Robust $\mathcal{H}_\infty$ Control of Networked Control Systems with Access Constraints and Packet Dropouts

Dongxiao Wu, Jun Wu and Sheng Chen

***Abstract*— We consider a class of networked control systems (NCSs) where the plant has time-varying norm-bounded parameter uncertainties, the network only provides a limited number of simultaneous accesses for the sensors and actuators, and the packet dropouts occur randomly in the network. For this class of NCSs with uncertainties and access constraints as well as packet dropouts, we derive sufficient conditions in the form of linear matrix inequalities that guarantee robust stochastic stabilisation and synthesis of $\mathcal{H}_\infty$ controller. An example is provided to illustrate our proposed method.**

***Index Terms*— Networked control systems, norm-bounded uncertainties, access constraints, packet dropouts, robust $\mathcal{H}_\infty$ control**

I. Introduction

Networked control systems (NCSs) have attracted much attention recently [1], [2], [3], [4], [5]. An NCS is a control system in which the feedback control loop is closed via a shared communication network. Compared to conventional point-to-point system connection, an NCS offers several advantages, such as low installation cost, reducing system wiring, simple system diagnosis and easy maintenance. However, some inherent shortcomings of the NCS, such as access constraints, packet dropouts and packet delays, will degrade performance of NCSs or even cause instability. Access constraints refer to scenarios where simultaneous access to all the sensors and actuators is lacked. The communication sequence policy under access constraints is first taken into account in a stabilisation problem [6]. This policy is further studied for LQG control [7], [8]. In [9], an $\mathcal{H}_\infty$ control synthesis with the periodic sequencing scheme is proposed. Due to network or system constraints, the multiple-packet transmission policy is often required. For example, a packet exceeding the maximum packet size is broken into multiple packets. In a distributed control system, each sensor or control signal is transmitted via the individual packet. The multiple-packet transmission policy is studied in [10], [11], [12].

Packet dropouts can randomly occur due to node failures or network congestion. Stochastic approaches are typically adopted to establish mean square stability [13], [14]. Under such a stochastic approach, the packet dropout process is usually modeled as a Bernoulli process or a Markov chain, and the system can be viewed as a special case of jump linear system [11], [15], [16]. The optimal LQG control and Kalman filter problems under lossy measurements are dealt in [3], [17]. In some other works [18], [19], the NCSs with arbitrary packet dropouts are modelled as switched systems. When the system has parameter uncertainties, the standard $\mathcal{H}_\infty$ control [20] cannot provide guaranteed $\mathcal{H}_\infty$ performance and stability. Robust $\mathcal{H}_\infty$ control has been investigated for both continuous-time and discrete-time systems [15], [21], [22], [23]. All these references only consider the systems with delays, such as state or network packet delays. In [9], the author studies $\mathcal{H}_\infty$ control problem under a periodic sequencing transmission policy and random packet dropouts for the nominal system. To the best of our knowledge, robust $\mathcal{H}_\infty$ control has not been studied for NCSs with access constraints and packet dropouts.

The novel contribution of this paper is that we study synthesis of robust stabilisation and design of $\mathcal{H}_\infty$ control for NCSs where the plant has time-varying norm-bounded uncertainties and the network has access constants as well as experiences random packet dropouts in both the sensor-to-controller (S/C) connection and controller-to-actuator (C/A) connection. The periodic communication sequence scheme is adopted to solve the problem of limited access, while the characteristics of packet dropouts is treated as random switchings by a memoryless process. The controller employs a plant model to estimate the plant state but when a multiple-packet state transmission succeeds, the corresponding part of the model state is updated by the plant state information. Sufficient conditions are derived for synthesising robust stochastic stabilisation controller and for designing robust $\mathcal{H}_\infty$ controller. These conditions are formulated in the form of linear matrix inequalities (LMIs) that can be solved by the existing numerical techniques [24]. Throughout this contribution we adopt the following notational conventions. $\mathbb{R}$ stands for real numbers and $\mathbb{N}$ for nonnegative integers. $\mathbf{W} > 0$ indicates that $\mathbf{W}$ is a positive-definite matrix. $\mathbf{I}$ and $\mathbf{0}$ represent the identity and zero matrices of appropriate dimensions, respectively, while $\ell_2[0,\infty)$ defines the space of square summable vector functions over $[0,\infty)$. The notation $*$ within a matrix denotes symmetric entries.

II. Problem formulation

The NCS $\hat{P}_K$, depicted in Fig. 1, contains a generalised discrete-time plant $\hat{P}$ and a discrete-time controller $\hat{K}$ with the control loop closed via a shared communication network.

D.-X. Wu and J. Wu are with State Key Laboratory of Industrial Control Technology, Institute of Cyber-Systems and Control, Zhejiang University, Hangzhou 310027, China. E-mails: {dxwu,jwu}@iipc.zju.edu.cn

S. Chen is with School of Electronics and Computer Science, University of Southampton, Highfield, Southampton SO17 1BJ, U.K. E-mail: sqc@ecs.soton.ac.uk

This work is supported by the National Natural Science Foundation of China (Grants No.60774001, No.60736021 and No.60721062), the 973 program of China (Grant No.2009CB320603), the 863 program of China (Grant No.2008AA042602), the 111 Project of China (Grant No.B07031), and the United Kingdom Royal Society and Royal Academy of Engineering.

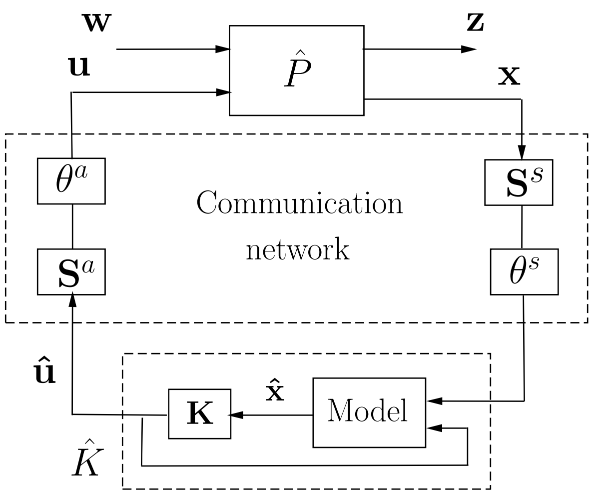

Fig. 1. Networked control system $\hat{P}_K$.

The plant $\hat{P}$ with parameter uncertainties is described by

$$\begin{cases} \mathbf{x}(k+1) = [\mathbf{A} + \mathbf{\Delta_A}(k)]\mathbf{x}(k) \\ \qquad\qquad +[\mathbf{B} + \mathbf{\Delta_B}(k)]\mathbf{u}(k) + \mathbf{B}_w\mathbf{w}(k), \quad \forall k \in \mathbb{N}, \\ \mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k), \end{cases} \tag{1}$$

where $\mathbf{x}(k) \triangleq [x_1(k) \ \cdots \ x_n(k)]^\mathsf{T} \in \mathbb{R}^n$, $\mathbf{u}(k) \triangleq [u_1(k) \ \cdots \ u_m(k)]^\mathsf{T} \in \mathbb{R}^m$ and $\mathbf{z}(k) \in \mathbb{R}^q$ are the state, input and controlled output vectors, respectively, $\mathbf{w}(k) \in \mathbb{R}^p$ is the disturbance input which belongs to $\ell_2[0,\infty)$, i.e., $\sum_{k=0}^{\infty} \mathbf{w}^\mathsf{T}(k)\mathbf{w}(k) < \infty$, $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_w$, $\mathbf{C}$ and $\mathbf{D}$ are the known constant matrices of appropriate dimensions, while $\mathbf{\Delta_A}(k)$ and $\mathbf{\Delta_B}(k)$ are the unknown matrices representing the time-varying parameter uncertainties which satisfy the following condition

$$[\mathbf{\Delta_A}(k) \ \mathbf{\Delta_B}(k)] = \mathbf{M} \ \mathbf{F}(k) \ [\mathbf{N}_a \ \mathbf{N}_b]. \tag{2}$$

Here $\mathbf{M}$, $\mathbf{N}_a$ and $\mathbf{N}_b$ are the known constant matrices of appropriate dimensions and $\mathbf{F}(k)$ is an unknown time-varying matrix with $\mathbf{F}^\mathsf{T}(k)\mathbf{F}(k) \leq \mathbf{I}$. The network communication is limited in the two ways: (i) due to access constraints it cannot offer simultaneous communication for all the sensors and actuators, and (ii) it imposes random packet dropouts.

The state of the plant is transmitted from the n sensors to the controller in the form of packets under a multiple-packet transmission policy where at any instant $k \in \mathbb{N}$, the state vector $\mathbf{x}(k)$ requires n packets for transmission, one packet for each element of $\mathbf{x}(k)$. However, due to the network access constraints, there are only s_σ available *output channels* in the S/C connection, where $1 \leq s_\sigma < n$. That is, only s_σ of the n state variables can be transmitted to the controller at instant k, while the other $n - s_\sigma$ state variables are ignored. Similarly, we assume that the m actuators share s_ρ *input channels* in the C/A connection to receive the control input vector from the controller, where $1 \leq s_\rho < m$. This specifies that only s_ρ of the m actuators can access the input channels simultaneously at instant k. Based on these assumptions on access constraints, we adopt the notations of a communication sequence [7] to describe the network access status of the plant state and input.

Definition 1: Let $\bar{M}, \bar{N} \in \mathbb{N}$ with $\bar{N} \leq \bar{M}$. An $\bar{M}$-to-$\bar{N}$ communication sequence is a map, $\bar{\boldsymbol{\sigma}}(k)$: $\mathbb{N} \mapsto \{0,1\}^{\bar{M}}$, satisfying $\|\bar{\boldsymbol{\sigma}}(k)\|^2 = \bar{N}$, $\forall k$.

Considering the S/C connection, let $\sigma_i(k)$, $1 \leq i \leq n$, denote the access status of the i-th state variable, $x_i(k)$, at k. If $\sigma_i(k) = 1$, the i-th state variable is transmitted to the controller; if $\sigma_i(k) = 0$, $x_i(k)$ is not transmitted to the controller. This n-to-s_σ communication sequence can be represented by

$$\boldsymbol{\sigma}(k) \triangleq [\sigma_1(k) \ \cdots \ \sigma_n(k)]^\mathsf{T}. \tag{3}$$

Similarly, the network access status of the plant's m inputs is represented by the m-to-s_ρ communication sequence

$$\boldsymbol{\rho}(k) \triangleq [\rho_1(k) \ \cdots \ \rho_m(k)]^\mathsf{T}. \tag{4}$$

Thus, the communication scheme in Fig. 1 is represented by the two memoryless system blocks $\mathbf{S}^s$ and $\mathbf{S}^a$ which are specified by the two diagonal matrices

$$\mathbf{S}_k^s \triangleq \mathrm{diag}\big[\sigma_1(k), \cdots, \sigma_n(k)\big], \tag{5}$$

$$\mathbf{S}_k^a \triangleq \mathrm{diag}\big[\rho_1(k), \cdots, \rho_m(k)\big], \tag{6}$$

respectively, for each $k \in \mathbb{N}$. To tackle the network access constraints, the *N-periodic sequence scheme* is employed which satisfies $\mathbf{S}_k^s = \mathbf{S}_{k+N}^s$ and $\mathbf{S}_k^a = \mathbf{S}_{k+N}^a$, $\forall k \in \mathbb{N}$, with a given period $N > 0$. Now define

$$\mathcal{N} \triangleq \{0, \cdots, N-1\},$$

$$\mathrm{mod}(k, N) \triangleq r \in \mathcal{N}, \ \forall k \in \mathbb{N}. \tag{7}$$

Then we have $\mathbf{S}_k^s = \mathbf{S}_r^s$ and $\mathbf{S}_k^a = \mathbf{S}_r^a$.

Network packet dropouts occur in both the S/C and C/A connections, which are represented by the system blocks θ^s and θ^a in Fig. 1. Let $\theta_k^s, \theta_k^a \in \{0,1\}$, $k \in \mathbb{N}$, be the indicators of the packet dropout in the S/C and C/A connections, respectively, where a value 0 indicates that the packet is dropped while a value 1 indicates that the packet is transmitted successfully. The protocol of the network is assumed to be TCP-like, in which there is acknowledgement (ACK) of received packets, i.e. at each instant k, the network sends an ACK signal to the controller to indicate whether the current control input is received or not by the actuator. Note that there is one step delay for the system block θ^a, since the controller receives θ_k^a at the time step $k+1$ rather than k. It is obvious that

$$(\theta_{k+1}^s, \theta_k^a) \in \hat{\mathcal{S}} \triangleq \{(0,0), (1,0), (0,1), (1,1)\}. \tag{8}$$

Next define the set $\mathcal{S} \triangleq \{1,2,3,4\}$ and the one to one mapping $f : \hat{\mathcal{S}} \to \mathcal{S}$ as

$$\theta_k = f(\theta_{k+1}^s, \theta_k^a) = \begin{cases} 1, & (\theta_{k+1}^s, \theta_k^a) = (0,0), \\ 2, & (\theta_{k+1}^s, \theta_k^a) = (1,0), \\ 3, & (\theta_{k+1}^s, \theta_k^a) = (0,1), \\ 4, & (\theta_{k+1}^s, \theta_k^a) = (1,1). \end{cases} \tag{9}$$

The network packet dropout process can be modelled by this *random switching* system, which is specified by the probability $p_i = \text{Prob}(\theta_k = i)$ for $i \in \mathcal{S}$.

The controller $\hat{K}$, similar to the one employed in [5], consists of the state feedback gain matrices $\mathbf{K}_r \in \mathbb{R}^{m\times n}$, $r \in \mathcal{N}$, and the plant model. Thus the controller output is given by

$$\hat{\mathbf{u}}(k) = \mathbf{K}_r\hat{\mathbf{x}}(k),\ \ r \in \mathcal{N}, \tag{10}$$

where $\hat{\mathbf{x}}(k) \in \mathbb{R}^n$ denotes the model state. Referring to Fig. 1, if the packet transmission is successfully in the C/A connection, $\mathbf{u}(k) = \mathbf{S}^a_k\hat{\mathbf{u}}(k)$. If the packet is lost, the actuator does nothing, i.e. $\mathbf{u}(k) = \mathbf{0}$. Thus we have

$$\mathbf{u}(k) = \bar{\mathbf{S}}^a_k\hat{\mathbf{u}}(k), \tag{11}$$

where

$$\bar{\mathbf{S}}^a_k \triangleq \theta^a_k\mathbf{S}^a_k. \tag{12}$$

Alternatively, if the packet is lost, the actuator may employ the previous control input, i.e., $\mathbf{u}(k) = \mathbf{u}(k-1)$. The analysis of this compensation scheme requires a different problem formulation and is not considered here. However, the both schemes are natural compensation methods for input packet dropout, and they are compared in [3], [8]. The plant model is given by

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k). \tag{13}$$

If the transmitted s_σ plant state variables, $\mathbf{S}^s_{k+1}\mathbf{x}(k+1)$, are dropped out, the model state vector $\hat{\mathbf{x}}(k+1)$ is updated purely by (13). Otherwise, if the transmission succeeds, the corresponding part of the model state, $\mathbf{S}^s_{k+1}\hat{\mathbf{x}}(k+1)$, is set to $\mathbf{S}^s_{k+1}\mathbf{x}(k+1)$, while the rest of the model states are still updated according to (13). Thus

$$\begin{aligned}&\hat{\mathbf{x}}(k+1)\\ &= \theta^s_{k+1}\mathbf{S}^s_{k+1}(\mathbf{A}+\boldsymbol{\Delta}_{\mathbf{A}}(k))\mathbf{x}(k) + \theta^s_{k+1}\mathbf{S}^s_{k+1}\mathbf{B}_w\mathbf{w}(k)\\ &\quad + \big(\bar{\mathbf{S}}^s_{k+1}\mathbf{A} + (\mathbf{B}+\theta^s_{k+1}\mathbf{S}^s_{k+1}\boldsymbol{\Delta}_{\mathbf{B}}(k))\bar{\mathbf{S}}^a_k\mathbf{K}_r\big)\hat{\mathbf{x}}(k)\\ &= \begin{cases}\mathbf{S}^s_{k+1}\mathbf{x}(k+1) + (\mathbf{I}-\mathbf{S}^s_{k+1}) & \\ \quad\times(\mathbf{A}\hat{\mathbf{x}}(k)+\mathbf{B}\mathbf{u}(k)), & \theta^s_{k+1}=1,\\ \mathbf{A}\hat{\mathbf{x}}(k)+\mathbf{B}\mathbf{u}(k), & \theta^s_{k+1}=0,\end{cases}\end{aligned} \tag{14}$$

where

$$\bar{\mathbf{S}}^s_{k+1} \triangleq \mathbf{I} - \theta^s_{k+1}\mathbf{S}^s_{k+1}. \tag{15}$$

Define $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$ and $\overline{\mathbf{x}}(k) \triangleq [\mathbf{x}^{\mathsf{T}}(k)\ \mathbf{e}^{\mathsf{T}}(k)]^{\mathsf{T}}$. The state-space equation for the NCS $\hat{P}_K$ is described by

$$\begin{bmatrix}\overline{\mathbf{x}}(k+1)\\ \mathbf{z}(k)\end{bmatrix} = \begin{bmatrix}\overline{\mathbf{A}}_{k,\theta_k} & \overline{\mathbf{B}}_{k,\theta_k}\\ \overline{\mathbf{C}}_{k,\theta_k} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\overline{\mathbf{x}}(k)\\ \mathbf{w}(k)\end{bmatrix}, \tag{16}$$

$\forall k \in \mathbb{N}$, $\theta_k \in \mathcal{S}$, where

$$\overline{\mathbf{A}}_{k,\theta_k} \triangleq \begin{bmatrix}\mathbf{A}+\boldsymbol{\Delta}_{\mathbf{A}}(k)+(\mathbf{B}+\boldsymbol{\Delta}_{\mathbf{B}}(k))\bar{\mathbf{S}}^a_k\mathbf{K}_r & -(\mathbf{B}+\boldsymbol{\Delta}_{\mathbf{B}}(k))\bar{\mathbf{S}}^a_k\mathbf{K}_r\\ \bar{\mathbf{S}}^s_{k+1}(\boldsymbol{\Delta}_{\mathbf{A}}(k)+\boldsymbol{\Delta}_{\mathbf{B}}(k)\bar{\mathbf{S}}^a_k\mathbf{K}_r) & \bar{\mathbf{S}}^s_{k+1}(\mathbf{A}-\boldsymbol{\Delta}_{\mathbf{B}}(k)\bar{\mathbf{S}}^a_k\mathbf{K}_r)\end{bmatrix}, \tag{17}$$

$$\overline{\mathbf{B}}_{k,\theta_k} \triangleq \begin{bmatrix}\mathbf{B}_w\\ \bar{\mathbf{S}}^s_{k+1}\mathbf{B}_w\end{bmatrix}, \tag{18}$$

$$\overline{\mathbf{C}}_{k,\theta_k} \triangleq \begin{bmatrix}\mathbf{C}+\mathbf{D}\bar{\mathbf{S}}^a_k\mathbf{K}_r & -\mathbf{D}\bar{\mathbf{S}}^a_k\mathbf{K}_r\end{bmatrix}. \tag{19}$$

Note that $\overline{\mathbf{A}}_{k,\theta_k}$ can be written as $\overline{\mathbf{A}}_{r,i} = \boldsymbol{\Phi}_{r,i} + \overline{\mathbf{M}}_{r+1}\overline{\mathbf{F}}(k)\boldsymbol{\Gamma}_{r,i}$, for $r \in \mathcal{N}$ and $i \in \mathcal{S}$, where

$$\boldsymbol{\Phi}_{r,i} = \begin{bmatrix}\mathbf{A}+\mathbf{B}\bar{\mathbf{S}}^a_r\mathbf{K}_r & -\mathbf{B}\bar{\mathbf{S}}^a_r\mathbf{K}_r\\ \mathbf{0} & \bar{\mathbf{S}}^s_{r+1}\mathbf{A}\end{bmatrix}, \tag{20}$$

$$\boldsymbol{\Gamma}_{r,i} = \begin{bmatrix}\mathbf{N}_a+\mathbf{N}_b\bar{\mathbf{S}}^a_r\mathbf{K}_r & -\mathbf{N}_b\bar{\mathbf{S}}^a_r\mathbf{K}_r\\ \mathbf{N}_a+\mathbf{N}_b\bar{\mathbf{S}}^a_r\mathbf{K}_r & -\mathbf{N}_b\bar{\mathbf{S}}^a_r\mathbf{K}_r\end{bmatrix}, \tag{21}$$

$$\overline{\mathbf{M}}_{r+1} = \text{diag}\big[\mathbf{M}, \bar{\mathbf{S}}^s_{r+1}\mathbf{M}\big], \tag{22}$$

$$\overline{\mathbf{F}}(k) = \text{diag}\big[\mathbf{F}(k), \mathbf{F}(k)\big]. \tag{23}$$

It is easy to see that $\overline{\mathbf{F}}^{\mathsf{T}}(k)\overline{\mathbf{F}}(k) \le \mathbf{I}$. Our objective is to establish criteria for synthesis of robust stochastic stabilisation control and to design appropriate robust $\mathcal{H}_\infty$ state feedback controllers that guarantee robust stochastic stability of the NCS $\hat{P}_K$.

III. Robust stabilisation

Definition 2: (See [15], [23]) The NCS $\hat{P}_K$ (16) with $\mathbf{w}(k) \equiv 0$ is said to be *robustly stochastically stable* if for any initial condition $\overline{\mathbf{x}}(0) \in \mathbb{R}^{2n}$,

$$\sum_{k=0}^{\infty} \mathsf{E}\left[\|\overline{\mathbf{x}}(k)\big|\overline{\mathbf{x}}(0)\|^2\right] < \infty \tag{24}$$

holds for all the admissible uncertainties $\boldsymbol{\Delta}_{\mathbf{A}}(k)$ and $\boldsymbol{\Delta}_{\mathbf{B}}(k)$, where $\mathsf{E}[\cdot]$ denotes the expectation.

The following lemma from [25] is useful for the proofs of our main results.

Lemma 1: Let $\mathbf{Z}$, $\mathbf{U}$, $\mathbf{H}$, $\mathbf{G}$ and $\tilde{\mathbf{F}}$ be the real matrices of appropriate dimensions such that $\mathbf{G} > 0$ and $\tilde{\mathbf{F}}^{\mathsf{T}}\tilde{\mathbf{F}} \le \mathbf{I}$. Then, for any scalar $\epsilon > 0$ such that $\mathbf{G} - \epsilon\mathbf{U}\mathbf{U}^{\mathsf{T}} > 0$, we have

$$\begin{aligned}&(\mathbf{Z}+\mathbf{U}\tilde{\mathbf{F}}\mathbf{H})^{\mathsf{T}}\mathbf{G}^{-1}(\mathbf{Z}+\mathbf{U}\tilde{\mathbf{F}}\mathbf{H})\\ &\qquad \le \mathbf{Z}^{\mathsf{T}}(\mathbf{G}-\epsilon\mathbf{U}\mathbf{U}^{\mathsf{T}})^{-1}\mathbf{Z} + \epsilon^{-1}\mathbf{H}^{\mathsf{T}}\mathbf{H}.\end{aligned}$$

Theorem 1: The NCS $\hat{P}_K$ (16) with $\mathbf{w}(k) \equiv 0$ is robustly stochastically stable if there exist scalars $\epsilon_i > 0$ for $i \in \mathcal{S}$, positive definite matrices $\mathbf{Q}_r > 0$ and $\mathbf{Y}_r$ for $r \in \mathcal{N}$ such that $\forall r \in \mathcal{N}$ the following LMIs are satisfied:

$$\begin{bmatrix}-\tilde{\mathbf{Q}}_r & \tilde{\boldsymbol{\Pi}}_{r,1} & \tilde{\boldsymbol{\Pi}}_{r,2} & \tilde{\boldsymbol{\Pi}}_{r,3} & \tilde{\boldsymbol{\Pi}}_{r,4}\\ * & \tilde{\boldsymbol{\Upsilon}}_{r+1,1} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ * & * & \tilde{\boldsymbol{\Upsilon}}_{r+1,2} & \mathbf{0} & \mathbf{0}\\ * & * & * & \tilde{\boldsymbol{\Upsilon}}_{r+1,3} & \mathbf{0}\\ * & * & * & * & \tilde{\boldsymbol{\Upsilon}}_{r+1,4}\end{bmatrix} \triangleq \boldsymbol{\Theta}_r < 0, \tag{25}$$

where r is defined in (7), $\overline{\mathbf{M}}_{r+1}$ and $\bar{\mathbf{S}}^s_{k+1}$ are given in (22) and (15), while

$$\tilde{\mathbf{Q}}_r = \text{diag}\big[\mathbf{Q}_r, \mathbf{Q}_r\big], \tag{26}$$

$$\tilde{\boldsymbol{\Upsilon}}_{r+1,i} = \text{diag}\Big[\epsilon_i\overline{\mathbf{M}}_{r+1}\overline{\mathbf{M}}^{\mathsf{T}}_{r+1} - \tilde{\mathbf{Q}}_{r+1},\ \ -\epsilon_i\mathbf{I}\Big], \tag{27}$$

$$\tilde{\boldsymbol{\Pi}}_{r,i} = \sqrt{p_i}\big[\tilde{\boldsymbol{\Phi}}^{\mathsf{T}}_{r,i}\ \ \tilde{\boldsymbol{\Gamma}}^{\mathsf{T}}_{r,i}\big], \tag{28}$$

$$\tilde{\boldsymbol{\Phi}}_{r,i} = \begin{bmatrix}\mathbf{A}\mathbf{Q}_r+\mathbf{B}\bar{\mathbf{S}}^a_r\mathbf{Y}_r & -\mathbf{B}\bar{\mathbf{S}}^a_r\mathbf{Y}_r\\ \mathbf{0} & \bar{\mathbf{S}}^s_{r+1}\mathbf{A}\mathbf{Q}_r\end{bmatrix}, \tag{29}$$

$$\tilde{\mathbf{\Gamma}}_{r,i} = \begin{bmatrix} \mathbf{N}_a\mathbf{Q}_r + \mathbf{N}_b\bar{\mathbf{S}}_r^a\mathbf{Y}_r & -\mathbf{N}_b\bar{\mathbf{S}}_r^a\mathbf{Y}_r \\ \mathbf{N}_a\mathbf{Q}_r + \mathbf{N}_b\bar{\mathbf{S}}_r^a\mathbf{Y}_r & -\mathbf{N}_b\bar{\mathbf{S}}_r^a\mathbf{Y}_r \end{bmatrix}, \quad (30)$$

with $i \in \mathcal{S}$. In this case, the state feedback gain matrices can be chosen as $\mathbf{K}_r = \mathbf{Y}_r\mathbf{Q}_r^{-1}$.

Proof Let $\mathbf{P}_r = \mathbf{Q}_r^{-1}$, then $\tilde{\mathbf{P}}_r = \tilde{\mathbf{Q}}_r^{-1}$. From (25), it is easy to show that

$$\mathbf{\Psi}_{r+1,i} \triangleq \tilde{\mathbf{P}}_{r+1}^{-1} - \epsilon_i\overline{\mathbf{M}}_{r+1}\overline{\mathbf{M}}_{r+1}^{\mathsf{T}} > 0, \ \forall i \in \mathcal{S}. \quad (31)$$

Now for the NCS $\hat{P}_K$, construct the Lyapunov function

$$V(k) \triangleq \overline{\mathbf{x}}^{\mathsf{T}}(k)\tilde{\mathbf{P}}_r\overline{\mathbf{x}}(k), \ \forall k \in \mathbb{N}. \quad (32)$$

Noticing $\epsilon_i > 0$, (22) and (31) as well as using Lemma 1, we have

$$\begin{aligned} &\mathsf{E}[V(k+1) - V(k)] \\ &= \overline{\mathbf{x}}^{\mathsf{T}}(k)\Big[\sum_{i\in\mathcal{S}} p_i\overline{\mathbf{A}}_{r,i}^{\mathsf{T}}\tilde{\mathbf{P}}_{r+1}\overline{\mathbf{A}}_{r,i} - \tilde{\mathbf{P}}_r\Big]\overline{\mathbf{x}}(k) \\ &= \overline{\mathbf{x}}^{\mathsf{T}}(k)\Big[\sum_{i\in\mathcal{S}} p_i(\mathbf{\Phi}_{r,i} + \overline{\mathbf{M}}_{r+1}\overline{\mathbf{F}}(k)\mathbf{\Gamma}_{r,i})^{\mathsf{T}}\tilde{\mathbf{P}}_{r+1} \\ &\quad \times (\mathbf{\Phi}_{r,i} + \overline{\mathbf{M}}_{r+1}\overline{\mathbf{F}}(k)\mathbf{\Gamma}_{r,i}) - \tilde{\mathbf{P}}_r\Big]\overline{\mathbf{x}}(k) \\ &\le \overline{\mathbf{x}}^{\mathsf{T}}(k)\hat{\mathbf{\Theta}}_r\overline{\mathbf{x}}(k), \end{aligned} \quad (33)$$

where

$$\hat{\mathbf{\Theta}}_r = \sum_{i\in\mathcal{S}} p_i\big(\mathbf{\Phi}_{r,i}^{\mathsf{T}}\mathbf{\Psi}_{r+1,i}^{-1}\mathbf{\Phi}_{r,i} + \epsilon_i^{-1}\mathbf{\Gamma}_{r,i}^{\mathsf{T}}\mathbf{\Gamma}_{r,i}\big) - \tilde{\mathbf{P}}_r. \quad (34)$$

On the other hand, pre- and post-multiplying (25) by $\mathrm{diag}\big[\tilde{\mathbf{P}}_r, \ \mathbf{I}\big]$ yields

$$\begin{bmatrix} -\tilde{\mathbf{P}}_r & \mathbf{\Pi}_{r,1} & \mathbf{\Pi}_{r,2} & \mathbf{\Pi}_{r,3} & \mathbf{\Pi}_{r,4} \\ * & \mathbf{\Upsilon}_{r+1,1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \mathbf{\Upsilon}_{r+1,2} & \mathbf{0} & \mathbf{0} \\ * & * & * & \mathbf{\Upsilon}_{r+1,3} & \mathbf{0} \\ * & * & * & * & \mathbf{\Upsilon}_{r+1,4} \end{bmatrix} < 0, \quad (35)$$

where

$$\begin{aligned} \mathbf{\Pi}_{r,i} &= \sqrt{p_i}[\mathbf{\Phi}_{r,i}^{\mathsf{T}} \ \ \mathbf{\Gamma}_{r,i}^{\mathsf{T}}], \ \forall i \in \mathcal{S}, \quad (36) \\ \mathbf{\Upsilon}_i &= \mathrm{diag}\big[-\mathbf{\Psi}_{r+1,i}, \ -\epsilon_i\mathbf{I}\big], \ \forall i \in \mathcal{S}, \quad (37) \end{aligned}$$

while $\mathbf{\Phi}_{r,i}$ and $\mathbf{\Gamma}_{r,i}$ are given in (20) and (21), respectively. By Schur complement, (35) implies that $\hat{\mathbf{\Theta}}_r < 0$. This together with (33) leads to

$$\begin{aligned} \mathsf{E}[V(k+1)] - V(k) &\le -\lambda_{\min}(-\hat{\mathbf{\Theta}}_r)\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k) \\ &\le -\tau\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k), \end{aligned} \quad (38)$$

where $\lambda_{\min}(-\hat{\mathbf{\Theta}}_r)$ denotes the minimal eigenvalue of $-\hat{\mathbf{\Theta}}_r$ and $\tau = \inf\{\lambda_{\min}(-\hat{\mathbf{\Theta}}_r), \, r \in \mathcal{N}\}$. From (38), we obtain

$$\begin{aligned} \mathsf{E}[V(T+1) - V(0)] &= \sum_{k=0}^{T}\Big(\mathsf{E}[V(k+1) - V(k)]\Big) \\ &\le -\tau\sum_{k=0}^{T}\mathsf{E}[\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k)] \end{aligned} \quad (39)$$

for any $T \ge 1$, which implies

$$\begin{aligned} \sum_{k=0}^{T}\mathsf{E}[\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k)] &\le \frac{1}{\tau}\big[\mathsf{E}[V(0)] - \mathsf{E}[V(T+1)]\big] \\ &\le \frac{1}{\tau}\mathsf{E}[V(0)]. \end{aligned} \quad (40)$$

Finally, from (40) we directly obtain

$$\sum_{k=0}^{\infty}\mathsf{E}[\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k)] \le \frac{1}{\tau}\mathsf{E}[V(0)] < \infty. \quad (41)$$

According to Definition 2, the NCS $\hat{P}_K$ is robustly stochastically stable. ■

IV. Robust $\mathcal{H}_\infty$ Control

Definition 3: (See [15], [23]) The NCS $\hat{P}_K$ (16) is said to be *robustly stochastically stable with disturbance attenuation level* $\gamma > 0$ if $\hat{P}_K$ is robustly stochastically stable and, for all nonzero $\mathbf{w}(k) \in \ell_2[0,\infty)$, the response $\{\mathbf{z}(k)\}$ under zero initial condition $\overline{\mathbf{x}}(0) = 0$ satisfies

$$\sum_{k=0}^{\infty}\mathsf{E}\Big[\mathbf{z}^{\mathsf{T}}(k)\mathbf{z}(k)\big|\overline{\mathbf{x}}(0) = 0\Big] < \gamma^2\Big[\sum_{k=0}^{\infty}\mathbf{w}^{\mathsf{T}}(k)\mathbf{w}(k)\Big]. \quad (42)$$

Theorem 2: Given a scalar $\gamma > 0$, the NCS $\hat{P}_K$ (16) is robustly stochastically stable with disturbance attenuation level γ, if there exist scalars $\epsilon_i > 0$ for $i \in \mathcal{S}$, positive definite matrices $\mathbf{Q}_r > 0$ and $\mathbf{Y}_r$ for $r \in \mathcal{N}$ such that $\forall r \in \mathcal{N}$ the following LMIs are satisfied:

$$\begin{bmatrix} \overline{\mathbf{Q}}_r & \mathbf{\Omega}_{r,1} & \mathbf{\Omega}_{r,2} & \mathbf{\Omega}_{r,3} & \mathbf{\Omega}_{r,4} \\ * & \mathbf{\Xi}_{r+1,1} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ * & * & \mathbf{\Xi}_{r+1,2} & \mathbf{0} & \mathbf{0} \\ * & * & * & \mathbf{\Xi}_{r+1,3} & \mathbf{0} \\ * & * & * & * & \mathbf{\Xi}_{r+1,4} \end{bmatrix} < 0, \quad (43)$$

where

$$\overline{\mathbf{Q}}_r = \mathrm{diag}\big[-\tilde{\mathbf{Q}}_r, \ -\gamma^2\mathbf{I}\big], \quad (44)$$

$$\mathbf{\Xi}_{r+1,i} = \mathrm{diag}\Big[\epsilon_i\overline{\mathbf{M}}_{r+1}\overline{\mathbf{M}}_{r+1}^{\mathsf{T}} - \tilde{\mathbf{Q}}_r, \ -\epsilon_i\mathbf{I}, \ -\mathbf{I}\Big], \quad (45)$$

$$\mathbf{\Omega}_{r,i} = \sqrt{p_i}\begin{bmatrix} \tilde{\mathbf{\Phi}}_{r,i}^{\mathsf{T}} & \tilde{\mathbf{\Gamma}}_{r,i}^{\mathsf{T}} & \tilde{\mathbf{C}}_{r,i}^{\mathsf{T}} \\ \overline{\mathbf{B}}_{r,i}^{\mathsf{T}} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (46)$$

$$\tilde{\mathbf{C}}_{r,i} = \begin{bmatrix} \mathbf{C}\mathbf{Q}_r + \mathbf{D}\theta_k^a\mathbf{S}_r^a\mathbf{Y}_r & -\mathbf{D}\theta_k^a\mathbf{S}_r^a\mathbf{Y}_r \end{bmatrix}, \quad (47)$$

with $i \in \mathcal{S}$, while $\tilde{\mathbf{Q}}_r$, $\tilde{\mathbf{\Phi}}_{r,i}$, $\tilde{\mathbf{\Gamma}}_{r,i}$, $\overline{\mathbf{B}}_{r,i}$ and $\overline{\mathbf{M}}_{r+1}$ are given in (26), (29), (30), (18) and (22), respectively. In this case, the state feedback gain matrices are given by $\mathbf{K}_r = \mathbf{Y}_r\mathbf{Q}_r^{-1}$.

Proof From (43), we can directly obtain

$$\begin{aligned} \mathbf{\Theta}_r &\le \mathbf{\Theta}_r + \sum_{i\in\mathcal{S}}\begin{bmatrix} \tilde{\mathbf{C}}_{r,i}^{\mathsf{T}} \\ \mathbf{0} \end{bmatrix}\begin{bmatrix} \tilde{\mathbf{C}}_{r,i} & \mathbf{0} \end{bmatrix} \\ &\quad + \frac{1}{\gamma^2}\begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{B}}_r \end{bmatrix}\begin{bmatrix} \mathbf{0} & \tilde{\mathbf{B}}_r^{\mathsf{T}} \end{bmatrix} < 0. \end{aligned} \quad (48)$$

where

$$\tilde{\mathbf{B}}_r^{\mathsf{T}} = \begin{bmatrix} \overline{\mathbf{B}}_{r,1}^{\mathsf{T}} & \mathbf{0} & \overline{\mathbf{B}}_{r,2}^{\mathsf{T}} & \mathbf{0} & \overline{\mathbf{B}}_{r,3}^{\mathsf{T}} & \mathbf{0} & \overline{\mathbf{B}}_{r,4}^{\mathsf{T}} & \mathbf{0} \end{bmatrix}, \quad (49)$$

while $\mathbf{\Theta}_r$ is defined in (25). Therefore, it follows from Theorem 1 that the NCS $\hat{P}_K$ with $\mathbf{w}(k) \equiv 0$ is robustly stochastically stable.

Next, we prove that $\hat{P}_K$ has the required noise attenuation level γ for all $\mathbf{w}(k) \in \ell_2[0, \infty)$. Let $\mathbf{P}_r = \mathbf{Q}_r^{-1}$, then $\tilde{\mathbf{P}}_r = \tilde{\mathbf{Q}}_r^{-1}$. Consider the Lyapunov function $V(k)$ as defined in (32) with the zero initial condition $\overline{\mathbf{x}}(0) = 0$ and $V(0) = 0$. It follows from (39) that for any $T \geq 1$

$$\sum_{k=0}^{T} \Big(\mathsf{E}[V(k+1)] - V(k)\Big) = \mathsf{E}[V(T+1)] \geq 0. \qquad (50)$$

Since $\epsilon_i > 0$ for $i \in \mathcal{S}$ and (31) is satisfied due to (43), according to Lemma 1 we have

$$\begin{aligned} &\mathsf{E}[V(k+1)] \\ &= \big[\overline{\mathbf{x}}^\mathsf{T}(k) \;\; \mathbf{w}^\mathsf{T}(k)\big] \mathbf{\Lambda}_r \big[\overline{\mathbf{x}}^\mathsf{T}(k) \;\; \mathbf{w}^\mathsf{T}(k)\big]^\mathsf{T} \\ &\leq \big[\overline{\mathbf{x}}^\mathsf{T}(k) \;\; \mathbf{w}^\mathsf{T}(k)\big] \tilde{\mathbf{\Lambda}}_r \big[\overline{\mathbf{x}}^\mathsf{T}(k) \;\; \mathbf{w}^\mathsf{T}(k)\big]^\mathsf{T}, \qquad (51) \end{aligned}$$

where

$$\begin{aligned} \mathbf{\Lambda}_r &= \sum_{i\in\mathcal{S}} p_i \big[\overline{\mathbf{A}}_{r,i} \;\; \overline{\mathbf{B}}_{r,i}\big]^\mathsf{T} \tilde{\mathbf{P}}_{r+1} \big[\overline{\mathbf{A}}_{r,i} \;\; \overline{\mathbf{B}}_{r,i}\big] \\ &= \sum_{i\in\mathcal{S}} p_i \left(\big[\mathbf{\Phi}_{r,i} \;\; \overline{\mathbf{B}}_{r,i}\big] + \overline{\mathbf{M}}_{r+1} \overline{\mathbf{F}}(k) \big[\mathbf{\Gamma}_{r,i} \;\; \mathbf{0}\big]\right)^\mathsf{T} \tilde{\mathbf{P}}_{r+1} \\ &\quad \times \left(\big[\mathbf{\Phi}_{r,i} \;\; \overline{\mathbf{B}}_{r,i}\big] + \overline{\mathbf{M}}_{r+1} \overline{\mathbf{F}}(k) \big[\mathbf{\Gamma}_{r,i} \;\; \mathbf{0}\big]\right), \qquad (52) \end{aligned}$$

$$\begin{aligned} \tilde{\mathbf{\Lambda}}_r = \sum_{i\in\mathcal{S}} p_i \Bigg(&\begin{bmatrix} \mathbf{\Phi}_{r,i}^\mathsf{T} \\ \overline{\mathbf{B}}_{r,i}^\mathsf{T} \end{bmatrix} \mathbf{\Psi}_{r+1,i}^{-1} \begin{bmatrix} \mathbf{\Phi}_{r,i} & \overline{\mathbf{B}}_{r,i} \end{bmatrix} \\ &+ \epsilon_i^{-1} \begin{bmatrix} \mathbf{\Gamma}_{r,i}^\mathsf{T} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{\Gamma}_{r,i} & \mathbf{0} \end{bmatrix} \Bigg). \qquad (53) \end{aligned}$$

Combining (16) and (51) yields

$$\begin{aligned} &\mathsf{E}\left[V(k+1)\right] - V(k) + \mathbf{z}^\mathsf{T}(k)\mathbf{z}(k) - \gamma^2 \mathbf{w}^\mathsf{T}(k)\mathbf{w}(k) \\ &\leq \big[\overline{\mathbf{x}}^\mathsf{T}(k) \; \mathbf{w}^\mathsf{T}(k)\big] \tilde{\mathbf{\Theta}}_r \big[\overline{\mathbf{x}}^\mathsf{T}(k) \; \mathbf{w}^\mathsf{T}(k)\big]^\mathsf{T}, \qquad (54) \end{aligned}$$

where

$$\begin{aligned} \tilde{\mathbf{\Theta}}_r &= \sum_{i\in\mathcal{S}} p_i \left(\tilde{\mathbf{\Lambda}}_r + \begin{bmatrix} \overline{\mathbf{C}}_{r,i}^\mathsf{T} \\ \mathbf{0} \end{bmatrix} \begin{bmatrix} \overline{\mathbf{C}}_{r,i} & \mathbf{0} \end{bmatrix} \right) \\ &\quad - \begin{bmatrix} \tilde{\mathbf{P}}_r & \mathbf{0} \\ \mathbf{0} & \gamma^2 \mathbf{I} \end{bmatrix} \\ &= \sum_{i\in\mathcal{S}} p_i \begin{bmatrix} \mathbf{\Phi}_{r,i} & \overline{\mathbf{B}}_{r,i} \\ \mathbf{\Gamma}_{r,i} & \mathbf{0} \\ \overline{\mathbf{C}}_{r,i} & \mathbf{0} \end{bmatrix}^\mathsf{T} \begin{bmatrix} \mathbf{\Psi}_{r+1,i}^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \epsilon_i^{-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \\ &\quad \times \begin{bmatrix} \mathbf{\Phi}_{r,i} & \overline{\mathbf{B}}_{r,i} \\ \mathbf{\Gamma}_{r,i} & \mathbf{0} \\ \overline{\mathbf{C}}_{r,i} & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{P}}_r & \mathbf{0} \\ \mathbf{0} & \gamma^2 \mathbf{I} \end{bmatrix}, \qquad (55) \end{aligned}$$

and $\mathbf{\Psi}_{r+1,i}$ is defined in (31). Pre- and post-multiplying (43) by diag$\big[\tilde{\mathbf{P}}_r, \; \mathbf{I}\big]$ as well as applying Schur complement yields

$$\tilde{\Theta}_r < 0. \qquad (56)$$

Let us define the performance function

$$J(T) = \sum_{k=0}^{T} \mathsf{E}\Big[\mathbf{z}^\mathsf{T}(k)\mathbf{z}(k) - \gamma^2 \mathbf{w}^\mathsf{T}(k)\mathbf{w}(k)\Big]. \qquad (57)$$

Then from (50), (54) and (57), we derive

$$\begin{aligned} J(T) &= \sum_{k=0}^{T} \mathsf{E}\Big[\big(\mathbf{z}^\mathsf{T}(k)\mathbf{z}(k) - \gamma^2 \mathbf{w}^\mathsf{T}(k)\mathbf{w}(k) \\ &\quad + V(k+1) - V(k)\big) - \big(V(k+1) - V(k)\big)\Big] \\ &\leq \sum_{k=0}^{T} \mathsf{E}\Big[\big[\overline{\mathbf{x}}^\mathsf{T}(k) \; \mathbf{w}^\mathsf{T}(k)\big] \tilde{\Theta}_r \big[\overline{\mathbf{x}}^\mathsf{T}(k) \; \mathbf{w}^\mathsf{T}(k)\big]^\mathsf{T}\Big] \\ &\quad - \mathsf{E}[V(T+1)]. \qquad (58) \end{aligned}$$

For all the $\mathbf{w}(k) \neq \mathbf{0}$, (56) and (58) yields

$$J(\infty) < 0. \qquad (59)$$

This completes the proof of Theorem 2. ■

V. A Numerical Example

To illustrate the effectiveness of the proposed approach, we considered the following uncertain NCS $\hat{P}_K$ of $\mathbf{x}(k) \in \mathbb{R}^3$, $\mathbf{u}(k) \in \mathbb{R}^2$, $\mathbf{z}(k) \in \mathbb{R}$ and $\mathbf{w}(k) \in \mathbb{R}$, with the following parameters

$$\mathbf{A} = \begin{bmatrix} -0.2 & 0 & 0.9 \\ 0.6 & -0.9 & 0.5 \\ 0.2 & -1 & 0 \end{bmatrix}, \; \mathbf{B} = \begin{bmatrix} 0.2 & 0.4 \\ 0.9 & 0.8 \\ 0.3 & 0.7 \end{bmatrix},$$

$$\mathbf{B}_w = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.2 \end{bmatrix}, \; \mathbf{M} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.2 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0.2 & 0.3 & 0.3 \end{bmatrix}, \; \mathbf{D} = \begin{bmatrix} 0.7 & 0.9 \end{bmatrix},$$

$$\mathbf{N}_a = \begin{bmatrix} 0.6 & 0.2 & 0.7 \end{bmatrix}, \; \mathbf{N}_b = \begin{bmatrix} 0.5 & 0.8 \end{bmatrix}.$$

The eigenvalues of the plant were -1.0554 and $-0.0233 \pm 0.6726\, i$. The plant was controlled via a shared communication network which had two output channels and one input channel, i.e., $s_\sigma = 2$ and $s_\rho = 1$. We fixed the period of the N-periodic sequence scheme to be $N = 3$. The deterministic *3-periodic* sequences were chosen as

$$\begin{aligned} \{\boldsymbol{\sigma}(1), \boldsymbol{\sigma}(2), \ldots\} &= \{[1,1,0]^\mathsf{T}, [1,0,1]^\mathsf{T}, [0,1,1]^\mathsf{T}, \ldots\}, \\ \{\boldsymbol{\rho}(0), \boldsymbol{\rho}(1), \ldots\} &= \{[1,0]^\mathsf{T}, [0,1]^\mathsf{T}, [0,1]^\mathsf{T}, \ldots\}. \end{aligned}$$

The probabilities of random switchings were $p_1 = 0.04$, $p_2 = p_3 = 0.16$ and $p_4 = 0.64$.

Our objective was to design the state feedback gain matrices $\mathbf{K}_1$, $\mathbf{K}_2$ and $\mathbf{K}_3$ such that, for all the admissible uncertainties, the NCS $\hat{P}_K$ was robustly stochastically stable with the specified disturbance attenuation level γ. Assuming $\gamma = 0.46$, we applied the Matlab LMI Control Toolbox to solve the LMIs (43) and obtained the following solution

$$\mathbf{Q}_1 = \begin{bmatrix} 1.3214 & 0.4491 & -0.1830 \\ 0.4491 & 0.5001 & 0.1157 \\ -0.1830 & 0.1157 & 1.0484 \end{bmatrix},$$

$$\mathbf{Q}_2 = \begin{bmatrix} 1.4708 & 0.5318 & -0.3939 \\ 0.5318 & 0.5096 & -0.0003 \\ -0.3939 & -0.0003 & 0.9746 \end{bmatrix},$$

$$\mathbf{Q}_3 = \begin{bmatrix} 1.4028 & 0.5284 & -0.2029 \\ 0.8284 & 0.6222 & 0.0020 \\ -0.2029 & 0.0020 & 0.8800 \end{bmatrix},$$

$$\mathbf{Y}_1 = \begin{bmatrix} -0.3317 & -0.0119 & -0.0156 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{Y}_2 = \begin{bmatrix} 0 & 0 & 0 \\ -0.0376 & 0.0255 & 0.0902 \end{bmatrix},$$

$$\mathbf{Y}_3 = \begin{bmatrix} 0 & 0 & 0 \\ -0.3496 & 0.0529 & -0.0798 \end{bmatrix},$$

$$\epsilon_1 = 4.0589,\ \epsilon_2 = 3.3769,\ \epsilon_3 = 4.3402,\ \epsilon_4 = 4.5363.$$

It followed from Theorem 2 that the robust $\mathcal{H}_\infty$ control problem was solvable with the state feedback gain matrices given by

$$\mathbf{K}_1 = \mathbf{Y}_1\mathbf{Q}_1^{-1} = \begin{bmatrix} -0.3876 & 0.3523 & -0.1214 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{K}_2 = \mathbf{Y}_2\mathbf{Q}_2^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ -0.0366 & 0.0884 & 0.0778 \end{bmatrix},$$

$$\mathbf{K}_3 = \mathbf{Y}_3\mathbf{Q}_3^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ -0.4557 & 0.4726 & -0.1969 \end{bmatrix}.$$

VI. CONCLUSIONS

We have studied synthesis of robust stabilisation control and design of $\mathcal{H}_\infty$ control for NCSs where the plant has time-varying norm-bounded uncertainties and the nextwork has limited access as well as imposes random packet dropouts in both the S/C and C/A connections. The N-periodic communication sequence scheme has been adopted to deal with access constraints, and the approach of random switchings has been utilised to model the packet dropout process. A smart controller has been employed, which estimates the plant state purely by a plant model when the state-information transmission fails but updates part of the state estimate using the available plant state information when the multiple-packet state transmission succeeds. Sufficient conditions have been derived in the form of solvable LMIs for synthesising robust stochastic stabilisation controller and for designing robust $\mathcal{H}_\infty$ controller. A numerical example has been included to illustrate our design method.

REFERENCES

[1] J.R. Moyne and D.M. Tilbury, "The emergence of industrial control networks for manufacturing control, diagnostics, and safety data," *Proc. IEEE*, vol.95, no.1, pp.29–47, 2007.

[2] J.P. Hespanha, P. Naghshtabrizi and Y.-G. Xu, "A survey of recent results in networked control systems," *Proc. IEEE*, vol.95, no.1, pp.138–162, 2007.

[3] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla and S.S. Sastry, "Foundations of control and estimation over lossy networks," *Proc. IEEE*, vol.95, no.1, pp.163–187, 2007.

[4] M. Tabbara and D. Nešić, "Input-output stability of networked control systems with stochastic protocols and channels," *IEEE Trans. Automatic Control*, vol.53, no.5, pp. 1160–1175, June 2008.

[5] L. Montestruque and P. Antsaklis, "Stability of model-based networked control systems with time-varying transmission times," *IEEE Trans. Automatic Control*, vol.49, no.9, pp.1562–1572, September 2004.

[6] R.W. Brockett, "Stabilization of motor networks," *Proc. IEEE Conf. Decision and Control*, Dec.13-15, 1995, vol.2, pp.1484–1488.

[7] D. Hristu-Varsakelis and L. Zhang, "LQG control of network control systems with access constraints and delays," *Int. J. Control*, vol.81, No.8, pp.1266–1280, 2008.

[8] D. Hristu-Varsakelis, "Short-period communication and the role of zero-order holding in networked control systems," *IEEE Trans. Automatic Control*, vol.53, no.5, pp.1285–1290, June 2008.

[9] H. Ishii, "$\mathcal{H}^\infty$ control with limited communication and message losses," *Systems and Control Letters*, vol.57, no.4, pp.322–331, 2008.

[10] W. Zhang, M.S. Branicky and S.M. Phillips, "Stability of networked control systems," *IEEE Control System Magazine*, vol.21, no.1, pp.84–99, February 2001.

[11] J. Wu and T. Chen, "Design of networked control systems with packet dropouts," *IEEE Trans. Automatic Control*, vol.52, no.7, pp.1314–1319, July 2007.

[12] S. Hu and W.-Y. Yan "Stability of networked control systems under a multiple-packet transmission policy," *IEEE Trans. Automatic Control*, vol.53, no.7, pp.1706–1711, August 2008.

[13] Y. Ji, H.J. Chizeck, X. Feng and K.A. Loparo, "Stability and control of discrete-time jump linear systems," *Control Theory Adv. Technol.*, vol.7, no.2, pp.247–270, 1991.

[14] O.L.V. Costa and M.D. Fragoso, "Stability results for discrete-time linear systems with Markovian jumping parameters," *J. Math. Analysis and Appl.*, vol.179, no.1, pp.154–178, 1993.

[15] E.K. Boukas and Z.K. Liu, "Robust $\mathcal{H}_\infty$ control of discrete-time Markovian jump linear systems with mode-dependent time-delays," *IEEE Trans. Automatic Control*, vol.46, no.12, pp.1918–1924, December 2001.

[16] P. Seiler and R. Sengupta, "An $\mathcal{H}_\infty$ approach to networked control," *IEEE Trans. Automatic Control*, vol.50, no.3, pp.356–364, March 2005.

[17] V. Gupta, B. Hassibi and R.M. Murray, "Optimal LQG control across packet-dropping links," *Systems and Control Letters*, vol.56, no.6, pp.439–446, 2007.

[18] M. Yu, L. Wang, T. Chu and G. Xie, "Stabilization of networked control systems with data packet dropout and network delays via switching system approach," *Proc. IEEE Conf. Decision and Control*, Dec.14-17, 2004, vol.4, pp.3539–3544.

[19] W. Zhang and L. Yu, "Output feedback stabilization of networked control systems with packet dropouts," *IEEE Trans. Automatic Control*, vol.52, no.9, pp.1705–1710, September 2007.

[20] K. Zhou, J.C. Doyle and K. Glover, *Robust and Optimal Control*. Prentice-Hall: Englewood Cliffs, NJ, 1995.

[21] S. Xu and T. Chen, "Robust $\mathcal{H}_\infty$ control for uncertain stochastic systems with state delay," *IEEE Trans. Automatic Control*, vol.47, no.12, pp.2089–2094, December 2002.

[22] D. Yue, Q.-L. Han and J. Lam "Network-based robust $\mathcal{H}_\infty$ control of systems with uncertainty," *Automatica*, vo.41, no.6, pp.999–1007, 2005.

[23] J. Wu, T. Chen and L. Wang, "Delay-dependent robust stability and $\mathcal{H}_\infty$ control for jump linear systems with delay," *Systems and Control Letters*, vol.55, no.11, pp.939–948, 2006.

[24] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishan, *Linear Matrix Inequalities in Systems and Control Theory*. SIAM: Philadelphia, PA, 1994.

[25] Y. Wang, L. Xie, and C. E. De Souza, "Robust control of a class of uncertain nonlinear systems," *Systems and Control Letters*, vol.19, no.2, pp.139–149, 1992.