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Design of Fixed-Point Processing Based LDPC Codes Using EXIT Charts

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Contents

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Introduction

- Low-Density Parity-Check (LDPC) codes.
  - LDPC codes [1] belong to a class of linear block codes, hence is defined by a Parity-Check Matrix (PCM);

$$\begin{bmatrix} 111010 \\ 101001 \\ 010101 \end{bmatrix}$$

  - Alternatively by a corresponding Tanner graph [2], on which the decoding process is based.

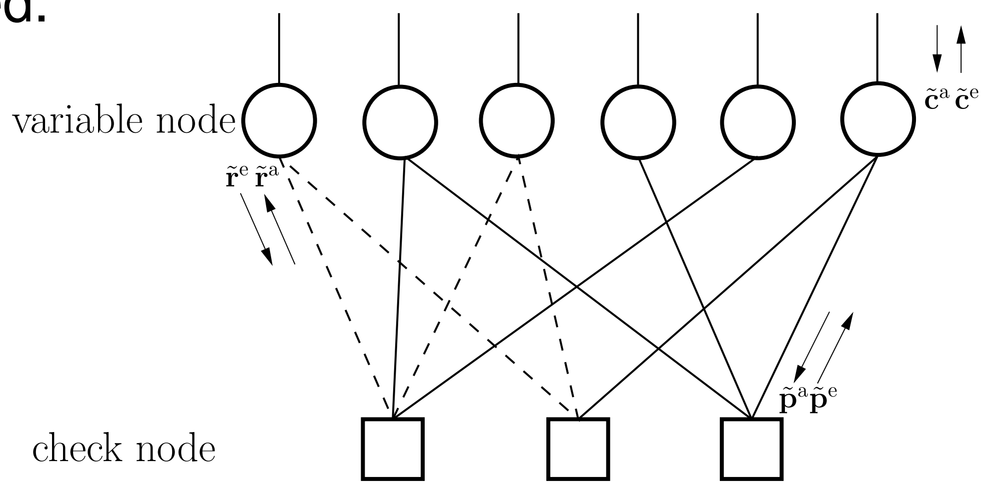


Figure 1: An example of a Tanner graph.

- Turbo-like decoder: a serial concatenation of two soft-input soft-output decoders, Check Node Decoder (CND) and Variable Node Decoder (VND).

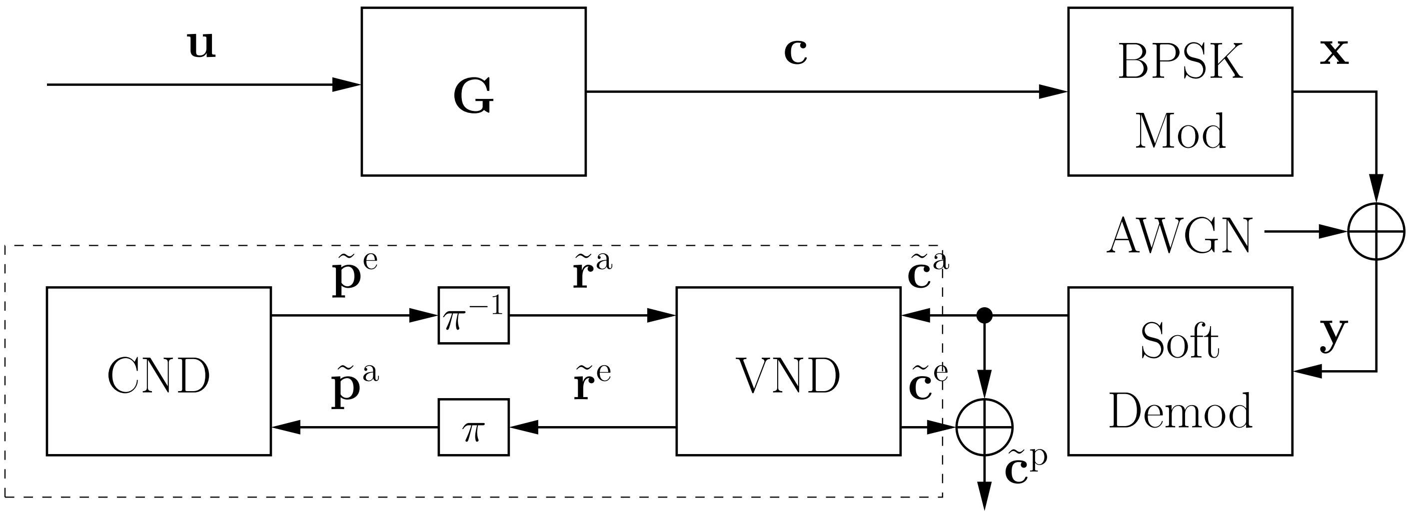


Figure 2: The schematic of encoder and decoder of LDPC codes.

  - The *a priori*, extrinsic and *a posteriori* messages are Log-Likelihood Ratios (LLRs).

- Iterative Decoding Algorithm.

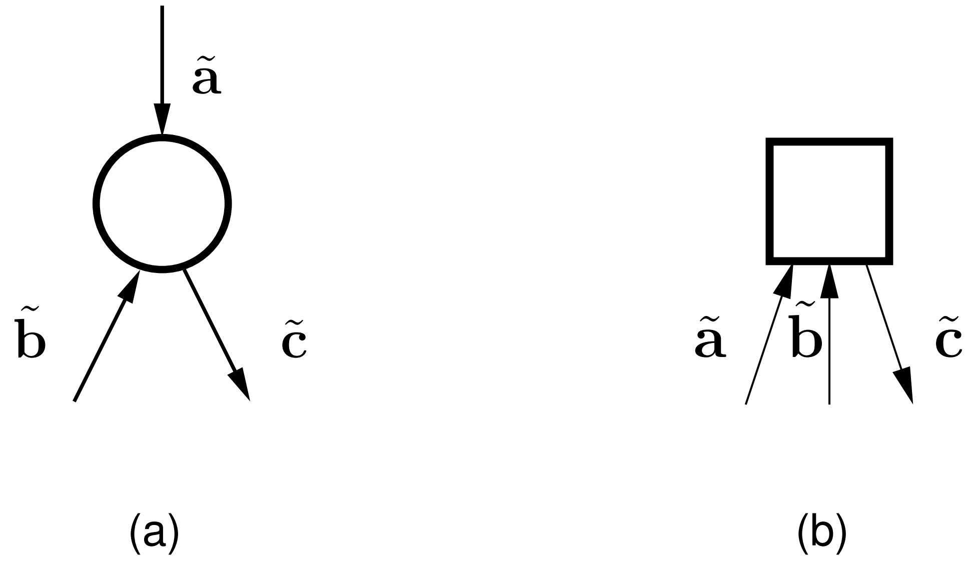


Figure 3: Basic operation units in (a) VND and (b) CND.

- VND:

$$\tilde{c} = \tilde{a} + \tilde{b} \tag{1}$$

- CND: replace $+$ by boxplus operator $\boxplus$ [3] in Eq (1).

$$\begin{aligned} \tilde{c} = \tilde{a} \boxplus \tilde{b} &= \operatorname{sign}(\tilde{a})\operatorname{sign}(\tilde{b}) \min(|\tilde{a}|, |\tilde{b}|) \\ &\quad + \log\left(1 + e^{-|\tilde{a}+\tilde{b}|}\right) - \log\left(1 + e^{-|\tilde{a}-\tilde{b}|}\right) \qquad (2) \\ &\approx \operatorname{sign}(\tilde{a})\operatorname{sign}(\tilde{b}) \min(|\tilde{a}|, |\tilde{b}|). \qquad (3) \end{aligned}$$

* Eq (2) is referred as the Log Sum-Product Algorithm (SPA) [4].
* The approximated version in Eq (3) is referred as the Min-Sum algorithm (MSA) [5].
* Correction function $\log\left(1+e^{-\tilde{x}}\right)$ is implemented by a Look-Up Table (LUT).

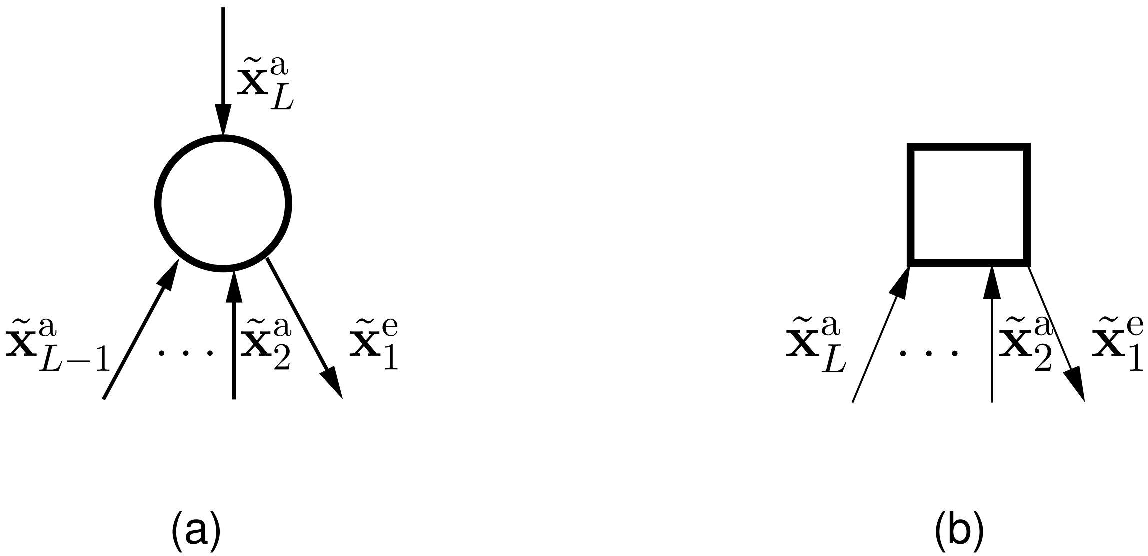


Figure 4: Decoding processings of a (a) VND and (b) CND with high degrees.

– For a VN connected to $L-1$ CNs $(L \geq 2)$ in the Tanner graph, the extrinsic message can be computed using forward-backward algorithm.

$$\tilde{x}_i^{\mathrm{e}} = \begin{cases} \tilde{b}_2 & \text{if } i = 1 \\ \tilde{f}_{i-1} + \tilde{b}_{i+1} & \text{if } 1 < i < L \\ \tilde{f}_{L-1} & \text{if } i = L \end{cases}, \tag{4}$$

$$\tilde{f}_i = \begin{cases} \tilde{x}_1^{\mathrm{a}} & \text{if } i = 1 \\ \tilde{x}_i^{\mathrm{a}} + \tilde{f}_{i-1} & \text{if } i > 1 \end{cases}, \tag{5}$$

$$\tilde{b}_i = \begin{cases} \tilde{x}_L^{\mathrm{a}} & \text{if } i = L \\ \tilde{x}_i^{\mathrm{a}} + \tilde{b}_{i+1} & \text{if } i < L \end{cases}. \tag{6}$$

– Likewise, the decoding in a CN connected to L VNs $(L \geq 2)$ can be obtained by replacing $+$ by $\boxplus$.

- Fixed-Point (FP) representation.
  - Notation $FP(x, y, z)$.
  - Two's complement [6]: from left to right, y integer bits including one sign bit (MSB) at the most left, the imaginary decimal point and z fraction bits.
  - Clipping is a usual method to control overflow. x represents the number of integer bits after clipped.
  - An example.

$$\begin{array}{lrrr} & yyyy.zz & 0100.10 & +4.50 \\ clipped & xxx.zz & 011.11 & +3.75 \end{array}$$

– When the decoder adopts FP representation, z decides the number of entries in LUT.

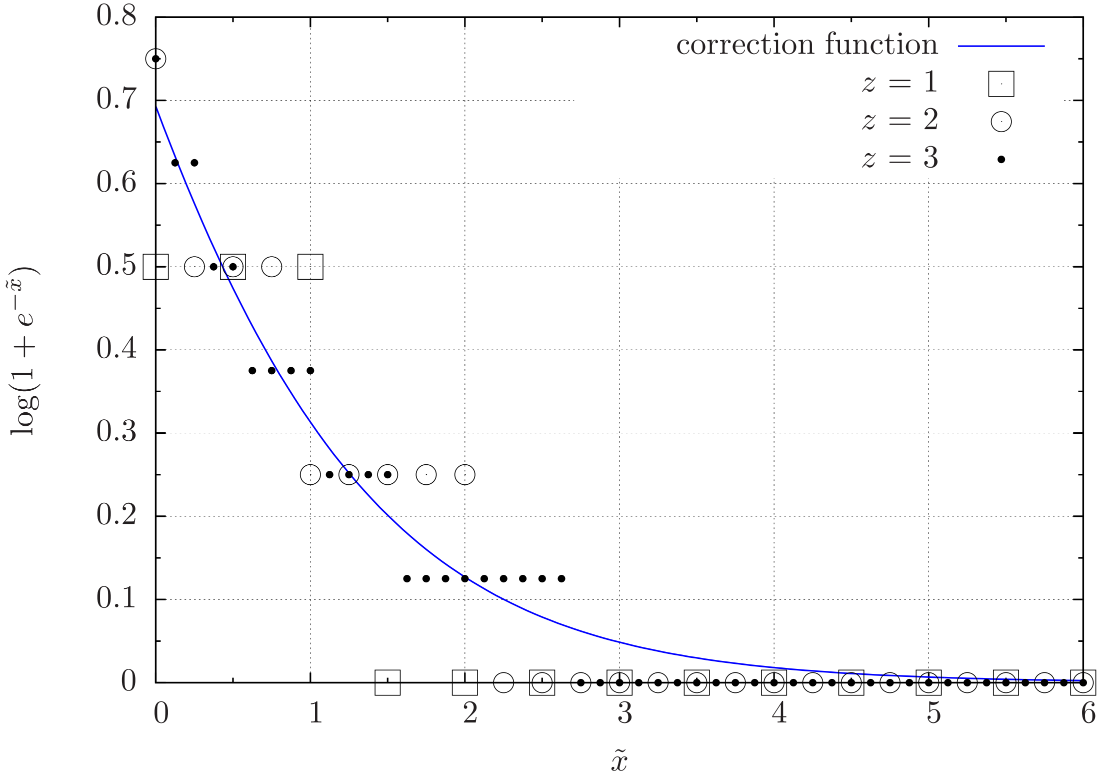
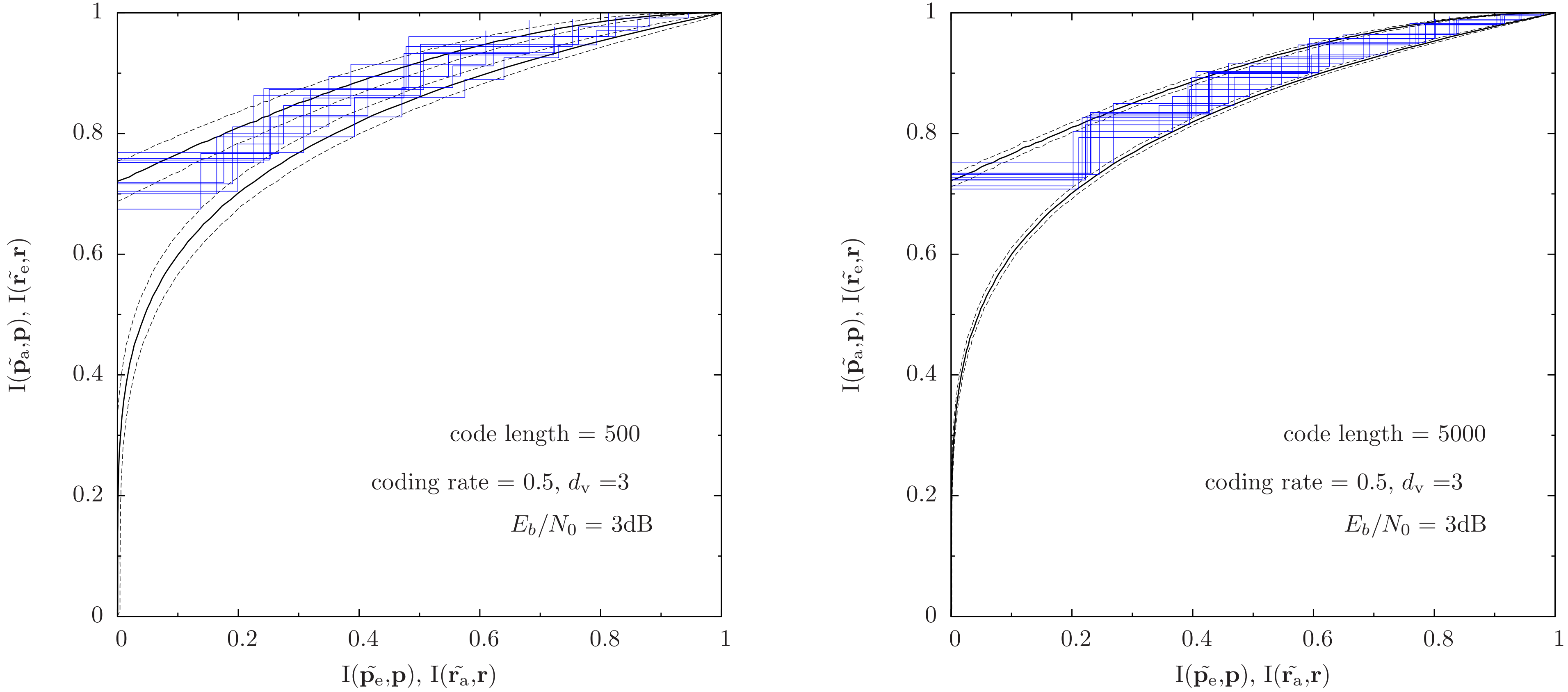


Figure 5: Correction function $\log\left(1 + e^{-\tilde{x}}\right)$ and its approximation by LUT.

- EXtrinsic Information Transfer (EXIT) charts.
  - EXIT charts [7] characterise the iteratively exchanged extrinsic information between the VND and CND. Mutual Information (MI) $I(\tilde{\mathrm{x}};\mathrm{x})$ is employed to quantify the reliability of information.
  - Histogram-based method [7, Eq (19)] of evaluating MI can accurately quantify the degradation imposed by sub-optimal decoding algorithms, which compares the histogram of LLR sequence $\tilde{\mathrm{x}}$ with the content of the bit sequence x.
  - EXIT charts can offer insights into performance degradation of iterative decoders.
  - By the aid of EXIT bands, more accurate prediction can be allowed, even when a short code is employed.

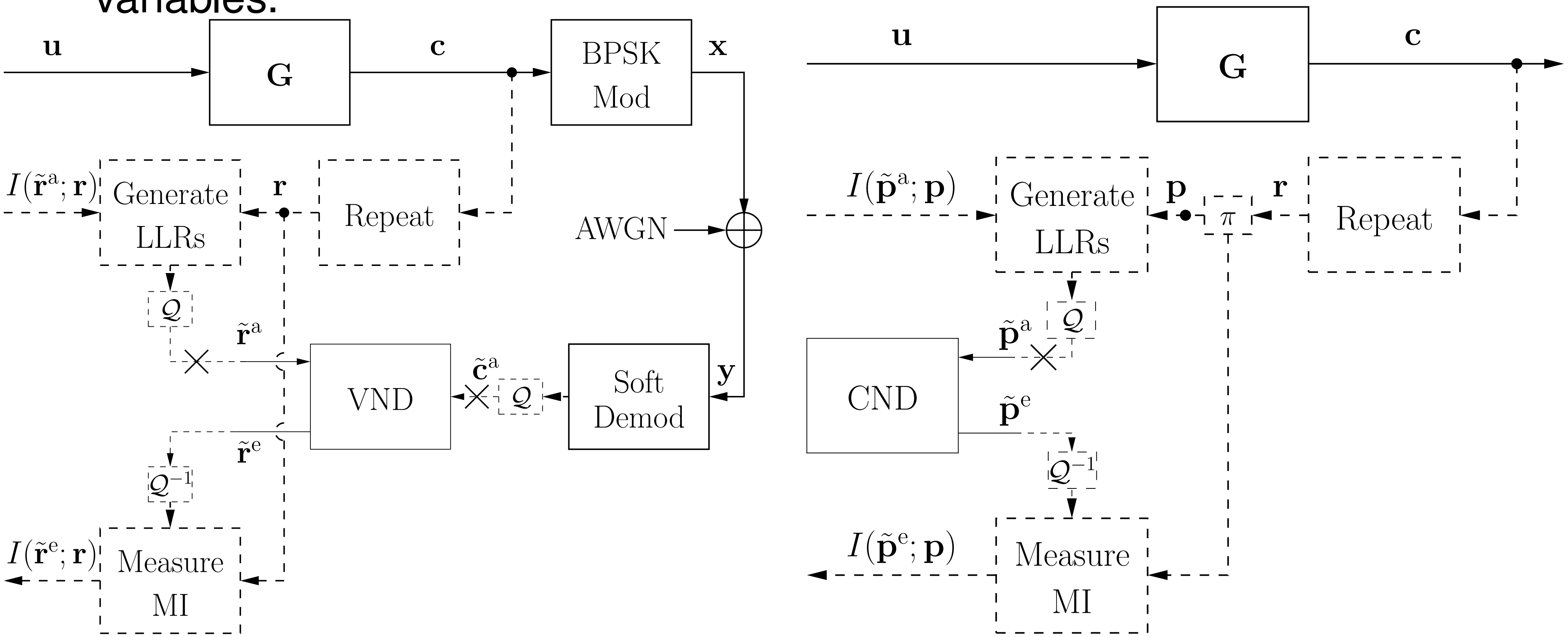
– An example of floating point EXIT charts and the bands.



(a) (b)

Figure 6: EXIT charts of two half-rate regular LDPC codes having code length of (a) 500 and (b) 5000 bits.

- Generations of EXIT functions for VND and CND, both of which operate on FP variables.



(a) VND (b) CND

Figure 7: schematics used to depict the generations of EXIT functions for the (a) VND, (b) CND, where $\times$ indicates clipping , $\mathcal{Q}$ represents conversion from floating point values to FP values and $\mathcal{Q}^{-1}$ is the inverse.

- Novel contributions of this paper:
  - We introduce the use of EXIT charts to determine the minimum Operand Width (OW), namely the value of x, y and z, of a LDPC decoder. This approach overcomes the time-consuming problem when bit-error ratio simulations are used to achieve comprehensive investigation, and it offers insights into the specific causes of the performance degradation encountered.
  - By the use of EXIT charts, a FP scheme having an overall OW of 6 bits is proposed.

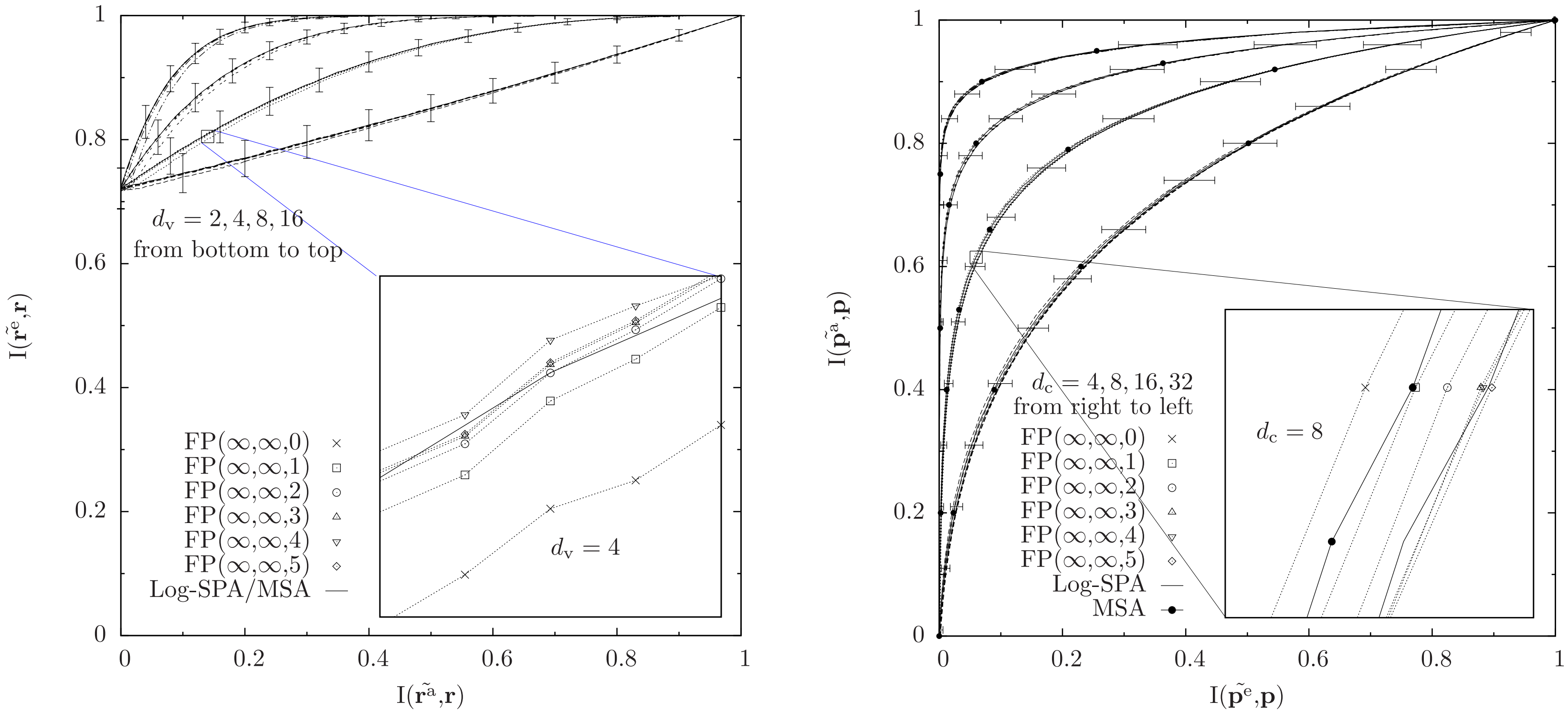
Simulation Results

- First x and z are separately considered, then y is decided by combined consideration of x and z.
  - Simulation parameters.

Table 1: Parameters used for FP EXIT chart simulations of LDPC codes.

clipped integer OW x	$1, 2, 3, 4, 5, 6$ bits
fraction OW z	$0, 1, 2, 3, 4, 5$ bits
integer OW y	$3, 4, 5,$ bits
infinite value ∞	32 bits
code length	500 bits
CN degree d_{c}	$4, 8, 16, 32$
VN degree d_{v}	$2, 4, 8, 16$
benchmarker	floating point Log-SPA and MSA

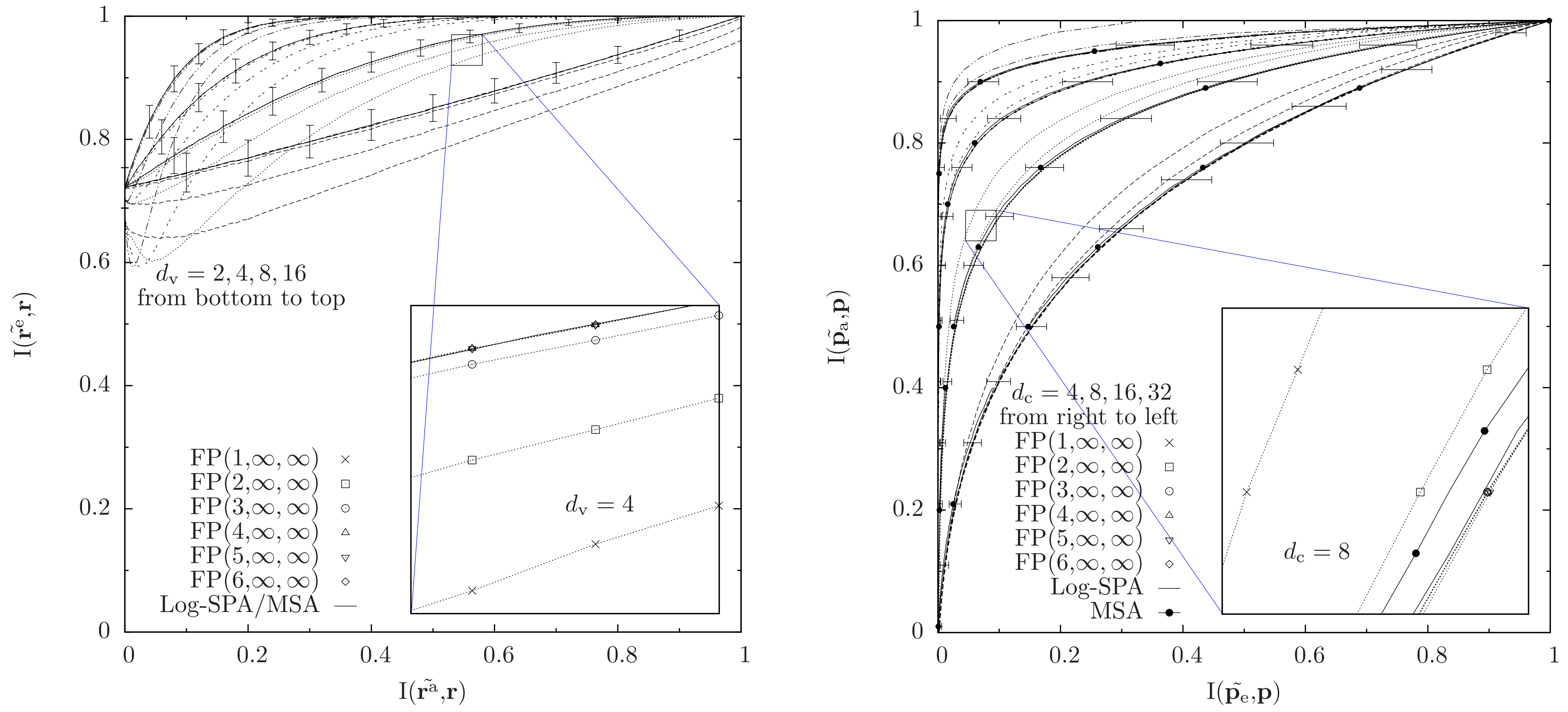
- Determining fraction OW z.



(a) VND (b) CND

Figure 8: The EXIT functions for FP implementations of LDPC codes employing various fraction OWs z, as well as VN and CN degrees, for communication over an AWGN channel having an E_b/N_0 of 3 dB.

- Determining clipped integer OW x.

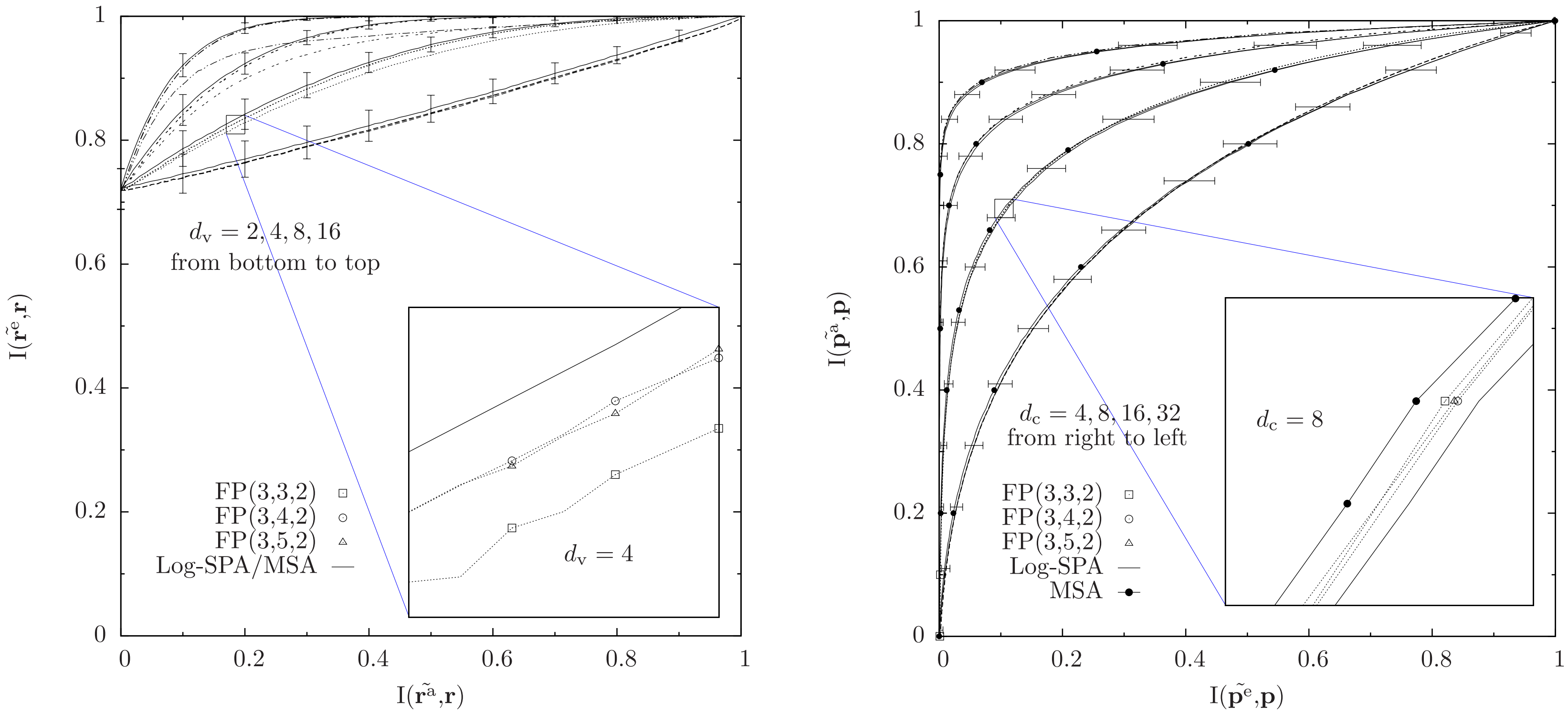


(a) VND (b) CND

Figure 9: The EXIT functions for FP implementations of LDPC codes employing various clipped integer OWs x, as well as VN and CN degrees, for communication over an AWGN channel having an E_b/N_0 of 3 dB.

- Decide fraction OW z.
  - CND: the degradation imposed by a fraction OW $z \geq 2$ is less than that imposed by MSA, and $z = 3$ approaches Log-SPA.
  - VND: $z \geq 1$ is acceptable, and $z = 2$ approaches Log-SPA.
  - To achieve better trade-off between complexity and performance, and simplify the decoder's architecture, z is selected as 2.
- Decide Clipped integer OW x.
  - CND: a clipped integer OW $x \geq 3$ is approaching Log-SPA.
  - VND: $x \geq 3$ approaches Log-SPA, but droops happen at low *a priori* mutual information region when $x = 1, 2$.
  - The minimum acceptable z is selected as 3.

- Determining integer OW x, based on the decisions of y and z.



(a) VND (b) CND

Figure 10: The EXIT functions for FP implementations of LDPC codes employing various integer OWs y, as well as VN and CN degrees, for communication over an AWGN channel having an E_b/N_0 of 3 dB.

- Decide integer OW y.
  - $x = 3$ and $z = 2$ are fixed, and y is considered from the range of $3, 4, 5$.
  - For both CND and VND, $y = 4$ is sufficient. FP$(3, 4, 2)$ achieves the most desirable trade-off.

- Explanation of the unusual ‘droop’ occured on VND EXIT functions.
  - Two conditions:
    a) integer OW clipped into too low value $x = 1, 2$,
    b) low *a priori* mutual information region $I(\tilde{\mathbf{r}}^{\mathrm{a}}; \mathbf{r}) < 0.1$.
  - Consequences:
    * Condition *a)*: Clipping makes the shape of histogram of channel output $\tilde{\mathbf{c}}^{\mathrm{a}}$ nearly two ‘spikes’ located at the upper and lower saturations limits.
    * Condition *b)*: The generated LLRs $\tilde{\mathbf{r}}^{\mathrm{a}}$ contains little information, hence most of them provide wrong indications of the content of the binary bits, which makes the two spikes less focused and more overlapped in the neighborhood of 0.
    * According to the histogram based MI mearsuring method, the resultant histogram conveys less MI even than the dual-‘spike’ like histogram.

- Various BER simulation results of FP based LDPC decoder as z or x varies, where the adopted LDPC code is half-rate regular and has code length of 500 bits and d_{v} of 3. The BER results proved our observations based on EXIT functions in Figure 8 and Figure 9.

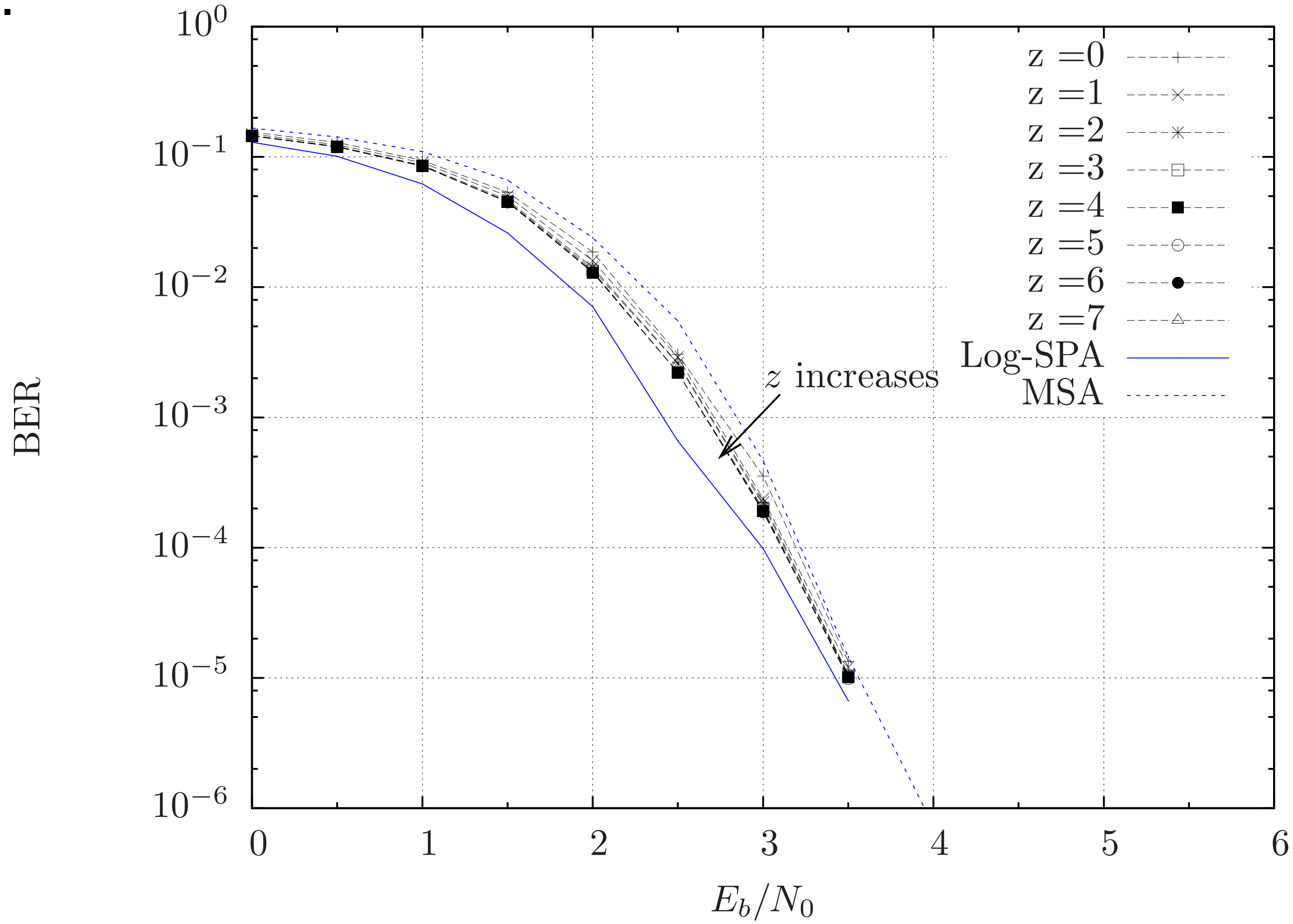


Figure 11: BER plots of LDPC codes using different fraction OWs.

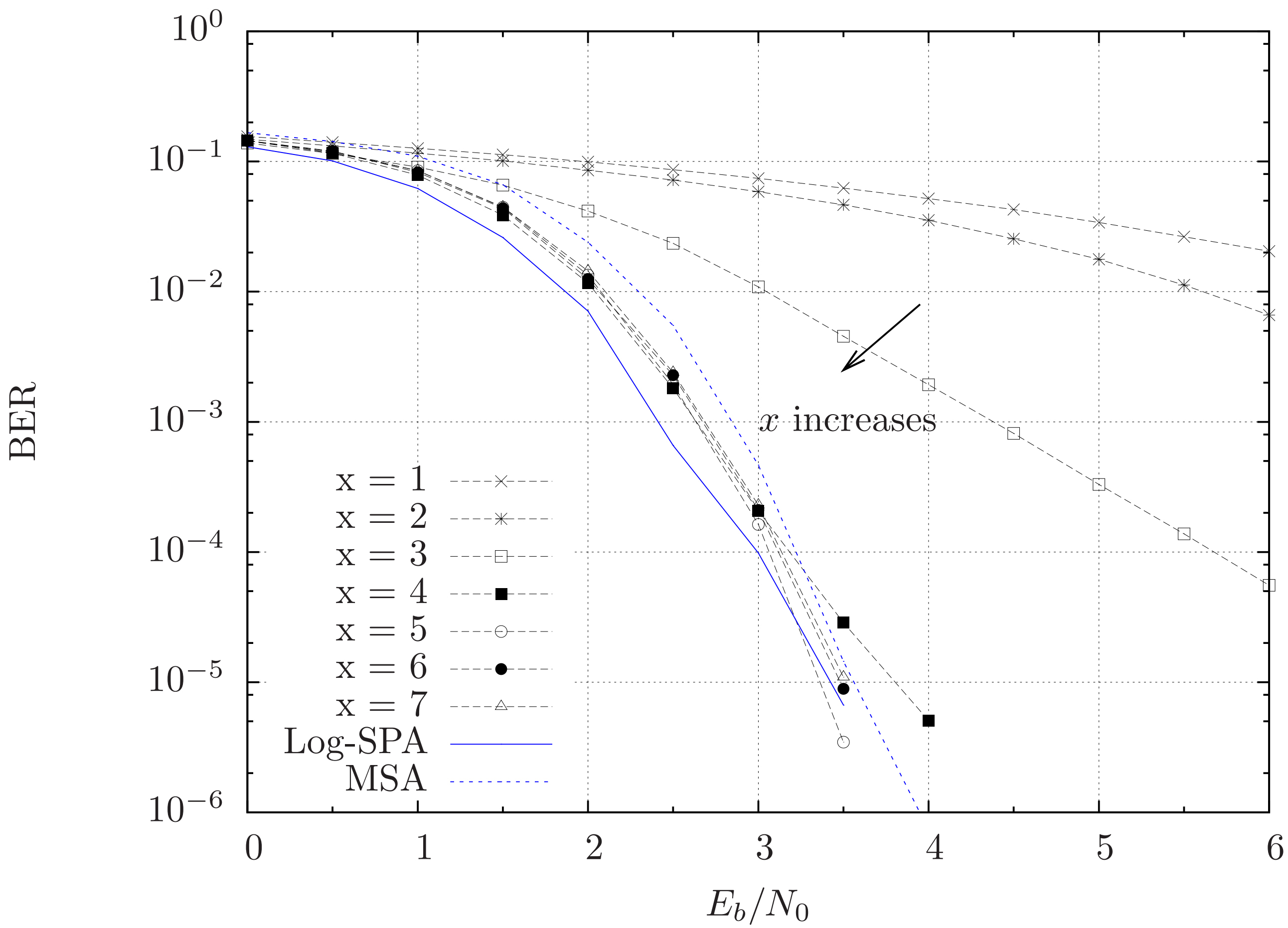


Figure 12: BER plots of LDPC codes using different clipped integer OWs.

Conclusions

- The use of EXIT charts is prosed to investigate the performance degradation caused by limited OWs used in LDPC decoders.
- Compared to BER simulations, EXIT charts overcome the time-consuming drawback, thus using EXIT charts, it is able to fully investigate the effects caused by limited OWs and others.
- EXIT charts are able to predict the convergence behaviour of turbo-like decoders, and provide extra insight into performance degradation.
- Based on investigation using EXIT charts, a scheme FP$(3, 4, 2)$ is proposed.

Thank you !