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Can Dependent Sources be Suppressed in Electrical Circuit Theory?

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Can dependent sources be suppressed in circuit theory? The standard answer to this question is an unequivocal "no"; all the reference texts agree on this point. An unpublished paper by Marshall Leach challenges this received wisdom, characterising it as a "misconception". I have analysed Leach's work carefully and I find his method—in which dependent sources are suppressed when applying superposition—both correct and well-motivated. However, on my interpretation, the proof on which he bases his method is erroneous in one important particular. This paper indicates how the proof can be corrected, and improves the presentation of the method relative to Leach's by giving an example more closely related to the proof. The significant implications of this work for the teaching of circuit theory to beginning students are discussed. Finally, a reconciliation of the two positions (which cannot both be completely right) is made, concluding that the standard method leads to correct results (as is well-known) but arguably has less claim to be called 'superposition'.

Keywords: linear circuit theory, principle of superposition, dependent sources, Thévenin and Norton theorems, education and teaching

1. Introduction

The principle of superposition, first pronounced by Helmholtz (1853), is one of the bedrocks of linear circuit theory. When I was an undergraduate circa 1970, I was taught never to suppress dependent sources in linear circuit analysis problems founded on superposition. I discovered that attempts to do so frequently led to obviously wrong answers when finding output impedances in Thévenin and Norton equivalent circuits. I was taught, and I learned, alternative methods that avoided this pitfall and produced satisfyingly correct results. I have now been teaching elementary circuit theory at undergraduate level for 34 years. Throughout this time, I have assiduously told my students never to suppress dependent sources; to do so is a gross error. Every text I have ever seen that addresses this point agrees.

The standard position is forcefully stated by Senturia and Wedlock (1975) in their highly-regarded text:

> "The answer is very simple: *never suppress a dependent source in a superposition problem*. The reason for this categorical statement ... is that ... to suppress a dependent source ... amounts to removing some elements from the network" (p. 81).

Senturia and Wedlock are slightly unusual in offering a reason for the categorical statement; most authors are content just to state it, in effect according it the status of 'received wisdom'. I have to admit at this point that I have often felt a slightly nagging worry when attempting to justify the "never suppress a dependent source" rule to my students, in that I do not find the above rationale of "amounts

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to removing ... elements" at all convincing. (An anonymous reviewer of this paper pointed out that "destructing the controlling-controlled pair means destructing the model of the physical phenomenon that is modelled"—an explanation that I find helpful, and which presumably captures rather better what Senturia and Wedlock were trying to say. We will return to this point later.) Still, I had seen enough examples of suppression of dependent sources leading to incorrect results (Senturia and Wedlock give one, reproduced below) that I have always felt secure in passing the received wisdom on to my students, justifying it as best I could.

What then are we to make of an unpublished paper from Leach (1994) that has been available on the web for many years? It characterises the standard position as a "misconception" and present examples showing how to suppress dependent sources in superposition problems so as to yield correct results. Leach gives both an intuitive argument and a proof in support of his unconventional (or even heretical) position. From the history of this paper (presented as a footnote at the beginning), we know that Leach has submitted this work to archival journals (*IEEE Transactions on Education*, *IEEE Transactions on Circuits and Systems*) on several occasions, but these submissions have all been rejected.

Several possibilities exist here:

- Leach is straightforwardly wrong. *Ab initio*, this has to be deemed very likely, and would certainly explain why his journal submissions have been rejected. Surely, the principle "never suppress a dependent source" is so well established, and so widely endorsed in the standard texts, that it is impervious to Leach's challenge?
- Leach is correct, but does this not lead to the inescapable conclusion that the standard position is somehow wrong? Yet it cannot be too badly wrong, since we know from long experience that it leads to absolutely correct results.
- There is some immediate possibility, such as both methods yield correct results but the theoretical justification for one or other (perhaps both) is flawed or invalid. But if both methods are correct, which one of them is truly 'superposition' and what is the status of the other?

The rest of the paper will consider these issues. To keep things clear and simple, I will restrict the treatment to resistive circuits and dc sources, although the generalisation to more complex cases of impedances and ac excitation should be obvious.

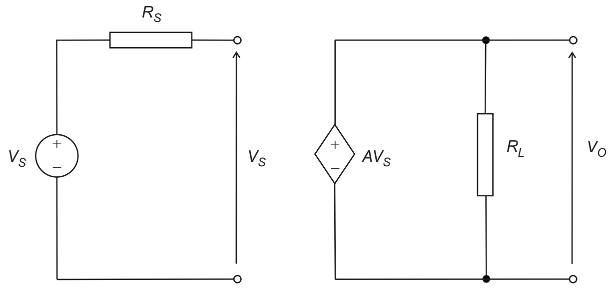
2. To Suppress or not to Suppress?

In this section, we set out and contrast the two contradictory positions to the suppression of dependent sources: namely, the standard 'received wisdom' position and Leach's.

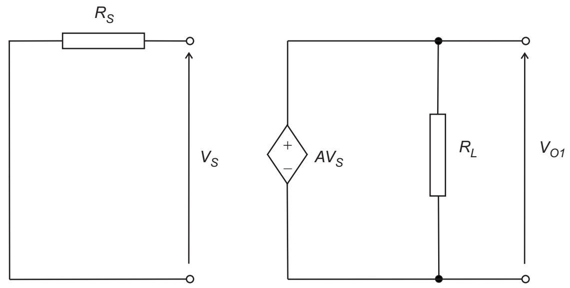
2.1 *The Standard Position*

Senturia and Wedlock (1975, pp. 81–82) give an example of how NOT to use superposition, which is reproduced here. Fig. 1(a) shows their original circuit, consisting of one independent voltage source and one dependent voltage source. (Note that the circuit symbols used here for independent and dependent sources are those employed by, e.g., van Valkenburg and Kinariwala, Figs. 2-36 and 2-37). It is clear that the output voltage is given by $V_0 = AV_S$.

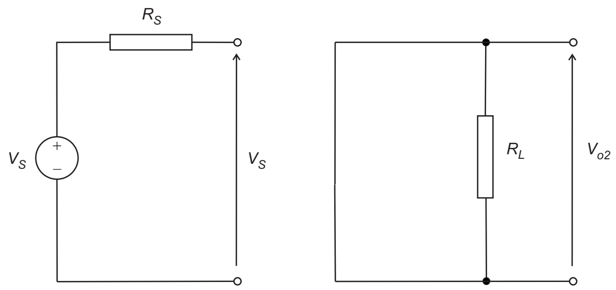
Fig. 1(b) shows the circuit with the independent source suppressed. In this case, V_S becomes zero as a consequence of the suppression. Hence, AV_S is zero and V_{01},



(a) Original circuit



(b) Independent source suppressed



(c) Dependent source suppressed

Figure 1. Effect of incorrectly suppressing a dependent source: $V_{01} + V_{02} = 0 \neq AV_S$.

i.e., the component of V_0 due to the independent source acting alone, is zero also. Fig. 1(c) shows the circuit with the dependent source suppressed—erroneously according to Senturia and Wedlock. In this case, V_{02} is zero as a consequence of the short circuit across R_L. Hence, by the superposition principle, $V_0 = V_{01} + V_{02} = 0$ giving an obviously incorrect result.

This seems pretty conclusive. At least, it did to me back in 1976 when I was preparing my first set of undergraduate lecture notes. One cannot suppress a dependent source: perhaps because it "amounts to removing some elements from the network", perhaps not.

2.2 *Leach's Method*

Leach cites 19 introductory texts on circuit analysis that either state that dependent sources must never be suppressed during superposition or, equivalently, that the superposition principle applies only to independent sources. He contends that this is a misconception and presents both an intuitive argument and a formal proof to support his position that dependent sources CAN be suppressed, "provided the controlling variable is not set to zero when the source is deactivated". He then presents several examples to illustrate the application and correctness of his method. Let us look at both the informal argument and the formal proof.

2.2.1 Leach's Informal Argument

Leach argues as follows. Consider a linear circuit containing both independent and dependent sources. Solve the circuit by any means other than superposition. This will yield values, D_i say, for each dependent source, i. Now solve the circuit using superposition, but this time treating each dependent source as an *in*dependent source of value D_i that can, according to received wisdom, be suppressed. This must give the same (correct) answer as in the first case. Hence, suppression of dependent sources is valid, contrary to the standard position. I find no fault with this argument.

But if it is correct, what are we to make of Senturia and Wedlock's counter-example in Fig. 1? Leach's answer would be that the controlling variable V_S has been erroneously set to zero when calculating V_{01} in Fig. 1(b). Instead of V_{01} taking the value 0, it should have been set to value AV_S, since this is its value (characterised as D_i above) in the full and final circuit, so yielding the correct answer.

At first sight, this seems a strange thing to do for two interdependent reasons:

(1) Notwithstanding the common notational difficulty encountered in superposition problems of distinguishing specific currents/voltages in the full circuit and the various versions of it in which sources are acting alone, V_S is incontrovertibly equal to zero in Fig. 1(b), so why should AV_S not be zero too in this circuit? But according to the informal argument, we must treat the dependent source as if was independent, with value AV_S (just as in Fig. 1(a)).
(2) Leach's argument appears to rely on wrongly 'importing' what we might call the final value of the dependent source (i.e., AV_S) found in the putative 'first' solution of the circuit of Fig. 1(a) into the *different* circuit of Fig. 1(b).

This concern is, I think, more apparent than real for the following reason. Circuit theory is, I believe, correctly viewed as a branch of applied mathematics. (I tell my students this.) In solving circuits, we make mathematical manipulations on an essentially abstract object: the circuit model. As Walker (1966) writes, "the general idea of a 'model' pervades the physical sciences, but often it is not acknowledged or recognized" (p. 95). Of course, the results of the manipulations on the model must be validly interpretable in terms of a real object, the physical circuit, in some important respects or there would be no interest in circuit theory for engineers. When we derive an equivalent circuit model of some device, we understand implicitly that the nodes and branches of that model do not have objective existence but are merely a convenient fiction. Provided we can characterise and/or predict terminal behaviour, via well-defined mathematical manipulations, we are content.

The standard position is, to my mind, predicated on what would 'really' happen if we suppressed dependent sources. But this is an over-interpretation: dependent sources are not 'real' in the sense that we could, e.g., put a voltmeter across their output terminals. What Leach does is to define, via his informal argument, the abstract operations required to produce correct answers to superposition problems. One of these operations is suppression of dependent sources; another is assignment to the dependent source of its value in the full circuit when it acts alone. These are, I believe, perfectly valid manipulations in the realm of applied mathematics; just like cross-multiplying algebraic equations or inverting a matrix.

Additional insight into Leach's informal argument can be gained by considering the anonymous reviewer's point, cited earlier, that: "Destructing the the controlling-controlled pair means destructing the model of the physical phenomenon that is modelled". This can be seen as a rationale for the dependent (controlled) source maintaining its value in the final circuit during superposition.

So let us proceed to examining the formal proof.

2.2.2 Leach's Formal Proof

This is said to be taken with modifications to account for dependent sources from Scott (1960). Assume a general set of node equations for any given linear circuit with current sources only (any voltage sources being first transformed into current sources) and n nodes at unknown potential:

$$\left.\begin{array}{l} I_1 = +G_{11}V_1 - G_{12}V_2 - \cdots - G_{1n}V_n \\ I_2 = -G_{12}V_1 + G_{22}V_2 - \cdots - G_{2n}V_n \\ \vdots \qquad\qquad\qquad \vdots \\ I_n = -G_{1n}V_1 - G_{2n}V_2 - \cdots + G_{nn}V_n \end{array}\right\} \quad \text{with } G_{ij} = G_{ji}$$

where I_j is the current into node j from current sources connected to that node. In matrix form, this can be written:

$$\boldsymbol{I} = \boldsymbol{GV} \tag{1}$$

By Cramer's rule:

$$V_1 = \frac{1}{\Delta}\begin{vmatrix} I_1 & -G_{12} & \cdots & -G_{1n} \\ I_2 & G_{22} & \cdots & -G_{2n} \\ \vdots & \vdots & & \vdots \\ I_n & -G_{2n} & \cdots & G_{nn} \end{vmatrix} \quad \begin{array}{c}\text{where } \Delta \text{ is} \\ \text{the determinant}\end{array} \quad \begin{vmatrix} G_{11} & -G_{12} & \cdots & -G_{1n} \\ -G_{12} & G_{22} & \cdots & -G_{2n} \\ \vdots & \vdots & & \vdots \\ -G_{1n} & -G_{2n} & \cdots & G_{nn} \end{vmatrix}$$

Without loss of generality, taking node 1 as an example, cofactor expansion yields the solution:

$$V_1 = I_1\frac{\Delta_{11}}{\Delta} - I_2\frac{\Delta_{12}}{\Delta} + I_3\frac{\Delta_{13}}{\Delta} - \cdots \tag{2}$$

where Δ_{ij} is the determinant formed by deleting row i and column j in Δ.

Leach now asserts that each term in (2) is "identical to the terms which would be written if only the source which generates that term is active and all other sources are deactivated". In this case, "the total response is written as the sum of the responses obtained with each source acting alone" and the principle of superposition is proved. Further, "because no assumption is made on the type of any source, the principle applies to both independent and dependent sources".

But Leach's basic assertion appears to me to be incorrect. It is apparent that each term in (2) involves any and all sources that contribute to the current at node 1. But if I interpret Leach correctly, he is saying that there is identity between the nodal-voltage terms in (2) and the components of the nodal voltages due to each source acting alone. Clearly, this cannot be the case. To illustrate this, and to help see how to repair the error in the proof, we now give a simple example.

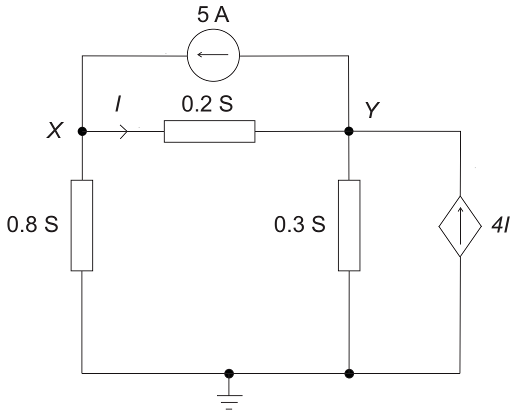


Figure 2. Illustrative example circuit with one independent current source and one current-controlled dependent current source.

3. Illustrative Example

Consider the example circuit in Fig. 2 (taken from my introductory Circuit Theory lecture slides). Note at the outset that any method of solution will easily yield the values:

$$V_X = 5\,\mathrm{V}, \quad V_Y = 0\,\mathrm{V}, \quad \text{and} \quad I = 1\,\mathrm{A} \tag{3}$$

also that this circuit is not amenable to a superposition solution via the standard method since it only contains a single independent source.

Writing the nodal voltage equations in matrix form:

$$\begin{bmatrix} 1 & -0.2 \\ -0.2 & 0.5 \end{bmatrix} \begin{bmatrix} V_X \\ V_Y \end{bmatrix} = \begin{bmatrix} 5 \\ -5 + 4I \end{bmatrix}$$

By Cramer's rule:

$$V_X = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 5 & -0.2 \\ -5+4I & 0.5 \end{vmatrix}}{\begin{vmatrix} 1 & -0.2 \\ -2 & 0.5 \end{vmatrix}}$$

$$= 5\frac{\Delta_{11}}{\Delta} - (-5+4I)\frac{\Delta_{12}}{\Delta} \qquad \text{with } \Delta = 0.46 \tag{4}$$

Leach's assertion is that these two terms, in Δ_{11} and Δ_{12}, are "identical to the terms which would be written if only the source which generates that term is active and all other sources are deactivated". But the term in Δ_{12} involves both sources. It is not the term that would be written if only the dependent source was active. So the term in Δ_{11}, even though it apparently only involves the 5 A source, cannot be the total contribution due to that source acting alone.

So what should the correct proof be? First, note that the linearity property of vector/matrix addition means that we are at liberty to decompose the column vector $\boldsymbol{I}$ in (1) into two parts in an infinite number of ways, for example:

$$\begin{bmatrix} 5 \\ -5+4I \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -5+4I \end{bmatrix} \tag{5}$$

$$= \begin{bmatrix} 2.5 \\ -5+I \end{bmatrix} + \begin{bmatrix} 2.5 \\ 3I \end{bmatrix} \tag{6}$$

$$= \begin{bmatrix} 5 \\ -5 \end{bmatrix} + \begin{bmatrix} 0 \\ 4I \end{bmatrix} \tag{7}$$

But only a very small subset of these decompositions have any sensible, physical interpretation in terms of the described circuit. For example, (5) represents the decomposition into two independent equations obtained by applying nodal analysis at each of nodes X and Y separately, (6) is a completely arbitrary decomposition having no real physical interpretation, and (7) is effectively the superposition of the independent and dependent sources in Fig. 2 according to Leach's method.

It should now be obvious how Leach's proof needs to be repaired. Each term in Δ_{ii} in (2) is, in fact, a superposition of the effect of (independent and dependent) sources connected to node i acting alone—subtly but importantly different to Leach's statement that equates the terms with sources acting alone. It should also be apparent that Leach's admonition "provided the controlling variable is not set to zero when the source is deactivated" is not quite complete; the error is to set the controlling variable to anything other than its value in the full, original circuit.

To complete the illustration, and to show better the workings of Leach's method and how it relates to the formal proof, let us calculate the *actual* contributions due to each source acting alone, by expanding out (4):

$$\begin{aligned} V_X &= 5\frac{0.5}{0.46} - (-5+4I)\frac{-0.2}{0.46} \\ &= \left(5\frac{0.5}{0.46} - \frac{1}{0.46}\right) + \frac{0.8I}{0.46} \\ &= \underbrace{3.261\,\text{V}}_{\text{due to 5 A source alone}} + \underbrace{1.739\,\text{V}}_{\text{due to } 4I \text{ source alone}} \qquad \text{with } I = 1\,\text{A, from (3)} \\ &= 5\,\text{V} \end{aligned}$$

The reader can easily confirm that 3.261 V and 1.739 V are exactly the values of the nodal voltage at X found with the dependent and independent sources, respectively, acting alone—in contradiction of the received wisdom that one can never suppress a dependent source. Further, there is no identity between these nodal voltages and the *actual* contributions due to each source acting alone; nor is there any reason to expect this. But this appears to contradict Leach's claim (or, at least, my interpretation of it) that there should be.

4. Dependent Sources and Thévenin/Norton Theorems

As is well known, dependent sources "introduce several complications into the problem of finding Thévenin or Norton equivalents" (Senturia and Wedlock 1975, p. 84). The received wisdom that one must never suppress a dependent source

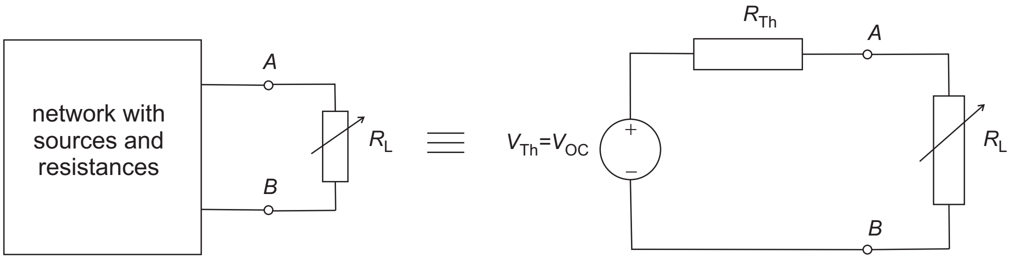


Figure 3. Thévenin equivalent circuit.

in superposition offers an apparently satisfying reason for these complications, simply by noting that the Thévenin/Norton theorems rest on superposition. See, for instance, the explanation of Senturia and Wedlock (1975, p. 55), which is related to the original proof of Thévenin, as outlined by Johnson (2003). But if, as Leach avers, it is perfectly valid to suppress dependent sources in superposition, why should there be any such "complications"? It was this question that originally made me doubt Leach's argument, until I was able to resolve it.

The answer is that whereas superposition underlies the proof of the Thévenin/Norton theorems, it is NOT used in the algorithm to find the equivalent resistance by suppressing internal sources. In its Thévenin guise, the theorem states that two circuits, as depicted in Fig. 3, will have the same terminal behaviour, at A and B, as the load R_L varies. That is, both circuits will have output voltage V_{AB} varying from the open-circuit voltage V_{OC} when R_L is infinite (an open-circuit) through to zero when R_L is zero (a short circuit), at which point $I_{AB} = I_{SC}$. Because the circuits are linear, their V_{AB} versus I_{AB} output characteristic is a straight line, and since any two lines that coincide at two points (here V_{OC} and I_{SC}) are identical, it follows that the two circuits are equivalent, in accordance with the theorem. But as we vary R_L to trace out the output characteristic, the controlling variables of any controlled sources will likewise vary. Hence, the algorithm of simultaneously suppressing all sources and evaluating the resistance between terminals A and B will not work for dependent sources. It is, in effect, a short-cut method for finding the (negative) slope of the V_{AB} versus I_{AB} output characteristic that relies on internal values of sources remaining fixed as R_L varies. This does not invalidate Leach's principle that dependent sources can be suppressed in superposition. In short, superposition solves a circuit in which everything is fixed, so that each dependent source can take its actual value in the full circuit when acting alone; Thévenin/Norton problems seek to characterise a circuit as some component (cf. the load resistance) varies.

5. Discussion and Implications for Teaching

Given the above analysis and discussion, it is now possible to adjudicate on the possibilities mentioned in Section 1 regarding the exact relation between the standard approach and Leach's method. It should be clear that both methods yield correct results. It is not the case that Leach's method must be wrong just because it is at variance with the standard approach, and the latter is known through long experience to give correct solutions. So how can apparently contradictory positions both be right? Either one can suppress dependent sources in superposition or one cannot. The answer is, as we have seen, that dependent sources can be suppressed. However, a method that denies this can still work, in spite of its essential premise being flawed, for the simple reason that the linearity property of vector/matrix addition means that we can 'superimpose' contributions to circuit voltages and currents in many ways (Section 3). For example, if we have a circuit with independent sources V_1, V_2, V_3, I_1 and I_2, there would be nothing to stop us solving the circuit by adding together partial solutions found (a) with V_2 and I_1 acting together and (b) with the remaining sources acting together. But this is not really 'superposition'. The essence of superposition is that a partial solution is found for *each* source acting alone. But the standard method fails to do this. From this perspective, Leach's method has more claim properly to be called 'superposition' just because each and every source is suppressed in turn.

These considerations produce something of a quandary for the teacher of elementary circuit theory. On the one hand, if we continue to tell students "never suppress dependent sources", they will not make an error in solving circuits. The rule will also serve to alert them to the complications of finding equivalent source resistances in Thévenin and Norton problems when dependent sources are present. But how can we justify this rule in a satisfying way? At worst, are we not telling them something wrong? On the other hand, it requires a degree of sophistication and facility with circuit theory to appreciate properly the arguments in Leach's work and their extensions as presented here. The beginning student is unlikely to possess such a degree of sophistication. Further, if we tell students that suppression of dependent sources is allowable, this contradicts what they will read in reference texts and requires the teacher to make a separate argument for disallowing suppression in Thévenin/Norton problems. But then, as we have seen, it really is a separate argument.

Recognising that his method often leads to much simpler solutions, Leach reports that "he has received nothing but positive responses from students" when teaching it. On balance, it does seem that the method is worth serious consideration as a way of teaching the important principle of superposition in a satisfying way.

6. Conclusions

Leach's unpublished work arguing that, contrary to received wisdom, dependent sources can be suppressed in superposition is both well-principled and well-motivated. This paper has identified an apparent error in Leach's proof of this fact, and indicated how this error could be corrected, as well as generally improving the presentation of the method. The essence of the method is that when acting alone dependent sources are treated as independent sources taking the current or voltage value that they assume in the full circuit. Whereas this may seem intuitively 'wrong', it has been shown here to be a simple consequence of the linearity of vector/matrix addition that underlies the principle of superposition.

The standard method, based on the received wisdom "never suppress a dependent source", also produces correct results—as indeed it must, given its long-standing acceptance. However, it has less claim to be considered truly 'superposition' since it fails to evaluate contributions to the circuit solution due to each and every source acting alone, as Leach's method does.

Acknowledgments

I am grateful to my colleague Dr. Nick Harris for pointing out the existence of Leach's work to me, and for challenging me to show where it was wrong. Thanks are also due to Dr. Marshall Leach for helpful discussions on the topic of superposition with dependent sources. Finally, I wish to thank the Journal's two anonymous reviewers for insightful criticisms that have encouraged me to improve this paper.

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