A Probabilistic Data Association Based MIMO Detector Using Joint Detection of Consecutive Symbol Vectors

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***Abstract*—A new probabilistic data association (PDA) approach is proposed for symbol detection in spatial multiplexing multiple-input multiple-output (MIMO) systems. By designing a joint detection (JD) structure for consecutive symbol vectors in the same transmit burst, more *a priori* information is exploited when updating the estimated posterior marginal probabilities for each symbol per iteration. Therefore the proposed PDA detector (denoted as PDA-JD detector) outperforms the conventional PDA detectors in the context of correlated input bit streams. Moreover, the conventional PDA detectors are shown to be a special case of the PDA-JD detector. Simulations and analyses are given to demonstrate the effectiveness of the new method.**

***Index Terms*—Gaussian approximation, MIMO detection, maximum likelihood (ML) detection, probabilistic data association (PDA)**

I. Introduction

The probabilistic data association (PDA) algorithm is a reduced complexity approximation to the *a posteriori* probability (APP) detector, and has attracted great interests for suboptimal detection of CDMA [1] and multiple-input multiple-output (MIMO) systems [2]-[4].

It is pointed out in [5] that if the MIMO detector is confronted by symbols that are correlated through some form of channel code, which is usually the practical case, it is suboptimal for the signal detector and channel decoder to operate separately and only on individual symbol vectors. To optimally detect the symbols, the detector should make decisions jointly on all the symbol vectors using knowledge of the correlations across symbol vectors introduced by the channel code, and the channel code should decode using likelihood information on all the symbol vectors obtained from the signal detector. Although this is computationally prohibitive, it is still valuable to make a tradeoff between the enhanced performance and complexity from the integral system point of view. The seminal iterative receiver that combines MIMO detector and a soft-input soft-output channel decoder to accomplish joint detection and decoding [6] is investigated from this motivation. In practice, a more simplified receiver where MIMO detector and soft channel decoder are concatenated in a serial style without feedback [7] is often adopted.

Since its output is probabilistic information, PDA algorithm is appropriate to play the role of "soft MIMO detector" in both the aforementioned receivers.

In this paper, we propose a new PDA based MIMO detector that achieves joint detection (JD) of consecutive symbol vectors in the same transmit burst, instead of the conventional strategy detecting the symbol vector in a burst one by one. The proposed PDA-JD detector exploits statistical correlations between consecutive symbol vectors, and leads to superior performance to conventional PDA MIMO detectors in the context of correlated input data symbols. The improved quality of soft reliability information provided for the concatenated channel decoder by PDA-JD detector can not only help reduce the complexity of both the iterative receiver and the serial receiver, but also abate error propagation in the latter scheme.

II. Basic System Model

Consider a spatial multiplexing MIMO system with N_T transmit antennas and N_R receive antennas (for generality, $N_R \geq N_T$ is not assumed here). The received baseband signal at each instant of time is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \qquad (1)$$

where $\mathbf{H}$ is the $N_R \times N_T$ channel matrix, $\mathbf{x}$ is the length N_T vector of transmit symbols taken from a modulation constellation $A = \{a_1, a_2, \cdots a_M\}$, and $\mathbf{n}$ is a length N_R zero mean complex circular symmetric Gaussian noise vector with covariance matrix $\sigma^2\mathbf{I}$, where $\mathbf{I}$ is an $N_R \times N_R$ identity matrix.

At the receiver, when the PDA-based algorithms are considered, the task of symbol detection is to estimate the posterior marginal symbol probabilities $\tilde{P}(x_j = a_m \mid \mathbf{y}) = \tilde{P}_m(x_j \mid \mathbf{y})$ for $j = 1, 2, \cdots, N_T$, $m = 1, 2, \cdots, M$ without making an exhaustive search in the space of all possible symbol combinations.

Define

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$$\bar{x}_k = \sum_{m=1}^{M} a_m \tilde{P}_m(x_k \mid \mathbf{y}), \quad (2)$$

$$\mathrm{V}\{x_k\} = \sum_{m=1}^{M} (a_m - \bar{x}_k)(a_m - \bar{x}_k)^* \tilde{P}_m(x_k \mid \mathbf{y}), \quad (3)$$

$$\underline{\mathrm{V}}\{x_k\} = \sum_{m=1}^{M} (a_m - \bar{x}_k)(a_m - \bar{x}_k)^T \tilde{P}_m(x_k \mid \mathbf{y}). \quad (4)$$

Based on (1), the non-decorrelated system model for PDA detectors is formulated as

$$\mathbf{y} = \mathbf{h}_j x_j + \sum_{k \neq j} \mathbf{h}_k x_k + \mathbf{n} \triangleq \mathbf{h}_j x_j + \mathbf{v}_j, \quad (5)$$

where $\mathbf{h}_j$ denotes the jth column of $\mathbf{H}$, and x_j the jth element of $\mathbf{x}$. The interference and noise term $\mathbf{v}_j$ in (5) is approximated as a multivariate colored Gaussian distributed random variable with mean $\mu(\mathbf{v}_j) = \sum_{k \neq j} \bar{x}_k \mathbf{h}_k$, covariance $\mathrm{V}(\mathbf{v}_j) = \sum_{k \neq j} \mathrm{V}\{x_k\} \mathbf{h}_k \mathbf{h}_k^H + \sigma^2 \mathbf{I}$, and pseudo covariance $\underline{\mathrm{V}}(\mathbf{v}_j) = \sum_{k \neq j} \underline{\mathrm{V}}(x_k) \mathbf{h}_k \mathbf{h}_k^T$. For more information about pseudo-covariance, see [8].

When $N_R \geq N_T$, the complexity of PDA detector can be reduced by adopting the decorrelated system model

$$\tilde{\mathbf{y}} = \mathbf{x} + \tilde{\mathbf{n}} = x_j \mathbf{e}_j + \sum_{k \neq j} x_k \mathbf{e}_k + \tilde{\mathbf{n}} \triangleq x_j \mathbf{e}_j + \tilde{\mathbf{N}}_j, \quad (6)$$

where $\tilde{\mathbf{y}} = (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \mathbf{y}$, $\tilde{\mathbf{n}}$ is a colored Gaussian noise with zero mean and covariance $\sigma^2 (\mathbf{H}^H \mathbf{H})^{-1}$, and $\mathbf{e}_k$ is a column vector with 1 in the kth position and 0 elsewhere. The interference and noise term $\tilde{\mathbf{N}}_j$ is still approximated as a colored Gaussian vector with mean $\mu(\tilde{\mathbf{N}}_j) = \sum_{k \neq j} \bar{x}_k \mathbf{e}_k$, covariance $\mathrm{V}(\tilde{\mathbf{N}}_j) = \sum_{k \neq j} \mathrm{V}\{x_k\} \mathbf{e}_k \mathbf{e}_k^T + \sigma^2 (\mathbf{H}^H \mathbf{H})^{-1}$, and pseudo covariance $\underline{\mathrm{V}}(\tilde{\mathbf{N}}_j) = \sum_{k \neq j} \underline{\mathrm{V}}(x_k) \mathbf{e}_k \mathbf{e}_k^T$. For simplicity, we will focus on the decorrelated system model in the following.

III. PDA-JD MIMO DETECTOR

A. On JD of Consecutive Symbol Vectors

Note that there are generally hundreds of symbol vectors in a single transmission burst of MIMO systems [9]. These symbol vectors are all received in a time interval as a block. Suppose the number of symbol vectors in a burst is denoted as L, then corresponding to each transmitted symbol vector, the received signals are $\mathbf{y}^{(1)} = \mathbf{H}^{(1)} \mathbf{x}^{(1)} + \mathbf{n}^{(1)}$, $\mathbf{y}^{(2)} = \mathbf{H}^{(2)} \mathbf{x}^{(2)} + \mathbf{n}^{(2)}$, $\cdots$, $\mathbf{y}^{(l)} = \mathbf{H}^{(l)} \mathbf{x}^{(l)} + \mathbf{n}^{(l)}$, $l = 1, 2, \cdots, L$, respectively. Conventionally the received symbol vectors are detected on an isolated vector-by-vector basis even in coded system, which is suboptimal, however, while soft information and correlations across symbol vectors introduced by some form of channel code are taken into consideration.

Our idea is to jointly detect an appropriate number of consecutive symbol vectors. To our best knowledge, such a detection strategy in MIMO systems has not been reported before.

Assume the number of symbol vectors detected simultaneously is denoted as d, which satisfies $d \in \{1, 2, \cdots, L\} \cap Nd = L$, with N being a positive integer. Then the basic system model (1) is redesigned as

$$\mathbf{r} = \mathbf{G}\mathbf{s} + \mathbf{u}, \quad (7)$$

where $\mathbf{r} = [(\mathbf{y}^{(1)})^T, \cdots, (\mathbf{y}^{(d)})^T]^T$, $\mathbf{s} = [(\mathbf{x}^{(1)})^T, \cdots, (\mathbf{x}^{(d)})^T]^T$, $\mathbf{u} = [(\mathbf{n}^{(1)})^T, \cdots, (\mathbf{n}^{(d)})^T]^T$, $\mathbf{G} = \begin{pmatrix} \mathbf{H}^{(1)} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathbf{H}^{(d)} \end{pmatrix}$, and $\mathbf{u}$ is still a zero mean complex circular symmetric Gaussian noise vector with covariance matrix

$$Cov(\mathbf{u}) = \mathrm{E}(\mathbf{u}\mathbf{u}^H) = \begin{pmatrix} \sigma_1^2 \mathbf{I} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma_d^2 \mathbf{I} \end{pmatrix},$$

in which $\sigma_{d'}^2$ is the power of the white noise $\mathbf{n}^{(d')}$ corresponding to the d'th symbol vector, $d' = 1, 2, \cdots, d$.

Similarly to (6), (7) can be reformulated as the decorrelated model

$$\tilde{\mathbf{r}} = \mathbf{s} + \tilde{\mathbf{u}} = s_{j'} \mathbf{e}_{j'} + \sum_{k' \neq j'} s_{k'} \mathbf{e}_{k'} + \tilde{\mathbf{u}} = s_{j'} \mathbf{e}_{j'} + \tilde{\mathbf{U}}_{j'}, \quad (8)$$

where $\tilde{\mathbf{r}} = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \mathbf{r}$

$$= \left(\left(\left(\mathbf{H}^{(1)H} \mathbf{H}^{(1)} \right)^{-1} \mathbf{H}^{(1)H} \mathbf{y}^{(1)} \right)^T, \cdots, \left(\left(\mathbf{H}^{(d)H} \mathbf{H}^{(d)} \right)^{-1} \mathbf{H}^{(d)H} \mathbf{y}^{(d)} \right)^T \right)^T$$

, $\tilde{\mathbf{u}}$ is Gaussian noise with zero mean and covariance

$$Cov(\tilde{\mathbf{u}}) = (\mathbf{G}^H \mathbf{G})^{-1} \mathbf{G}^H \begin{pmatrix} \sigma_1^2 \mathbf{I} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma_d^2 \mathbf{I} \end{pmatrix} \mathbf{G} (\mathbf{G}^H \mathbf{G})^{-1}$$

$$= \begin{pmatrix} \sigma_1^2 \left(\mathbf{H}^{(1)H} \mathbf{H}^{(1)} \right)^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma_d^2 \left(\mathbf{H}^{(d)H} \mathbf{H}^{(d)} \right)^{-1} \end{pmatrix},$$

$j', k' = 1, 2, \cdots, d \times N_T$. Since the elements of $\mathbf{H}^{(d')}$ are i.i.d. complex Gaussian, the rank of $\mathbf{H}^{(d')}$ is almost always N_T, i.e. $\mathbf{H}^{(d')}$ has full column rank. So the Hermitian matrix $\mathbf{H}^{(d')H} \mathbf{H}^{(d')}$ is almost definitely invertible.

Notice that the above derivation does not make the assumption that $\mathbf{H}$ is quasi-static, which means the derivation makes sense for the fast variant channel as well where $\mathbf{H}$ changes randomly from one symbol vector to another. When quasi-static channel is taken into account, we have $\mathbf{H}^{(1)} = \mathbf{H}^{(2)} = \cdots = \mathbf{H}^{(d)} = \mathbf{H}$. Then $\tilde{\mathbf{r}}$ and $Cov(\tilde{\mathbf{u}})$ are rewritten respectively as

$$\left(\left(\left(\mathbf{H}^H \mathbf{H} \right)^{-1} \mathbf{H}^H \mathbf{y}^{(1)} \right)^T, \cdots, \left(\left(\mathbf{H}^H \mathbf{H} \right)^{-1} \mathbf{H}^H \mathbf{y}^{(d)} \right)^T \right)^T$$

and $\begin{pmatrix} \sigma_1^2\left(\mathbf{H}^H\mathbf{H}\right)^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma_d^2\left(\mathbf{H}^H\mathbf{H}\right)^{-1} \end{pmatrix}$. Furthermore, if the Gaussian noise $\mathbf{n}^{(d')}$ is considered i.i.d. between different d', $Cov(\tilde{\mathbf{u}})$ can be simplified as

$$\begin{pmatrix} \sigma^2\left(\mathbf{H}^H\mathbf{H}\right)^{-1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \sigma^2\left(\mathbf{H}^H\mathbf{H}\right)^{-1} \end{pmatrix}.$$

From (8) we can clearly see that when any symbol $s_{j'}$ is detected, the dimension of the corresponding interference is $d \times N_T - 1$ instead of $N_T - 1$ in the conventional PDA detectors. And the conventional PDA detectors are a special case of the PDA-JD detector when $d = 1$.

B. PDA Algorithm Derivation Based on JD

For any element $s_{j'}$, we associate a probability vector $\tilde{\mathbf{P}}(j')$ whose *m*th element $\tilde{P}_m(s_{j'} \mid \mathbf{r})$, is the current estimate of the posterior probability that $s_{j'} = a_m$, $m = 1,2,\cdots,M$. Following the PDA principle ("Gaussian approximation") in [4], the pseudo noise $\tilde{\mathbf{U}}_{j'}$ is approximated by a complex Gaussian distribution with matched mean $\sum_{k'\neq j'} \tilde{s}_{k'}\mathbf{e}_{k'}$, covariance matrix $\mathbf{\Omega}_{j'} \triangleq \sum_{k'\neq j'} \omega_{k'}\mathbf{e}_{k'}\mathbf{e}_{k'}^T + Cov(\tilde{\mathbf{u}})$ and pseudo-covariance matrix $\mathbf{\Xi}_{j'} \triangleq \sum_{k'\neq j'} \xi_{k'}\mathbf{e}_{k'}\mathbf{e}_{k'}^T$, where

$$\tilde{s}_{k'} = \sum_{m=1}^{M} a_m \tilde{P}_m(s_{k'} \mid \mathbf{r}), \tag{9}$$

$$\omega_{k'} = \sum_{m=1}^{M} (a_m - \tilde{s}_{k'})(a_m - \tilde{s}_{k'})^* \tilde{P}_m(s_{k'} \mid \mathbf{r}), \tag{10}$$

$$\xi_{k'} = \sum_{m=1}^{M} (a_m - \tilde{s}_{k'})(a_m - \tilde{s}_{k'})^T \tilde{P}_m(s_{k'} \mid \mathbf{r}). \tag{11}$$

Here $\tilde{P}_m(s_{k'} \mid \mathbf{r})$ is initialized as a uniform distribution and will be replaced with an updated value where the pseudo-covariance typically does not vanish.

Let $\mathbf{w} = \tilde{\mathbf{r}} - s_{j'}\mathbf{e}_{j'} - \sum_{k'\neq j'} \tilde{s}_{k'}\mathbf{e}_{k'}$ and

$$\phi_m(s_{j'}) \triangleq \exp\left(-\begin{pmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{pmatrix}^T \mathbf{\Lambda}_{j'} \begin{pmatrix} \Re(\mathbf{w}) \\ \Im(\mathbf{w}) \end{pmatrix}\right), \tag{12}$$

$$\mathbf{\Lambda}_{j'} \triangleq \begin{pmatrix} \Re\left(\mathbf{\Omega}_{j'} + \mathbf{\Xi}_{j'}\right) & -\Im\left(\mathbf{\Omega}_{j'} - \mathbf{\Xi}_{j'}\right) \\ \Im\left(\mathbf{\Omega}_{j'} + \mathbf{\Xi}_{j'}\right) & \Re\left(\mathbf{\Omega}_{j'} - \mathbf{\Xi}_{j'}\right) \end{pmatrix}^{-1} \tag{13}$$

$$= \left(\begin{pmatrix} \Re\left(Cov(\tilde{\mathbf{u}})\right) & -\Im\left(Cov(\tilde{\mathbf{u}})\right) \\ \Im\left(Cov(\tilde{\mathbf{u}})\right) & \Re\left(Cov(\tilde{\mathbf{u}})\right) \end{pmatrix} + \mathbf{Q}\right)^{-1},$$

where $\Re(\bullet)$ and $\Im(\bullet)$ are the real and imaginary part of a complex variable respectively, and

$$\mathbf{Q} = \sum_{k'\neq j'} (\omega_{k'} + \Re(\xi_{k'}))\mathbf{e}_{k'}\mathbf{e}_{k'}^T + \sum_{k'\neq j'} (\omega_{k'} - \Re(\xi_{k'}))\mathbf{e}_{k'+d\times N_T}\mathbf{e}_{k'+d\times N_T}^T + \sum_{k'\neq j'} \Im(\xi_{k'})\mathbf{e}_{k'}\mathbf{e}_{k'+d\times N_T}^T + \sum_{k'\neq j'} \Im(\xi_{k'})\mathbf{e}_{k'+d\times N_T}\mathbf{e}_{k'}^T.$$

Since it is assumed that the transmitted symbols have equal *a priori* probabilities, the estimated posterior marginal symbol probability is given as

$$\tilde{P}_m(s_{j'} \mid \tilde{\mathbf{r}}) = \frac{\tilde{p}_m(\tilde{\mathbf{r}} \mid s_{j'})P(s_{j'} = a_m)}{\sum_{m=1}^{M} \tilde{p}_m(\tilde{\mathbf{r}} \mid s_{j'})P(s_{j'} = a_m)} \approx \frac{\phi_m(s_{j'})}{\sum_{m=1}^{M} \phi_m(s_{j'})}. \tag{14}$$

To sum up, the algorithm proceeds as follows.

1) Determine the optimal detection sequence according to the ordering criterion proposed for the decision feedback detector in [10], and denote the sequence as $\left\{n_{j'}\right\}_{j'=1}^{d\times N_T}$.

2) Initialization: compute $\tilde{\mathbf{r}}$, set $\tilde{P}_m(s_{j'} \mid \mathbf{r})$ as a uniform distribution for $\forall j' = 1,2,\cdots,d\times N_T$, $\forall\, m = 1,2,\cdots,M$, i.e. $\tilde{P}_m(s_{j'} \mid \mathbf{r}) = 1/M$; set the iteration counter $z = 1$.

3) Initialize $j' = 1$.

4) Based on the current values of $\left\{\tilde{\mathbf{P}}(n_{k'})\right\}_{n_{k'}\neq n_{j'}}$, compute $\tilde{P}_m(s_{n_{j'}} \mid \mathbf{r})$ via (9) ~ (14), and set the results equal to the corresponding elements of $\tilde{\mathbf{P}}(n_{j'})$.

5) If $j' < d\times N_T$, let $j' = j' + 1$ and go to step 4). Otherwise, go to step 6).

6) If $\forall j'$, $\tilde{\mathbf{P}}(j')$ has converged, or the iteration counter has come to a certain given number, go to step 7). Otherwise, let $z = z + 1$ and return to step 3).

7) For $j' = 1,2,\cdots,d\times N_T$, make a decision $\hat{s}_{j'}$ for $s_{j'}$ via $\hat{s}_{j'} = a_l$, $l = \arg\max_{m'=1,2,\cdots,M} \left\{\tilde{P}_{m'}(s_{j'} \mid \mathbf{r})\right\}$.

Then $\hat{\mathbf{s}} = \{\hat{s}_{j'} \mid j' = 1,2,\cdots,d\times N_T\}$ is obtained.

8) Finally, we have $\mathbf{x}^{(d')} = \{\hat{s}_{j'} \mid j' = (d'-1)\times N_T + 1, (d'-1)\times N_T + 2, \cdots, d'\times N_T\}$, $d' = 1,2,\cdots,d$.

From the above algorithm procedure, we can get the perspective that the essence of the PDA-JD detector is it exploits more *a priori* information when estimating the posterior marginal probabilities of symbols being detected. One needs to compute at least $d\times N_T\times(M-1)$ probabilities in each iteration.

Because we are interested in the fundamental property of the PDA-JD detector, the refinements such as complexity reduction techniques proposed in [1], and conversion of the complex-valued signal model into an equivalent real-valued signal model for QAM constellation in [2], are not incorporated into the above basic algorithm procedure, though they both make sense in this PDA-JD detector.

C. Complexity Analysis

Taking QAM modulation as an example, the complexity of the proposed PDA-JD detector using the matrix speed-up tactic (the Sherman–Morrison–Woodbury formula) of [1] is approximately $O((2dN_T)^3/d) = O(8d^2 N_T{}^3)$ real operations for updating the "inverse of the complete-covariance matrix" $\mathbf{\Lambda}_{j'}$ per symbol vector per iteration. This is actually the worst case complexity, since the size of the problem may be lowered by the use of successive cancellation [1]. What is more, the best record of matrix exponent for the complexity of matrix inverse is 2.376 up to now [11] and many mathematicians are trying to prove the conjecture that the value can be reduced to 2.

By comparison, the corresponding complexity is approximately $O(N_T^3)$ [1] and $O((2N_T)^3) = O(8N_T{}^3)$ [2][4] real operations when BPSK and QAM modulation is used respectively.

It is clear that N_T plays a dominant role in the complexity of all PDA based MIMO detectors, especially for the system with large N_T. Therefore the PDA-JD detector enjoys almost the same order polynomial complexity of $N_T{}^3$ as the conventional PDA detectors do, when d is properly determined by the constraint length of specific channel code scheme and simulations, to be an affordable constant coefficient.

For the overall complexity of the PDA-JD detector, many techniques can be used to further reduce it. For example, when the system model is decoupled into real and imaginary parts like [2], the number of probabilities that are evaluated per transmit symbol per iteration is reduced to at least $2(\sqrt{M}-1)$ and $\sqrt{2M} + \sqrt{M/2} - 2$ for square QAM and rectangle QAM respectively. While complex system model is applied, both the values are $M-1$.

IV. Simulation Results and discussions

A. Simulation Results

We demonstrate the power of the PDA-JD detector using computer simulations for a V-BLAST system which utilizes $N_T = N_R = 2$ antennas and QPSK modulation scheme. The entries of the MIMO channel are chosen as i.i.d., zero mean, standard complex normal random variables, which remain constant for the duration of the burst with length $L = 64$ but change randomly from one burst to another. A rate 1/2 convolutional code with polynomials (177, 133) is applied to introduce correlation across symbols. In order to demonstrate the error probability improvement brought by PDA-JD detector, which is far less obvious when entangled with channel decoder, we focus on the part from the input to the output of soft MIMO detector as shown in [6]and [7], and evaluate the bit error performance at the output of PDA-JD detector directly. This is a generally accepted manner in research of MIMO detectors.

The performances of the PDA-JD detector, the conventional PDA detector proposed in [4] and the ML detector are compared in Fig. 1. Both the PDA detectors do not use the optimal detection sequence and refinements we have mentioned in the above section. This is done in order to provide the same comparison conditions as in [4]. It is easy to find the advantage of the PDA-JD detector over the conventional PDA detector. When $d = 8$ and $d = 2$, the BER performance of the proposed PDA-JD detector is nearly 2 dB and more than 1 dB better than that of the conventional PDA detector respectively at BER=10^{-3}.

Compared with ZF-OSIC [9], the PDA-JD detector substantially improves the BER performance. This can be seen in Fig. 2. For ZF-OSIC detector, when the number of transmit antennas is equal to that of receive antennas, the performance of the larger one is much worse than that of the smaller one except in very high SNR regimes. But if the PDA-JD detector is invoked, the performance gap between the larger one and the smaller one shrinks. Furthermore, the "cross-point" in ZF-OSIC curve is 18 dB while in PDA-JD curve it dramatically decreases to 10 dB. Hence, we can conclude that the PDA-JD detector prefers larger MIMO systems in terms of BER performance, which is a natural result of the fact that it exploits more *a priori* information when detecting each symbol of the symbol sequence.

Fig. 3 shows the complexity of the PDA-JD and the conventional PDA detectors (the special case of $d = 1$ in the PDA-JD detector). It is evident that in both detectors N_T imposes dominant influence on the complexity. This observation is consistent with the theoretical result $O(8d^2 N_T{}^3)$ and $O(8N_T{}^3)$. Taking the constraint length 7 of (177, 133) into account, the more appropriate value of d is 2, which gives more favorable tradeoff between the BER performance and the computational complexity than other values of d.

B. Some Discussions

High-quality probabilistic data offered by soft MIMO detector like PDA-JD detector is capable of giving more reliable *a priori* information to its following channel decoder in both receivers in [6]and [7], and thus helps reduce the total complexity accumulated through round by round iterations for the whole receiver. Additionally, for the straightforward serial receiver, the soft error decisions made by the soft MIMO detector flow into the after channel decoder and induce serious error propagation. However, by using the PDA-JD detector, this problem can be mitigated.

In [2], it is shown that real-valued signal model based PDA detector gives an improved error probability and reduced complexity over the direct PDA approach based on complex-valued signal model. The former processes the components of complex signal sequentially, and the latter processes both the real and imaginary part in parallel. In fact, the sequential style corresponds to "Gauss-Seidel iteration", while the parallel is "Jacobi iteration". Gauss-Seidel is generally lot better than Jacobi in terms of speed of convergence. Therefore, when the real valued signal model is employed in the PDA-JD detector, the real-valued version of PDA-JD detector has a reduced complexity and improved error

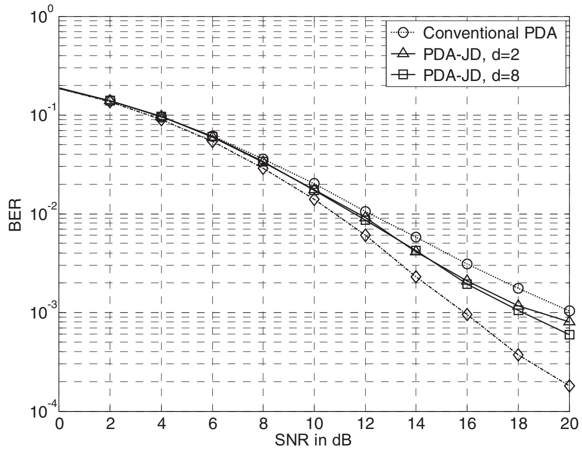

Fig. 1. Comparison of bit error rates of the PDA-JD, the conventional PDA, and the ML detectors for V-BLAST with QPSK and $N_T = N_R = 2$

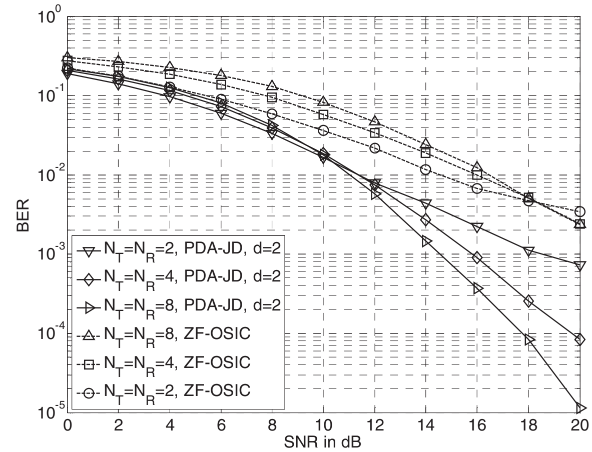

Fig. 2. Comparison of bit error rates of the PDA-JD and ZF-OSIC detectors for V-BLAST with QPSK and equal number of antennas at transceiver

performance than the complex-valued PDA-JD detector, which is validated by our numerous simulation results (not shown for page limitation). On the other hand, both the PDA-JD detector and the GPDA detector in [2] proceeds in a parallel style to expand the dimension of symbol vector, and the former operates on consecutive symbol vectors while the latter only focuses on the interior of one symbol vector

PDA based algorithms fall into the category of "belief-propagation" based iterative methods, and most of the algorithms in this category have not yet been well understood. The performance analysis and convergence property of PDA based algorithms are still open problems and will be addressed in our future research.

V. Conclusion

We proposed a novel PDA detector using joint detection of consecutive symbol vectors in the same transmit burst for spatial multiplexing MIMO systems. Our analyses and computer simulations show that the PDA-JD detector significantly outperforms the conventional PDA detector especially in high SNR regimes in the context of correlated input bit streams, and its worst-case complexity is almost the same order polynomial of $N_T^{\,3}$ as that of the conventional PDA detectors. In addition the conventional PDA detectors are in fact a special case of the PDA-JD detector. Simulation results also show that the PDA-JD detector prefers MIMO systems with large number of transmit antennas in terms of BER performance.

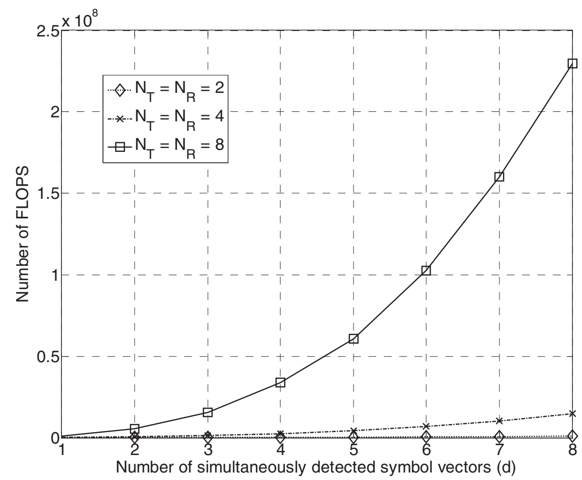

Fig. 3. Comparison of complexity of the PDA-JD and the conventional PDA detectors (the special case of $d = 1$ in the PDA-JD detector)

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