# Joint Channel Impulse Response and Noise-Variance Estimation for OFDM/SDMA Systems Based on Expectation Maximization

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***Abstract*—A joint channel impulse response (CIR) and noise-variance estimation scheme is proposed for multi-user Multiple-Input-Multiple-Output (MIMO) Orthogonal Frequency-Division Multiplexing / Space Division Multiple Access (OFDM/SDMA) systems, which is based on the Expectation Maximization (EM) algorithm. Multiple users communicating over time-invariant and/or time-variant channels are considered in this paper. Channel estimation becomes quite challenging in this scenario, since an increased number of independent transmitter-receiver links having different statistical characteristics have to be simultaneously estimated for each subcarrier. The proposed EM-based joint CIR and noise-variance estimator designed for multi-user MIMO OFDM/SDMA system is shown to simultaneously estimate the time-invariant and time-variant CIRs as well as noise-variance.**

## I. Introduction

Orthogonal Frequency-Division Multiplexing (OFDM) [1], constitute a promising technique of combating the detrimental effects of multipath induced delay spread in high-data-rate transmission. In recent years various smart antenna designs have attracted substantial research interests, because they are capable of mitigating the deleterious effects of multipath fading on the desired signal and of suppressing the interfering signals, thereby increasing the achievable performance of wireless systems [2]. Specifically, smart antenna-assisted Space Division Multiple Access (SDMA) is capable of achieving a high spectral efficiency by supporting a multiplicity of users within the same frequency band and facilitating the separation of their signals based on their unique, user-specific channels.

As a beneficial combination, OFDM/SDMA systems have attracted substantial interests [1–4]. Typical OFDM/SDMA systems employ an array of antennas at the base station, which detects the received signal of multiple single-antenna aided user terminals. As a result, a substantially improved system capacity is achieved, despite employing low-complexity user terminals [2, 5]. However, the performance of these systems is critically dependent on the precision of the channel knowledge. Furthermore, while exploiting the joint benefits of OFDM and SDMA, their combination faces new challenges, because a significantly increased number of independent transmitter-receiver channel links have to be estimated simultaneously for each subcarrier, while the interfering signals of the other transmitters have to be suppressed [3].

Acknowledgments: The financial support of the China Scholarship Council (CSC) and the EU under the auspices of the OPTIMIX project as well as of the EPSRC, UK is gratefully acknowledged.

Over the past decade, intensive research efforts have been devoted to developing effective approaches for channel estimation or symbol detection for transmitter- and/or the receiver-diversity aided systems. The powerful expectation maximization (EM) technique employs probabilistic information about the data symbols in order to assist in channel estimation, which has been shown to strike an attractive trade-off between the performance attained and the complexity imposed. Two EM algorithms, the classic EM and space-alternating generalized EM (SAGE), were compared in terms of their convergence rates in [6]. To improve the attainable receiver performance, the authors of [7] introduced two unbiased EM-type algorithms, namely the unbiased EM (UEM) and the unbiased expectation-conditional-maxization (UECM) algorithms, both of which relied on viewing the EM channel impulse response (CIR) estimator as least suqares (LS) estimator. As a further advance, the authors of [8] derived an EM-based maximum a posteriori (MAP) channel estimation method which avoided the inversion of large matrices, hence the receiver's complexity had been reduced.

The channel estimation techniques found in the open literature were typically developed under the assumption that all the channels are statistically similar to each other, either time-invariant or time-variant. However, in practice the mobility of the different user terminals is independent of each other, hence naturally, they may encounter different channels. A practical scenario is that some of the users may be motionless, while other users may be moving. **Hence we solve the open problem of jointly estimating the channels of both stationary and roaming OFDM/SDMA users having time-invariant and time-variant channels, respectively.**

The rest of this paper is organized as follows. The classic system model of multi-user MIMO OFDM/SDMA is described in Section II. The proposed EM-based joint CIR and noise-variance estimation scheme proposed for a multi-user MIMO OFDM/SDMA system is elaborated on Section III. Our simulation results and discussions are presented in Section IV, and our conclusions are offered in Section V.

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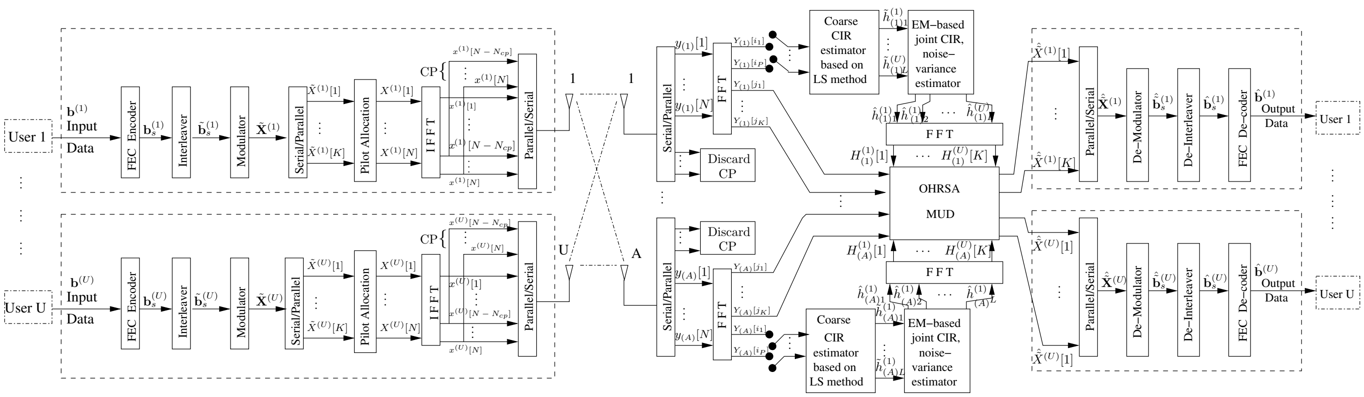


Fig. 1. Uplink system model for Multi-user MIMO SDMA/OFDM

## II. System Model

The OFDM/SDMA system considered supports $U$ users simultaneously transmitting in the uplink (UL) to the base station (BS) as seen in Fig. 1. Each of the users has a single antenna, while the BS has an array of $A$ antennas. It is assumed that a Time Division Multiple Access (TDMA) protocol manages the division of the available time-domain (TD) resources into OFDM/SDMA time slots (TS) [2, 5].

More specially, all of the $U$ users transmit independent data streams, which are denoted by $\mathbf{b}^{(u)}, u = 1, 2, \cdots, U$. The information block $\mathbf{b}^{(u)}$ is encoded by a user-specific FEC encoder and interleaved by the interleaver as portraited in Fig. 1. The information bits output by the interleaver are grouped and mapped to a stream of modulated data symbols, each forming a complex number. The modulated data $\mathbf{X}^{(u)}[k], k = 1, 2, \cdots, K$ of Fig. 1 are then serial to parallel (S/P) converted and the frequency-domain (FD) pilots are embedded into certain subcarriers. The parallel modulated data (including the pilots) are further processed by the inverse fast fourier transform (IFFT) to form a set of OFDM symbols. The baseband TD model of the $n$-th sample of the $m$-th OFDM symbol of user $u$ can be formulated as

$$x^{(u)}[m, n] = \frac{1}{N} \sum_{k=1}^{N} X^{(u)}[m, k] e^{j2\pi knT_s/T}, \tag{1}$$

where $T_s$ is the OFDM sampling interval and $T = NT_s$ is the time duration of an OFDM symbol without the CP. After concatenating the cyclic prefix (CP) of $N_{cp}$ samples, the TD signal is transmitted through a multipath fading channel and contaminated by the receiver's additive white Gaussian noise (AWGN).

At the BS, the CP is discarded from every OFDM symbol and the resultant signal is fed into the corresponding fast Fourier transform (FFT) based receiver of Fig. 1. Let $Y_{(a)}[s, n]$ denote the signal received by the $a$-th receiver antenna element in the $n$-th subcarrier of the $s$-th OFDM symbol, which is given as the superposition of the different users' channel-impaired received signal contributions plus the AWGN, which is expressed as [2]:

$$Y_{(a)}[s, n] = \sum_{u=1}^{U} H_{(a)}^{(u)}[s, n] X^{(u)}[s, n] + W_{(a)}[s, n], \tag{2}$$

where $H_{(a)}^{(u)}[s, n]$ denotes the frequency domain channel transfer factors (FD-CHTFs) of the link between the $u$-th user and the $a$-th receiver antenna in the $n$-th subcarrier of the $s$-th OFDM symbol, which can be expressed as

$$H_{(a)}^{(u)}[s, n] = \sum_{l=1}^{L_{ua}} h_{(a)}^{(u)}[s, l] F_N^{nl}, \tag{3}$$

where $h_{(a)}^{(u)}[s, l] = h_{(a)}^{(u)}[sT_f, n(T_s/N)]$ and $F_N = exp[-j(2\pi/N)]$. In the above expression $T_f$ is the block length, $T_f = T_s + T_g$, with $T_g$ being the duration of the CP.

Upon invoking vector notations, the set of equations constituted by Equation (2) for $n = 1, 2, \cdots, N$ can be rewritten as:

$$\mathbf{Y}_{(a)}[s] = \mathbf{X}^T[s] \mathbf{H}_{(a)}[s] + \mathbf{W}_{(a)}[s], \tag{4}$$

where the superscript $^T$ of $[\cdot]^T$ denotes the transpose, while $\mathbf{Y}_{(a)}[s] \in \mathbb{C}^{N \times 1}$, $\mathbf{W}_{(a)}[s] \in \mathbb{C}^{N \times 1}$, $\mathbf{X}[s] \in \mathbb{C}^{UN \times N}$ and $\mathbf{H}_{(a)}[s] \in \mathbb{C}^{UN \times 1}$, are column vectors hosting the subcarrier-related variables $Y_{(a)}[s, n], H_{(a)}^{(u)}[s, n]$ and $W_{(a)}[s, n]$, respectively.

To simplify our notation without any loss of generality, we will omit the receiver antenna's index $a$ from now on, and the discrete-model of the received signal associated with one of the BS antennas can be rewritten as:

$$\mathbf{Y}[s] = \mathbf{X}^T[s] \mathbf{H}[s] + \mathbf{W}[s]. \tag{5}$$

## III. EM-Based Channel Estimation for Multi-user MIMO OFDM/SDMA Systems

The EM algorithm [9] constitutes an iterative technique of finding the ML estimates of parameters that is particularly attractive, when direct access to the data necessary to make an estimate is unavailable, or when some of the data are missing. The EM algorithm can be broken down into two primary steps as the followings:

*Expectation-Step (***E-Step***)*: Determine the average log-likelihood function of the complete data as follows:

$$Q(\mathcal{B}|\mathcal{B}^{(p)}) = E\left\{\log f(\mathcal{X}|\mathcal{B})|\mathcal{Y}, \mathcal{B}^{(p)}\right\}, \tag{6}$$

*Maximization-Step (***M-Step***)*: Maximize the average loglihood function of the complete data over all possible values of $\mathcal{B}$, which is formulated as:

$$\hat{\mathcal{B}}^{(p+1)} = \arg\max_{\mathcal{B}} \; Q(\mathcal{B}|\mathcal{B}^{(p)}). \tag{7}$$

where $\mathcal{X}$ denote the "complete" data which can be separate into two components, $\mathcal{X} = (\mathcal{Y}, \mathcal{Z})$. The observation vector $\mathcal{Y}$ is referred to as the "incomplete" data within the EM framework and $\mathcal{Z}$ is called as "missing" data.

Although the following derivation of the proposed scheme is based on a simple multi-user MIMO SDMA/OFDM system supporting two users, our technique may be directly extended to the general scenario of a multi-user MIMO SDMA/OFDM system supporting $U$ users with the aid of $A$ antennas at the BS ($U > 2, A > 2$). Assuming that the channel of one of the users is static over the period of $S$ consecutive OFDM symbols while that of the other user is time-variant from symbol to symbol, but remains unchanged within an OFDM symbol. Considering that there are two users, we can rewrite Equation (5) as:

$$\bar{\mathbf{Y}}[s] = \bar{\mathbf{X}}^{(1)}[s]\bar{\mathbf{F}}^{(1)}\mathbf{h}^{(1)}[s] + \bar{\mathbf{X}}^{(2)}[s]\bar{\mathbf{F}}^{(2)}\mathbf{h}^{(2)}[s] + \bar{\mathbf{W}}[s]. \quad (8)$$

Here we only consider the pilot tones, namely,

$$\bar{\mathbf{Y}}[s] = \Big[Y[s,i_1], Y[s,i_2], \cdots, Y[s,i_P]\Big]^T, \quad (9)$$

$$\bar{\mathbf{F}}^{(u)} = \Big[\mathbf{F}^{(u)T}[i_1,:], \cdots, \mathbf{F}^{(u)T}[i_P,:]\Big]^T, \quad (10)$$

$$\bar{\mathbf{X}}^{(u)}[s] = diag\Big\{X^{(u)}[s,i_1], \cdots, X^{(u)}[s,i_P]\Big\}, \quad (11)$$

$$\bar{\mathbf{W}}[s] = \Big[W[s,i_1], W[s,i_2], \cdots, W[s,i_P]\Big]^T, \quad (12)$$

where $\mathbf{F}^{(u)}[i_p,:]$ denotes the $i_p$th row of the unitary discrete Fourier Transform (DFT) matrix $\mathbf{F}^{(u)}$ [2], and $\{i_1, i_2, \cdots, i_P\}$ are the pilot tone positions, while $P$ is the number of pilot tones.

Let us assume in Equation (8) that the FD-CHTF of user-1 is time-invariant, while that of user-2 is time-variant, both of which are determined by the initial channel estimation with the aid of the method in [10]. Since the CIR taps are unkonwn, we treat $\mathbf{h}^{(1)}[s]$ as a $(L_0{\times}1)$ vector of fixed elements to be estimated over the period of $S$ OFDM symbols, where $L_0$ is the numbers of CIR taps. Again, we have $\mathbf{h}^{(1)}[s] = \mathbf{h}^{(1)}$ for $s = 1, 2, \cdots, S$, and $\mathbf{h}^{(2)}[s]$ is treated as a $(L_0{\times}1)$-element vector of random values, whose distribution obeys $N(0, \mathbf{\Omega})$. The number of CIR taps $L_0$ is assumed to be higher than the actual number of taps, regardless of the user index. We emphasize that $\mathbf{h}^{(2)}[s]$ is independent of the noise vector $\bar{\mathbf{W}}[s]$, which has a distribution of $N(0, \sigma_n^2\mathbf{I}_P)$. Here, $\sigma_n^2$ is the noise-variance to be estimated and $\mathbf{I}_P$ is the $(P \times P)$-element identity matrix having values of unity on the main diagonal and zeros elsewhere.

Following the terminology of the EM algorithm [9], we view $\mathcal{Y} = \left[\bar{\mathbf{Y}}^T[1], \bar{\mathbf{Y}}^T[2], \cdots, \bar{\mathbf{Y}}^T[S]\right]^T$ as "incomplete" data, $\mathcal{X} = \left[\mathcal{X}^T[1], \mathcal{X}^T[2], \cdots, \mathcal{X}^T[S]\right]^T$ as "complete" data, and $\mathcal{Z} = \left[\mathbf{h}^{(2)T}[1], \mathbf{h}^{(2)T}[2], \cdots, \mathbf{h}^{(2)T}[S]\right]^T$ as the "missing" data vector, respectively. More explicitly, $\mathcal{X}[s]$ is defined as:

$$\mathcal{X}[s] = \begin{bmatrix} \bar{\mathbf{Y}}[s] \\ \mathbf{h}^{(2)}[s] \ , \end{bmatrix} \quad (13)$$

which has a multivariate normal distribution with a mean of

$$\mu_{\mathcal{X}[s]} = \begin{bmatrix} \mathbf{A}^{(1)}[s]\mathbf{h}^{(1)} \\ \mathbf{0} \end{bmatrix}, \ s = 1, 2, \cdots, S \quad (14)$$

and covariance matrix of

$$\mathbf{\Sigma}_s = \begin{bmatrix} \mathbf{\Sigma}_{\bar{\mathbf{Y}}[s]} & \mathbf{\Sigma}_{\bar{\mathbf{Y}}[s]\mathbf{h}^{(2)}[s]} \\ \mathbf{\Sigma}_{\mathbf{h}^{(2)}[s]\bar{\mathbf{Y}}[s]} & \mathbf{\Sigma}_{\mathbf{h}^{(2)}[s]} \end{bmatrix}, \quad (15)$$

where we have

$$\mathbf{\Sigma}_{\bar{\mathbf{Y}}[s]} = \mathbf{A}^{(2)}[s]\mathbf{\Omega}\mathbf{A}^{(2)H}[s] + \sigma_n^2\mathbf{I}_P, \quad (16)$$

$$\mathbf{\Sigma}_{\bar{\mathbf{Y}}[s]\mathbf{h}^{(2)}[s]} = \mathbf{A}^{(2)}[s]\mathbf{\Omega}, \quad (17)$$

$$\mathbf{\Sigma}_{\mathbf{h}^{(2)}[s]\bar{\mathbf{Y}}[s]} = \mathbf{\Sigma}^H_{\bar{\mathbf{Y}}[s]\mathbf{h}^{(2)}[s]} = \mathbf{\Omega}\mathbf{A}^{(2)H}[s], \quad (18)$$

$$\mathbf{\Sigma}_{\mathbf{h}^{(2)}[s]} = \mathbf{\Omega}. \quad (19)$$

where $\mathbf{A}^{(u)}[s] = \bar{\mathbf{X}}^{(u)}[s]\bar{\mathbf{F}}^{(u)}, u = 1, 2.$

The vector $\mathcal{B}$, consisting of $\mathbf{h}^{(1)}, \sigma_n^2$ and $\mathbf{\Omega}$, denotes a parameter to be estimated. Hence the log-likelihood probability density function of the "complete" data is given by

$$\log f(\mathcal{X}|\mathcal{B}) = -\frac{(P+L_0)S}{2}\log(2\pi) - \frac{1}{2}\sum_{s=1}^{S}\Big[\log|\mathbf{\Sigma}_s| + \left(\mathcal{X}[s] - \mu_{\mathcal{X}[s]}\right)^H \mathbf{\Sigma}_s^{-1}\left(\mathcal{X}[s] - \mu_{\mathcal{X}[s]}\right)\Big]. \quad (20)$$

Since we wish to maximize $Q(\mathcal{B}|\mathcal{B}^{(p)})$ with respect to $\mathcal{B}$ in Equation (20), we can omit the expected value of the constant $-\frac{(P+L_0)S}{2}\log(2\pi)$, because it does not depend on $\mathcal{B}$. Hence we can formulate the so-called **E-Step** as

$$Q(\mathcal{B}|\mathcal{B}^{(p)}) = -\frac{1}{2}\sum_{s=1}^{S}\Big[\log\left|\widehat{\mathbf{\Sigma}}_s^{(p)}\right| + \left(\mathcal{X}[s] - \hat{\mu}^{(p)}_{\mathcal{X}[s]}\right)^H \widehat{\mathbf{\Sigma}}_s^{(p)-1}\left(\mathcal{X}[s] - \hat{\mu}^{(p)}_{\mathcal{X}[s]}\right)\Big], \quad (21)$$

where we have

$$\hat{\mu}^{(p)}_{\mathcal{X}[s]} = \begin{bmatrix} \mathbf{A}^{(1)}[s]\hat{\mathbf{h}}^{(1)(p)} \\ \mathbf{0} \end{bmatrix}, \ s = 1, 2, \cdots, S. \quad (22)$$

$$\widehat{\mathbf{\Sigma}}_s^{(p)} = \begin{bmatrix} \mathbf{A}^{(2)}[s]\widehat{\mathbf{\Omega}}^{(p)}\mathbf{A}^{(2)H}[s] + \hat{\sigma}_n^{2(p)}\mathbf{I}_P & \mathbf{A}^{(2)}[s]\widehat{\mathbf{\Omega}}^{(p)} \\ \widehat{\mathbf{\Omega}}^{(p)}\mathbf{A}^{(2)H}[s] & \widehat{\mathbf{\Omega}}^{(p)} \end{bmatrix}. \quad (23)$$

The ensuring **M-Step** aims to calculate the new estimates for the channel taps $\hat{\mathbf{h}}^{(1)(p+1)}, \hat{\sigma}_n^{2(p+1)}$ and $\widehat{\mathbf{\Omega}}^{(p+1)}$ of the $(p+1)$st iteration that maximizes $Q(\mathcal{B}|\mathcal{B}^{(p)})$, given $\mathcal{B}^{(p)}$. The $(p+1)$st iteration estimates for $\hat{\mathbf{h}}^{(1)(p+1)}, \hat{\sigma}_n^{2(p+1)}$ and $\widehat{\mathbf{\Omega}}^{(p+1)}$ can be obtained by the direct differentiation of $Q(\mathcal{B}|\mathcal{B}^{(p)})$, which may be expressed as

$$\hat{\mathbf{h}}^{(1)(p+1)} = \left(\sum_{s=1}^{S}\mathbf{A}^{(1)H}[s]\mathbf{A}^{(1)}[s]\right)^{-1}\sum_{s=1}^{S}\mathbf{A}^{(1)H}[s] \cdot \left(\bar{\mathbf{Y}}[s] - \mathbf{A}^{(2)}[s]\hat{\mathbf{h}}^{(2)(p)}[s]\right), \quad (24)$$

$$\widehat{\mathbf{\Omega}}^{(p+1)} = \frac{1}{S}\sum_{s=1}^{S}E\left\{\mathbf{h}^{(2)}[s]\mathbf{h}^{(2)H}[s]\middle|\bar{\mathbf{Y}}[s], \mathcal{B}^{(p)}\right\}, \quad (25)$$

$$\hat{\sigma}_n^{2(p+1)} = \frac{1}{SP}\sum_{s=1}^{S}E\left\{\bar{\mathbf{W}}^H[s]\bar{\mathbf{W}}[s]\middle|\bar{\mathbf{Y}}[s], \mathcal{B}^{(p)}\right\}. \quad (26)$$

In each iteration, the updated estimate of $\hat{\mathbf{h}}^{(2)(p+1)}[s]$ is also obtained automatically as a by-product, which is given by

$$\hat{\mathbf{h}}^{(2)(p+1)}[s] = E\left\{\mathbf{h}^{(2)}[s]\middle|\bar{\mathbf{Y}}[s], \mathcal{B}^{(p+1)}\right\}. \quad (27)$$

After further manipulations, we arrive at the following more concrete expressions derived from Equations (25), (26) and (27),

$$\widehat{\mathbf{\Omega}}^{(p+1)} = \frac{1}{S}\sum_{s=1}^{S}\left(\Psi^{(p)-1}[s] + \hat{\mathbf{h}}^{(2)(p)}[s]\hat{\mathbf{h}}^{(2)(p)H}[s]\right), \quad (28)$$

$$\hat{\sigma}_n^{2(p+1)} = \frac{1}{SP}\sum_{s=1}^{S}\left(tr\left\{\mathbf{A}^{(2)H}[s]\Psi^{(p)-1}[s]\mathbf{A}^{(2)}[s]\right\} + \widehat{\bar{\mathbf{W}}}^{(p)H}[s]\widehat{\bar{\mathbf{W}}}^{(p)}[s]\right), \quad (29)$$

$$\hat{\mathbf{h}}^{(2)(p+1)}[s] = \hat{\sigma}_n^{-2(p+1)}\Psi^{(p+1)-1}[s]\mathbf{A}^{(2)H}[s]\Big(\bar{\mathbf{Y}}[s] - \mathbf{A}^{(1)}[s]\hat{\mathbf{h}}^{(1)(p+1)}\Big), \quad (30)$$

where we have

$$\Psi^{(p)}[s] = \hat{\sigma}_n^{(p)-2}\mathbf{A}^{(2)H}[s]\mathbf{A}^{(2)}[s] + \widehat{\mathbf{\Omega}}^{(p)-1}, \quad (31)$$

$$\widehat{\bar{\mathbf{W}}}^{(p)}[s] = \Big(\bar{\mathbf{Y}}[s] - \mathbf{A}^{(1)}[s]\hat{\mathbf{h}}^{(1)(p)} - \mathbf{A}^{(2)}[s]\hat{\mathbf{h}}^{(2)(p)}[s]\Big). \quad (32)$$

## IV. Simulation Results and Discussion

We constructed a multi-user MIMO OFDM/SDMA uplink system to demonstrate the efficiency of the proposed EM-based joint CIR and noise-variance estimation scheme. At the BS, we employ the Optimised hierarchy reduced search algorithm multi-user detection (OHRSA-MUD) [11] to separate the signals of the simultaneous users. Similar to [6], each of these users' channel bandwidth is 800kHz, and is divided into 128 QPSK/QAM-modulated subcarriers. To ensure that the consecutive OFDM symbol do not interfere with each other, a CP of 32 samples is employed as the guard interval for each OFDM symbol. Different users may employ different modulation schemes, but for simplicity, in this paper, we assume that all users employ QPSK modulation. Moreover, all of the users' data was protected by a convolutional FEC encoder before interleaving. The negative exponentially decaying Rayleigh fading CIR has $L$ taps, with delays of $0, 1, \cdots, (L-1)$ samples and a delay profile specified by $E\{\alpha_l^2\} = exp(-l/10)$. The amplitude $\alpha_l$ of each path is independent of those of the others. More specifically, one of the user's FD-CHTF is assumed to be time-invariant over a frame-duration of $50$ consecutive OFDM symbols, while the other user's channel was assumed to be time-variant from symbol to symbol over 50-symbol frame-duration. The number of channel taps was $L_1 = 5$ and $L_2 = 6$. The stopping criterion of the EM algorithm is that both $\|\mathbf{h}^{(1)(p+1)} - \mathbf{h}^{(1)(p)}\|^2 \leq 10^{-4}$ and $\|\mathbf{h}^{(2)(p+1)} - \mathbf{h}^{(2)(p)}\|^2 \leq 10^{-4}$ were met.

Fig. 2 shows the attainable mean square error (MSE) performance for a time-invariant slow-fading channel having a constant fading envelope for 50 consecutive OFDM symbols versus the SNR for different channel estimation schemes. As expected, the proposed EM-based joint channel estimation scheme performs close to the Cramer-Rao Lower Bound (CRLB), which is significantly better than the performance of the EM channel estimation methods of [6, 8]. More specifically, the SNR gain is up to $18dB$ for the range of $SNR < 5dB$ and about $10dB$ in the range of $SNR > 5dB$.

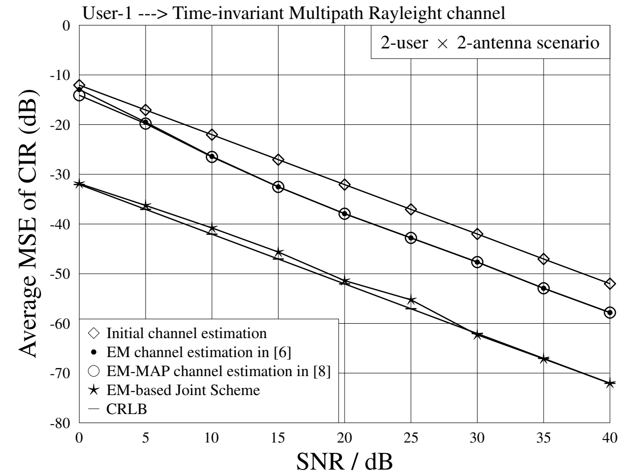


Fig. 2. MSE performance for a time-invariant channel, which has a constant envelope for 50 consecutive OFDM symbols

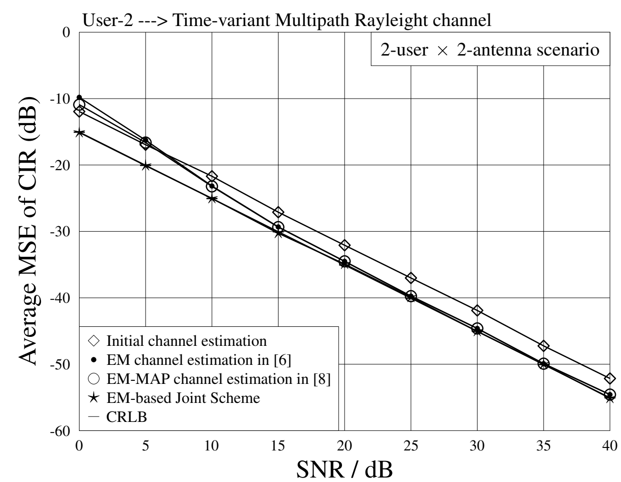


Fig. 3. MSE performance for a time-variant channel, which varies from OFDM symbol to OFDM symbol over a frame-duration

By contrast, in Fig. 3, we characterize the achievable MSE performance for the time-variant channel. Observe from this figure that similarly to the time-invariant scenario of Fig. 2, the proposed EM-based channel estimation scheme is capable of approaching the CRLB both in the low and high SNR range. By contrast, the EM-based channel estimation methods of [6] and [8] approach the CRLB over the more limited range of $SNR > 20dB$. We emphasize that we assumed the noise-variance $\sigma_n^2$ can be perfectly estimated by the EM-MAP algorithm of [8]. If $\sigma_n^2$ cannot be accurately estimated, the performance of the EM-MAP algorithm will degrade, especially for low SNRs.

In order to provide an overall impression, we evaluate the system's bit-error-ratio (BER) in Fig. 4 both with and without convolutional FEC coding, as shown using solid and dashed lines, respectively. Observe in Fig. 4 that our scheme approaches the BER performance of the ideal case associated

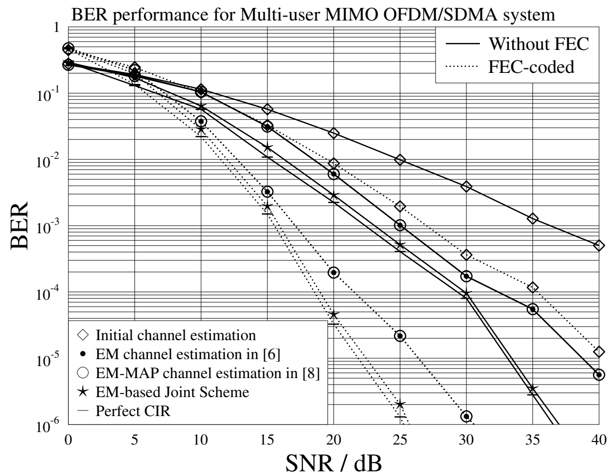


Fig. 4. BER performance for various channel estimation algorithms, when one of the user's channel is assumed to be time-invariant over a frame-duration of 50 consecutive OFDM symbols while the other user's channel is assumed to be time-variant from OFDM symbol to OFDM symbol over a frame-duration

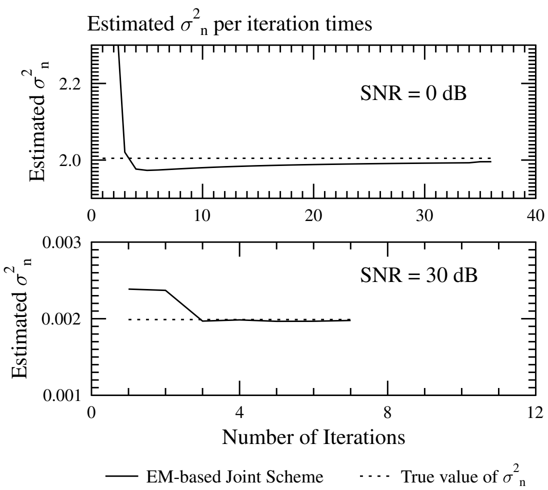


Fig. 5. Iterative convergence for the estimated $\sigma_n^2$ versus iteration index

with perfect channel information, both with and without FEC coding.

In Fig. 5, we characterize the estimated $\hat{\sigma}_n^2$ versus the number of iterations at the SNRs of $0dB$ and $30dB$. In our investigations the initial value of $\sigma_n^2$ is set higher than the actual $\sigma_n^2$ value given by Equation (16). We observe that the estimated $\hat{\sigma}_n^2$ rapidly converges to the true value of $\sigma_n^2$, which is indicated by the dashed line in Fig. 5.

## V. Conclusion

In this paper, we proposed a joint CIR and noise-variance estimation scheme based on the EM algorithm and designed for multi-user MIMO OFDM/SDMA systems. We considered a more realistic scenario, where some users are stationary and hence their channel is time-invariant while the CIR of other user is time-variant. The proposed EM-based channel estimation technique is capable of approaching the CRLB for both time-invariant and time-variant channels. Our simulations demonstrated that the proposed EM-based channel estimation schemes are capable of attaining a BER performace close to the ideal scenario associated with perfect channel information, both with and without FEC coding. Our simulation results also demonstrated that the estimated noise-variance $\hat{\sigma}_n^2$ rapidly converges to the true value of $\sigma_n^2$.

## References


[1] M. Jiang and L. Hanzo, "Multiuser MIMO-OFDM for next-generation wireless systems," *Proceedings of the IEEE*, vol. 95, no. 7, pp. 1430–1469, 2007.

[2] L. Hanzo, M. Münster, B. J. Choi, and T. Keller, *OFDM and MC-CDMA for broadband multi-user communications, WLANs, and broadcasting*. Piscataway, NJ: IEEE Press, 2003.

[3] M. Jiang, J. Akhtman, and L. Hanzo, "Iterative joint channel estimation and multi-User detection for multiple-antenna aided OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 8, pp. 2904–2914, 2007.

[4] X. Dai, "Pilot-aided OFDM/SDMA channel estimation with unknown timing offset," *IEE Proceedings-Communications*, vol. 153, no. 3, pp. 392–398, 2006.

[5] P. Vandenameele, L. Van Der Perre, M. Engels, B. Gyselinckx, and H. De Man, "A combined OFDM/SDMA approach," *IEEE Journal on Selected Areas in Communications*, vol. 18, no. 11, pp. 2312–2321, 2000.

[6] Y. Z. Xie and C. N. Georghiades, "Two EM-type channel estimation algorithms for OFDM with transmitter diversity," *IEEE Transactions on Communications*, vol. 51, no. 1, pp. 106–115, 2003.

[7] X. Wautelet, C. Herzet, A. Dejonghe, J. Louveaux, and L. Vandendorpe, "Comparison of EM-based algorithms for MIMO channel estimation," *IEEE Transactions on Communications*, vol. 55, no. 1, pp. 216–226, 2007.

[8] J. Gao and H. Liu, "Low-complexity MAP channel estimation for mobile MIMO-OFDM systems," *IEEE Transactions on Wireless Communications*, vol. 7, no. 3, pp. 774–780, 2008.

[9] T. Moon, "The expectation-maximization algorithm," *IEEE Signal processing magazine*, vol. 13, no. 6, pp. 47–60, 1996.

[10] Y. Li, N. Seshadri, and S. Ariyavisitakul, "Channel estimation for OFDM systems with transmitter diversity in mobile wireless channels," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 3, pp. 461–471, 1999.

[11] J. Akhtman, A. Wolfgang, S. Chen, and L. Hanzo, "An optimized-hierarchy-aided approximate log-MAP detector for MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 6, no. 5, pp. 1900–1909, 2007.