An Extended Spectral Angle Map for Hyperspectral and Multispectral Imaging

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Abstract: We present an extension to the widely used spectral angle metric, calculating an azimuthal angle around a reference vector. We demonstrate that it provides additional information, thus improving the classification ability of the spectral angle.

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1. Introduction

The spectral angle[1] is a common metric in spectral imaging. It treats spectra as vectors in a high-dimensional space allowing the scalar product of, and hence the angle between, a test vector and reference vector to be calculated (see equation 1 where the test spectrum, *t*, and the reference spectrum, *r*, are each considered as *N* dimensional vectors). Smaller angles indicate more similar vectors.

$$\theta = \arccos\left(\frac{\sum_{i=1}^{N} t_i r_i}{\sqrt{\sum_{i=1}^{N} t_i^2}\sqrt{\sum_{i=1}^{N} r_i^2}}\right) \qquad (1)$$

This metric is simple and computationally inexpensive to calculate. It is easily understood as an analogue of a generalised distance between two points, indeed for small angles it is approximately equal to the Euclidean distance between two points[2]. Further, it is intensity independent as all test vectors k*t* subtend the same angle from *r*; as such illumination changes across an image do not affect this measurement.

For a given reference vector however, the angle θ does not specify a unique test vector, or even a unique test unit-vector. This is clear in two dimensions where there are exactly two test vectors for each θ, one between the reference vector and each axis. In three dimensions the vectors lying on the surface of a cone around the reference vector all subtend the same angle, in principle an infinite number of vectors. In a discrete space, as the number of dimensions increases, the number of degenerate test vectors also increases, although in more than three dimensions this is very difficult to visualise.

We suggest then that as well as calculating the spectral angle, a set of azimuthal angles should also be calculated so as to reduce this degeneracy. In section 2 we set out the method for this and in section 3 we show how this can be applied to a real dataset.

2. Method

In a three dimensional space, we have a set of vectors $\boldsymbol{T}$, at an angle θ from a reference vector, $\boldsymbol{R}$, forming a cone. We can rotate the co-ordinate space (x,y,z) → (x', y', z'), such that $\boldsymbol{R}=R\mathbf{z}'$, the projection of $\mathbf{T}$ onto the x'y' plane is now a circle. Each member of $\boldsymbol{T}$ can now be specified by considering the angles that this projection subtends from the x' and y' axes.

In an arbitrary *N*+1 dimensions, $\boldsymbol{T_i} = \sum_{j=1}^{N+1} T_{ij}\, x_j$, and if we rotate the co-ordinate space such that $\boldsymbol{R}=R\boldsymbol{x'_1}$, then the projection of $\boldsymbol{T}$ onto the *N* dimensional subspace orthogonal to $\boldsymbol{R}$ is $\boldsymbol{\tau_i} = \sum_{j=2}^{N} T'_{ij}\, x'_j$. For each $\boldsymbol{\tau}_i$, we can now calculate the set of angles φ_k between the vector and relevant axis x'_k (equation 2).

$$\varphi_{ik} = \arccos\left(\frac{T'_{ik}}{\sqrt{\sum_{j=2}^{N+1} T'^2_{ij}}}\right) \qquad (2)$$

3. Application and Discussion

We demonstrate this technique with an example using a publicly available multispectral image from The University of Eastern Finland[3]. The image in question is available as "Landscape.zip". The image is a multispectral image with seven channels of data, and shows a landscape scene of a tree against a backdrop of a field. A monochrome version of the image is shown in figure 1.

Figure1 – Monochrome landscape image

We calculated the spectral angle of each pixel in the image with respect to a common reference (a "grey" pixel, i.e. with the same value in each channel). We then calculated a histogram of the spectral angle values and and fitted Gaussian peaks to the histogram. We then classified each pixel by assigning it to the peak to which it most strongly belonged, the result of this is shown in figure 2. We can see from this image that the road/track, which shows up most as red, blue and green in the lower portion of the image, and the longer grass in the central third of the image, which shows up as predominantly pink, are clearly distinguished. However, this classification assigns the tree and the areas of shorter grass the same category (the yellow areas of the image).

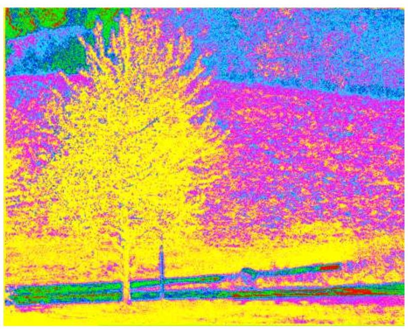

Figure 2 – Classification of pixels using the spectral angle metric

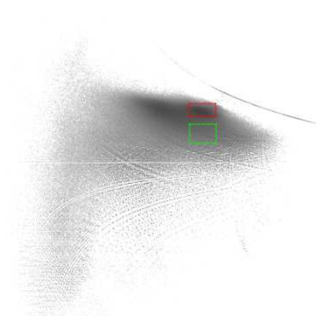

Figure 3 – A plot showing spectral angle (horizontal axis) against azimuthal angle φ_2 (vertical axis)

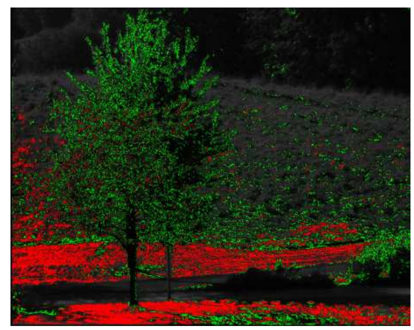

Figure 4 – Spatial configuration of pixels from the red and green boxes shown on figure 3.

We then plotted the distribution of spectral angle against the azimuthal angle φ_2 (the angle from the $\boldsymbol{x'}_2$ axis), as shown in figure 3. From this we can see that at any given spectral angle, there is a spread of azimuthal angles. The darker areas on this plot show areas of higher pixel densities and so these are the interesting cases to examine further. The red box on the image highlights one such area, the green box highlights pixels with the same spectral angle as those in the red box, but a different azimuthal position. Figure 4 shows the pixels in these highlighted areas on the spatial image. On this image we can clearly see the pixels in the red box (red areas of the image) correspond well to the areas of shorter grass, and those in the green box correspond well to the leaves of the tree.

4. Conclusion

We have presented an extension to the spectral angle metric in order to address the limits placed on this measure by the fact that a single angle does not uniquely specify a spectrum. By applying this technique to a real multispectral image we have demonstrated that pixels corresponding to different materials which were inseparable by spectral angle alone can be separated by this azimuthal position metric.

5. References

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