i, ($1 \leq i \leq k - 1$) b's optimal choice is either the issue with highest weight for a or else the one with the lowest weight from the remaining issues $I - \{\cup_{j=1}^{i-1} AB^g_j\}$. Hence, as mentioned in the statement of this theorem, we let $\cup_{i=1}^{k-1} AB^g_i = \{1, \ldots, p, q, \ldots, m\}$ where ($1 \leq p < q \leq m$) and ($p + m - q + 1 = k - 1$). This implies that $w^a_p < w^a_{AB^g_k} < w^a_q$. Also, as before, the equilibrium allocation for k issues for $t = n - 1$ must give a a utility of $Y = \delta^{n-1}\sum_{i=1}^{k} w^a_{AB^g_i}$. We find a's equilibrium shares in this allocation by considering the following three possible cases (CK.1 that corresponds to Equation 1 being true, CK.2 that corresponds to Equation 1 being false and Equation 2 being true, or CK.3 that corresponds to Equations 1 and 2 being false) that can arise:

CK.1 *The cumulative utility Y can be given to a just from the pies in $\{j, \ldots, m\}$ (where $q \leq j \leq m$) so we have[5]:*

$$\delta^{n-2} \sum_{r=j}^{m} w_{\alpha_r}^a \geq \delta^{n-1} \sum_{i=1}^{k} w_{AB_i^g}^a \tag{11}$$

If $x_{AB_k^g}^a$ denotes a's equilibrium share for AB_k^g for $t = n - 1$, then as per Section 3.2, we have:

$$x_j^a = \left(\delta \sum_{i=1}^{k} w_{AB_i^g}^a - \sum_{r=j+1}^{m} w_r^a\right)/w_j^a \tag{12}$$

Here, x_j^a is minimized if $w_{AB_k^g}^a$ is minimized. So b's optimal choice for AB_k^g is the issue with lowest weight from the remaining issues $I - \{\cup_{i=1}^{k-1} AB_i^g\}$, i.e., $AB_k^g = p+1$. Moreover Equation 11 is true iff $AB_k^g = p+1$ because, in $I - \{\cup_{i=1}^{k-1} AB_i^g\}$, $p+1$ is the issue with least weight.

CK.2 *The entire utility Y cannot be given just from the pies in $\{q, \ldots, m\}$ but can be given from $\{q, \ldots, m\}$ together with AB_k^g, i.e., $\exists AB_k^g \in I - \{\cup_{i=1}^{k-1} AB_i^g\}$ such that:*

$$\delta^{n-2} \sum_{r=q}^{m} w_r^a < \delta^{n-1} \sum_{i=1}^{k} w_{AB_i^g}^a, \quad \text{and} \tag{13}$$

$$\delta^{n-1}\left(w_{AB_k^g}^a x_{AB_k^g}^a + \sum_{r=q}^{m} w_r^a\right) \geq \delta^{n-2} \sum_{i=1}^{k} w_{AB_i^g}^a \tag{14}$$

If $x_{AB_k^g}^a$ denotes a's equilibrium share for AB_k^g for $t = n - 1$, then as per Section 3.2, we have:

$$x_{AB_k^g}^a = \frac{\delta \sum_{i=1}^{k-1} w_{AB_i^g}^a - \sum_{r=q}^{m} w_r^a}{w_{AB_k^g}^a} + \delta \tag{15}$$

Here, if Equation 3 is true, then $x_{AB_k^g}^a$ is minimized if $w_{AB_k^g}^a$ is maximized, i.e., $AB_k^g = p+1$. But if Equation 3 is false, $x_{AB_k^g}^a$ is minimized if $w_{AB_k^g}^a$ is minimized, i.e., $AB_k^g = q - 1$.

CK.3 *The cumulative utility Y cannot be given just from the pies in $\{AB_k^g, q, \ldots, m\}$ but can be given from $\{j, \ldots, r, AB_k^g, q, \ldots, m\}$ where $1 \leq j \leq p$, so we have:*

$$\delta^{n-2}\left(w_{AB_k^g}^a + \sum_{r=q}^{m} w_r^a\right) < \delta^{n-1} \sum_{i=1}^{k} w_{AB_i^g}^a, \quad \text{and}$$

---

[5] Here, a's share for each of the pies $j+1, \ldots, m$ is one, for pie j it is x_j^a, and for each of the pies $1, \ldots, j-1$, is zero.

$$\delta^{n-1}\Big(\sum_{r=j}^{p} w_r^a + w_{AB_k^g}^a + \sum_{r=q}^{m} w_r^a\Big) \geq \delta^{n-2} \sum_{i=1}^{k} w_{AB_i^g}^a$$

If $x_{AB_k^g}^a$ denotes a's equilibrium share for AB_k^g for $t = n-1$, then as per Section 3.2, we have:

$$x_j^a = \Big(\delta \sum_{i=1}^{k} w_{AB_i^g}^a - \sum_{r=q}^{m} w_r^a - w_{AB_k^g}^a - \sum_{r=j+1}^{p} w_r^a\Big)/w_j^a$$

Here x_j^a is minimized when $w_{AB_k^g}^a$ is maximized. So AB_k^g must be the issue with highest weight from the remaining issues $I - \{\cup_{i=1}^{k-1} AB_i^g\}$, i.e., $AB_k^g = q - 1$.

Thus, for the case CK.1, $AB_k^g = p + 1$. For CK.2, $AB_k^g = p + 1$ or $AB_k^g = q - 1$ (depending on Equation 3). And for CK.3, $AB_k^g = q - 1$.

In the same way, we can obtain b's optimal agenda for the case where a is the offering agent at $t = n - 1$. □

5   Optimal Agendas: Incomplete Information

We will now show how to find optimal agendas for the incomplete information setting of Section 3.2. To this end, Theorem 3 gives a's optimal agenda and Theorem 4 that for b. Finally, Theorem 5 gives the time complexity of computing these optimal agendas. Let the m issues $\{1, \ldots, m\}$ be such that: $ew_1^a \leq ew_2^a \leq \ldots \leq ew_m^a$. Here, agent a has different expected weights for different issues, but b has the same expected weight for all the m issues.

Theorem 3. *In a setting where agent a has different expected weights for different issues but, for b, the set of possible weights for an issue and the associated probabilities are the same for all the m issues, agent a's optimal agenda of size g is a set of g issues that are associated with the g highest expected weights (i.e., $AA^g = \{m - g + 1, \ldots, m\}$).*

Proof. As Theorem 1, with weights replaced with corresponding expected weights and utilities with corresponding expected utilities. □

Theorem 4. *In a setting where agent a has different expected weights for different issues but, for b, the set of possible weights for an issue and the associated probabilities are the same for all the m issues, b's optimal agenda AB^g (for $g \geq 2$) is obtained as in Theorem 2 with agent's weights replaced with corresponding expected weights, and utilities replaced with corresponding expected utilities.*

Proof. As Theorem 2, with weights replaced with corresponding expected weights and utilities with corresponding expected utilities. □

Theorem 5. *For the complete information setting, the time taken to compute AA^g is $\mathcal{O}(g)$ and to compute AB^g is $\mathcal{O}(mg^2)$. For the incomplete information setting, the time to compute AA^g is $\mathcal{O}(Tg)$ and to compute AB^g is $\mathcal{O}(Tmg^2)$.*

Proof. For the complete information setting, as per Theorem 1, AA^g is the set of g issues with highest weight in w^a. We therefore need to choose the last g issues from α which takes time $\mathcal{O}(g)$. For AB^g, as per Theorem 2, determining AB_k^g $(1 \le k \le g)$ requires evaluating the conditions in Equations 1, 2, and 3. Consider Equation 2 which requires one comparison between $w_Z^a + \sum_{i=q}^m w_i^a$ and $\delta \sum_{i=1}^{k-1} w_{AB_i^g}^a + \delta w_Z^a$. Computation of these two terms to compare requires no more than k additions because the summation variable i varies between 1 and k. Also, Z can vary at most between 1 and m. So finding AB_k^g takes $\mathcal{O}(mk)$ time, and to find AB_k^g for $1 \le k \le g$, it takes $\sum_{k=1}^g \mathcal{O}(mk) = \mathcal{O}(mg^2)$ time. Note that we considered only Equation 2 because the time to evaluate the condition in Equations 1 or 3 is no more than the time taken to evaluate the condition in Equation 2.

It follows that, for the incomplete information setting, if there are T possible utility function pairs, the time to compute AA^g is $\mathcal{O}(Tg)$ and to compute AB^g is $\mathcal{O}(Tmg^2)$.
□

6   Conclusions and Future Work

This paper presented polynomial time methods for finding each agent's optimal agenda for the PDP. The polynomial time complexity of our methods makes it easy for automating the process of choosing an agent's optimal agenda, and thereby reducing human involvement during negotiation.

Possible avenues for future research include extending the current analysis to scenarios where both agents have different weights for different issues. Also, this paper focussed on one specific incomplete information setting. In future, we will extend this analysis to other possible incomplete information settings.

Bibliography

[1] M. Bac and H. Raff. Issue-by-issue negotiations: the role of information and time preference. *Games and Economic Behavior*, 13:125–134, 1996.

[2] L. A. Busch and I. J. Horstman. A comment on issue-by-issue negotiations. *Games and Economic Behavior*, 19:144–148, 1997.

[3] T. H. Cormen, C. E. Leiserson, R. L Rivest, and C. Stein. *An introduction to algorithms*. The MIT Press, Cambridge, Massachusetts, 2003.

[4] S. S. Fatima, M. Wooldridge, and N. R. Jennings. Optimal agendas for multi-issue negotiation. In *Proceedings of the Second International Conference on Autonomous Agents and Multi-Agent Systems*, pages 129–136, Melbourne, Australia, July 2003.

[5] S. S. Fatima, M. Wooldridge, and N. R. Jennings. An agenda based framework for multi-issue negotiation. *Artificial Intelligence Journal*, 152(1):1–45, 2004.

[6] S. S. Fatima, M. Wooldridge, and N. R. Jennings. Multi-issue negotiation with deadlines. *Journal of Artificial Intelligence Research*, 27:381–417, 2006.

[7] C. Fershtman. The importance of the agenda in bargaining. *Games and Economic Behavior*, 2:224–238, 1990.

[8] C. Fershtman. A note on multi-issue two-sided bargaining: bilateral procedures. *Games and Economic Behavior*, 30:216–227, 2000.

[9] D. Fudenberg and J. Tirole. Sequential bargaining with incomplete information. *Review of Economic Studies*, 50:221–247, 1983.

[10] S. Kraus. *Strategic negotiation in multi-agent environments*. The MIT Press, Cambridge, Massachusetts, 2001.

[11] D. A. Lax and J. K. Sebenius. Dealcrafting: The substance of three dimensional negotiations. *Negotiation Journal*, 18(1):5–28, 2002.

[12] M. J. Osborne and A. Rubinstein. *A Course in Game Theory*. The MIT Press, 1994.

[13] J. S. Rosenschein and G. Zlotkin. *Rules of Encounter*. MIT Press, 1994.

[14] J. K. Sebenius. Negotiation arithmetic: Adding and subtracting issues and parties. *International Organization*, 37(2):281–316, 1983.