Harmony Search Aided Iterative Channel Estimation, Multiuser Detection and Channel Decoding for DS-CDMA

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Abstract—A novel Multiuser Detection (MUD) scheme is proposed for DS-CDMA systems employing the so-called Harmony Search (HS) algorithm, which is a novel meta-heuristic optimisation method. We specifically design the HS aided MUD for the communications problem considered and apply it in an iterative joint Channel Estimation (CE), MUD and channel decoding framework. The simulation results demonstrate that a near-single-user performance can be achieved by the proposed algorithm while avoiding the excessive-complexity full-searchbased optimum detection even in overloaded DS-CDMA systems. Moreover, the HS algorithm can be efficiently applied in the Expectation Maximisation (EM) based CE framework.

I. INTRODUCTION

Optimum Maximum *A posterior* Probability (MAP) based Joint Detection and Decoding (JDD) scheme for near-singleuser detection in a high-dimensional and highly-correlated coded DS-CDMA system can be achieved by reducedcomplexity Iterative Detection and Decoding (IDD) upon exchanging extrinsic information in terms of Log-Likelihood Ratio (LLR) between the receiver components without a significant performance compromise [1], where the associated decoupling of the JDD receiver into components is facilitated by the inclusion of an interleaver employed between the detector and decoder.

In the context of the MultiUser Detection (MUD) scheme, the optimum Bayesian MAP detector has an excessive computational complexity. Alternatively, stochastic global optimisation techniques may be pursued in order to reduce the complexity, while still capturing the MAP solution with a high probability using for example the well-known Genetic Algorithms (GA) [2] or the Swarm Intelligence (SI) algorithms [3]. Apart from these algorithms, imitating the improvisation process of musicians, a new naturally-inspired metaheuristic optimisation method was proposed recently, leading to the so-called Harmony Search (HS) algorithm [4].

In addition to detection and decoding, the data-aided Channel Estimation (CE) may be incorporated in the aforementioned IDD framework [2], [5], so that the estimated channel parameters can be iteratively improved by the error correction decoder. In particular, the well-established Expectation Maximisation (EM) algorithm [6] provides a general framework for solving such problems. In this paper, the key idea of our HS aided EM algorithm is that instead of using the true *a posteriori* distribution of the transmitted data, given the observation and the current channel estimate, we use the *a posteriori* distribution based on the data estimates provided by the HS algorithm.

The novel contribution of our paper is that we design a HS based global optimisation method for iterative joint CE, MUD and channel decoding.

The remainder of this paper is organised as follows. In Section II, we briefly review the original HS algorithm. In Section III we specifically design the HS algorithm for iterative joint CE, MUD and channel decoding. We then characterise its performance in Section IV and conclude in Section V.

II. HARMONY SEARCH ALGORITHM

A. Objective Function and Analogy

Consider the optimisation of $f(\mathbf{x})$, where $f(\cdot)$ is the *Fitness Function* (FF), $\mathbf{x} = [x_1, \ldots, x_K]^T$ is the set of *candidates* containing K legitimate variables chosen from a discrete or continuous alphabet $\mathcal{A}_k, k \in [1, K]$, where the superscript $(\cdot)^T$ denotes transpose. In our MUD problem, K is the number of users and **x** hosts one of the 2^K possible legitimate K-user BPSK modulated candidate vector.

We now briefly review the HS algorithm introduced in [4] and relate it to our K-user MUD context in Section III. When a musician improvises, the aesthetic quantification (FF) results from a set of pitches produced by the music instruments (variables) involved. The musician seeks to produce aesthetically pleasing harmony (the optimum K-user vector) as determined by his/her aesthetic perception inferred from rehearsals (iterations).

B. Algorithm in Step

There is a range of parameters associated with the HS algorithm [4]. The harmony memory size M specifies the number of initial K-user harmony candidates stored in the Harmony Memory Matrix (HMM), which represents the initial population size. The harmony memory activation probability P_{ma} specifies the typically less than unity probability of a new K-user candidate being selected from the HMM rather than randomly, where the latter has a probability of $(1 - P_{ma})$. The pitch adjustment probability P_{pa} specifies the chance of

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changing the pitch, i.e. that of opting for a newly generated Kuser candidate from the HMM. Finally, Q represents the total number of iterations or improvisations carried out throughout the HS algorithm. More specifically, the HS algorithm can be summarized as follows:

Step 1 - Initialisation: We initialize the HMM $\mathbf{X}^0 = [\mathbf{x}_1^0, \dots, \mathbf{x}_M^0]$, each variable $x_{m,k}^0, m \in [1, M], k \in [1, K]$ is generated randomly from a uniform distribution based on its legitimate alphabet \mathcal{A}_k , where $x_{m,k}^0$ denotes the *k*th variable of the *m*th *K*-user harmony candidate vector \mathbf{x}_m^0 in the HMM.

Step 2 - Improvisation: At iteration $q \in [1, Q]$, the new K-user harmony candidate \mathbf{x}^{new} is generated using an appropriate combination of the following HS operations: memory activation, pitch adjustment and random selection. More particularly, a new K-user harmony vector \mathbf{x}^{new} may be randomly selected from the alphabet $x_k^{new} \in \mathcal{A}_k, k \in [1, K]$ with a probability of $(1 - P_{ma})$ or selected from the HMM \mathbf{X}^{q-1} with a probability of P_{ma} according to $x_k^{new} = x_{m_i,k}^{q-1}, m_i \sim$ $\mathcal{U}[1, M], k \in [1, K]$, which implies inheriting the kth bit of one of the M candidate vectors in the (q-1)th iteration. Once a K-user candidate was selected from \mathbf{X}^{q-1} , then a further pitch adjustment characterised by a step of Δ may be applied with a pitch adjustment probability of P_{pa} , where the specific value of each variable $x_k^{new}, k \in [1, K]$ of the new K-user harmony candidate is tuned to match the neighbouring values in its legitimate candidate solution alphabet A_k .

Step 3 - Updating: The new harmony candidate \mathbf{x}^{new} generated replaces the worst harmony of the HMM \mathbf{X}^{q-1} , provided that its score measured in terms of the FF is better than that of the worst harmony in \mathbf{X}^{q-1} . Otherwise, no changes are made in the HMM, namely we have $\mathbf{X}^q = \mathbf{X}^{q-1}$.

III. HARMONY SEARCH AIDED ITERATIVE RECEIVER

A. System Model

We consider a rate-R coded BPSK modulated K-user DS-CDMA system employing user-specific N_c -chip random spreading sequences. The discrete-time system model can be written as:

$$\mathbf{Y} = \mathbf{CHX} + \mathbf{N},\tag{1}$$

where $\mathbf{Y} \in \mathcal{C}^{N_c \times N}$, $\mathbf{X} \in \mathcal{R}^{K \times N}$ and $\mathbf{N} \in \mathcal{C}^{N_c \times N}$ denotes the matrix of received signal samples, transmitted symbols and noise samples of a given transmission frame having a block length of N bits. Furthermore, $\mathbf{C} \in \mathcal{R}^{N_c \times K}$ denotes the DS-CDMA spreading matrix and $\mathbf{H} = \text{diag}[h_1, \ldots, h_k]$ contains the block-invariant complex channel to be estimated.

Fig 1 shows the iterative CE, MUD and channel decoding, where each CE update will be based on the soft output of the IDD scheme and the new CE results will again be used by the IDD. We will first design the HS aided MUD algorithm assuming perfect channel knowledge and then demonstrate that our HS-aided MUD algorithm can be efficiently applied in the EM based channel estimation framework.

B. Optimum Soft MUD

Assuming that a long interleaver is employed between the MUD and channel decoder, which decorrelates the soft



Fig. 1. Iterative channel estimation, multiuser detection and channel decoding, where ENC is short for encoding.

information exchanged during the consecutive iterations, we focus on the *n*th symbol interval $\mathbf{y}_n = \mathbf{CHx}_n + \mathbf{n}_n$ and drop the symbol index *n*. In the BPSK modulated system, the MUD delivers soft information in terms of *extrinsic* LLRs denoted by $\mathcal{L}^e(\mathbf{x})$ to the outer channel decoder based on the observation of the input *a priori* LLRs denoted by $\mathcal{L}^a(\mathbf{x})$. Since all users' information is independent of each other, the *extrinsic* LLR of the *k*th user is given by $\mathcal{L}^e(x_k) = \mathcal{L}(x_k) - \mathcal{L}^a(x_k)$, where the *a posteriori* LLR $\mathcal{L}(x_k)$ is given by:

$$\mathcal{L}(x_k) = \ln \frac{P[x_k = +1 | \mathbf{y}, \mathcal{L}^a(\mathbf{x})]}{P[x_k = -1 | \mathbf{y}, \mathcal{L}^a(\mathbf{x})]}$$

=
$$\ln \frac{\sum_{\forall \mathbf{x}_{-k}} P[x_k = +1, \mathbf{x}_{-k} | \mathbf{y}, \mathcal{L}^a(\mathbf{x})]}{\sum_{\forall \mathbf{x}_{-k}} P[x_k = -1, \mathbf{x}_{-k} | \mathbf{y}, \mathcal{L}^a(\mathbf{x})]}, \quad (2)$$

where \mathbf{x}_{-k} denotes the *K*-user vector with the *k*th element x_k excluded. We note that the optimum Bayesian approach aided summation leads to prohibitive complexity by considering all possible 2^{K-1} such vectors. This motivates the low-complexity HS algorithm.

C. Harmony Search Aided MUD

The idea of the HS algorithm is that, in order to approximate the Bayesian optimum of Eq. (2), we gather a sufficient number of K-user candidate vectors after Q iterations in the final HMM, each contributing significantly (in enchanting harmony) to the evaluation of the overall summation, where the significance is quantified by the FF.

1) Fitness Function: The FF to be evaluated in the HS is defined as the joint APP of the K-user transmitted vector \mathbf{x} based on the observation \mathbf{y} and the *a priori* LLRs $\mathcal{L}^{a}(\mathbf{x})$ provided by the channel decoder, which may be expressed as:

$$f(\mathbf{x}) = P[\mathbf{x}|\mathbf{y}, \mathcal{L}^{a}(\mathbf{x})]$$

$$\propto \ln p(\mathbf{y}|\mathbf{x}) + \ln P[\mathbf{x}|\mathcal{L}^{a}(\mathbf{x})]$$

$$= -||\mathbf{y} - \mathbf{CH}\mathbf{x}||^{2}/2\sigma^{2} + \sum_{\forall k} \ln P^{a}(x_{k}), \quad (3)$$

where in the second line, we exploited the Bayes rule and monotonic nature of the FF, leading to a numerically efficient log-domain expression, while in the third line, we exploited the fact that each user's bits are independent of each other, leading to $\ln P[\mathbf{x}|\mathcal{L}^{a}(\mathbf{x})] = \sum_{\forall k} \ln P^{a}(x_{k})$, where $P^{a}(x_{k})$ of Eq. (3) denoting the *a priori* probability of each user x_{k} may be expressed as $P^{a}(x_{k} = +1) = 1/[1 + e^{-\mathcal{L}^{a}(x_{k})}]$.

2) Naive Transplanting: We now approximate Eq. (2) by collecting a sufficient number of significant K-user candidate vectors x, where the 'collection' is followed by a set of HS rules introduced in Section II and the 'significance' is quantified by the FF introduced above.

However, direct employment of the HS algorithm results in a poor performance, since in the particular MUD problem, only binary data are considered, which is incompatible with the pitch adjustment step of the original proposal [4]. Thus, instead of 'tuning' the kth binary value of x_k^{new} to its opposite based on the predefined pitch adjustment probability of P_{pa} in an binary manner, we propose to take the soft information associated with x_k^{new} into consideration for determining P_{pa} . As a result, the specific value of x_k^{new} will be toggled to the opposite binary value during each pitch adjustment step based on the probability containing its soft information.

3) Pitch Adjustment: After randomly selecting a K-user base harmony vector \mathbf{x}^b at iteration q from the HMM \mathbf{X}^{q-1} with a probability of P_{ma} , the pitch adjustment is carried out by generating the kth variable x_k^{new} based on the marginal APP $P(x_k^b|\mathbf{y}, \mathbf{x}_{-k}^b)$ of the base harmony vector. This marginal APP represents the soft information of x_k^{new} and acts as the replacement of P_{pa} in the original HS proposal [4]. The LLR of this marginal APP may be conveniently evaluated as:

$$\mathcal{L}_{pa} = \ln \frac{P\left[x_{k}^{b} = +1 | \mathbf{y}, \mathbf{x}_{-k}^{b}, \mathcal{L}^{a}(x_{k})\right]}{P\left[x_{k}^{b} = -1 | \mathbf{y}, \mathbf{x}_{-k}^{b}, \mathcal{L}^{a}(x_{k})\right]}$$
$$= \ln \frac{p(\mathbf{y} | x_{k}^{b} = +1, \mathbf{x}_{-k}^{b})}{p(\mathbf{y} | x_{k}^{b} = -1, \mathbf{x}_{-k}^{b})} + \mathcal{L}^{a}(x_{k})$$
(4)

Hence we arrive at $P_{pa}(x_k^{new} = +1) = 1/1 + e^{-\mathcal{L}_{pa}}$. As a result, the P_{ma} -based *memory activation* and the P_{pa} -based *pitch adjustment* merge into a single joint step.

The automatically updated marginal APP is capable of providing a sufficiently high decision reliability, which implies that the selection of a new K-user harmony vector from the HMM is more appropriate than a random choice. Hence, we may set the probability $P_{ma} = 1$ and avoid the operation of random selection. Importantly, ignoring the associated random selection does not limit the exploration capability of the algorithm, because if we have a sufficiently high M, randomly selected base vectors have first been generated before fine-tuning the pitch adjustment. In summary, the pseudo-code of our HS-aided MUD algorithm is shown in Table I.

4) Soft Output: After Q iterations, a list of K-user candidate vectors with reasonably good fitness was generated. Then Eq. (2) is evaluated based on the K-user candidate vectors in the final HMM \mathbf{X}^Q , which can be expressed as:

$$\mathcal{L}(x_k) = \ln \frac{\sum_{\mathbf{x}_{-k} \in \mathbf{X}^Q} P\left[x_k = +1, \mathbf{x}_{-k} | \mathbf{y}, \mathcal{L}^a(\mathbf{x})\right]}{\sum_{\mathbf{x}_{-k} \in \mathbf{X}^Q} P\left[x_k = -1, \mathbf{x}_{-k} | \mathbf{y}, \mathcal{L}^a(\mathbf{x})\right]}$$
$$= \ln \sum_{\mathbf{x}_{-k} \in \mathbf{X}^Q} e^{f(\mathbf{x}_{k+1})} - \ln \sum_{\mathbf{x}_{-k} \in \mathbf{X}^Q} e^{f(\mathbf{x}_{k-1})}$$
(5)

where $f(\mathbf{x}_{k+})$ and $f(\mathbf{x}_{k-})$ represent the FF function of a given candidate vector \mathbf{x} having its kth entry equals to +1 and -1, respectively. When considering the *extrinsic* LLR $\mathcal{L}^e(x_k)$, the corresponding *extrinsic* FF value $f^e(\mathbf{x})$ is substituted in Eq. (5), which may be given by $f^e(\mathbf{x}) = f(\mathbf{x}) - \ln P^a(x_k)$.

D. Harmony Search Based Channel Estimation

Let us now demonstrate that our HS-aided MUD algorithm can be efficiently applied in the EM based CE framework.

TABLE I PSEUDO-CODE OF THE HS-AIDED MUD

```
/* Initial HMM Generation */
Initialise \mathbf{X}^0 Define FF of Eq (3) Compute f(\mathbf{x}_m^0), m \in [1, M]
/* Improvisation Loop */
for q = 1, \ldots, Q do
        /* New Harmony Candidate Generation */
                 /* Memory Activation */
                for k = 1, \dots, K do

x_k^b = x_{m_i,k}^{q-1}, m_i \sim \mathcal{U}[1, M]
                 end for
                 /* Pitch Adjustment */
                 for k = 1, \ldots, K do
                         Compute pitch adjustment probability based on Eq (4)
                         if \mathcal{U}(0,1) \leq P_{pa}(x_k^{new} = +1) do x_k^{new} = +1
                         else x_k^{new} = -1
                         end if
                         Set \mathbf{x}^{b} = [x_{1}^{new}, \dots, x_{k}^{new}, x_{k+1}^{b}, \dots, x_{K}^{b}]
                 end for
                 Set \mathbf{x}^{new} = \mathbf{x}^b
        /* New HMM Generation */
        Find \epsilon = \arg \min f(\mathbf{x}_{\epsilon}^{q-1}), \epsilon \in [1, M]
        \begin{split} &\text{if } f(\mathbf{x}^{new}) > f(\mathbf{x}^{q-1}_{\epsilon}) \text{ and } f(\mathbf{x}^{new}) \neq f(\mathbf{x}^{q-1}_{m}), \forall m \text{ do} \\ &\mathbf{x}^{q}_{\epsilon} = \mathbf{x}^{new}, \mathbf{x}^{q}_{m} = \mathbf{x}^{q-1}_{m}, \forall m \neq \epsilon \\ &\text{else } \mathbf{X}^{q} = \mathbf{X}^{q-1} \end{split}
        end if
end for
```

We introduce the notation $\mathbf{h} = [h_1, \ldots, h_k]^T$ and consider the entire observation frame of received samples \mathbf{Y} . The optimal estimate of the channel conditioned on the observation of \mathbf{Y} is given by $\mathbf{h}^* = \arg \max_{\mathbf{h}} p(\mathbf{Y}|\mathbf{h})$. However, the explicit expression of the likelihood function $p(\mathbf{Y}|\mathbf{h})$ is unknown, since the transmitted data \mathbf{X} is unknown. We thus resort to the so-called EM algorithm [6], which iteratively finds the optimal solution for \mathbf{h} . Given the channel-contaminated received data \mathbf{Y} that we do know, we can find *a posteriori* probabilities for the transmitted data \mathbf{X} , given the previously estimated value of \mathbf{h}^{q-1} . Then, for each detected set of \mathbf{X} , we can thus calculate an expected value of the likelihood function with the aid of the channel-contaminated received data \mathbf{Y} and the previous channel estimate.

1) Expectation: More formally, we commence from the following expectation calculation:

$$\Omega(\mathbf{h}, \mathbf{h}^{q-1}) = \operatorname{E}\left[\ln p(\mathbf{X}, \mathbf{Y}|\mathbf{h}) | \mathbf{Y}, \mathbf{h}^{q-1}\right]$$
$$= \sum_{\forall \mathbf{X}} P(\mathbf{X}|\mathbf{Y}, \mathbf{h}^{q-1}) \ln p(\mathbf{X}, \mathbf{Y}|\mathbf{h}), \quad (6)$$

which means the log-domain likelihood function of the complete data $p(\mathbf{X}, \mathbf{Y}|\mathbf{h})$ averaged over all possible 2^{KN} number of transmitted data sets \mathbf{X} .

We rewrite Eq. (6) as $\ln p(\mathbf{\tilde{X}}, \mathbf{Y}|\mathbf{h})$, which can be further reformulated as:

$$\ln p(\tilde{\mathbf{X}}, \mathbf{Y} | \mathbf{h}) \propto \ln p(\mathbf{Y} | \tilde{\mathbf{X}}, \mathbf{h}) P(\tilde{\mathbf{X}} | \mathbf{h})$$

$$\propto -\sum_{n=1}^{N} ||\mathbf{y}_{n} - \mathbf{C} \chi_{n} \mathbf{h}||^{2} / 2\sigma^{2}$$

$$\propto (2 \operatorname{Re} \{ \mathbf{r}^{\mathrm{H}} \mathbf{h} \} - \mathbf{h}^{\mathrm{H}} \mathbf{R} \mathbf{h}) / 2\sigma^{2}, \quad (7)$$

where \mathbf{y}_n and $\chi_n = \text{diag}[\tilde{x}_{1,n}, \dots, \tilde{x}_{K,n}]$ denote the received signal vector and the expected value of the transmitted vector

during the *n*th symbol interval, respectively. We also removed the contribution of $P(\tilde{\mathbf{X}}|\mathbf{h})$ in the second line based on the fact that the estimated transmitted symbol $\tilde{\mathbf{X}}$ is independent of the channel **h**. Furthermore, we have $\mathbf{r} = \sum_{n=1}^{N} \chi_n \mathbf{C}^T \mathbf{y}_n$ and $\mathbf{R} = \sum_{n=1}^{N} \chi_n \mathbf{C}^T \mathbf{C} \chi_n$, where the entries $\tilde{x}_{k,n}$ of χ_n are given by the soft output values of the HS MUD:

$$\tilde{x}_{k,n} = \sum_{\forall \mathbf{X}} x_{k,n} P(\mathbf{X}|\mathbf{Y}, \mathbf{h}^{q-1})$$

$$\propto \sum_{\forall \mathbf{x}_{-k,n}} P(x_{k,n} = +1, \mathbf{x}_{-k,n} | \mathbf{y}_n, \mathbf{h}^{q-1})$$

$$-P(x_{k,n} = -1, \mathbf{x}_{-k,n} | \mathbf{y}_n, \mathbf{h}^{q-1}).$$
(8)

The key idea of our HS aided EM algorithm is that instead of using the true *a posteriori* distribution of the transmitted data **X**, given both the observation **Y** and the channel estimate **h**, we use the approximated *a posteriori* distribution based on the *K*-user harmony vectors in the final HMM, which are believed to contribute significantly to the true *a posteriori* distribution. We may now utilize the set of significantly contributing *K*-user vectors stored in the final HMM instead of summing them over all possible 2^{K-1} number of values $\mathbf{x}_{-k,n}$. Hence, Eq. (5) may now be directly applied for the calculation of the soft value $\tilde{x}_{k,n}$ and we arrive at $\tilde{x}_{k,n} = \tanh[\mathcal{L}(x_{k,n})]$.

2) *Maximisation:* We may now maximize the expected value of Eq. (6) in the sense of the Minimum Mean Square Error (MMSE), which is explicitly given by [7]:

$$\mathbf{h}^{q} = \arg \max_{\mathbf{h}} \Omega(\mathbf{h}, \mathbf{h}^{q-1}) = \mathbf{R}^{-1} \mathbf{r}.$$
 (9)

However, the EM based blind-type CE suffers from the classic phase ambiguity of π in the BPSK modulated system considered. Hence, a few initial training symbols are required to remove the phase ambiguity of π before activating the blind EM algorithm, leading to a semi-blind CE. Since the EM algorithm is a data-aided scheme, the length of the initial training sequence is typically short compared to a pilot-aided CE. In this case, we may use the rough initial CEs based on the training symbols to initialise the receiver, while the subsequent channel updates of Eq. (9) can be generated by combining the data-aided and pilot-aided channel estimates. More explicitly, we have:

$$\mathbf{h}^{q} = \left[\mathbf{R} + \mathbf{R}^{p}\right]^{-1} \left(\mathbf{r} + \mathbf{r}^{p}\right), \qquad (10)$$

where the calculation of \mathbf{R}^p and \mathbf{r}^p is based on the true value of the pilot symbols.

IV. PERFORMANCE EVALUATION

A. Parameters and Complexity

Consider an outer rate R = 1/3 repetition coded DS-CDMA system employing user-specific random spreading sequences of length $N_c = 7$, where the information frame length was $N_i = 512$ and the number of iterations between the MUD and the soft decoder was set to $I_{IDD} = 10$. The outer repetition code employed is known as being capable of generating the highest extrinsic information in interference-limited scenarios compared to other channel codes [8]. Furthermore, we define the so-called normalised system-load $\beta = K/N_c$ as the ratio of the number of users supported to the spreading sequence length employed. An AWGN channel was assumed and a uniformly distributed block-invariant channel phase noise was imposed, namely when we had $h_k = e^{j\theta_k}, \theta_k \in [-\pi, \pi)$. Apart from employing the EM based CE algorithm using the abovementioned random phase-noise based false-locking model, we will also use an idealized benchmarker, where we assume perfect knowledge of the channel's phase noise.

We measure the complexity of our HS-aided MUD in terms of the required FF evaluations. Generating the *a posteriori* LLRs given by Eq. (2) for K users requires $Q_{opt} = K \times 2^K$ evaluations of Eq. (3), while the HS algorithm requires:

$$Q_{HS} = Q \times 2 \times K + Q + M + 2 \times M \times K \quad (11)$$

evaluations of the FF of Eq. (3), In detail, it includes M FF evaluations of the initial HMM, and a further evaluation at the end of each of the Q improvisations as well as $(2 \times K)$ evaluations, when generating the marginal APP based pitch adjustment of each of the Q improvisations. In addition, the soft output generation requires a further $(2 \times M \times K)$ evaluations of Eq. (5).

B. Simulation Results

1) Effects of the Number of Improvisations: Fig 2 shows the effects of varying the number of improvisations from Q =2, 5, 8, 10, 20, when the normalised system load was $\beta = 4$ and M = 10 and we had $Q_{HS} = 684, 855, 1026, 1140, 1710$, while $Q_{MAP} = 28 \times 2^{28}$. It demonstrated that in this overloaded scenario, the HS-aided MUD was capable of mitigating the detrimental effect of the high correlations of random spreading sequences and of attaining a performance, which was within $E_b/N_0 = 0.5$ dB from the single-user performance measured at the Bit Error Ratio (BER) of $P_e = 10^{-5}$.

2) Effects of System Load: Fig 3 shows the effects of varying the normalised system load β , when we have Q = 20 improvisations. As shown in this figure, having an almost unprecedented system load as high as $\beta = 6$ is possible for the HS aided MUD. More specifically, when the normalised system load is $\beta = 6$, and $Q_{HS} = 2550$ FF evaluations are used instead of $Q_{MAP} = 42 \times 2^{42}$, the performance is only about $E_b/N_0 = 1$ dB away from the single user performance measured at the BER of $P_e \approx 2 \times 10^{-5}$. We also note that the HS parameters were kept the same for both $\beta = 6$ and $\beta = 3$, which implies that proposed HS-aided algorithm is capable of achieving a near-single-user performance without the excessive complexity of the optimum detector and that within limits, the complexity of the algorithm is reasonably independent of β .

3) HS assisted EM based CE: Fig 4 shows the achievable performance of our HS assisted EM based CE for a DS-CDMA system having a normalised system load of $\beta = 4$, when the channel's phase noise was unknown to the receiver. The number of iterations between the CE and IDD was set to $I_{EM} = 5$. Fig 4 demonstrates that employing training sequences alone, i.e. without a data-aided mode, failed to generate sufficiently accurate phase estimates. On the other hand, the semi-blind



Fig. 2. BER performance of HS-aided MUD of a K = 28-user R = 1/3-repetition coded BPSK modulated DS-CDMA system using $N_c = 7$ -chip random sequences and I = 10 iterations.



Fig. 3. BER performance of HS-aided MUD of a R = 1/3-repetition coded BPSK modulated DS-CDMA system using $N_c = 7$ -chip random sequences for supporting the system load of $\beta = 3, 4, 5, 6, 7$.

data-aided EM approach, which jointly considered the pilotand data-aided channel estimates become capable of acquiring accurate phase noise estimates. Furthermore, the longer the training sequences employed, the faster the convergence to the best possible performance associated with perfect knowledge of the channel's phase noise, when modelling the effects of false-locking. Furthermore, T = 16 pilots are required in the data-aided EM based CE to achieving convergence, while employing T = 64 pilot-aided CE can not lead to convergence.

Remarks: In comparison to other Evolutionary Algorithms (EAs), which require the tuning of a range of parameters, as long as the initial HMM size of M is sufficiently large, the HS-aided algorithm requires the tuning of a single parameter, namely of the number of improvisations Q. This property is acquired as a benefit of the marginal APP-based pitch adjustment step, which equips it with the capability of attaining convergence from a Bayesian inference point of view. At the



Fig. 4. BER performance of HS-assisted EM based CE algorithm of a R = 1/3-repetition coded BPSK modulated DS-CDMA system using $N_c = 7$ -chip random sequences for supporting a system load of $\beta = 4$.

same time, the entire search space is visited by the HS MUD with the aid of the randomly generated base vector from the set of M rather diverse candidate vectors at each improvisation.

V. CONCLUSION

In this paper, we proposed a novel HS-aided MUD for DS-CDMA systems and developed it into a joint iterative CE, MUD and channel decoding framework. We used the APP based soft information as a variant of the P_{pa} in the original HS-aided algorithm, which led to a low-complexity MUD approaching the single-user performance even for DS-CDMA systems supporting an extremely high normalised system load of $\beta = 6$. Moreover, our HS-aided MUD can be efficiently combined with the EM based CE framework.

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