MIMO-OFDM for LTE, WIFI and WIMAX
Coherent versus Non-Coherent and Cooperative Turbo-Transceivers

by

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We dedicate this monograph to the numerous contributors of this field, many of whom are listed in the Author Index

The MIMO capacity theoretically increases linearly with the number of transmit antennas, provided that the number of receive antennas is equal to the number of transmit antennas. With the further proviso that the total transmit power is increased proportionately to the number of transmit antennas, a linear capacity increase is achieved upon increasing the transmit power. However, under realistic conditions the theoretical MIMO-OFDM performance erodes and hence to circumvent this degradation, our monograph is dedicated to the design of practical coherent, non-coherent and cooperative MIMO-OFDM turbo-transceivers...

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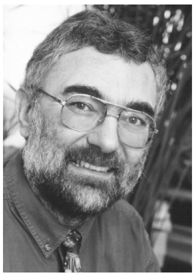
About the Authors



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[1] For detailed contents and sample chapters please refer to http://www-mobile.ecs.soton.ac.uk

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Preface

The rationale and structure of this volume is centred around the following ’story-line’. The conception of *parallel transmission of data* over dispersive channels dates back to the seminal paper of Doelz *et al.* published in 1957, leading to the OFDM philosophy, which has found its way into virtually all recent wireless systems, such as the WiFi, WiMax, LTE and DVB as well as DAB broadcast standards. Although *MIMO techniques* are significantly ’younger’ than OFDM, they *also reached a state of maturity* and hence the family of recent wireless standards includes the optional employment of MIMO techniques, which motivates the joint study of OFDM and MIMO techniques in this volume.

The research of MIMO arrangements was motivated by the observation that the MIMO capacity increases linearly with the number of transmit antennas, provided that the number of receive antennas is equal to the number of transmit antennas. With the further proviso that the total transmit power is increased proportionately to the number of transmit antennas, a linear capacity increase is achieved upon increasing the transmit power. This is beneficial, since according to the classic Shannon-Hartley law the achievable channel capacity increases only logarithmically with the transmit power. Hence *MIMO-OFDM may be considered a ’green’ transceiver solution.*

Therefore this volume sets out to explore the recent research advances in MIMO-OFDM techniques as well as their limitations. The basic types of multiple antenna-aided OFDM systems are classified and their benefits are characterised. Spatial Division Multiple Access (SDMA), Spatial Division Multiplexing (SDM) and space-time coding MIMOs are addressed. We also argue that *under realistic propagation conditions*, when for example the signals associated with the MIMO elements become correlated owing to shadow fading, *the predicted performance gains may substantially erode.* Furthermore, owing to the limited dimensions of shirt-pocket-sized handsets, the employment of multiple antenna elements at the mobile station is impractical.

Hence in practical terms only the family of distributed MIMO elements, which relies on the cooperation of potentially single-element mobile stations is capable of eliminating the correlation of the signals impinging on the MIMO elements, as it will be discussed in the book. The topic of *cooperative wireless communications* cast in the context of distributed MIMOs has recently attracted substantial research interests, but nonetheless, it *has numerous open problems, before all the idealized simplifying assumptions currently invoked in the literatue are eliminated.*

On a more technical note, *we aim for achieving a near-capacity MIMO-OFDM performance*, which requires sophisticated designs, as detailed below:

- A high throughput may be achieved with the aid of a high number of MIMO elements, but this is attained at a *potentially high complexity, which exponentially increases as a function of both the number of MIMO elements as well as that of the number of bits per symbol*, when using a full-search based Maximum Likelihood (ML) multi-stream/multi-user detector.

- In order to approach the above-mentioned near-capacity performance, whilst circumventing the problem of an exponentially increasing complexity, *we design radical multi-stream/multi-user detectors, which ’capture’ the ML solution with a high probability at a fraction of the ML-complexity.*

- This ambitious design goal is achieved with the aid of sophisticated *soft-decision-based Genetic Algorithm (GA) assisted MUDs or new sphere detectors, which are capable of operating in the high-importance rank-deficient scenarios*, when the number of transmit antennas may be as high as twice the number of receiver antennas.

- The achievable gain of space-time codes is further improved with the aid of *sphere-packing modulation, which allows us to design the space-time symbols of multiple transmit antennas jointly*, whilst previous designs made no effort to do so. Naturally, this joint design no longer facilitates low-complexity single-stream detection, but our sphere-decoders allow us to circumvent this increased detection complexity.

- Sophisticated *joint coding and modulation schemes* are used, which accommodate the parity bits of the channel codec without bandwidth extension, simply by extending the modulation alphabet.

- Estimating the MIMO channel for a high number of transmit and receive antennas becomes extremely challenging, since we have to estimate $N_t \cdot N_r$ channels, although in reality we are only interested in the data symbols, but not the channel. *This problem becomes even more grave in the context of the above-mentioned rank-deficient scenarios, since we have to estimate more channels, than the number of received streams.* Finally, the pilot overhead imposed by estimating $N_t \cdot N_r$ channels might become prohibitive, which erodes the attainable throughput gains.

- In order to tackle the above-mentioned challenging channel estimation problem, we designed *new iterative joint channel estimation and data detection techniques.* More explicitly, provided that a powerful MIMO MUD, such as the above-mentioned GA-aided or sphere-decoding based MUD is available for delivering a sufficiently reliable first data estimate, the power of decision-directed channel estimation may be invoked, which exploits that after a first tentative data decision - in the absence of decision errors - the receiver effectively knows the transmitted signal and hence may now exploit the presence of 100% pilot information for generating a more accurate channel estimate. Again, this design philosophy is detailed in the book in great depth in the context of joint iterative channel estimation and data detection.

- Although the number of studies/papers on cooperative communications increased exponentially over the past few years, most *investigations stipulate the simplifying assumption of having access to perfect channel information* - despite the fact that as detailed under the previous bullet-point, this is an extremely challenging task even for co-located MIMO elements.

- Hence it is necessary to design new non-coherently detected cooperative systems, which can dispense with the requirement of channel estimation, despite the typical 3 dB performance loss of differential detection. It is demonstrated in the book that *the low-complexity non-coherent detector's potential performance penalty can in fact be recovered with the aid of jointly detecting a number of consecutive symbols with the aid of the so-called multiple-symbol differential detector*, although this is achieved at the cost of an increased complexity.

- *Hence the proposed sphere-detector may be invoked again, but now as a reduced-complexity multiple-symbol differential detector*.

- The above-mentioned cooperative systems require *specifically designed resource allocation*, including the choice of the relaying protocols, the selection of the cooperating partners and the power-control techniques.

- It is demosntrated that when the available relaying partners are roaming close to the source, decode-and-forward (DF) is the best cooperating protocol, which avoids potential error-precipitation. By contrast, in case the cooperating partners roam closer to the destination, then the amplify-and-forward (AF) protocol is preferred for the same reasons. *These complementary features suggest the emergence of a hybrid DF/AF protocol*, which is controlled with the aid of our novel resource-allocation techniques.

- The book is concluded by outlining a variety of promising *future research directions.*

Our intention with the book is:

1. First, to pay tribute to all researchers, colleagues and valued friends, who contributed to the field. Hence this book is dedicated to them, since without their quest for better MIMO-OFDM solutions this monograph could not have been conceived. They are too numerous to name here, hence they appear in the author index of the book. Our hope is that the conception of this monograph on the topic will provide an adequate portrayal of the community's research and will further fuel this innovation process.

2. We expect to stimulate further research by exposing open research problems and by collating a range of practical problems and design issues for the practitioners. The coherent further efforts of the wireless research community is expected to lead to the solution of the range of outstanding problems, ultimately providing us with flexible coherent- and non-coherent detection aided as well as cooperative MIMO-OFDM wireless transceivers exhibiting a performance close to information theoretical limits.

List of Symbols

$(\cdot)[n,k]$	The indices indicating the k^{th} subcarrier of the n^{th} OFDM symbol
$(\cdot)^T$	The transposition operation
$(\cdot)^{\boldsymbol{H}}$	Hermitian transpose
$(\cdot)^*$	Complex conjugate
$\Im$	The imaginary component of a complex number
$\Re$	The real component of a complex number
$\mathcal{I}\{\cdot\}$	Imaginary part of a complex value
$\mathcal{I}$	Mutual information,sort
π	The ratio of the circumference of a circle to the diameter
$\mathcal{R}\{\cdot\}$	Real part of a complex value
$\exp(\cdot)$	The exponential operation

$\mathbf{A}^{(l)}$	The remaining user set for the l^{th} iteration of the subcarrier-to-user assignment process
$\boldsymbol{A}^{\mathtt{T}}$	Matrix/vector transpose
$\boldsymbol{A}^{\mathtt{H}}$	Matrix/vector hermitian adjoint, *i.e.* complex conjugate transpose
$\boldsymbol{A}^*$	Matrix/vector/scalar complex conjugate
$\boldsymbol{A}^{-1}$	Matrix inverse
$\boldsymbol{A}^+$	Moore-Penrose pseudoinverse
$\mathtt{tr}(\boldsymbol{A})$	Trace of matrix, *i.e.* the sum of its diagonal elements
α_P	The user load of an L-user and P-receiver conventional SDMA system

B_{T}	The overall system throughput in bits per OFDM symbol
$(i_{\mathrm{ce}}, i_{\mathrm{det}}, i_{\mathrm{dec}})$	Number of (channel estimation,detection,decoding) iterations
E_b	Energy per transmitted bit
E_s	Energy per transmitted M-QAM symbol
L_f	Number of data-frames per transmission burst
N_d	Number of data SDM-OFDM symbols per data-frame
N_p	Number of pilot SDM-OFDM symbols in burst preamble
T	OFDM symbol duration
T_s	OFDM FFT frame duration
f_{D}	Maximum Doppler frequency
K	Number of OFDM subcarriers
B	Signal bandwidth
β	RLS CIR tap prediction filter forgetting factor
$\mathcal{C}$	Uncostrained capacity
f_{c}	Carrier frequency
η	PASTD aided CIR tap tracking filter forgetting factor
γ	OHRSA search resolution parameter
m_{t}	Number of receive antennas
n_{r}	Number of transmit antennas
ν_τ	OFDM-symbol-normalized PDP tap drift rate
ρ	OHRSA search radius factor parameter
σ_w^2	Gaussian noise variance

$\tau_{\rm rms}$	RMS delay spread
ε	Pilot overhead
ζ	MIMO-CTF RLS tracking filter forgetting factor
b_{l,m_B}	The $(m_B)^{th}$ bit of the l^{th} user's transmitted symbol
r	Size of the transmitted bit-wise signal vector $\mathbf{t}$
$\hat{b}_s^{(l)}[n,k]$	The l^{th} user's detected soft bit
$\hat{\mathbf{b}}_s^{(l)}$	The detected soft bit block of the l^{th} user
$\mathbf{b}^{(l)}$	The information bit block of the l^{th} user
$\mathbf{b}_s^{(l)}$	The coded bit block of the l^{th} user

$\mathbb{C}$	The complex space
$\mathbb{C}^{(x\times y)}$	The $(x\times y)$-dimensional complex space
$\mathbf{CC}(n,k,K)$	Convolutional codes with the number of input bits k, the number of coded bits n and the constraint length K
$\boldsymbol{I}$	Identity matrix
$\mathcal{H}$	Hadamard matrix
$\mathcal{L}$	Log Likelihood Ratio value
$\mathcal{M}$	Set of M-PSK/M-QAM constellation phasors
$c_{g_l}(t)$	The DSS signature sequence assigned to the l^{th} user and associated with the g^{th} DSS group
$\bar{\mathbf{c}}_{G_q}$	The $(1\times L_q)$-dimensional DSS code vector
$\check{\mathbf{c}}_{G_q}$	The $(G_q\times 1)$-dimensional DSS code vector
$\mathbf{c}_g$	The spreading code sequence associated with the g^{th} DSS group
$\mathbf{c}$	The user signature vector
$\mathbf{c}^{(l)}$	the l^{th} user's code sequence
$\mathbf{c}_{g_l}$	The DSS code vector for the l^{th} user in the g^{th} DSS group

$\check{\mathbf{s}}$	*A priori* signal vector estimate
$\hat{\mathbf{s}}$	*A posteriori* signal vector estimate
$\hat{\mathbf{x}}$	Unconstrained *a posteriori* signal vector estimate
$\mathbf{H}$	Subcarrier-related MIMO CTF matrix
$\mathbf{d}$	Transmitted bit-wise signal
$\mathbf{s}$	Transmitted subcarrier-related SDM signal
$\mathbf{t}$	Transmitted subcarrier-related bit-wise SDM signal
$\mathbf{y}$	Received subcarrier-related SDM signal
$\mathbf{w}$	Gaussian noise sample vector
$\tilde{\mathbf{s}}$	Soft-information aided signal vector estimate
$\Delta_{p,(y,x)}^{(l)}[n,k]$	The random step size for the $(p,l)^{th}$ channel gene during step mutation associated with the x^{th} individual of the y^{th} generation

ϵ	The pilot overhead

F_D	The OFDM-symbol-normalized Doppler frequency
$\mathtt{Cov}\{\cdot,\cdot\}$	Covariance of two random variables
$\mathtt{Var}\{\cdot\}$	Variance of a random variable
$\mathtt{E}\{\cdot\}$	Expectation of a random variable
$\mathtt{Ei}\{\cdot\}$	Exponential integral
$\mathtt{JacLog}(\cdot)$	Jacobian logorithm
κ	Channel estimation efficiency criteria
$\|\cdot\|_2$	Second order norm
$\mathtt{P}\{\cdot\}$	Probability density function
$\mathrm{rms}\{\cdot\}$	Root mean square value
f_d'	Normalized Doppler frequency
f_c	Carrier frequency
f_d	Maximum Doppler frequency
f_q	Carrier frequency associated with the q^{th} sub-band

$f_{(y,x)}$	The fitness value associated with the x^{th} individual of the y^{th} generation
G	The number of DSS user groups in a DSS/SSCH system
G_q	The total number of different DSS codes used by the users activating the q^{th} subcarrier
$\Gamma_\tau(t)$	The rectangular pulse within the duration of $[0,\tau)$
$H_p^{(l)}$	The FD-CHTF associated with the l^{th} user and the p^{th} receiver antenna element
$H_{p,q}^{(l)}$	The FD-CHTF associated with the specific link between the l^{th} user and the p^{th} receiver at the q^{th} subcarrier
$H_p^{(l)}[n,k]$	The true FD-CHTF associated with the channel link between the l^{th} user and the p^{th} receiver
$\hat{H}_p^{(l)}[n,k]$	The improved *a posteprioriI* FD-CHTF estimate associated with the channel link between the l^{th} user and the p^{th} receiver
$\mathbf{H}$	The FD-CHTF matrix
$\mathbf{H}^{(l)}$	The FD-CHTF vector associated with the l^{th} user
$\mathbf{H}_{g,q}^{(l)}$	The $(P \times 1)$-dimensional FD-CHTF vector associated with the transmission paths between the l^{th} user's transmitter antenna and each element of the P-element receiver antenna array, corresponding to the g^{th} DSS group at the q^{th} subcarrier
$\mathbf{H}_p$	The p^{th} row of the FD-CHTF matrix $\mathbf{H}$
$\mathbf{H}_{g,q}$	The $(P \times l_g)$-dimensional FD-CHTF matrix associated with the g^{th} DSS group at the q^{th} subcarrier
$\mathbf{H}_{p,g,q}$	The p^{th} row of the FD-CHTF matrix $\mathbf{H}_{g,q}$ associated with the g^{th} DSS group at the q^{th} subcarrier
$\mathbf{H}_p[n,k]$	The initial FD-CHTF estimate matrix associated with all the channel links between each user and the p^{th} receiver
$\bar{\mathbf{H}}_{p,q}$	The L_q users' $(L_q \times L_q)$-dimensional diagonal FD-CHTF matrix associated with the q^{th} subcarrier at the p^{th} receiver
$\bar{\mathbf{H}}_p[n,k]$	The diagonal FD-CHTF matrix associated with all the channel links between each user and the p^{th} receiver
$\tilde{\mathbf{H}}[n,k]$	The trial FD-CHTF matrix of the GA-JCEMUD
$\tilde{\mathbf{H}}_{(y,x)}[n,k]$	The FD-CHTF chromosome of the GA-JCEMUD individual associated with the x^{th} individual of the y^{th} generation
$\tilde{H}_{p,(y,x)}^{(l)}[n,k]$	The $(p,l)^{th}$ channel gene of the GA-JCEMUD FD-CHTF chromosome associated with the x^{th} individual of the y^{th} generation
$\tilde{H}_p^{(l)}[0,k]$	The initial FD-CHTF estimate associated with the channel link between the l^{th} user and the p^{th} receiver at the k^{th} subcarrier in the first OFDM symbol duration
$\tilde{h}_p^{(l)}[n,k]$	The initial estimate of the CIR-related taps associated with the channel link between the l^{th} user and the p^{th} receiver
$\mathbf{I}$	Identity matrix
K_0	The range of CIR-related taps to be retained
L	Number of simultaneous mobile users supported in a SDMA system
L_q	The number of users that activate the q^{th} subcarrier
$\mathcal{L}_{l,m_B}$	The LLR associated with the $(m_B)^{th}$ bit position of the l^{th} user's transmitted symbol
$\Lambda_q^{(l)}(t)$	The subcarrier activation function
l_g	The number of users in the g^{th} DSS group
λ_{max}	The maximum mutation step size of the step mutation
M_{WHT}	The WHT block size
$\mathcal{M}^L$	The set consisting of 2^{mL} number of $(L \times 1)$-dimensional trial vectors
$\mathcal{M}_{l,m_B,b}^L$	The specific subset associated with the l^{th} user, which is constituted by those specific trial vectors, whose l^{th} element's $(m_B)^{th}$ bit has a value of b

$\mathcal{M}_c$	The set containing the 2^m number of legitimate complex constellation points associated with the specific modulation scheme employed
m_B	The bit position of a constellation symbol
$\overline{\mathrm{MSE}}$	The average FD-CHTF estimation MSE
$\overline{\mathrm{MSE}}[n]$	The average FD-CHTF estimation MSE associated with the n^{th} OFDM symbol
N_T	The total number of OFDM symbols transmitted
$n_p(t)$	The AWGN at the p^{th} receiver
$n_{p,q}$	The noise signal associated with the q^{th} subcarrier at the p^{th} receiver
$\bar{\mathbf{n}}_{p,q}$	The $(G_q \times 1)$-dimensional effective noise vector associated with the q^{th} subcarrier at the p^{th} receiver
$\mathbf{n}$	Noise signal vector
ω_{ij}	The cross-correlation coefficient of the i^{th} DSS group's and the j^{th} DSS group's signature sequence
$\Omega(\cdot)$	The GA's joint objective function for all antennas
$\Omega_{g,q}(\cdot)$	The GA's joint objective function for all antennas associated with the g^{th} DSS group at the q^{th} subcarrier
$\Omega_{p,g,q}(\cdot)$	The GA's objective function associated with the g^{th} DSS group of the p^{th} antenna at the q^{th} subcarrier
$\Omega_p(\cdot)$	The GA's objective function associated with the p^{th} antenna
$\Omega_{y,T}$	The maximum GA objective score generated by evaluating the T individuals in the mating pool
P	Number of receiver antenna elements employed by the BS in SDMA systems
P_T	Transmitted signal power
$\tilde{p}_{mt}^{(ij)}$	The normalized mutation-induced transition probability
$p_{\overline{mt}}^{(ij)}$	The 1D transition probability of mutating from a 1D symbol s_{Ri} to another 1D symbol s_{Rj}
$p_{mt}^{(ii)}$	The original legitimate constellation symbol's probability of remaining unchanged
$p_{mt}^{(ij)}$	The mutation-induced transition probability, which quantifies the probability of the i^{th} legitimate symbol becoming the j^{th}
p_m	The mutation probability, which denotes the probability of how likely it is that a gene will mutate
$\Phi(\cdot)$	The cost function of the OHRSA MUD
$\Phi_i(\cdot)$	The cumulative sub-cost function of the OHRSA MUD at the i^{th} recursive step
$\varphi^{(l)}$	The l^{th} user's phase angle introduced by carrier modulation
$\phi(\cdot)$	The sub-cost function of the OHRSA MUD
$Q(x)$	The Q-function
$\mathbf{Q}_L$	The L-order full permutation set
Q_c	The number of available subcarriers in conventional or SSCH systems
Q_f	The number of available sub-bands in SFH systems
Q_g	The number of subcarriers in a USSCH subcarrier group
$\mathbf{q}_k$	The subcarrier vector generated for the k^{th} subcarrier group
$\mathfrak{q}^{(l)}$	The USSCH pattern set of the l^{th} user
R	Code rate
$\mathbf{R}_\mathbf{n}$	The $(P \times P)$-dimensional covariance matrix
$\bar{\mathbf{R}}_{G_q}$	The $(G_q \times L_q)$-dimensional cross-correlation matrix of the L_q users' DSS code sequences
$r_p(t)$	The received signal at the p^{th} receiver
$r_{p,q}$	The discrete signal received at the q^{th} subcarrier of the p^{th} receiver during an OFDM symbol duration
$x_{p,g}(t)$	The despread signal of the g^{th} DSS group at the p^{th} receiver

$\hat{s}_i^{(l)}$	The i^{th} constellation point of $\mathcal{M}_c$ as well as a possible gene symbol for the l^{th} user
$s'^{(l)}_{g_l,q}(t)$	The transmitted signal at the q^{th} subcarrier associated with the l^{th} user in the g^{th} DSS group
$s^{(l)}$	The transmitted signal of the l^{th} user at a subcarrier
$s^{(l)}_{g_l,q}(t)$	The information signal at the q^{th} subcarrier associated with the l^{th} user in the g^{th} DSS group
s_{Ri}	The i^{th} 1D constellation symbol in the context of real axis
$\bar{\mathbf{s}}_q$	The L_q users' $(L_q \times 1)$-dimensional information signal vector
$\check{\mathbf{s}}$	The candidate trial vector
$\check{\mathbf{s}}_i$	The sub-vector of $\check{\mathbf{s}}$ at the i^{th} OHRSA recursive step
$\hat{\mathbf{s}}^{(l)}$	The l^{th} user's estimated information symbol block of the FFT length
$\hat{\mathbf{s}}_{\mathbf{w}}^{(l)}$	The estimated l^{th} user's WHT-despreading signal block
$\hat{\mathbf{s}}_{\mathbf{w},0}^{(l)}$	The estimated l^{th} user's WHT-despread signal block
$\hat{\mathbf{s}}_{\mathbf{GA}}$	The estimated transmitted symbol vector detected by the GA MUD
$\hat{\mathbf{s}}_{\mathbf{GA}_{g,q}}$	The GA-based estimated $(l_g \times 1)$-dimensional signal vector associated with the g^{th} DSS group at the q^{th} subcarrier
$\hat{\mathbf{s}}_{\mathbf{MMSE}_{g,q}}$	The MMSE-based estimated $(l_g \times 1)$-dimensional signal vector associated with the g^{th} DSS group at the q^{th} subcarrier
$\tilde{\mathbf{s}}[n,k]$	The trial data vector of the GA-JCEMUD
$\tilde{\mathbf{s}}_{(y,x)}$	The x^{th} individual of the y^{th} generation
$\tilde{\mathbf{s}}_{(y,x)}[n,k]$	The symbol chromosome of the GA-JCEMUD individual associated with the x^{th} individual of the y^{th} generation
$\mathbf{s}$	Transmitted signal vector
$\mathbf{s}^{(l)}$	The l^{th} user's information symbol block of the FFT length
$\mathbf{s}_{\mathbf{w}}^{(l)}$	The l^{th} user's WHT-spread signal block
$\mathbf{s}_{\mathbf{w},0}^{(l)}$	The l^{th} user's WHT-spreading signal block
$\mathbf{s}_g$	The $(l_g \times 1)$-dimensional trial symbol vector for the GA's objective function associated with the g^{th} DSS group
$\tilde{s}^{(l)}_{(y,x)}[n,k]$	The l^{th} symbol gene of the GA-JCEMUD symbol chromosome associated with the x^{th} individual of the y^{th} generation
σ_l^2	Signal variance associated with the l^{th} user
σ_n^2	Noise variance
T_h	The FH dwell time
$\mathbf{TC}(n,k,K)$	Turbo convolutional codes with the number of input bits k, the number of coded bits n and the constraint length K
T_r	The reuse time interval of hopping patterns
T_c	The DSS chip duration
$\mathbf{U}_{\mathbf{WHT}_K}$	The K-order WHT matrix
$u_{g_l}[c]$	The c^{th} element of the g^{th} row in the $(G \times G)$-dimensional WHT matrix, which is associated with the l^{th} user
$\mathbf{V}$	The upper-triangular matrix having positive real-valued elements on the main diagonal
v	CM code memory
W	System bandwidth
W_{sc}	Subcarrier bandwidth
$\mathbf{W}_{\mathbf{MMSE}}$	The MMSE-based weight matrix
$\mathbf{W}_{\mathbf{MMSE}_{g,q}}$	The MMSE-based $(P \times l_g)$-dimensional weight matrix associated with the g^{th} DSS group at the q^{th} subcarrier
X	GA population size
x_p	The received signal at the p^{th} receiver at a subcarrier

$\bar{\mathbf{x}}_{p,q}$	The despread signal associated with the q^{th} subcarrier at the p^{th} receiver
$\mathbf{x}$	Received signal vector
$\mathbf{x}_p$	The received symbol block of the FFT length at the p^{th} receiver
$\mathbf{x}_{g,q}$	The $(P \times 1)$-dimensional despread signal vector associated with the g^{th} DSS group at the q^{th} subcarrier
Y	Number of GA generations