Soft-decision Star-QAM aided BICM-ID

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***Abstract*— Differentially detected non-coherent Star Quadrature Amplitude Modulation (Star-QAM) is ideal for low-complexity wireless communications, since it dispenses with high-complexity channel estimation. We conceive soft-decision based demodulation for 16-level Star-QAM (16-StQAM), which is then invoked for iterative detection aided Bit-Interleaved Coded Modulation (BICM-ID). It is shown that the proposed 16-StQAM based BICM-ID scheme achieves a coding gain of approximately 14 dBs in comparison to the 16-level identical-throughput Differential Phase-Shift Keying (16DPSK) assisted BICM scheme at a bit error ratio of 10^{-6}.**

***Index Terms*— Soft-decision, Iterative detection, Star QAM, BICM-ID, Correlated Rayleigh fading channel**

I. INTRODUCTION

Coherent detection aided Quadrature Amplitude Modulation (QAM) requires accurate Channel State Information (CSI) in order to avoid false-phase locking, especially when communicating over Rayleigh fading channels [1]–[4]. As a remedy, differentially detected non-coherent Star-QAM was proposed in [5] in order to dispense with high-complexity CSI estimation. More specifically, 16-level Star-QAM (16-StQAM) is based on two concentric 8-level Phase-Shift Keying (8PSK) constellations having two different amplitudes. Differential detection has also been investigated recently in wireless relay networks [6]–[8]. The significance of this low-complexity detection method may be expected to increase in the cooperative communications era, since it might be unrealisitic to expect from a relay station constituted by a cooperating mobile phone to estimate the channel of the link it is relaying [7], [8].

Star-QAM schemes having more than two PSK constellations are also referred to as Differential Amplitude and Phase-Shift Keying (DAPSK) schemes [9], [10]. The authors of [9], [10] have further improved the performance of DAPSK/Star-QAM schemes [9], [10]. However, despite its attractive performance versus complexity characteristics, soft-decision based demodulation has not been conceived for these Star-QAM and DAPSK schemes. This also implies that without soft-decision based demodulation, the potential power of sophisticated channel coding or coded modulation schemes cannot be fully exploited. Hence, when channel coding is incorporated into Star-QAM as in [5], its performance is far from the channel capacity due to the employment of hard-decision based demodulation. More specifically, powerful channel coding, such as Bit-Interleaved Coded Modulation (BICM) [11], [12] and Iteratively-Detected BICM (BICM-ID) [13], [14] heavily relies on the exploitation of soft-decision based demodulation.

Our novel contribution is that we will first derive the soft-decision demodulation formula for 16-StQAM. Secondly, the performance benefits of using this new formula will be demonstrated in the context of BICM and BICM-ID schemes invoked for communications over correlated Rayleigh fading channels. Note, however, that the proposed soft-decision based 16-StQAM demodulation principles may be readily extended to DAPSK schemes having more than two concentric PSK constellations. This letter is organised as follows. In Section II, the soft-decision demodulation of 16-StQAM will be presented. Our results will be discussed in Section III and our conclusions are offered in Section IV.

II. SYSTEM MODEL AND ANALYSIS

Fig. 1 shows the simplified schematic of the proposed 16-StQAM aided BICM-ID scheme. A sequence of 3-bit information symbols is encoded by a rate-3/4 BICM encoder for yielding a sequence of 4-bit coded symbols. The Most Significant Bit (MSB) of the 4-bit encoded symbol will be used for selecting the amplitude of the Phase-Shift-Keying (PSK) ring, while the remaining 3 bits will be used for selecting the phase of the complex-valued 16-StQAM symbol x_k, where the subscript k denotes the symbol index. The BICM-encoded 16-StQAM symbol is corrupted by both

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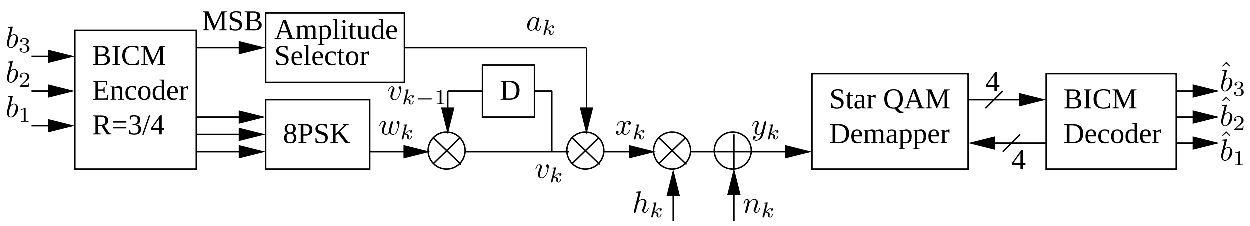


1: The schematic of the 16-StQAM aided BICM-ID scheme, where the parallel bit interleavers between the encoder/decoder and mapper/demapper are not shown for avoiding obfuscating details.

the Rayleigh fading channel h_k and the Additive White Gaussian Noise (AWGN) n_k, when it is transmitted to the receiver, as shown in Fig. 1. Iterative detection is then carried out by exchanging extrinsic information between the 16-StQAM soft demapper and BICM decoder based on the received sequence $\{y_k\}$ without the any need for CSI.

A. Star-QAM Mapper

As seen in Fig. 1, the 16-StQAM mapper consists of three components, namely the amplitude selector, the 8PSK mapper and a differential encoder. The 8PSK mapper and the differential encoder jointly form a conventional 8-level DPSK (8DPSK) mapper. The MSB of the BICM-encoded symbol, namely b_3, is used for selecting one of the two possible amplitudes. The remaining 3 bits, namely $b_2\ b_1\ b_0$, are used by the 8DPSK mapper. Note that similar to any DPSK scheme, we insert a reference symbol at the beginning of each frame before the 16-StQAM mapper.

1) Amplitude Selection: The MSB, b_3, is used for selecting the amplitude of the PSK ring, a_k. The two possible amplitude values are denoted as $a^{(1)}$ and $a^{(2)}$, respectively. When the MSB of the kth BICM-encoded symbol is given by $b_3 = 0$, the amplitude of the PSK ring will remain the same as that of the previous value $a_k = a_{k-1}$. The amplitude of the PSK ring will be switched to another value, if $b_3 = 1$. This amplitude selection mechanism may be referred to as 2-level Differential Amplitude Shift Keying (2DASK). After normalisation for maintaining a symbol energy of unity, we have $a^{(1)} = 1/\sqrt{2.5}$ and $a^{(2)} = 2/\sqrt{2.5}$. The amplitude value of the reference symbol is given by $a_0 = a^{(1)}$.

2) Phase Selection: The kth differentially encoded symbol v_k can be expressed as:

$$v_k = v_{k-1} w_k, \tag{1}$$

where $x_k = \mu(b_2\ b_1\ b_0)$ is the kth 8PSK symbol based on the 8PSK mapping function of $\mu(.)$, while v_{k-1} is the $(k-1)$st 8DPSK symbol and $|v_k|^2 = 1$. The reference symbol for the 8DPSK part is given by $v_0 = \mu(0\ 0\ 0)$.

The kth 16-StQAM symbol is then given by:

$$x_k = a_k v_k, \tag{2}$$

where $a_k \in \{a^{(1)}, a^{(2)}\}$.

B. Star-QAM Soft Demapper

The soft-decision based 16-StQAM block is placed in front of the BICM decoder of Fig. 1. The kth received symbol may then be written as:

$$y_k = h_k x_k + n_k = h_k a_k v_k + n_k \ , \tag{3}$$

where h_k is the Rayleigh fading channel's coefficient, while n_k represents the AWGN having a variance of $N_0/2$ per dimension. Assuming a slow Rayleigh fading channel, where $h_k \approx h_{k-1}$, we can rewrite (3) using (1) as:

$$\begin{aligned} y_k &= h_{k-1} a_k v_{k-1} w_k + n_k \ , \\ &= \frac{a_k}{a_{k-1}} \left(y_{k-1} - n_{k-1}\right) w_k + n_k \ , \\ &= p_k y_{k-1} w_k + \tilde{n}_k \ , \end{aligned} \tag{4}$$

where $p_k = \frac{a_k}{a_{k-1}}$ is the ratio of the kth and $(k-1)$st amplitudes, while $\tilde{n}_k = -\frac{a_k}{a_{k-1}} n_{k-1} w_k + n_k$ is the effective noise.

1) Amplitude Detection: Three amplitude ratios can be derived from the two PSK ring amplitudes of 16-StQAM as follows:

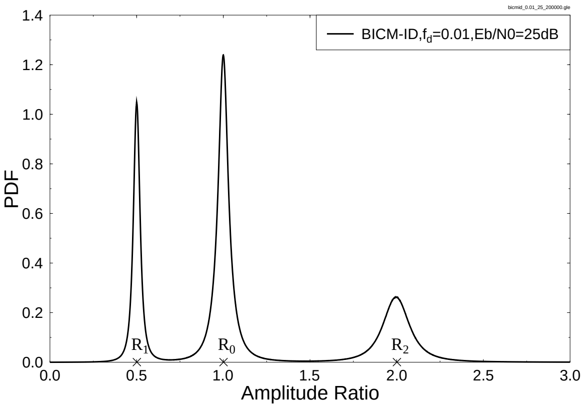
$$p_k = \begin{cases} R_0 = \dfrac{a^{(1)}}{a^{(1)}} \text{ or } \dfrac{a^{(2)}}{a^{(2)}} = 1 \\ R_1 = \dfrac{a^{(1)}}{a^{(2)}} \\ R_2 = \dfrac{a^{(2)}}{a^{(1)}} \ . \end{cases} \tag{5}$$

When the noise power is low, the amplitude ratio p_k may be approximated as:

$$\frac{|y_k|}{|y_{k-1}|} = \frac{|h_k a_k v_k + n_k|}{|h_{k-1} a_{k-1} v_{k-1} + n_{k-1}|} \ , \tag{6}$$

$$\begin{aligned} &\approx \frac{|a_k|}{|a_{k-1}|} \ , \\ &\approx p_k \ . \end{aligned} \tag{7}$$

Fig. 2 shows the Probability Density Function (PDF) of the received signal amplitude ratios $\frac{|y_k|}{|y_{k-1}|}$. It becomes plausible from Fig. 2 that the PDF peak, which is characteristic of each amplitude ratio experiences a different noise variance, although all the 16-StQAM symbols experience the same AWGN at the same E_b/N_0 value of 25 dB.



2: The PDF of the received signal amplitude ratios of 16StQAM $\frac{|y_k|}{|y_{k-1}|}$ based on (6), when communicating over correlated Rayleigh fading channels having an E_b/N_0 of 25 dB.

2) Probability Computation: The effective noise variance of $\tilde{n}_k$ in (4) depends on the amplitude ratio used at time instant k, which can be computed as:

$$\widetilde{N}_0 \;=\; N_0 + |p_k|^2|w_k|^2 N_0 = N_0(1+|p_k|^2)\ , \quad (8)$$

where $\widetilde{N}_0 = 2N_0 = N_0^{(0)}$ if $b_3 = 0$, while $\widetilde{N}_0 = (1+R_1^2)N_0 = N_0^{(1)}$ or $\widetilde{N}_0 = (1+R_2^2)N_0 = N_0^{(2)}$ for $b_3 = 1$. Based on (4) we can express the probability of receiving y_k conditioned on the transmission of b_0, b_1, b_2 and b_3 as follows:

$$P(y_k|w^{(m)}, b_3=0) \;=\; \frac{1}{\pi N_0^{(0)}} e^{\frac{-\left|y_k - y_{k-1}R_0 w^{(m)}\right|^2}{N_0^{(0)}}} \quad (9)$$

$$P(y_k|w^{(m)}, b_3=1) \;=\; \frac{1}{\pi N_0^{(1)}} e^{\frac{-\left|y_k - y_{k-1}R_1 w^{(m)}\right|^2}{N_0^{(1)}}} + \frac{1}{\pi N_0^{(2)}} e^{\frac{-\left|y_k - y_{k-1}R_2 w^{(m)}\right|^2}{N_0^{(2)}}} \quad (10)$$

where $w^{(m)} = \mu(b_2\ b_1\ b_0)$ and μ is the conventional 8PSK mapping function. However, when the *a priori* bit probabilities $P^a(b_i)$ become available from the BICM decoder, the extrinsic bit probability that can be gleaned from the 16-StQAM demapper becomes:

$$P^e(b_i = b) \;=\; \sum_{w^{(m)} \in \chi(i,b)} \Big(P(y_k|w^{(m)}, b_3=0) + P(y_k|w^{(m)}, b_3=1) \Big) \prod_{\substack{j=0 \\ j\neq i}}^{3} P^a(b_j),$$
$$\text{for } i \in \{0,1,2\},\quad b \in \{0,1\}\ , \quad (11)$$

where b_i denotes the ith coded bit of the symbol and $\chi(i,b)$ is the set of constellation points having the ith bit set to b. The extrinsic bit probability of the MSB may be formulated as:

$$P^e(b_3 = b) \;=\; \sum_{w^{(m)}}^{\text{all}} P(y_k|w^{(m)}, b_3 = b) \prod_{j=0}^{2} P^a(b_j), \quad (12)$$

where the summation term considers all possible 8PSK constellation points, because the MSB b_3 influences only the amplitude selection. The extrinsic bit probabilities can then be employed for generating the Log-Likelihood Ratios (LLRs) [15] of all BICM-coded bits, which are then fed back to the BICM decoder.

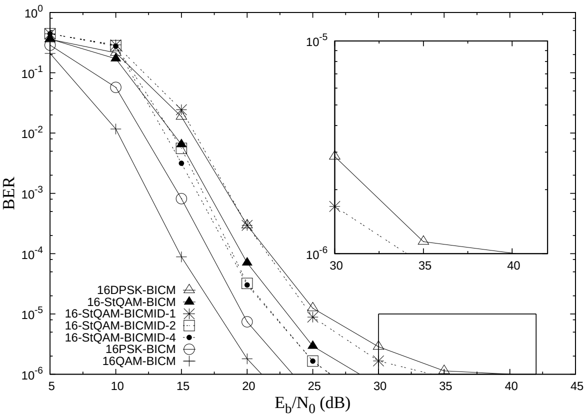
III. SIMULATION RESULTS

Monte-Carlo simulations have been performed for characterising the proposed soft-decision based 16-StQAM demodulation technique in the context of BICM and BICM-ID coding schemes. The simulation parameters are shown in Table I.

Coded Modulation	BICM	BICM-ID
Modulation Scheme	16-StQAM, 16PSK 16QAM, 16DPSK	16-StQAM
Mapper type	Gray-labelled	Set-Partitioned
Number of iterations	1	1,2,4
Code Rate	3/4	
Code Memory	3	
Code Polynomial (octal)	$G = [4\ 4\ 4\ 4\ ;\ 0\ 6\ 2\ 4\ ;\ 0\ 2\ 5\ 6\ ]$	
Decoder type	Approximate Log-MAP	
Symbols per frame	1,200	
Number of frames	20,000	
Channel	Correlated Rayleigh channel	
Normalised Doppler Frequency (f_d)	0.01	

I: Simulation parameters. Note that we declare 'an iteration' being completed when both the demapper and decoder were activated once.

Fig. 3 portrays the E_b/N_0 performance of the 16DPSK aided BICM, 16-StQAM assisted BICM, 16PSK aided BICM, 16QAM BICM and 16-StQAM based BICM-ID schemes, when communicating over correlated Rayleigh fading channels. Solid lines are used for illustrating the performance of Gray-labelled BICM, while the dotted lines represent the Set-Partitioning (SP) based 16-StQAM BICM-ID. As seen from Fig. 3, the 16DPSK-BICM scheme suffers from a high BER



3: BER versus E_b/N_0 performance of the 16DPSK-BICM, 16-StQAM-BICM, 16-StQAM-BICM-ID, 16PSK-BICM and 16QAM-BICM schemes. The simulation parameters are shown in Table I

floor, since the minimum Euclidean distance of a 16-point constellation ring is lower than that of the classic square 16QAM or 16-StQAM schemes. The 16-StQAM-BICM scheme outperforms the 16DPSK-BICM scheme by approximately 12 dBs at a BER of 10^{-6}. The coherently detected 16QAM-BICM and 16PSK-BICM are considered here as our benchmark schemes, while assuming perfect CSI. During the $first\ iteration$, the SP-based 16-StQAM-BICM-ID scheme performs worse than the Gray-labelled 16-StQAM-BICM, since the SP-based mapper has a lower minimum Euclidean distance compared to that of the Gray-label-based mapper. Note that both the 16-StQAM-BICM-ID and 16-StQAM-BICM schemes use the bit-probabilities of (9) and (10) during the first iteration. However, after the $second\ iteration$ the 16-StQAM-BICM-ID outperforms the non-iterative 16-StQAM-BICM by approximately 2 dB with the aid the extrinsic bit-probabilities of (11) and (12).

IV. CONCLUSIONS

In this paper, soft-decision based demodulation was conceived for 16-StQAM in order to enable the employment of power-efficient channel codes and coded modulation. The performance of soft-decision 16-StQAM assisted BICM and BICM-ID schemes was investigated, when communicating over correlated Rayleigh fading channels. The proposed soft-decision aided 16-StQAM demodulation techniques can be extended for assisting DAPSK schemes having more than two PSK constellations.

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