Ant-Colony Based Near-ML Space-Time Multiuser Detection for the STBC Assisted DS-CDMA Uplink

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Abstract— In this contribution, a novel Ant Colony Optimization (ACO) based Space-Time (ST) Multiuser Detection (MUD) algorithm was proposed in order to improve the Bit Error Ratio (BER) versus Signal-to-Noise Ratio (SNR) performance achieved by the Space-Time Block Code (STBC) assisted two transmit antenna aided fully loaded DS-CDMA system directly employing an ACO based MUD algorithm. Our simulation results demonstrate that the proposed ACO based MUD algorithm enables the fully loaded system to approach the single user performance associated with a second-order diversity gain. Furthermore, the complexity of the improved ACO based MUD algorithm is a factor of $10^{15}$ lower than that of the Maximum Likelihood (ML) MUD, when $K = 32$ users are supported by the STBC $G_2$ assisted DS-CDMA systems employing Gold codes having a length of 31 chips.

I. INTRODUCTION

Ant Colony Optimization (ACO) was inspired by studying the foraging behavior of ants. It was first invoked by Colorni, Dorigo and Maniezzo [1] in 1991. This population-based approach has recently been applied to a large number of so-called Non-deterministic Polynomial (NP)-hard combinatorial optimization problems [1]–[5]. These investigations motivated its employment in near-Maximum Likelihood (ML) Multi-User Detection (MUD) aided Multiple Access (MA) systems [6]–[9], [11], [12].

The ACO based MUD of both [9], [11], [12] and of [6]–[8] are capable of achieving a lower BER at a lower complexity, than the Genetic Algorithm (GA)-based MUDs of [13]–[15]. The authors are capable of achieving a lower BER at a lower complexity, than the Maximum A posteriori (MAP) systems [6]–[9], [11], [12].

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where we have

\[
C = \begin{bmatrix} C_1, & C_2, & \ldots, & C_K \end{bmatrix}
\]

\[
H = \text{diag} \left\{ H_1, H_2, \ldots, H_K \right\}
\]

\[
v = \begin{bmatrix} v_1^T, & v_2^T, & \ldots, & v_K^T \end{bmatrix}^T.
\]

III. ANT-COLONY OPTIMIZATION AIDED SPACE-TIME MUD

A. Traditional ACO Aided ST-MUD

The MUD’s goal is to identify the \(2K\)-element ML data-vector of the twin-antenna-aided \(K\) users at the BS with a high probability, despite searching through only a small fraction of the \(2^{2K}\)-element search-space.

TABLE I: The \((2 \times 2K)\)-element route table of the multi-user STBC decoder employing the ACO principle of [11], [12], when the system supports \(K\) users and each MS is equipped with two transmit antennas.

<table>
<thead>
<tr>
<th>(s_i^{(1)})</th>
<th>(j)</th>
<th>(s_i^{(2)})</th>
<th>(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

1) Route Table: The interpretations of the number of rows and columns associated with the \((2 \times 2K)\)-element route table are the same, as in the MC DS-CDMA system of [11]. More explicitly, the rows in Tab. I represent the legitimate BPSK modulated \(1\) and \(-1\) symbols, while the columns of the route-table are the \(2K\)-element twin-antenna symbols of the \(K\) users. In terms of ACO parlance, the two rows of the table provide the two options for an artificial ant to consider during its passage through the set of legitimate \(2K\)-element data vectors. In Tab. I, the legitimate symbols \(1\) and \(-1\) are represented by \(s_1^{(1)}\) and \(s_2^{(1)}\), respectively.

The symbol \(s_i^{(m)}\) is introduced in order to represent the \(i\)th legitimate BPSK symbol combination in the full set containing all the \(M = 2^m\) \(m\)-bit symbol combinations. The index \(i\) is defined as the decimal interpretation of the binary symbol vector plus one. The binary symbol vector is obtained by assigning \(+1\) to \(0\) and \(-1\) to \(1\). The symbol in the \(j\)th column and \(i\)th row of the route table is denoted by \(b_{ji} = s_i^{(1)}\) for \(j = 1, \ldots, 2K\) and \(i = 1, 2\).

2) Cellular Likelihood: The cellular likelihood \(l(b_{ji})\) was determined such that the BPSK symbol \(b_{ji}\) was transmitted as the \(j\)th element of the \(2K\)-element vector, which corresponds to the \(j\)th row and the \(i\)th column of the \((2 \times 2K)\)-element route table adopted by the traditional hard-output ACO algorithm, which may be formulated as [9]:

\[
l(b_{ji}) = 2R\{\xi_{ji}y_{ji}\} - \xi b_{ji}y_{ji}, \quad j = 1, 2, \ldots, 2K; \quad i = 1, 2;
\]

where \(y_{ji}\) is the \(j\)th element of the \((2K \times 1)\)-element matched filter output vector \(y\) given by

\[
y = H^H C^T r = \xi H^H C^T CHv + H^H C^T n
\]

\[
= \xi RV + \bar{n}.
\]

Furthermore, \(R\) in Eq.(6) is the \((2K \times 2K)\)-element correlation matrix of the composite matrix \(CH\), where we have

\[
R = H^H C^T CH
\]

\[
= \begin{bmatrix} \Sigma_1 & \rho_{12} \Lambda_{12} & \cdots & \rho_{1K} \Lambda_{1K} \\
\rho_{12} \Lambda_{21} & \Sigma_2 & \cdots & \rho_{2K} \Lambda_{2K} \\
\vdots & \vdots & \ddots & \vdots \\
\rho_{1K} \Lambda_{K1} & \rho_{2K} \Lambda_{K2} & \cdots & \Sigma_K \\
\end{bmatrix},
\]

and \(\Sigma_k = \left( \sum_{i=1}^{2} |h_{ki}|^2 \right) I_2\), while \(\Lambda_{kj} = H_i^H H_j\) for \(k = 1, \ldots, K, j = 1, \ldots, K\) and \(j \neq k\). Finally, \(I_2\) represents a \((2 \times 2)\)-element identity matrix and the correlation coefficient \(\rho_{kj}\) between \(c_i\) and \(c_j\) is given by \(\rho_{kj} = c_i^* c_j / N_r = \sum_{n=1}^{N_r} c_{kn} c_{jn}/N_r\) for \(k = 1, \ldots, K, j = 1, \ldots, K\) and \(j \neq k\).

Therefore, \(R_{ji}\) in Eq.(6) is the \(j\)th diagonal element of \(R\), whose value is \(\sum_{i=1}^{2} |h_{ki}|^2\).

3) Vector Likelihood: The likelihood of a particular \((2K \times 1)\)-element \(K\)-user twin-antenna vector \(s_i^{(2K)}\) in the full set \(\mathcal{S}^{(2K)}\) containing all the \(2^{2K}\) vectors is given by

\[
\mathcal{L}(s_i^{(2K)}) = 2R\{\xi s_i^{(2K)^T} y\} - \xi^2 s_i^{(2K)^T} R s_i^{(2K)}, \quad i = 1, 2, \ldots, 2K
\]

where \(s_i^{(2K)}\) is the route followed by one of the \(\zeta\) ants in the route-table, which is formed by concatenating the \(2K\) passages chosen by that particular ant, yielding:

\[
s_i^{(2K)} = [s_i^{(1)} \ s_i^{(2)} \ \cdots \ s_i^{(2K)}]^T
\]

\[
\forall j = 1, 2, \ldots, 2K : t_j \in \{1, 2\}.
\]

4) Traditional Algorithm of Determining \(N\): In the benchmark ACO-aided ST-MUD algorithm characterized by Alg. 1, both the probability matrix \(p\) and the integer matrix \(N\) have the same size as the \((2 \times 2K)\)-element route table. Each element of the matrix \(N\) represents the number of ants choosing the corresponding entry in the route-table. For each of the \(2K\) columns in the route-table, the sum of the numbers of ants choosing the two entries should be equivalent to the total number of artificial ants \(\zeta\) adopted by the algorithm, yielding \(N_{j1} + N_{j2} = \zeta\), for \(j = 1, \ldots, 2K\).

Firstly, \(N_{j1}\) number of ants choose the route table entry associated with the higher probability, where \(N_{j1}\) is calculated as the rounded value of \((\zeta \times p_{j1})\). Similarly, \(N_{j2}\) represents the number of ants, which choose the other route table entry, namely that associated with the lower probability, where we have \(N_{j2} = \zeta - N_{j1}\).

The interpretations of the ‘sort’ function and ‘\(\cdot\)’ function of Alg. 1 are provided in the Appendix. The ACO algorithm portrayed above is then embedded into the STBC MUD and the achievable performance of the system considered is characterized in Fig. 1. As observed in Fig. 1, the BER versus SNR curve of the system employing the ACO-aided hard-decision STBC MUD exhibits an error-floor. Hence, in the next section we will propose a novel ACO-aided MUD algorithm, which will enable the fully-loaded system supporting as many users as the number of chips in the spreading sequence to approach the BER performance achieved by the single-user system.

B. Improved ACO aided ST-MUD

In this section, novel ACO-based hard-output ST-MUD algorithm is proposed in order to provide an improved BER performance at the cost of a moderate complexity increase. To elaborate a little further, the proposed algorithm is different from the traditional ACO based

Algorithm 1: Traditional Algorithm of Determining \(N\)

input: \((2 \times 2K)\)-element double precision matrix \(p\)
output: \((2 \times 2K)\)-element integer matrix \(N\)

for \(j = 1, 2, \ldots, 2K\) do

\(p'_j = \text{sort}(p_j)\)

\(i = \text{sort}(p'_j, p_j)\)

\(N_{j1} = [\zeta \times p'_{j1}]\)

\(N_{j2} = \zeta - N_{j1}\)
TABLE II: \((4 \times K)\)-element route table of the MU STBC decoder employing the improved ACO algorithm, when the system supports \(K\) users and each MS is equipped with two transmit antennas.

\[
\begin{array}{cccc}
  \text{\textbf{i}} & \text{\textbf{j}} & \text{\textbf{v}_1} = \begin{bmatrix} v_{11}^T \end{bmatrix} & \text{\textbf{v}_2} = \begin{bmatrix} v_{21}^T \end{bmatrix} & \cdots & \text{\textbf{v}_K} = \begin{bmatrix} v_{K1}^T \end{bmatrix} \\
  1 & 1 & 1^+ & \text{s}_1 = s_1^{(1)} & \text{s}_2 = s_1^{(2)} & \cdots & \text{s}_K = s_1^{(K)} \\
  2 & 2 & 2^+ & \text{s}_1 = s_2^{(1)} & \text{s}_2 = s_2^{(2)} & \cdots & \text{s}_K = s_2^{(K)} \\
  3 & 4 & 1^+ & \text{s}_1 = s_3^{(1)} & \text{s}_2 = s_3^{(2)} & \cdots & \text{s}_K = s_3^{(K)} \\
  4 & 4 & 1^+ & \text{s}_1 = s_4^{(1)} & \text{s}_2 = s_4^{(2)} & \cdots & \text{s}_K = s_4^{(K)} \\
\end{array}
\]

1) Route Table: The route table in the proposed ACO based ST-MUD algorithm has \((4 \times K)\)-element, where we have \(m = 2\). More explicitly, the fourth entries of each column correspond to the \((M = 2^m = 4)\) legitimate \((m = 2)\)-symbol combinations, as indicated in Tab. II, which should be contrasted to Table I.

2) Cellular Likelihood: As shown in Subsec. III-A, for \(m = 2\) the submatrix \(R_{ij}, j = 1, \ldots, K\) is a diagonal matrix. Therefore, the cellular likelihood \(l_{ji}\) of the \((i, j)\)th cell can be expressed as

\[
l_{ji} = 2 \Re \{e^{(l_{ji})} \} - \xi^2 s_i^{(2)} R_{ij} s_j^{(2)} = l_{ji1} + l_{ji2}, \quad l_{ji1}, l_{ji2} \in \{1, 2\}, \quad j = 1, 2, 3, 4; \quad (11)
\]

subject to \(\varphi_j = (2j - 1), \quad l_i = 2j\) and \(s_j^{(2)} = [s_j^{(1)} s_j^{(2)}]^T\).

3) Vector Likelihood: The formula quantifying the vector likelihood adopted by the improved ACO based ST-MUD algorithm remains the same as in Eq. (9), which is the one adopted by the traditional ACO based ST-MUD algorithm. The only difference lies in the formation of the \((2K \times 1)\)-element candidate vector \(s_i^{(2K)}\) that concatenates all the \(K\) number of 2-by-1 passages pursued by the same ant, yielding

\[
s_i^{(2K)} = [s_{i1} \quad s_{i2} \quad \cdots \quad s_{iK}]^T \\
\forall j = 1, \ldots, K, \quad i \in \{1, 2, 3, 4\}. \quad (12)
\]

4) Improved Algorithm of Determining \(N\): Both the probability matrix \(p\) and the integer matrix \(N\) have the same size as the \((4 \times K)\)-element route table. However, the traditional MUD used in Alg. 1 may become unable to provide an integer-valued matrix \(N\) having reasonable entries for the improved ACO based ST-MUD algorithm. For example, assume that the probabilities of choosing the four entries associated with the \(j\)th column, \(j = 1, 2, \ldots, K\) are given by the probability vector \(p_j = [0.36, 0.27, 0.15, 0.22]\), and \(\zeta = 10\) artificial ants are used by the ACO aided ST-MUD algorithm. Then we have \(N_j = \lfloor p_j \times \zeta \rfloor = [3.6, 2.7, 1.5, 2.2]\). According to the traditional MUD of Alg. 1, the number of ants \(N_j\) choosing the four entries in the \(j\)th column of the route-table are determined as the rounded integers of \(N_j\), yielding \(N_j = [4, 3, 2, 2]\), where the sum of the four integer elements equals to 11 > \(\zeta\). In order to determine the \(2^m\) integers in a more beneficial manner, an improved algorithm of deriving \(N\) from \(p\) is proposed for the scenarios associated with \(m > 1\), as detailed in the flow-chart of Alg. 2.

In order to illustrate the advantages of the new algorithm, the same probability vector \(p_j = [0.36, 0.27, 0.15, 0.22]\) will be used as the input vector in the following example. Firstly, the value of each element in \(N_j\) is directly given by the integer part of each element in the vector \(\hat{N}_j\), yielding \(N_j = [3, 2, 1, 2]\). Then, if the sum of all the four integer elements in the vector \(N_j\) is less than \(\zeta\), the \(j\)th loop has to be continued, since some of the ants have not as yet carried out their decisions. More quantitatively, \(\sum_{i=1}^{4} N_{ji}\) ants have already made their decisions, but the remaining \((\zeta - \sum_{i=1}^{4} N_{ji})\) ants have to make their impending choices among the four cells in the current column. The decision is made according to the value of the decimal part of the four elements of \(N_j\), which is represented by the \((4 \times 1)\)-element vector \(\tilde{N}_j\). In the example, we have \(\tilde{N}_j = [0.6, 0.7, 0.5, 0.2]\).

In order to carry out this decision, the four elements in \(\tilde{N}_j\) will be ranked in ascending order according to their values. Secondly, the cell associated with the value having the highest rank will be selected as the decision of an ant randomly chosen from the remaining \((\zeta - \sum_{i=1}^{4} N_{ji})\) ants, yielding the vector with the updated values \(N_j = [3, 3, 1, 2]\). Then the cells will be selected from the queue constructed according to the rank of their associated value in \(N_j\). In the given example, the output vector will then be further updated as \(\hat{N}_j = [4, 3, 1, 2]\). The selection process will not be curtailed, until all the \(\zeta\) ants have made their unique decisions. As the sum of all the elements in \(\hat{N}_j\) in the given example is equivalent to \(\sum_{i=1}^{4} N_{ji} = \zeta = 10\), the process can be curtailed and the final output is the vector \(\hat{N}_j = [4, 3, 1, 2]\).

All in all, compared to the traditional ACO based MUD of Alg. 1, the improved MUD of Alg. 2 has not increased the number of cells contained by the route table nor has it increased the complexity of calculating the cellular likelihood. Additionally, the same formula was adopted by both algorithms to calculate the vector likelihood.

Despite these identical properties, the improved ACO based MUD algorithm increases the number of candidate vectors appearing in the search pool by reducing the difference between the probabilities associated with each cell within a column, while still making them equally informative. Therefore a better BER performance is expected at the price of a moderately increased complexity imposed by calculating the vector likelihood associated with the increased number of vectors. The simulation results quantifying both the achievable BER performance as a function of the complexity imposed will be detailed in the next section.

IV. SIMULATION RESULTS

Two system scenarios are considered in this section, both of which are STBC assisted two transmit antenna aided DS-CDMA systems.
For the sake of convenience, the fully loaded DS-CDMA system supporting $K = 7$ users and employing a set of $m$ sequences having a length of $N_s = 7$ is referred to as the $m7K7$ system and that supporting $K = 32$ users with the aid of a set of Gold codes having a length of $N_s = 31$ is referred to as the $G31K32$ system. Apart from using a different number of artificial ants $\zeta$, all the other parameters characterizing the ACO algorithms are identical in the traditional and in the proposed ST-MUD algorithms, which are listed in Tab. III.

**Table III: Basic simulation parameters used in Section IV.**

<table>
<thead>
<tr>
<th>ACO parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial pheromone $\tau$</td>
<td>0.01</td>
</tr>
<tr>
<td>Evaporation rate $\rho$</td>
<td>0.5</td>
</tr>
<tr>
<td>Maximum number of iterations $\Xi$</td>
<td>10</td>
</tr>
<tr>
<td>Weight of pheromone $\alpha$</td>
<td>1</td>
</tr>
<tr>
<td>Weight of intrinsic affinity $\beta$</td>
<td>6</td>
</tr>
<tr>
<td>Weight of the elite ant $\sigma$</td>
<td>8</td>
</tr>
</tbody>
</table>

Fig. 1: Comparison of the BER performances achieved by the STBC assisted two transmit antenna rendered DS-CDMA UL employing different detection schemes, while seven users are supported by the $m$ sequence with a length of seven.

The complexity imposed is quantified in terms of the number of Floating point Operations Per Second (FLOPS) within a STBC block duration, divided by the number of users $K$. Since both the channel quality and the AWGN vector encountered vary from block to block, the number of times we have to evaluate the $2K$-element vector likelihood, which predetermines the total complexity imposed by the ACO algorithm is different in the consecutive block durations. Hence, to characterize the time-variant complexity of the ACO-aided ST-MUD algorithm more accurately, the PDF of the complexity was plotted in Fig. 2 and Fig. 4 at different SNRs ranging from 0dB to 30dB. As seen from Fig. 2 (a) to Fig. 2 (d) and Fig. 4 (a) to Fig. 4 (d), when the SNR increases from 0dB to 30dB, gradually decreasing complexities are imposed by both the traditional and the improved ACO ST-MUD algorithms in both the $m7K7$ system and the $G31K32$ system.

As seen from Fig. 1, the BER performance achieved by the $m7K7$ system employing the MMSE ST-MUD algorithm is better than that of the traditional ACO ST-MUD algorithm, which is achieved as a benefit of the MMSE MUD’s higher complexity, as revealed by their improved BER performance. Furthermore, in the $G31K32$ system compared to that in the $m7K7$ system. However, the BER performance achieved by the system employing the proposed ACO algorithm is still better than that of the traditional ACO-aided ST-MUD and it is lower than that of the MMSE ST-MUD algorithm, as shown in Fig. 2 (e). Most of the time, the complexity of the proposed ACO algorithm is 1000 times lower than that of the ML algorithm.

In the $G31K32$ system, the BER versus SNR curve achieved by the MMSE algorithm exhibits an error floor, as shown in Fig. 3. The difference between the BER performances achieved by the traditional and the proposed ACO ST-MUD algorithm is lower in the $G31K32$ system compared to that in the $m7K7$ system. However, the BER performance achieved by the system employing the proposed ACO algorithm is still better than that of the traditional ACO ST-MUD.

Returning to the complexity issues, as shown in Fig. 4 (e), the number of FLOPS to be carried out by the ACO ST-MUD algorithms are significantly lower than that of the MMSE ST-MUD despite their improved BER performance. Furthermore, in the $K = 32$ user-scenarios both ACO algorithms impose a factor $10^{18}$ lower complexity than the ML algorithm, while allowing the G31K32 system to perform similarly to the single-user system.

**V. Conclusion**

We proposed a novel ACO based ST-MUD algorithm applied in the STBC assisted two transmit antenna aided DS-CDMA system. The advantages of the improved algorithm over the traditional algorithm were detailed and the BER performance provided by the improved ACO algorithm was characterized by simulation in the context of fully loaded systems employing different spreading codes. Finally, the PDF of the complexity imposed by a range of detection algorithms, including both the improved and the traditional ACO algorithms was...
Fig. 3: Comparison of the BER performances achieved by the STBC assisted two transmit antenna rendered DS-CDMA UL employing different detection schemes, while thirty-two users are supported by Gold codes having a length of thirty-one.

 portrayed in Fig. 2 and Fig. 4 in order to demonstrate the advantages of the improved ACO aided ST-MUD algorithm.

Fig. 4: PDF of quantified complexity over different SNR associated with different detection schemes in the STBC assisted two transmit antenna rendered DS-CDMA UL employing different ACO aided ST-MUD algorithms.

REFERENCES


APPENDIX

The output of a’ of the function a’ = sort(a) holds all the elements of a in an ascending order according to their value. Assuming that we have a = [0.3 0.2 0.4 0.1], we arrive at a’ = sort(a) = [0.4 0.3 0.2 0.1].

Also assuming that a is a (J × 1) vector, and b is a (J × 1) vector containing all the J elements in a, which are arranged in a different order than in a, i.e., i = (a, b) becomes a (J × 1) integer vector. The jth element of i, i_j, is the index of the jth element of b, b_j, in the vector a. Let us give a simple example of the ‘sort’ function. Then, given a = [1.0 0.2 0.3 2.4], b = [0.3 0.2 1.0 2.4] and i = i(a, b), we have i = [3 2 1 4].