Amplify-and-Forward Relaying Aided Reed-Solomon Coded Hybrid-ARQ Relying on Realistic Channel Estimation

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Abstract— **Channel estimation (CE) plays an important role in determining the achievable performance of coherently detected communications systems. In this paper, the impact of imperfect CE on Reed-Solomon coded Hybrid Automatic-Repeat-Request (ReS/H-ARQ) systems is investigated for transmission over correlated Rayleigh fading channels. The proposed scheme invokes Amplify-and-Forward relaying, where the benefits of multiple cooperative stations are also quantified. Both the corresponding bit error probability and goodput are characterized. The system parameters are adjusted for maximizing the attainable system performance. An optimum pilot power allocation scheme is proposed, which reduces the required bit-energy by 4 dB for the (255/223) Reed-Solomon code defined over the Galois field $GF(256)$ without reducing the goodput.**

I. Introduction

Due to their better power efficiency, coherent detection schemes are employed in many contemporary digital wireless communication systems, such as in the 3G DS/CDMA, HSPA [1] and IEEE 802.11a standards, for example. Coherent detectors typically estimate the channel state information (CSI) using pilot sequences, which are known to both the transmitter and receiver. Naturally, the channel estimates acquired at the receiver are typically imperfect, resulting in a degraded performance.

Cooperative communications has attracted a lot of attention [2]–[4], since it is capable of creating a virtual multiple-input-multiple-output (MIMO) from the single-antenna-aided mobile stations (MS). The impact of imperfect channel estimates has been studied in a number of papers [5], [6]. The authors of [5] have proposed a channel estimator for Amplify-and-Forward (AF) relaying, while a single-relay-aided scenario relying on both orthogonal and non-orthogonal cooperative protocols was considered in [6]. These contributions, however, only considered uncoded systems, albeit all contemporary communication systems employ forward-error-correction (FEC) coding [7].

The authors of [8] investigated the effects of channels estimation on Hybrid Automatic-Repeat-reQuest (H-ARQ) systems, relying on error-detection rather than FEC codes. Against the above-mentioned background, in this paper we derived Bit Error Probability (BEP) and goodput equations in order to analyse the attainable system performance. Furthermore, the most appropriate number of pilot symbols and the corresponding pilot power were determined.

The rest of this paper is organized as follows. In Section II, our AF relaying aided Reed-Solomon (ReS) Coded H-ARQ system is described. Section III characterizes the channel estimation error imposed in correlated Rayleigh fading channels, followed by the definition of the Accepted-Packet-Error-Ratio (APER) and goodput. The proposed system's BEP and goodput expressions are also derived in this section. Section IV provides our numerical results and discussions, while our concluding remarks are offered in Section V.

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II. System Model

We will consider an H-ARQ scheme operating with the assistance of ReS coding. At the source station (SS), the information bits are grouped into blocks of m bits first, generating a symbol. Then a group of k information symbols is forwarded to an (n, k) ReS encoder, which is defined over the finite Galois field $GF(2^m)$ and has the code rate of $R = k/n$. Subsequently, N_p pilot bits, which are known to both the transmitter and receiver, are inserted into the encoded bit stream. The pilot symbol spacing T_p will be detailed in Section III-A. No error detection code is required as a benefit of the ReS code's capability of both error detection and correction [9], provided that the code is sufficiently long [9]. As a result, each transmitted packet includes k m-bit information symbols, $(n - k)$ parity symbols and N_p pilot bits. The packet's structure is shown in Fig. 1. Following modulation, the packet is transmitted to relay stations (RSs).

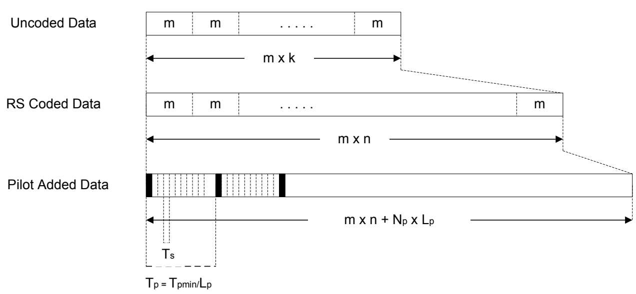

Fig. 1. Packet Structure

The classic Selective Repeat ARQ (SR-ARQ) scheme employing packet buffers at all the stations is used. The channels are assumed to be correlated Rayleigh fading. Then the signals received at the RSs may be expressed as

$$y_{R_l}[k] = G_{SR_l} h_{SR_l}[k] x[k] + n_{SR_l}[k], \tag{1}$$

where k is the symbol index, $l = 1, 2, ..., L$ is the RS index; $x[k]$ is the transmit signal at the SS with energy $\sqrt{E_S}$ and $y_{R_l}[k]$ is the signal received at the RS R_l; $n_{SR_l}[k]$ represents the zero-mean complex-valued AWGN with a variance of σ_n^2; $h_{SR_l}[k]$ represents the channel between the SS and the RS R_l, modelled as a wide-sense stationary (WSS) zero-mean complex Gaussian (ZMCG) random process with variance $\sigma_{SR_l}^2$; and G_{SR_l} is the path-loss-related power gain for the l^{th} Source-to-Relay (SR_l) link.

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The RSs amplify $y_{R_l}[k]$ and forward them to the Destination Station (DS), which receives

$$\begin{aligned} y_D[k] &= \sum_{l=0}^{L} \omega_l y_{R_l D}[k] \\ &= \sum_{l=0}^{L} \omega_l \Big\{ G_{R_l D} A_{R_l}[k] h_{R_l D}[k] y_{R_l}[k] + n_{R_l D}[k] \Big\} \\ &= \sum_{l=0}^{L} \omega_l \Big\{ G_{R_l D} A_{R_l}[k] h_{R_l D}[k] h_{SR_l}[k] x[k] \\ &\quad + G_{R_l D} A_{R_l}[k] h_{R_l D}[k] n_{SR_l}[k] + n_{R_l D}[k] \Big\} \\ &= \sum_{l=0}^{L} \omega_l \Big\{ h_l[k] x[k] + G_{R_l D} A_{R_l}[k] h_{R_l D}[k] n_{SR_l}[k] \\ &\quad + n_{R_l D}[k] \Big\}, \end{aligned} \tag{2}$$

where, again, $y_D[k]$ is the signal received at the DS; ω_l is the maximum ratio combining receiver's weighting coefficient; $h_{R_l D}[k]$ is the channel between the l^{th} Relay-to-Destination ($R_l D$) link, modelled as a WSS ZMCG process with a variance of $\sigma^2_{R_l D}$ and $n_{R_l D}[k]$ is the zero-mean complex AWGN with a variance of σ_n^2; finally, $G_{R_l D}$ is the path-loss-related power gain for the l^{th} $R_l D$ link. The overall relay channel is represented by $h_l[k] = G_{R_l D} A_{R_l}[k] h_{R_l D}[k] h_{SR_l}[k]$, where $A[k]$ is the fixed relay gain, which is expressed as [5]

$$A_{R_l}[k] = \sqrt{\frac{E_{R_l}}{E\big[|y_{R_l}[k]|^2\big]}} = \sqrt{\frac{G_{SR_l} E_{R_l}}{G_{SR_l} E_S \sigma^2_{SR_l} + \sigma_n^2}}. \tag{3}$$

In Eq. (2), the direct Source-to-Destination (SD) link is represented by $l = 0$, where we have $h_{R_0}[k] = h_{SD}[k]$, $G_{R_0 D} = G_{SR_0} = 1$, $A_{R_l}[k] = 1$, $n_{SR_0}[k] = n_{SD}[k]$ and $n_{R_0 D}[k] = 0$.

At the receiver, the pilot symbols are recovered first in order to estimate the complex-valued channel envelope. Then, the encoded bits are demodulated with the aid of the estimated channel coefficients before being passed to the ReS decoder. The decoder evaluates the ReS code's syndromes and checks for errors. If errors are detected, the error correction process is activated. Provided that all errors were successfully corrected, a positive acknowledgement (ACK) is returned to the transmitter, requesting a new packet. Otherwise, a negative ACK is sent to request retransmissions.

III. Impact of Imperfect CE on ReS coded H-ARQ Systems

A. Channel Estimation

1) Pilot Insertion Period: To estimate the CSI, a certain number of pilot symbols have to be inserted into the data symbol stream. However, the pilot-overhead has to be minimized in oder to avoid wasting power as well as to prevent the reduction of the effective data rate. According to the Nyquist sampling theorem, the minimum period for pilot insertion in the correlated fading channel must satisfy [5]

$$T_p \le \left(T_{pmin} = \frac{1}{2F_{max} T_s} \right), \tag{4}$$

where F_{max} is the maximum Doppler frequency of the fading channel $h_l[k]$ and T_s is the modulated symbol duration. When considering a down-link scenario of the maximum Doppler frequency F_{max} for an AF scheme having a fixed gain and using stationary relays is equal to [5]

$$F_{max} = f_{SR_l} + f_{R_l D}, \tag{5}$$

while that for mobile relays becomes

$$F_{max} = 2f_{SR_l} + f_{R_l D}, \tag{6}$$

where f_{SR_l} and $f_{R_l D}$ are the maximum Doppler frequencies of the SR_l and $R_l D$ links, respectively. Let us now define the pilot oversampling factor as the ratio between the actual number of inserted pilots N_p and the minimum number of required pilots N_{pmin} of an ReS-codeword, where the latter is determined by the Nyquist theorem:

$$L_p = \frac{N_p}{N_{pmin}} = \frac{T_{pmin}}{T_p}. \tag{7}$$

2) Channel Estimation Error: To recover the channel coefficients, the inserted pilot symbols are fed into the channel estimator, such as a Wiener filter [10], for generating the estimated version $\hat{h_l}[k]$ of $h[k_l]$. According to [11], the filter outputs, which are estimated from M_1 preceding and M_2 succeeding pilot symbols, are expressed as

$$\hat{h_l}[k] = \sum_{i=-M_1}^{M_2} w_i^* h_l[k-i], \tag{8}$$

where the asterisk superscript denotes complex conjugation and w_i represents the filter coefficients. As quantified in [12], the resultant Mean-Square-Error (MSE) of the channel estimates in a correlated fading channel is obtained as

$$E[|\hat{h_l} - h_l|^2] = \sigma^2_{e_l} = \frac{\sigma^2_{\hat{h}_l}}{1 + \frac{1}{2F_{max}T_p} \cdot \frac{E_p}{N_0}}, \tag{9}$$

where $\sigma^2_{\hat{h}_l}$ is the variance of the overall relay channel $\hat{h}_l$. Then Eq. (2) can be rewritten as

$$\begin{aligned} y_D[k] &= \sum_{l=0}^{L} \omega_l \Big\{ \hat{h}_{R_l}[k] x[k] + z_{R_l D}[k] \Big\}, & (10) \\ z_l[k] &= e_l[k] x[k] & (11) \\ &\quad + \sqrt{G_{R_l D}} A_{R_l}[k] h_{R_l D}[k] n_{SR_l}[k] + n_{R_l D}[k], \end{aligned}$$

where $z_l[k]$ is the total AWGN imposed on the received signal, which has the variance defined as

$$\sigma_{z_l} = E_S \sigma^2_{e_l} + G_{R_l D} A^2_{R_l} \sigma^2_{R_l D} \sigma^2_n + \sigma^2_n. \tag{12}$$

B. APER and Goodput

The performance of an ARQ system is typically evaluated in terms of two basic parameters, namely its reliability and throughput. More explicitly, when using FEC schemes, the reliability of the system may be quantified in terms of the APER, which is defined in [13] as

$$P_E = \frac{P_{ue}}{1 - P_{de}}, \tag{13}$$

where P_{ue} is the probability of an undetected packet error and P_{de} is the probability of a detected packet error (or probability of retransmission).

The corresponding throughput η may be expressed as [13]

$$\eta = R_e(1 - P_{de}) = \frac{km}{nm + N_p}(1 - P_{de}), \tag{14}$$

where $R_e = \frac{km}{nm+N_p}$ denotes the effective rate of each packet, since the code-rate of k/n is further reduced by the pilots. Additionally, the probability of retransmission may be expressed by subtracting the probability P_{ue} of an undetected packet error event with the aid of the probability P_e that a received packet contains at least one symbol error. Hence, we have

$$P_{de} = P_e - P_{ue}. \tag{15}$$

To jointly evaluate both the APER and the achievable throughput, the so-called goodput may be considered. The goodput is defined as the ratio between the expected number of correctly received information bits and the number of bits transmitted in a given period of time. In other words, the goodput reflects the ratio of correctly received packets in the total throughput. Hence, the goodput η_g may be expressed as

$$\eta_g = (1 - P_E)\eta. \tag{16}$$

C. Analysis of CE Error on ReS coded H-ARQ systems

Owing to its direct impact on all of the above-mentioned probabilities, the BEP p_e will be evaluated first. As mentioned in [14], the PDF-based approach of BEP approximation has limitations, especially in multi-path fading scenarios. Hence, instead of using the Q-function based solution, we invoke the Moment Generation Function (MGF) based technique to approximate the BEP. According to [14], the BEP of M-PSK modulated transmission may be obtained as

$$p_e = \frac{2}{max(log_2 M, 2)} \cdot \frac{1}{\pi} \int_0^{(M-1)\pi/M} \prod_{l=0}^{L} M_{\gamma_l}\left(s = \frac{1}{sin^2(\theta)}\right) d\theta, \tag{17}$$

where M_{γ_l} is the MGF of γ_l, which is expressed as [5]

$$M_{\gamma_l}(s) = \frac{1}{\gamma_l s} exp\Big(\frac{1}{\gamma_l s}\Big) \int_{\gamma_l s}^{\infty} \frac{e^{-x}}{x} dx, \tag{18}$$

and

$$\gamma_l = \frac{E_s |\hat{h}_l[k]|^2}{\sigma_{z_l}^2} = \frac{E_S^2(\sigma_{h_l}^2 - \sigma_{e_l}^2)}{\sigma_{z_l}^2}. \tag{19}$$

is the received symbols SNR, which is conditioned upon $\hat{h}_l[k]$.

A ReS-coded symbol becomes erroneous, when one or more of its m bits is incorrectly received. Thus, the probability of an erroneous ReS-coded symbol is obtained as

$$p_s = 1 - (1 - p_e)^m. \tag{20}$$

Next, we will evaluate the probability of an undetectable error P_{ue}. An (n, k) ReS decoder, designed to correct $t = \lfloor \frac{n-k}{2} \rfloor$ symbol errors, makes an incorrect codeword decision, when there are more than t symbol errors in a received packet. According to [9], the probability of an undetectable ReS-codeword error is

$$\begin{aligned} P_{ue} &= \sum_{h=d}^{n} \left[\binom{n}{h} (2^m - 1) \sum_{j=0}^{h-d} (-1)^j \binom{h-1}{j} (2^m)^{h-d-j} \right] \\ &\cdot \sum_{s=0}^{\frac{d-1}{2}} \sum_{g=h-s}^{h+s} \sum_{z=z_{min}}^{z=z_{max}} \binom{h}{h-s+z} \binom{s-z}{g-h+s-2z} \\ &\cdot \binom{n-h}{z} (2^m - 2)^{g-h+s-2z} (2^m - 1)^z \\ &\cdot \frac{1}{(2^m - 1)^g} [1 - (1 - p_e)^m]^g [(1 - p_e)^m]^{n-g}, \end{aligned} \tag{21}$$

where we have $d = n - m + 1$, $z_{min} = max\{0, g - h\}$ and $z_{max} = \lfloor \frac{g-h+s}{2} \rfloor$.

Furthermore, according to [15], the probability P_{de} of retransmission for a given received packet is

$$\begin{aligned} P_{de} &= 1 - P_{ue} - \sum_{h=0}^{\frac{n-k}{2}} \binom{n}{h} p_s^h (1 - p_s)^{n-h} \\ &= 1 - P_{ue} - \sum_{h=0}^{\frac{n-k}{2}} \binom{n}{h} [1 - (1 - p_e)^m]^h [(1 - p_e)^m]^{n-h}. \end{aligned} \tag{22}$$

Parameters	Values
m/k/n	8/223/255
Modulation type	BPSK
Relay type	fixed
N^o of relays	L=2
Normalized d_{SR_l}/d_{R_lD}	0.5/0.5
Data-pilot power allocation scheme	Equal
Source's transmit power	$E_{sS} = E_{pS} = \frac{mk}{mn+N_p} \cdot \frac{E_b}{2}$
Relays' transmit power	$E_{sR_l} = E_{pR_l} = \frac{mk}{mn+N_p} \cdot \frac{E_b}{2L}$
Channel model	correlated Rayleigh fading
Normalized Doppler freq. of SD link	$f_{SD}T_s$ = 0.002
Normalized Doppler freq. of SR$_l$ link	$f_{SR_l}T_s$ = 0.001
Normalized Doppler freq. of R$_l$D link	$f_{R_lD}T_s$ = 0.001
Pilot oversampling factor	L_p = 10

TABLE I

BASIC PARAMETERS OF ReS CODED H-ARQ SYSTEM

Substituting Eqs. (13) and (14) into Eq. (15) yields an explicit formula for the goodput as follows

$$\begin{aligned} \eta_g &= \frac{km}{nm + N_p} \left(1 - \frac{P_{ue}}{1 - P_{de}}\right) (1 - P_{de}) \\ &= \frac{km}{nm + N_p} \sum_{h=0}^{\frac{n-k}{2}} \binom{n}{h} [1 - (1 - p_e)^m]^h [(1 - p_e)^m]^{n-h}. \end{aligned} \tag{23}$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we investigate the achievable system performance of diverse network configurations. The basic parameters of Table I are employed, unless otherwise stated. To characterize the ReS/H-ARQ system's overall performance, let us consider the achievable goodput.

The effects of different Doppler frequencies are shown in Fig. 2. In this case, the normalized Doppler frequencies of the SR link as well as the RD link were set to {0.001, 0.005, 0.01, 0.02, 0.03}. According to the analysis in the previous section, when the channel was less correlated, the BEP was reduced. This was because each ReS-coded word experienced more-or-less random uniformly distributed errors, which was more beneficial for the ReS code than having some near-error-free and some badly contaminated codewords. On the other hand, the number of pilots has to be increased in order to adequately sample the higher-Doppler channel. As a consequence, the goodput of the entire system was actually reduced. Quantitatively, the goodput was reduced by a factor of two in Fig.2, when the normalized Doppler frequency was increased from 0.001 to 0.03.

In [5], the authors characterized both the effects of pilot spacing and those of the number of pilots separately. In fact, both of these effects can be treated jointly as that of the pilot power. Therefore, below we characterize the effects of different pilot powers. As shown in Section III-A, both the number of pilots and their spacing are related to the pilot oversampling factor of Eq. (7). The same power is assigned to all the data and pilot symbols. Hence, we will study the effect of the pilot oversampling factor L_p instead of the pilots' power.

It is plausible that increasing the pilot oversampling factor L_p, or - equivalently - the number of pilots, is expected to reduce the MSE. However, this automatically reduces the useful data symbols' energy at a fixed total power budget. Consequently, the BEP would be increased. Hence, the optimal pilot oversampling factor L_opt has to be determined.

In a mobile relaying aided network, the available number of cooperating nodes, their position and channel characteristics are time-variant. Thus, optimizing the pilot oversampling factor for the SS is feasible. As shown in Eq.(17), the BEP is a monotonically decreasing function of the instantaneous received SNR γ. Therefore, we have to find the highest value of γ in Eq.(19) in order to minimize the BEP. As demonstrated in the Appendix, the optimal pilot oversampling factor L_{popt} should be set to

$$L_{popt} = \sqrt{\frac{\sigma_{e0}^2 \cdot mk \cdot mn \cdot \frac{E_b}{N_0} + (mn)^2}{mk \cdot \frac{1}{2F_{max}T_{pmin}} \cdot \frac{E_b}{N_0} \cdot N_p + N_p^2}}. \tag{24}$$

The goodput performance of the ReS coded AF relaying aided H-ARQ system is further characterized in Fig. 3, but in contrast to Fig. 2, it is now parameterized by L_p. According to the figure, the optimized value of L_p shifts the goodput curve to the left, which is illustrated by the asterisk-dashed line, but its maximum value is lower than those of $L_p = 1$ and $L_p = 5$. This can be explained by the fact that upon minimizing the BEP by optimizing L_p, the effective rate R_e is also reduced. Thus, the goodput of the optimized scenario is also reduced. This problem can be overcome by optimizing the L_p value in Eq. (23) instead of that in Eq. (17). The results of this optimization process are also shown in Fig. 3. Clearly, the optimized goodput curve represented by the bold continuous line in the figure indeed reaches the maximum achievable goodput value. Of unity, the number of pilot symbols per ReS codeword versus E_b/N_0 curves seen in Fig. 4 provide a clearer view. During the BEP optimization, the SS kept the number of pilot symbols constant, even when the BEP was low. By contrast, the number of pilots was reduced during the goodput optimization, resulting in an increased goodput.

Fig. 5 characterizes the achievable goodput performance for three different ReS codeword lengths, when the code rate was fixed at 0.87. It was found from Fig. 5 that a shorter ReS codeword length of 63 symbols provided a higher goodput in the lower E_b/N_0 region, namely below 10 dB, while the longer codeword of 255 symbols proved to be more efficient in the rest of E_b/N_0 region. This trend may be explained by the characteristic behaviour of the component $\sum_{h=0}^{\frac{n-k}{2}} \binom{n}{h}[1-(1-p_e)^m]^h[(1-p_e)^m]^{n-h}$ in Eq. (23).

The code rate of $R = k/n$ has a substantial impact on the achievable system performance. Reducing the coding rate provides system with a better chance to correct errors imposed during transmissions, but reduces the effective throughput. Therefore, selecting the most appropriate code rate is necessary. Fig. 6 and Fig. 7 characterize the goodput performance of our ReS coded H-ARQ system, when the code rate is varied from 0.3 to 0.98 in steps of 0.02. It can be observed that the system performs worse, when the code rate is lower than 0.6. Depending on the channel quality, the optimal code rate may be selected from the curve, which ranges from 0.6 to 1, as seen in Fig. 7.

V. Conclusion

In this paper, we have investigated the impact of imperfect CE on the ReS/H-ARQ scheme operating in an AF relaying aided network for transmission over correlated Rayleigh fading channels. The system's BEP performance and goodput were derived analytically in order to evaluate the achievable performance. Based on these expressions, the pilot-versus-data symbol power allocation was optimized in order to minimize the BEP and to maximize the system's goodput. Furthermore, an optimum pilot power allocation scheme was proposed, which reduced the required bit-energy by 4 dB for the (255/223) Reed-Solomon code defined over the Galois field (256) without reducing the goodput.

Appendix I

Expressing the value of γ from Eq. (19) for the SS, we have

$$\gamma = \frac{\frac{E_{sS}}{N_0}\sigma_{h_{sd}}^2 \cdot \frac{1}{2F_{max}T_p} \cdot \frac{E_{sS}}{N_0}}{\frac{E_{sS}}{N_0}\sigma_{h_{sd}}^2 + 1 + \frac{1}{2F_{max}T_p} \cdot \frac{E_{sS}}{N_0}}. \tag{25}$$

If the pilot symbol energy and the encoded data symbol energy are set to be equal, then the received SNR of the SD link can be expressed as

$$\frac{E_{sS}}{N_0} = \frac{E_{pS}}{N_0} = \frac{mk}{mn + N_{pmin}L_p} \cdot \frac{E_b}{N_0}. \tag{26}$$

Upon taking into account $T_p = T_{pmin}/L_p$, Eq. (25) can be rewritten as

$$\gamma = \frac{\left(\frac{mk}{mn+N_{pmin}L_p} \cdot \frac{E_b}{N_0}\right)^2 \cdot \frac{1}{2F_{max}T_{pmin}} \cdot \sigma_{e0}^2 L_p}{\frac{mk}{mn+N_{pmin}L_p} \cdot \frac{E_b}{N_0} \cdot \left(\sigma_{e0}^2 + \frac{1}{2F_{max}T_{pmin}} \cdot L_p\right) + 1}. \tag{27}$$

Setting the derivative of γ with respect to L_p equal to zero and solving the resultant equation, we can obtain the optimal value L_{popt} as

$$L_{popt} = \sqrt{\frac{\sigma_{e0}^2 \cdot mk \cdot mn \cdot \frac{E_b}{N_0} + (mn)^2}{mk \cdot \frac{1}{2F_{max}T_{pmin}} \cdot \frac{E_b}{N_0} \cdot N_p + N_p^2}}. \tag{28}$$

References

[1] L. Hanzo, J. S. Blogh, and S. Ni, *3G, HSPA and FDD versus TDD Networking: Smart Antennas and Adaptive Modulation*. Wiley, 2nd ed., Feb. 2008.

[2] A. Nosratinia, T. E. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, vol. 42, pp. 74–80, Oct. 2004.

[3] L. Hanzo, O. Alamri, M. El-Hajjar, and N. Wu, *Near-Capacity Multi-Functional MIMO Systems: Sphere-Packing, Iterative Detection and Cooperation*. John Wiley - IEEE Press, May 2009.

[4] R. Zhang and L. Hanzo, "Coding schemes for energy efficient multi-source cooperation aided uplink transmission," *IEEE Signal Processing Letters*, vol. 16, pp. 438–441, May 2009.

[5] C. S. Patel and G. L. Stuber, "Channel estimation for amplify and forward relay based cooperation diversity systems," *IEEE Transactions on Wireless Communications*, vol. 6, pp. 2348–2356, June 2007.

[6] B. Gedik and M. Uysal, "Impact of imperfect channel estimation on the performance of amplify-and-forward relaying," *IEEE Transactions on Wireless Communications*, vol. 8, pp. 1468–1479, March 2009.

[7] L. Hanzo, C. H. Wong, and M. S. Yee, *Adaptive Wireless Transceivers: Turbo-Coded, Space-Time Coded TDMA, CDMA and OFDM Systems*. Wiley-IEEE Press, 2002.

[8] L. Cao, P. Y. Kam, and M. Tao, "Impact of imperfect channel state information on ARQ schemes over Rayleigh fading channels," in *Proceedings of IEEE International Conference on Communications (ICC '09)*, pp. 1–5, 14–18 June 2009.

[9] R. Steele and L. Hanzo, *Mobile Radio Communications: Second and Third-generation Cellular and WATM Systems*. John Wiley - IEEE Press, 2nd ed., May 1999.

[10] L. Hanzo, S. X. Ng, T. Keller, and T. Webb, *Quadrature Amplitue Modulation: From Basics to Adaptive Trellis-Coded, Turbo-Equalized and Space-time Coded OFDM. CDMA and MC-CDMA Systems*. Wiley, 3 ed., Sept. 2004.

[11] S. Haykin, *Adaptive Filter Theory*. Prentice Hall, 4th ed., 2002.

[12] J. Baltersee, G. Fock, and H. Meyr, "An information theoretic foundation of synchronized detection," *IEEE Transactions on Communications*, vol. 49, pp. 2115–2123, Dec. 2001.

[13] S. Wicker, *Error Control Systems for Digital Communication and Storage*. Prentice Hall, Oct 1994.

[14] M. K. Simon and M.-S. Alouini, *Digital communication over fading channels*. Wiley-IEEE Press, 2nd ed., Nov. 2004.

[15] S. B. Wicker, "Reed-Solomon error control coding for Rayleigh fading channels with feedback," *IEEE Transactions on Vehicular Technology*, vol. 41, pp. 124–133, May 1992.

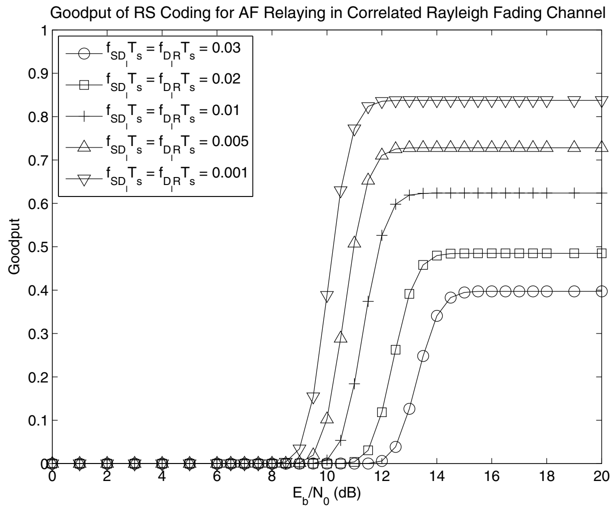

Fig. 2. Effect of different normalized Doppler frequencies on the achievable goodput in ReS coded H-ARQ using AF relaying where we have $f_{SR_l}T_s = f_{R_lD}T_s = \{0.001, 0.005, 0.01, 0.02, 0.03\}$, $f_{SD}T_s = 2f_{SR_l}T_s$, the remaining parameters are provided in Table I.

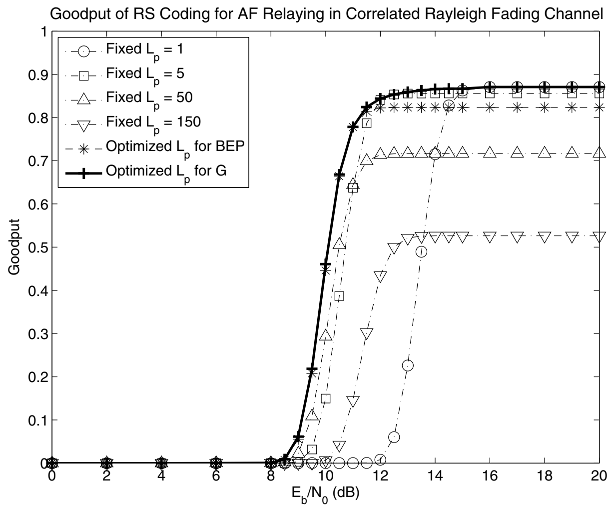

Fig. 3. Effect of the pilot oversampling factor L_p on the achievable goodput in ReS coded H-ARQ using AF relaying where we have $L_p = \{1, 3, 50, 150, L_{popt}\ for\ BEP, L_{popt}\ for\ G\}$, the remaining parameters are provided in Table I.

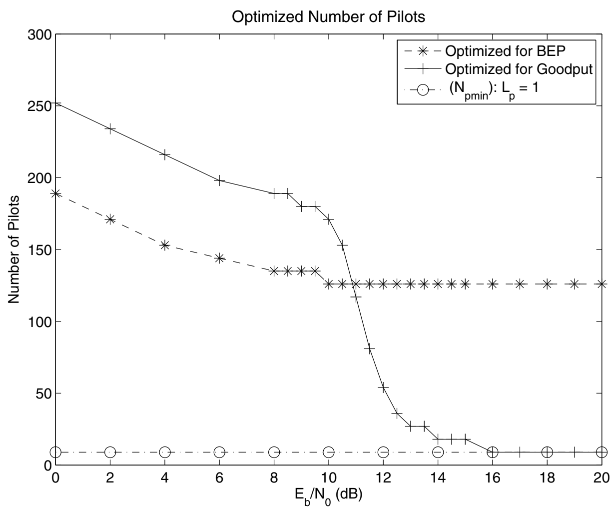

Fig. 4. Optimized number of pilots in ReS coded H-ARQ using AF relaying where we have $L_p = \{L_{popt}\ for\ BEP, L_{popt}\ for\ G, 1\}$, the remaining parameters are provided in Table I.

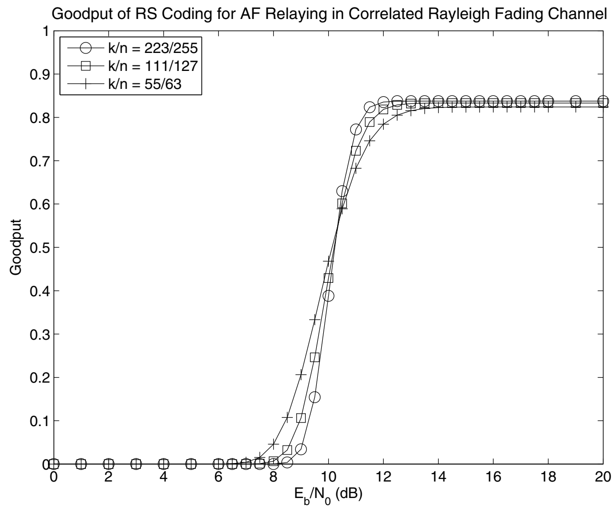

Fig. 5. Effect of codeword length on the achievable goodput in ReS coded H-ARQ using AF relaying where we have $k/n = \{223/255, 111/127, 55/63\}$, the remaining parameters are provided in Table I.

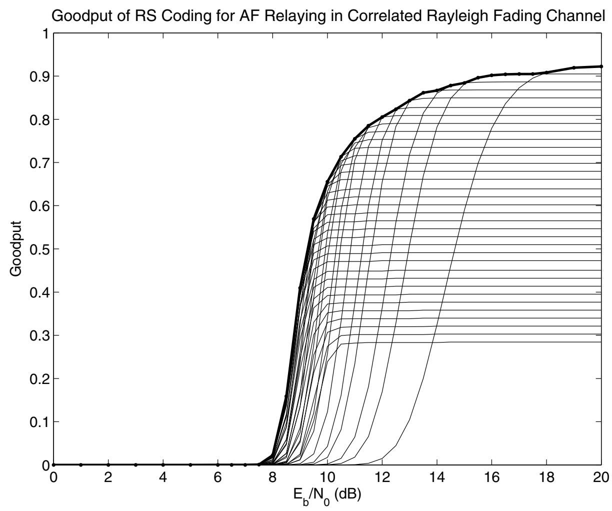

Fig. 6. Effect of code rate on the achievable goodput in ReS coded H-ARQ using AF relaying where we have $k/n = 0.3 \div 0.98$ in steps of 0.02, the remaining parameters provided in Table I.

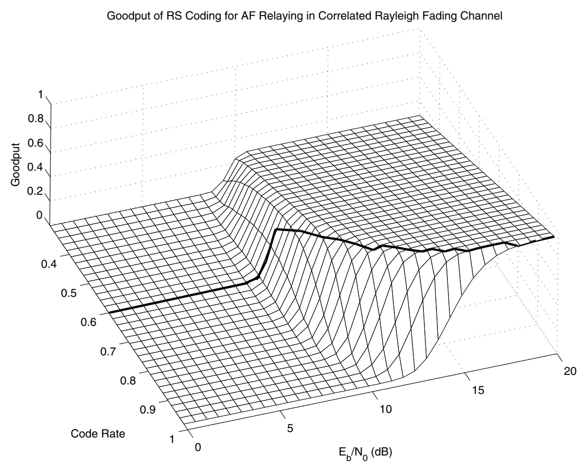

Fig. 7. Effect of code rate on the achievable goodput in ReS coded H-ARQ using AF relaying where we have $k/n = 0.3 \div 0.98$ in steps of 0.02, the remaining parameters provided in Table I.