Receiver Multiuser Diversity Aided Multi-Stage MMSE Multiuser Detection for DS-CDMA and SDMA Systems Employing I-Q Modulation

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***Abstract*— The so-called receiver multiuser diversity aided multi-stage minimum mean-square error multiuser detector (RMD/MS-MMSE MUD), which was proposed previously by the author, is investigated in the context of the direct-sequence code-division multiple-access (DS-CDMA) and space-division multiple-access (SDMA) systems that employ in- and quadrature-phase (I-Q) modulation schemes. A detection scheme is studied, which is operated in real domain in the principles of successive interference cancellation (SIC). The concept of *noise recognition factor* (NRF) is proposed for explaining the efficiency of SIC-type detectors and also for motivating to design other high-efficiency detectors. The achievable bit error rate (BER) performance of the RMD/MS-MMSE MUD is investigated for DS-CDMA and SDMA systems of either full-load or overload, when communicating over Rayleigh fading channels for the SDMA and over either additive white Gaussian noise (AWGN) or Rayleigh fading channels for the DS-CDMA. The studies and performance results show that the RMD/MS-MMSE MUD is a highly promising MUD. It has low implementation complexity and good error performance. Furthermore, it is a high-flexibility detector suitable for various communication systems operated in different communication environments.**

I. Introduction

Due to the exponentially dependent complexity of the optimum maximum-likelihood (ML)-MUD (or maximum *a-posteriori* (MAP)-MUD) [1, 2], since its invention by Verdu in 1983 [3], researchers in wireless communications have made a lot of efforts in order to find the possible low-complexity MUD algorithms that are capable of achieving near-optimum BER performance. Here, by low-complexity, we mean those MUD algorithms having polynomially dependent, but preferably linearly dependent complexity. So far, a huge number of MUD algorithms have been proposed, as shown, e.g., in [2, 4–6] and the references there in. However, most MUD algorithms so far proposed either are still too complex to be implemented in practice or achieve much worse BER performance than the ML-MUD [2, 4–6].

Towards the above problem, the author of this paper has studied from different perspectives [7–10] the ML-MUD as well as a range of other MUDs, and a MUD algorithm has been designed, which makes it possible to achieve near-optimum BER performance but with linearly dependent detection complexity [9, 10]. In more detail, in [7], the statistics of both the MMSE- and ML-MUDs have been studied from various aspects, in order to find the hints for design of high-efficiency and low-complexity MUDs. In the light of the V-BLAST systems [11, 12], in [8], a multi-stage (MS)-MMSE MUD and two types of reliability measurement schemes, namely the Type-L and Type-A schemes, have been proposed for multiantenna multiple-input multiple-output (MIMO) systems. Furthermore, the Type-L and Type-A schemes have been analyzed and compared with the well-known Type-γ scheme [11, 12], which measures the reliabilities based on signal-to-interference-plus-noise ratio (SINR). Our studies demonstrate that the Type-A scheme converges to the Type-L scheme and both of them converge to the optimum reliability measurement, as the MIMO system becomes larger. By contrast, as the size of a MIMO system increases, the type-γ scheme becomes less and less efficient.

Following [8], in [9, 10], the conditions for a SIC-type MUD to attain optimum BER performance have been studied. The concept of receiver multiuser diversity (RMD) has been introduced for explaining the diversity of reliabilities when measured based on the Type-L scheme. The studies show that RMD exists in both additive white Gaussian noise (AWGN) and fading channels and the reliabilities of different users are usually highly diverse, which are hence beneficial to using the SIC-type MUDs. Therefore, in [9, 10] a so-called RMD/MS-MMSE MUD has been proposed and investigated in the context of both DS-CDMA and SDMA systems. For the DS-CDMA systems, both AWGN and Rayleigh fading channels have been considered, while for the SDMA systems uncorrelated Rayleigh fading channels have been assumed. Furthermore, the BER performance of both the full-load and overload DS-CDMA and SDMA systems has been studied, where full-load means that K of the number of users equals N of the spreading factor of DS-CDMA systems or equals N of the number of receive antennas of SDMA systems, while overload means $K > N$. Our analysis and performance results show that the RMD/MS-MMSE MUD is a very general and high-efficiency MUD. It is suitable for nearly any communications scenarios experiencing multiuser or multi-symbol interference, it is suitable for detection in either fading or non-fading channels and it is an efficient detector for under-load ($K < N$), full-load or overload systems. The BER performance achieved by the RMD/MS-MMSE MUD converges to the optimum as the system size increases. Aided by the RMD/MS-MMSE MUD, a full-load DS-CDMA or SDMA system of moderate size is generally capable of achieving the BER performance similar to that of the ML-MUD. Within the BER range of interest, an overload DS-CDMA or SDMA system using the RMD/MS-MMSE MUD to support $K = 2N$ users still significantly outperforms a corresponding DS-CDMA or SDMA system using the conventional MMSE-MUD to support $K = N$ users. Furthermore, the RMD/MS-MMSE MUD's complexity is linearly proportional to K of the number of users supported.

However, the BER performance of the RMD/MS-MMSE MUD has so far only been studied in the context of the binary phase-shift keying (BPSK) baseband modulation [9, 10]. Therefore, based on the insights gained from [9, 10], in this contribution, we extend our studies to the systems using non-binary I-Q modulations (or M-ary quadrature amplitude modulation (MQAM)). Except the non-binary modulation, here we consider the same scenarios, including DS-CDMA/SDMA, fading/non-fading, full-load/overload, etc., and the same assumptions, such as random spreading for DS-CDMA, uncorrelated fading for SDMA, etc., as that considered in [9, 10]. Furthermore, in this paper, we introduce a novel concept of *noise recognition factor (NRF)* ϱ and explain that an efficient SIC-type MUD should have a NRF of $\varrho = 1$. Based on this observation, finally, a RMD/MS-MMSE MUD having

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a NRF of $\varrho = 1$ is proposed and studied associated with both DS-CDMA and SDMA systems.

II. MMSE Detection and Reliability Measurement

The MIMO equation for both DS-CDMA and SDMA systems can be expressed as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{1}$$

where $\boldsymbol{y}$ and $\boldsymbol{n}$ are N-length complex-valued observation vector and noise vector, $\boldsymbol{x} = \boldsymbol{x}^{(I)} + j\boldsymbol{x}^{(Q)} = [x_1, x_2, \cdots, x_K]^T$ contains the data symbols transmitted by the K users, $x_k = x_k^{(I)} + jx_k^{(Q)}$, and $\boldsymbol{H} = [\boldsymbol{h}_1, \boldsymbol{h}_2, \cdots, \boldsymbol{h}_K]$ is an $(N \times K)$ matrix. For DS-CDMA systems, the matrix $\boldsymbol{H}$ can be written as $\boldsymbol{H} = \boldsymbol{C}\boldsymbol{A}$, where $\boldsymbol{C}$ is an $(N \times K)$ spreading matrix and $\boldsymbol{A} = \text{diag}\{a_1, a_2, \cdots, a_K\}$ is an $(K \times K)$ diagonal matrix with $a_k = e^{j\theta_k}$ for AWGN channels and obeying independently identically distributed (iid) complex Gaussian distribution with zero mean and $E[|a_k|^2] = 1$ for Rayleigh fading channels. For SDMA systems, each element of $\boldsymbol{H}$ is iid complex Gaussian distributed with zero mean and a variance of $1/N$. For both DS-CDMA and SDMA, we assume that $E[|x_k|^2] = 1$ and the matrix $\boldsymbol{H}$ is normalized to satisfy $E[\boldsymbol{H}^H\boldsymbol{H}] = \boldsymbol{I}$. Correspondingly, the noise vector $\boldsymbol{n}$ is a multivariate complex Gaussian noise vector distributed with zero mean and a covariance matrix $E\left[\boldsymbol{n}\boldsymbol{n}^H\right] = 2\sigma^2\boldsymbol{I}_N$, where for DS-CDMA $\sigma^2 = 1/(2\gamma)$ with γ representing the average signal-to-noise ratio (SNR) per symbol, while for SDMA $\sigma^2 = 1/(2N\gamma)$ with γ denoting the average SNR per symbol per receive antenna.

In [13], an equivalent real-valued MIMO model has been proposed for equalization in V-BLAST systems. Based on the N-length observation vector $\boldsymbol{y}$ as seen in (1), an equivalent $2N$-length real-valued vector can be formed, which satisfies the MIMO equation

$$\boldsymbol{y}_R = \boldsymbol{H}_R\boldsymbol{x}_R + \boldsymbol{n}_R \tag{2}$$

where

$$\begin{aligned} \boldsymbol{y}_R &= \left[\Re\{\boldsymbol{y}^T\}, \Im\{\boldsymbol{y}^T\}\right]^T, \ \boldsymbol{x}_R = \left[(\boldsymbol{x}^{(I)})^T, (\boldsymbol{x}^{(Q)})^T\right]^T \\ \boldsymbol{H}_R &= \begin{bmatrix} \Re\{\boldsymbol{H}\} & -\Im\{\boldsymbol{H}\} \\ \Im\{\boldsymbol{H}\} & \Re\{\boldsymbol{H}\} \end{bmatrix}, \ \boldsymbol{n}_R = \begin{bmatrix} \Re\{\boldsymbol{n}\} \\ \Im\{\boldsymbol{n}\} \end{bmatrix} \end{aligned} \tag{3}$$

When the MMSE-MUD is applied based on (2), the decision variable vector $\boldsymbol{z}$ for the K users or, specifically, the decision variable $z_k^{(I)}$ (or $z_k^{(Q)}$) for user $x_k^{(I)}$ ($x_k^{(Q)}$) can be expressed as

$$\begin{aligned} \boldsymbol{z} &= \boldsymbol{W}^T\boldsymbol{y}, \\ z_k^{(\cdot)} &= \boldsymbol{w}_k^{(\cdot)T}\boldsymbol{y}, \ k = 1, 2, \ldots, K \end{aligned} \tag{4}$$

where $(\cdot)$ is for (I) or (Q), while $\boldsymbol{W}$ and $\boldsymbol{w}_k^{(\cdot)}$ optimized in MMSE sense are given by

$$\begin{aligned} \boldsymbol{W} &= \left(\boldsymbol{H}_R\boldsymbol{H}_R^T + 2\sigma^2\boldsymbol{I}_{2N}\right)^{-1}\boldsymbol{H}_R, \\ \boldsymbol{w}_k^{(\cdot)} &= \frac{\boldsymbol{R}_k^{-1}\boldsymbol{h}_k^{(\cdot)}}{1 + \boldsymbol{h}_k^{(\cdot)T}\boldsymbol{R}_k^{-1}\boldsymbol{h}_k^{(\cdot)}}, \ k = 1, 2, \ldots, K \end{aligned} \tag{5}$$

where $\boldsymbol{R}_k$ is the autocorrelation matrix of interference-plus-noise expressed as $\boldsymbol{R}_k = \boldsymbol{H}_R\boldsymbol{H}_R^T + 2\sigma^2\boldsymbol{I}_{2N} - \boldsymbol{h}_k^{(\cdot)}\boldsymbol{h}_k^{(\cdot)T}$.

According to [4, 14], $z_k^{(\cdot)}$ can be approximated as the Gaussian random variable having the PDF

$$\begin{aligned} f(z_k^{(\cdot)}|x_k^{(\cdot)}) &= \frac{1}{\sqrt{2\pi}\sigma_k^{(\cdot)}}\exp\left(-\left[z_k^{(\cdot)} - m_k^{(\cdot)}\right]^2/2\sigma_k^{(\cdot)2}\right), \\ k &= 1, 2, \ldots, K \end{aligned} \tag{6}$$

where $\sigma_k^{(\cdot)2} = (\sigma_k^{(\cdot)})^2$, the mean and variance are given by

$$m_k^{(\cdot)} = \frac{\bar{\gamma}_k^{(\cdot)}}{1 + \bar{\gamma}_k^{(\cdot)}}x_k^{(\cdot)}, \ \sigma_k^{(\cdot)2} = \frac{\bar{\gamma}_k^{(\cdot)}}{2(1 + \bar{\gamma}_k^{(\cdot)})^2} \tag{7}$$

respectively. In (7), $\bar{\gamma}_k^{(\cdot)} = \boldsymbol{h}_k^{(\cdot)T}\boldsymbol{R}_k^{-1}\boldsymbol{h}_k^{(\cdot)}$ represents the instantaneous SINR for detection of $x_k^{(\cdot)}$.

A. Reliability Measurement

According to the studies in [8, 9], we know that, in order for the RMD/MS-MMSE MUD to achieve the near-optimum error performance, the reliabilities of the K users must be evaluated in an optimum way based on, for example, the MAP principles, so as to minimize the probability of error. In general, let the signal set of a M-ary communication system be given by $\mathcal{S} = \{s_0, s_1, \ldots, s_{M-1}\}$ and let the observation for detection be z. Then, according to the MAP principles, the estimate to the transmit symbol, say x, can be decided as

$$\hat{x} = \arg\max_{s_i \in \mathcal{S}}\{f(s_i|z)\} \tag{8}$$

This decision rule results in an error probability of detection [15]

$$\begin{aligned} p_e &= 1 - \max_{s_i \in \mathcal{S}}\{f(s_i|z)\} \\ &= 1 - \frac{\max_{s_i \in \mathcal{S}}\{\pi_i f(z|s_i)\}}{\sum_{s_j \in \mathcal{S}}\pi_j f(z|s_j)} \end{aligned} \tag{9}$$

where $\pi_j, \ j = 0, \ldots, M-1$, are the *a-priori* probabilities and, from the first to the second equation, we applied $f(z) = \sum_{s_j \in \mathcal{S}}\pi_j f(z|s_j)$.

Equation (9) explicitly shows that the error probability of detection decreases as the value of the term

$$L = \frac{\max_{s_i \in \mathcal{S}}\{\pi_i f(z|s_i)\}}{\sum_{s_j \in \mathcal{S}}\pi_j f(z|s_j)} \tag{10}$$

increases. In other words, the detection becomes more reliable as the value of L increases. Therefore, the reliability of the detection can be measured based on (10) in the sense of MAP or of minimum symbol error probability.

For the conventional MMSE-MUD operated in complex domain, the decision variable for $x_k = x_k^{(I)} + jx_k^{(Q)}$ is $z_k = z_k^{(I)} + jz_k^{(Q)}$ and the reliability for detection of x_k can be evaluated based on z_k. Furthermore, for the I-Q modulation schemes, $x_k^{(I)}$ and $x_k^{(Q)}$ can be detected separately for the sake of reducing the detection complexity [1]. However, the reliability of x_k does not reflect exactly the reliabilities of $x_k^{(I)}$ and $x_k^{(Q)}$. This is because the decision variable z_k of x_k includes two noise samples, expressed, say, by $n_k^{(I)}$ and $n_k^{(Q)}$. Therefore, the reliability measurement based on z_k is only optimum in terms of x_k, but not necessary optimum for $x_k^{(I)}$ and $x_k^{(Q)}$.

In order to understand the philosophy and gain further insight from it, we define a so-called *noise recognition factor* (NRF) as

$$\varrho = \frac{\max\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2\}}{\sum_{j=1}^{M}\sigma_j^2} \tag{11}$$

where $\sigma_1^2, \sigma_2^2, \ldots, \sigma_M^2$ denote the variances of the noise samples invoked and M is the number of noise samples. Physically, (11) reflects fact that the overall behavior of the noise samples is dominated by the one having the largest variance. When $\sigma_1^2 = \sigma_2^2 = \ldots = \sigma_M^2$, (11) reduces to

$$\varrho = 1/(\text{number of invoked noise samples}) \tag{12}$$

[1]For MQAM, x_k can be any of the M possible symbols, while $x_k^{(I)}$ (or $x_k^{(Q)}$) is one of the $\sqrt{M}$ possible symbols.

Based on the above definition, the reliability measurement based on z_k corresponds to a NRF of $\varrho = 1/2$. However, if we measure independently the reliabilities of $x_k^{(I)}$ and $x_k^{(Q)}$ based on $z_k^{(I)}$ and $z_k^{(Q)}$, then, each of these two measurements includes only one noise sample, resulting in a NRF of $\varrho = 1$. Since the reliability measurement having $\varrho = 1$ is capable of revealing the true and real-time reliability of a symbol being detected, we can be implied that it should outperform the other reliability measurements having $\varrho < 1$ in terms of the resultant error performance, when they are invoked in the SIC-type MUDs. Note that, in literature, the reliability for the SIC-type MUDs is usually directly measured by the SINR [11, 16, 17], or by its modified versions [18–20]. Since this class of reliability measurements contain an infinite number of noise samples, hence, their NRF is $\varrho = 0$. This explains why the RMD/MS-MMSE MUD, which uses the reliability measurement of $\varrho = 1$, is so efficient that it outperforms the existing MUDs of its kind [4, 11, 16–21].

Consequently, given the decision variables $z_k^{(I)}$ ($z_k^{(Q)}$) as shown in (4) for $x_k^{(I)}$ ($x_k^{(Q)}$), where $x_k^{(I)}$ ($x_k^{(Q)}$) $\in \mathcal{S}' = \{s_0, \ldots, s_{\sqrt{M}-1}\}$, the reliability of $x_k^{(I)}$ ($x_k^{(Q)}$) can be evaluated by

$$L_k^{(\cdot)} = \frac{\max_{s_i \in \mathcal{S}'}\{\pi_i f(z_k^{(\cdot)}|s_i)\}}{\sum_{s_j \in \mathcal{S}'} \pi_j f(z_k^{(\cdot)}|s_j)},\ k = 1, 2, \ldots, K \tag{13}$$

Finally, when applying (6) into (13) and assuming that the transmit symbols obey iid distribution, the reliability for detection of $x_k^{(I)}$ or $x_k^{(Q)}$ can be expressed as

$$L_k^{(\cdot)} = \frac{\max_{s_i \in \mathcal{S}'}\left\{\exp\left[-\left(\frac{1+\bar{\gamma}_k^{(\cdot)}}{\sqrt{\bar{\gamma}_k^{(\cdot)}}} z_k^{(\cdot)} - \sqrt{\bar{\gamma}_k^{(\cdot)}} s_i\right)^2\right]\right\}}{\sum_{s_j \in \mathcal{S}'} \exp\left[-\left(\frac{1+\bar{\gamma}_k^{(\cdot)}}{\sqrt{\bar{\gamma}_k^{(\cdot)}}} z_k^{(\cdot)} - \sqrt{\bar{\gamma}_k^{(\cdot)}} y s_j\right)^2\right]},\quad k = 1, 2, \ldots, K \tag{14}$$

From (13) or (14), we can find that computing the kth user's reliability requires to evaluate $2\sqrt{M}$ exponential functions, half for the real part and half for the imaginary part. By contrast, if based on (10), computing the kth user's reliability requires to evaluate M exponential functions. Hence, by detecting $x_k^{(I)}$ and $x_k^{(Q)}$ separately, we can not only enhance the detection performance but also reduce significantly the detection complexity.

III. RMD/MS-MMSE Detection Algorithm

The RMD/MS-MMSE MUD considered in this section is similar to that proposed in [9], where BPSK baseband modulation was assumed. When the I-Q modulation is employed, according to our analysis in Section II, the real and imaginary parts of $x_1, x_2, \ldots, x_K$ are detected separately in the principles of SIC, similar to the detection employed in the V-BLAST systems [11, 16, 17]. The RMD/MS-MMSE MUD is divided into $2K$ detection stages. Each stage detects half a symbol, such as $x_k^{(I)}$ or $x_k^{(Q)}$, of a user, which is the most reliable of those having not been detected. The reliabilities are measured according to (14).

Let $\boldsymbol{y}_R^{(0)} = \boldsymbol{y}_R$, $\boldsymbol{W}^{(0)} = \boldsymbol{W}$, while $\boldsymbol{y}_R^{(s)}$ and $\boldsymbol{W}^{(s)}$ be the modified observation vector and weight matrix achieving MMSE detection after the sth stage interference cancellation. Then, the detection procedure of the RMD/MS-MMSE MUD can be described as the following steps.

Initialization:

$$\boldsymbol{y}_R^{(0)} = \boldsymbol{y}_R,\ \boldsymbol{W}^{(0)} = \boldsymbol{W}. \tag{15}$$

Detection: for $s = 1, 2, \ldots, 2K$, execute

1) *Forming decision variables*:

$$\boldsymbol{z}^{(s)} = \left(\boldsymbol{W}^{(s-1)}\right)^T \boldsymbol{y}_R^{(s-1)}. \tag{16}$$

2) *Determining the most reliable*:
For the user symbols $k'_1, k'_2, \ldots, k'_{2K-s+1}$ that have not been detected, compute their reliabilities according to (14), and find the most reliable as

$$k_I^{(s)} (\text{or } k_Q^{(s)}) = \arg\max_{k'_i}\{L_{k'_1}^{(\cdot)}, \ldots, L_{k'_{2K-s+1}}^{(\cdot)}\}. \tag{17}$$

3) *Detection of the most reliable*:
Based on the decision variable $z_{k_I^{(s)}}^{(s)}$ (or $z_{k_Q^{(s)}}^{(s)}$), detect the corresponding real (or imaginary) symbol $\hat{x}_{k_I^{(s)}}^{(I)}$ (or $\hat{x}_{k_Q^{(s)}}^{(Q)}$).

4) *Interference cancellation*:

$$\begin{aligned} \boldsymbol{y}_R^{(s)} &= \boldsymbol{y}_R^{(s-1)} - \boldsymbol{h}_{k_I^{(s)}}^{(I)} \hat{x}_{k_I^{(s)}}^{(I)}, \\ \text{or } \boldsymbol{y}_R^{(s)} &= \boldsymbol{y}_R^{(s-1)} - \boldsymbol{h}_{k_Q^{(s)}}^{(Q)} \hat{x}_{k_Q^{(s)}}^{(Q)}. \end{aligned} \tag{18}$$

5) *Update*:

$$\begin{aligned} \boldsymbol{W}^{(s)} &= \left[\boldsymbol{W}^{(s-1)} + \frac{\boldsymbol{w}_{k_I^{(s-1)}} \boldsymbol{h}_{k_I^{(s)}}^{(I)T} \boldsymbol{W}^{(s-1)}}{1 - \boldsymbol{h}_{k_I^{(s)}}^{(I)T} \boldsymbol{w}_{k_I^{(s-1)}}}\right] \boldsymbol{P}^{(s)}, \\ \text{or } \boldsymbol{W}^{(s)} &= \left[\boldsymbol{W}^{(s-1)} + \frac{\boldsymbol{w}_{k_Q^{(s-1)}} \boldsymbol{h}_{k_Q^{(s)}}^{(Q)T} \boldsymbol{W}^{(s-1)}}{1 - \boldsymbol{h}_{k_Q^{(s)}}^{(Q)T} \boldsymbol{w}_{k_I^{(s-1)}}}\right] \boldsymbol{P}^{(s)}. \end{aligned} \tag{19}$$

Note that, in (19), $\boldsymbol{w}_{k_I^{(s-1)}}$ ($\boldsymbol{w}_{k_Q^{(s-1)}}$) and $\boldsymbol{h}_{k_I^{(s)}}^{(I)}$ ($\boldsymbol{h}_{k_Q^{(s)}}^{(Q)}$) correspond to the symbol detected at the $(s-1)$th detection stage, $\boldsymbol{P}^{(s)}$ is a permutation matrix obtained from $\boldsymbol{I}_{2N}$ by deleting the columns corresponding to the symbols having been detected.

For the implementation of the RMD/MS-MMSE MUD as above described, the algorithms, e.g., in [22–26], proposed for the V-BLAST systems can be modified for the RMD/MS-MMSE MUD. Additionally, for evaluation of the reliabilities, as shown in (14), $\bar{\gamma}_k^{(\cdot)}$ needs to be computed at each iteration for all the undetected symbols. It can be shown that $\bar{\gamma}_k^{(\cdot)}$ after the sth detection stage can be expressed in the form of [4]

$$\bar{\gamma}_k^{(\cdot)} = \boldsymbol{h}_k^{(s)T} \boldsymbol{R}_k^{-1} \boldsymbol{h}_k^{(s)} = \frac{1}{1 - \boldsymbol{h}_k^{(s)T} \boldsymbol{w}_k^{(s)}} - 1 \tag{20}$$

for an undetected symbol k. For all the undetected symbols, we have

$$\bar{\gamma}_k^{(\cdot)} = \frac{1}{1 - (\boldsymbol{H}^{(s)T} \boldsymbol{W}^{(s)})_{k^{(\cdot)}, k^{(\cdot)}}} - 1 \tag{21}$$

where $\boldsymbol{H}^{(s)}$ corresponds to the undetected symbols after the sth detection stage, while $(\boldsymbol{A})_{k,k}$ denotes the (k,k)th element of $\boldsymbol{A}$. Based on the above formulas and with the aid of the algorithms proposed in [22–26], it can be shown that, the complexity of the RMD/MS-MMSE MUD for a given MQAM modulation is on the order of $\mathcal{O}\left(c_1 K N + c_2 N^2\right)$ per user, where c_1 and c_2 are certain constants. Therefore, when given N, the complexity of the RMD/MS-MMSE MUD increases only linearly with K of the number of users supported.

Note again that, in [13], an real-valued MIMO model has been proposed for equalization in V-BLAST systems. The studies show that, by detecting separately the real and imaginary parts of the transmit symbols, a performance gain can be attained in comparison with the conventional V-BLAST systems [11, 12], which detect the real and imaginary parts of the transmit symbols simultaneously. However, since the detector in [13] uses a reliability measurement scheme having a NRF of $\varrho = 0$, the performance improvement over the conventional V-BLAST systems is marginal. In comparison with the detection scheme proposed in [13], our RMD/MS-MMSE MUD uses the reliability measurement with a NRF of $\varrho = 1$. It significantly outperforms the detection scheme proposed in [13] in terms of their achievable error performance.

IV. Performance Results

In this section, the BER performance of the SDMA and DS-CDMA systems employing two baseband modulation schemes, namely the QPSK (4QAM) and 16QAM, are evaluated. As in [9], the SDMA and DS-CDMA systems may be full-load ($K = N$) or overload ($K > N$). The BER performance for the SDMA systems is shown in Fig. 1 for QPSK and Fig. 4 for 16QAM. For the DS-CDMA systems, the BER performance over AWGN channels is depicted in Fig. 2 for QPSK and Fig. 5 for 16QAM, while the BER over flat Rayleigh fading channels is depicted in Fig. 3 for QPSK and Fig. 6 for 16QAM. For the sake of comparison, in Figs. 1 - 3, the single-user BER bound and the BER of the conventional MMSE-MUD for the full-load SDMA or DS-CDMA systems are illustrated.

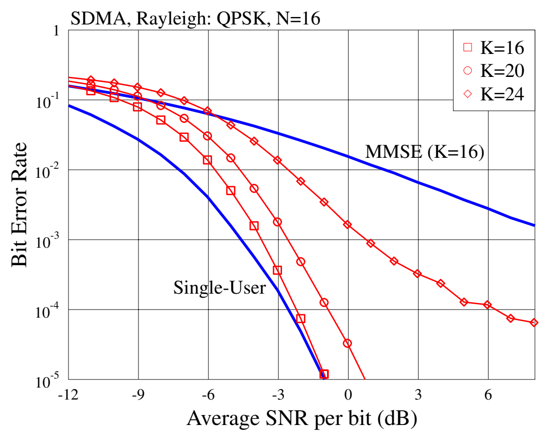

Fig. 1. BER versus average SNR per bit performance of the SDMA systems using QPSK, when communicating over Rayleigh fading channels.

From the BER results shown in Figs. 1-6, we can derive the following observations.

- First, for the full-load SDMA systems and the full-load DS-CDMA systems communicating over either AWGN or flat Rayleigh fading channels, and for the QPSK modulation scheme, the RMD/MS-MMSE MUD is generally capable of achieving the BER that is very close to the single-user BER bound. This observation becomes more declared when the SNR increases. As seen in Figs. 1 - 3, when the SNR is sufficiently high, the BER of the full-load systems converges to the single-user BER bound.
- Second, when QPSK is considered, the RMD/MS-MMSE MUD is a highly effective detector for the overload SDMA or overload DS-CDMA systems. As the results in Figs. 1, 2 and 3 show, the BER of the RMD/MS-MMSE MUD supporting $K = 3N/2$ users is still much better than that of the conventional MMSE-MUD supporting $K = N$ users, provided that the SNR per bit is a reasonable value.
- Third, when the constellation size increases from $M = 4$ to $M = 16$, the BER performance of the full-load systems moves

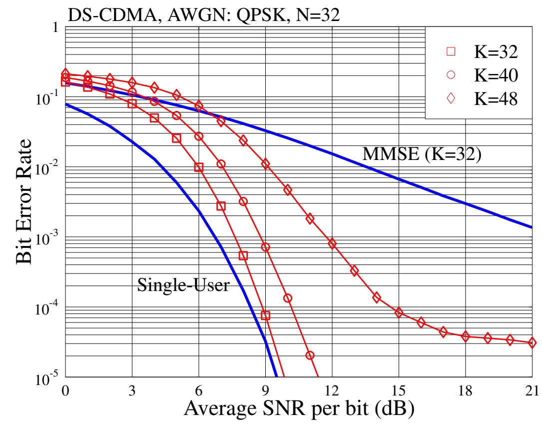

Fig. 2. BER versus SNR per bit performance of the DS-CDMA systems using QPSK and binary random spreading sequences, when communicating over AWGN channels.

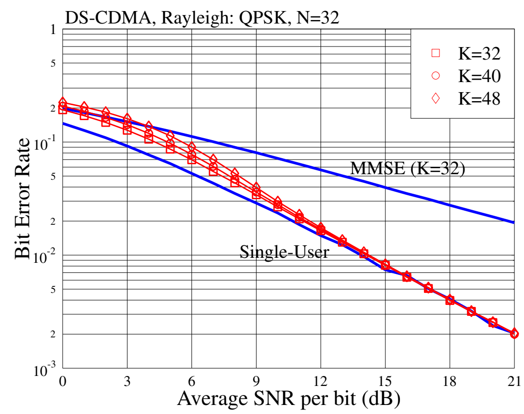

Fig. 3. BER versus average SNR per bit performance of the DS-CDMA systems using QPSK and binary random spreading sequences, when communicating over flat Rayleigh fading channels.

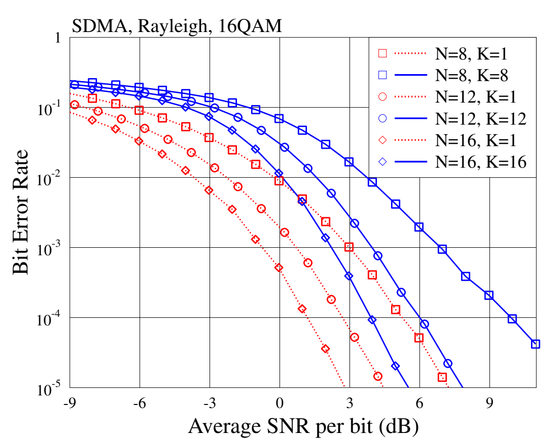

Fig. 4. BER versus average SNR per bit performance of the SDMA systems using 16QAM, when communicating over Rayleigh fading channels.

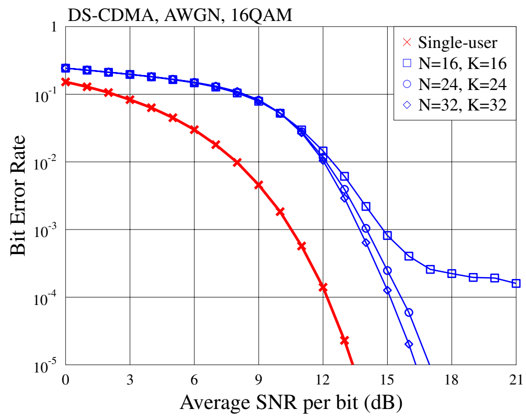

Fig. 5. BER versus SNR per bit performance of the DS-CDMA systems using 16QAM and binary random spreading sequences, when communicating over AWGN channels.

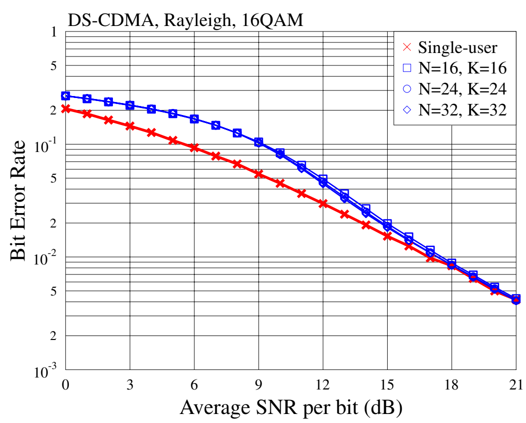

Fig. 6. BER versus average SNR per bit performance of the DS-CDMA systems using 16QAM and binary random spreading sequences, when communicating over flat Rayleigh fading channels.

away from the single-user BER bound. However, as shown in Figs. 4 - 6, when the system size increases, as predicted, the BER performance becomes closer to the single-user BER bound.

In **conclusion**, from the studies in [8–10] and this paper, we can be convinced that the RMD/MS-MMSE MUD is a highly promising detection scheme. It has low-complexity, can achieve the BER performance close to the optimum and can be applied for nearly any communications scenarios, such as, SDMA, CDMA, or any other MIMO systems experiencing multiuser interference, inter-carrier interference, inter-symbol interference, etc. Furthermore, it is highly effective, when operated in either AWGN or fading environments.

References

[1] S. Verdu, "Minimum probability of error for asynchronous Gaussian multiple-access channels," *IEEE Transactions on Information Theory*, vol. 32, no. 1, pp. 85 – 96, January 1986.
[2] S. Verdu, *Multiuser Detection*. Cambridge University Press, 1998.
[3] S. Verdu, "Minimum probability of error for asynchronous multiple access communication systems," in *IEEE Military Communications Conference*. IEEE, October 1983, pp. 213–219.
[4] L.-L. Yang, *Multicarrier Communications*. Chichester, United Kingdom: John Wiley, 2009.
[5] X. Wang and H. V. Poor, *Wireless Communication Systems - Advanced Techniques for Signal Reception*. Prentice Hall, 2003.
[6] F. Hasegawa and et.al, "Speed and accuracy comparison of techniques for multiuser detection in synchronous CDMA," *IEEE Transactions on Communications*, vol. 52, no. 4, pp. 540 – 545, April 2004.
[7] L.-L. Yang, "Minimum mean-square error and maximum likelihood multiuser detection: Statistical properties and applications," in *IEEE 69th Veh. Tech. Conf.: VTC2009-Spring*, 26–29 April 2009.
[8] L.-L. Yang, "Using multi-stage MMSE detection to approach optimum error performance in multiantenna MIMO systems," in *IEEE 70th Veh. Tech. Conf.: VTC2009-Fall*. IEEE, 20–23 Sept. 2009.
[9] L.-L. Yang, "Receiver multiuser diversity aided multi-stage MMSE multiuser detection: A low-complexity detector fast-converging to the optimum," in *IEEE 71th Veh. Tech. Conf.: VTC2010-Spring*. IEEE, 16–19 May 2010.
[10] L.-L. Yang, "Receiver multiuser diversity aided multi-stage minimum mean-square error detection for heavily loaded DS-CDMA and SDMA systems," *IEEE Transactions on Communications (to appear)*, pp. –, 2010.
[11] P. Wolniansky, G. Foschini, G. Golden, and R. Valenzuela, "V-BLAST: an architecture for realizing very high data rates over the rich-scattering wireless channel," in *International Symposium on Signals, Systems, and Electronics (ISSSE)*. IEEE, 1998, pp. 295–300.
[12] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multiple antennas," *Bell Labs Technical Journal*, pp. 41–59, Autumn 1996.
[13] R. Fischer and C. Windpassinger, "Real versus complex-valued equalization in V-BLAST systems," *Electronics Letters*, vol. 39, no. 5, pp. 470 – 471, March 2003.
[14] H. V. Poor and S. Verdu, "Probability of error in MMSE multiuser detection," *IEEE Transactions on Information Theory*, vol. 43, no. 3, pp. 858–871, May 1997.
[15] C. W. Baum and M. B. Pursley, "Bayesian methods for erasure insertion in frequency-hop communication system with partial-band interference," *IEEE Trans. on Commun.*, vol. 40, no. 7, pp. 1231–1238, July 1992.
[16] G. Foschini, G. Golden, R. Valenzuela, and P. Wolniansky, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE Journal on Selected Areas in Communications*, vol. 17, no. 11, pp. 1841 – 1851, November 1999.
[17] G. Golden, C. Foschini, R. Valenzuela, and P. Wolniansky, "Detection algorithm and initial laboratory results using V-BLAST space-time communication architecture," *IEE Electronics Letter*, vol. 35, no. 1, pp. 14 – 16, January 1999.
[18] H. Elders-Boll and G. Alirezaei, "On the influence of substream ordering for MIMO decision-feedback detectors with higher order modulation," in *The 5th International Symposium on Wireless Personal Multimedia Communications*, October 2002, pp. 691–695.
[19] X. Zhu and R. D. Murch, "Layered space-time equalization for wireless MIMO systems," *IEEE Transactions on Wireless Communications*, vol. 2, no. 6, pp. 1189 – 1203, November 2003.
[20] ——, "Layered space-frequency equalization in a single-carrier MIMO system for frequency-selective channels," *IEEE Transactions on Wireless Communications*, vol. 3, no. 3, pp. 701 – 708, May 2004.
[21] A. Duel-Hallen, "A family of multiuser decision-feedback detectors for asynchronous code-division multiple-access channels," *IEEE Trans. on Commun.*, vol. 43, no. 2/3/4, pp. 421 – 434, Feb./Mar./Apr. 1995.
[22] J. Benesty, Y. Huang, and J. Chen, "A fast recursive algorithm for optimum sequential signal detection in a BLAST system," *IEEE Transactions on Signal Processing*, vol. 51, no. 7, pp. 1722 – 1730, July 2003.
[23] K. Kusume, M. Joham, W. Utschick, and G. Bauch, "Cholesky factorization with symmetric permutation applied to detecting and precoding spatially multiplexed data streams," *IEEE Trans. on Signal processing*, vol. 55, no. 6, pp. 3089 – 3103, Junr 2007.
[24] T. Liu and Y.-L. Liu, "Modified fast recursive algorithm for efficient MMSE-SIC detection of the V-BLAST system," *IEEE Trans. on Wireless Commun.*, vol. 7, no. 10, pp. 3713 – 3717, October 2008.
[25] B. Hassibi, "An efficient square-root algorithm for BLAST," in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP'2000)*. IEEE, June 2000, pp. II737 – II740.
[26] L. Szczecinski and D. Massicotte, "Low complexity adaptation of MIMO MMSE receivers, implementation aspects," in *IEEE Global Telecommunications Conference (GLOBECOM'05)*. IEEE, December 2005, pp. 2332– 2337.