Approximate Minimum BER Power Allocation for MIMO-THP System

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Abstract—**This paper proposes a transmit power allocation (TPA) scheme based on multiple-input multiple-output (MIMO) Tomlinson-Harashima precoding (THP) structure, where a TPA matrix is introduced to the conventional MIMO-THP. We analyze the influence of the introduced TPA matrix on the performance of MIMO-THP. The proposed TPA scheme invokes the minimum average uncoded bit-error rate (BER) criterion subjected to a sum-power constraint. During the derivation, we consider the effects of precoding loss factor on the TPA scheme and obtain a closed-form expression of the TPA. Compared to existing TPA methods for MIMO-THP systems, the proposed scheme reduces processing complexity and improves the BER performance.**

Keywords- *multiple-input multiple-output (MIMO);Tomlinson-Harashima precoding (THP); Minimum bit-error rate (MBER); transmit power allocation (TPA).*

I. INTRODUCTION

Nowadays, multiple-input multiple-output (MIMO) system has been considered as one of the most important techniques owing to the significantly higher capacity improvement than single-input single-output (SISO) wireless channel [1], [2].The Vertical Bell Laboratories Layered Space-Time (V-BLAST) system, originally proposed in [3], is one of the most prominent schemes to achieve this higher spectral efficiency. Ginis and Cioffi [4] proved that V-BLAST receiver processing is equivalent to the MIMO Decision Feedback Equalization (DFE) structure.

Above methods require channel state information (CSI) only at the receiver side. In some cases, if the communication environment is slowly time varying, the CSI at transmitter is possible obtained by feedback in frequency-division duplex (FDD) systems or the reciprocal principle in time-division duplex (TDD) systems. Fischer et al. [5] first introduce the Tomlinson-Harashima precoding (THP) structure to eliminate the spatial interference in MIMO channel. This scheme moves the feedback filters of MIMO-DFE to the transmitter to avoid the error propagation, which is similar to the original THP used in dispersive SISO channel [6], [7]. Because the feed-forward filters are still at the receivers, it should be pointed out that the received signals of the method [5] must be cooperatively processed. Modulo operators in MIMO-THP systems result in

a transmit power increase, which is quantified by precoding loss [5], [8], [11].

In order to make full use of CSI, transmit power allocation (TPA) schemes can achieve a performance improvement for MIMO systems. From the point view of maximizing channel capacity, TPA strategies were derived in [9], [10] for V-BLAST systems and [11], [12] for MIMO-THP systems. Alternatively, from the point view of minimizing error rate performance, TPA strategies were derived in [13], [14] for V-BLAST systems and [15] for MIMO-THP systems. The scheme in [15], which approximately minimizes the average Symbol-Error Rate (SER) under sum-power constraint, achieves better performance than MIMO-THP systems without TPA. However, the existing scheme needs iterative procedure and does not consider the precoding loss which is not negligible at lower constellations.

In this paper, we bring a TPA matrix into the conventional MIMO-THP system and analyze the modification of MIMO-THP structure employed with TPA scheme first. Then an effective TPA method that minimizes the bit-error rate (BER) performance under sum-power constraint is proposed. This method is capable of achieving further performance improvement because the precoding loss factor is considered in the derivations. When employed to uncoded QAM, a tighter approximation expression to the complementary cumulative distribution function (Q-function) is introduced. Finally, a closed-form expression of TPA scheme is derived.

The rest of this paper is organized as follows. The modified MIMO-THP systems for TPA are described in section II. In section III, the proposed TPA scheme is derived for this modified MIMO-THP structure in perfect CSI case. Simulation results are given in Section IV and conclusions are followed in Section V.

II. MODIFIED MIMO-THP SYSTEM

A. System Model and Assumptions

In this paper, we assume the transmitter has perfect knowledge of the CSI and consider the MIMO system equipped with N_T transmit antennas and N_R receive antennas. For simplicity, we assume both sides of the system have equal
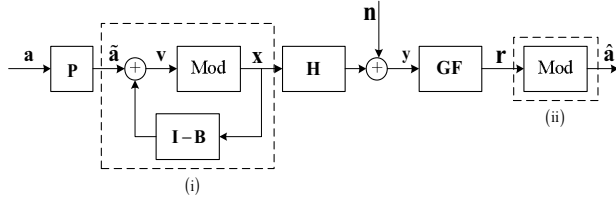
Figure 1.   Block diagram of Modified MIMO-THP for TPA.

Figure 2.   Alternative linear representations of modulo operators in Fig. 1.

antennas number (i.e. $N_T = N_R = K$). Let $\mathbf{x} = [x_1,...,x_K]^T$ and $\mathbf{y} = [y_1,...,y_K]^T$ represent the transmitted and the received signals, respectively. The system model can be written as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n} . \tag{1}$$

$\mathbf{H} = [h_{ij}]$ is the $K \times K$ channel gain matrix. The channel gain from transmit antenna j to receive antenna i is denoted by h_{ij} and assumed to be normalized i.i.d. zero-mean complex Gaussian random variables(i.e. flat fading case). $\mathbf{n} = [n_1,...,n_K]^T$ denotes the channel noise, which is uncorrelated white Gaussian noise with noise power σ_n^2 per dimension (i.e. $\mathbf{R_n} = \mathrm{E}[\mathbf{nn}^H] = \sigma_n^2 \mathbf{I}$). It is assumed that the transmitter knows the noise variance σ_n^2.

The definitions of some symbols and operators used in this paper are listed below.

$\mathbf{a}$: Boldface small symbol represents a vector

$\mathbf{H}$: Boldface capital symbol represents a matrix

$diag[x_1, x_2,...,x_D]$: A diagonal matrix with diagonal entries $x_1, x_2,...,x_D$

$diag(\mathbf{A})$: A vector made by the diagonal entries of $\mathbf{A}$

$[a_{ij}]$: The entry at the i th row and j th column of $\mathbf{A}$

$\mathrm{tr}(\cdot)$: Trace of a matrix

$\mathrm{E}(\cdot)$: Expectation

$(\cdot)^T$: Transpose

$(\cdot)^H$: Conjugate transpose

B.   Modified THP Structure

We first review the MIMO-THP scheme presented in [5] with some modifications. The overall system structure is illustrated as a block diagram in Fig. 1, where the TPA matrix $\mathbf{P}$ is introduced to the structure in [5]. Here $\mathbf{I}$ is identity matrix, $\mathbf{B}$ is unit lower triangular (lower left triangular matrix with ones on the main diagonal), $\mathbf{F}$ is unitary, $\mathbf{G}$ and $\mathbf{P}$ are diagonal matrices. $\mathbf{a} = [a_1,...,a_K]^T$ is the data vector. Each $a_k, k = 1,...,K$ uses the same M-ary square constellation (M is
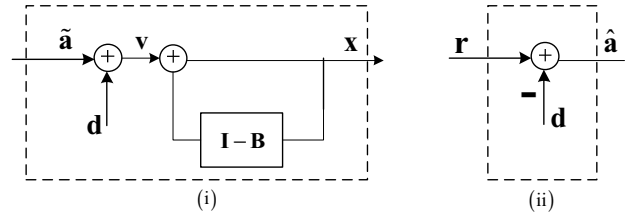
squared number) and has unity power (i.e. $\sigma_{a_k}^2 = \sigma_a^2 = 1, \forall k$). $\mathbf{P} = diag\left[\sqrt{P_1},...,\sqrt{P_K}\right]$ is the TPA matrix. $\tilde{\mathbf{a}} = \mathbf{Pa}$ is the power loaded data vector and the variance of each component is $\sigma_{\tilde{a}_k}^2 = P_k \cdot \sigma_{a_k}^2 = P_k, k = 1,...,K$. The feedback matrix $\mathbf{B}$ and feed-forward matrix $\mathbf{GF}$ can be calculated under ZF or MMSE criteria. If we use ZF criterion, the filter matrices $\mathbf{G}$, $\mathbf{B}$ and $\mathbf{F}$ can be obtained by performing QL-factorization to $\mathbf{H}$ [5], i.e.

$$\mathbf{H} = \mathbf{QS} , \tag{2.1}$$

$$\mathbf{G} = diag\left[s_{11}^{-1},...,s_{KK}^{-1}\right], \tag{2.2}$$

$$\mathbf{B} = \mathbf{GS} , \tag{2.3}$$

$$\mathbf{F} = \mathbf{Q}^{-1} = \mathbf{Q}^H . \tag{2.4}$$

$\mathbf{S} = [s_{ij}]$ is a lower triangular matrix, $\mathbf{Q}$ is a unitary matrix. In this case the covariance matrix of noise $\tilde{\mathbf{n}} = \mathbf{GFn}$ is

$$\begin{aligned}\mathbf{R_{\tilde{n}}} &= \mathrm{E}[\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H] = \sigma_n^2 \mathbf{GG}^H \\ &= diag\left[\sigma_n^2 / s_{11}^2,...,\sigma_n^2 / s_{KK}^2\right].\end{aligned} \tag{3}$$

On an alternative criterion, the MMSE based solution has also been given in [5]. The error is defined as

$$\mathbf{e} = \mathbf{r} - \mathbf{v} = \mathbf{GFy} - \mathbf{Bx} , \tag{4}$$

and the covariance matrix of error is $\mathbf{R_e} = \mathrm{E}[\mathbf{ee}^H]$ .By minimizing the mean-square error $\mathrm{tr}(\mathbf{R_e})$, we perform a (modified) Cholesky factorization of $\mathbf{H}^H \mathbf{H}$ to obtain the filter matrices, i.e.

$$\mathbf{H}^H \mathbf{H} + \varsigma \mathbf{I} = \mathbf{S}'^H \mathbf{S}' , \tag{5.1}$$

$$\mathbf{G}' = diag\left[s_{11}'^{-1},...,s_{KK}'^{-1}\right], \tag{5.2}$$

$$\mathbf{B}' = \mathbf{G}'\mathbf{S}' , \tag{5.3}$$

$$\mathbf{F}' = \mathbf{S}'^{-H} \mathbf{H}^H , \tag{5.4}$$

where $\varsigma = \sigma_n^2 / \sigma_a^2 = \sigma_n^2$ and $\mathbf{S}' = [s_{ij}']$ is a lower triangular matrix. The covariance matrix of the error vector $\mathbf{e}$ is

$$\begin{aligned}\mathbf{R_e} &= \mathrm{E}[\mathbf{ee}^H] = diag\left[\sigma_{e_1}^2,...,\sigma_{e_K}^2\right] \\ &= diag\left[\sigma_n^2 / s_{11}'^2,...,\sigma_n^2 / s_{KK}'^2\right].\end{aligned} \tag{6}$$

From (3) and (6), we can see that the structures of noise and error covariance matrices are similar. Thus, the transmit power can be calculated as the same manner both in ZF and MMSE cases.

C. Equivalent Sub-channel Model

By considering Fig.1, Fig.2 and properties of THP matrices, the detected signal vector $\hat{\mathbf{a}}$ is given as

$$\hat{\mathbf{a}} = Mod_{\mathbf{t}}(\tilde{\mathbf{a}} + \mathbf{d} + \mathbf{I}' + \tilde{\mathbf{n}}), \qquad (7)$$

and can be decoupled into K parallel streams [15]

$$\hat{a}_k = Mod_{t_k}(\tilde{a}_k + d_k + I'_k + \tilde{n}_k), \forall k, \qquad (8)$$

where $\mathbf{I}' = [I'_1, ..., I'_K]^T$ is the residual spatial Co-Channel Interference(CCI) after MMSE criterion on THP filters (for ZF case $\mathbf{I}' = \mathbf{0}$), The modulo operator is introduced to reduce the transmit signal power, where $\mathbf{t} = [t_1, ..., t_K]$ is the modulo bound vector. $Mod_{t_k}(\bullet)$ is modulo t_k operator to bound its output on $T_k = [-t_k/2, t_k/2) + j \cdot [-t_k/2, t_k/2), \forall k$ and is defined for a complex variable c as

$$Mod_{t_k}(c) \triangleq c - \left\lfloor \frac{Re(c)}{t_k} + \frac{1}{2} \right\rfloor t_k - j \left\lfloor \frac{Im(c)}{t_k} + \frac{1}{2} \right\rfloor t_k, \qquad (9)$$

where the floor operator $\lfloor \bullet \rfloor$ rounds the argument to the nearest integer towards minus infinity. Note that the modulo operator can be represented as a linear model [11] and generates a data vector $\mathbf{v} = \tilde{\mathbf{a}} + \mathbf{d}$. $\mathbf{d} = [d_1, ..., d_K]^T$ is the modulo alphabet which can be eliminated at the receivers by another modulo operator as explicated in Fig.2.

In optimum case, the spatial CCI approximates to zero ($I'_k \to 0$). For clarity, we ignore the truncation effect [17] on noise $\tilde{\mathbf{n}}$, and the parallel K streams (8) can be simplified as

$$\hat{a}_k = \tilde{a}_k + \tilde{n}_k = \sqrt{P_k} a_k + \tilde{n}_k, k = 1, ..., K. \qquad (10)$$

Note that the transmit power is successfully introduced to each equivalent sub-channel, we may use the TPA scheme to achieve better performance.

III. TRANSMIT POWER ALLOCATION

We first analyze the precoding loss of MIMO-THP system in section III.A. Followed in section III.B, a TPA scheme is derived by minimizing the BER performance. In the derivations, we assume that ZF-THP is used for transmission, and the derivations of MMSE case are straightforward. Finally, the results of section III.B are applied to an uncoded quadrature amplitude modulation (QAM) in section III.C.

A. Precoding Loss Analysis

Because of modulo operator in THP, the transmitted symbols x_k are assumed to be uniformly distributed, bounded on enlarged range T_k. As a result, the transmit power will be increasing and is quantified by the precoding loss, which is usually defined for square QAM as $\gamma \triangleq E[|x|^2]/E[|\tilde{a}|^2] = M/(M-1)$ [5], [8], [11]. But in fact, because $\mathbf{B}$ is a unit lower triangular matrix, the first element $\tilde{a}_1$ of power loaded data vector $\tilde{\mathbf{a}}$ is directly passed through modulo operator without being interfered by other power loaded data components. Consequently, the distribution of x_1 is the same with $\tilde{a}_1$ rather than the uniform distribution bounded on T_1. We redefine the precoding loss as

$$\gamma_k = \begin{cases} 1 & , k = 1 \\ \gamma = M/(M-1), & k = 2, ..., K \end{cases}. \qquad (11)$$

From the analysis above, we note that the precoding loss is negligible for moderate and large modulation sizes, but it should be considered for the small modulation sizes as well. Thus the exact power of x_k is given by

$$\sigma_{x_k}^2 = \gamma_k \sigma_{\tilde{a}_k}^2 = \begin{cases} \sigma_{\tilde{a}_k}^2 = P_k & , k = 1 \\ \gamma \sigma_{\tilde{a}_k}^2 = \gamma P_k, & k = 2, ..., K \end{cases}, \qquad (12)$$

and the corresponding bound region t_K can be calculated as

$$\frac{t_k^2}{6} = \sigma_{x_k}^2 = \gamma_k P_k. \qquad (13).$$

B. Derivation of TPA Scheme

To derive a TPA scheme, we will express the BER of each transmitted symbol as a function of transmit power P_k, and find the TPA matrix $\mathbf{P}$ by means of minimizing the average BER. As discussed in section II, from the equivalent model in (10) and the noise covariance matrix (3), the postdetection signal to interference and noise ratio (SINR) ρ_k of the k-th sub-channel can be expressed as

$$\rho_k = \frac{\sigma_{\tilde{a}_k}^2}{\sigma_{\tilde{n}_k}^2} = \frac{P_k s_{kk}^2}{\sigma_n^2}. \qquad (14)$$

The average uncoded BER $\bar{P}_b$ is then expressed as an arithmetic mean of the BER for every sub-channel

$$\bar{P}_b = \frac{1}{K} \sum_{k=1}^{K} P_b(\rho_k), \qquad (15)$$

where $P_b(\rho_k)$ is the BER of the k-th sub-channel. Hence the TPA scheme that approximately minimizes the average uncoded BER (AMBER-TPA) under sum-power constraint is given by

$$\{P_k\}_{k=1}^{K} = \arg \min_{P_1, ..., P_K} \frac{1}{K} \sum_{k=1}^{K} P_b(\rho_k)$$
$$\text{s.t.} \quad \sum_{k=1}^{K} \gamma_k P_k = P_T \qquad (16)$$

where the sum-power constraint is denoted as

$$P_T = \sum_{k=1}^{K} \sigma_{x_k}^2 = \sum_{k=1}^{K} \gamma_k P_k = K . \tag{17}$$

We may use the Lagrange multiplier method to deal with this problem in (16) and the cost function may be expressed as

$$L(P_1,...,P_k,\lambda) = \frac{1}{K} \sum_{k=1}^{K} P_b(\rho_k) + \lambda(\sum_{k=1}^{K} \gamma_k P_k - K) . \tag{18}$$

From $\partial L / \partial P_k = 0$, we get a set of K equations

$$\frac{\partial P_b(\rho_k)}{\partial P_k} = -\lambda K \gamma_k, k=1,...,K . \tag{19}$$

Solving $K+1$ equations in (17) and (19), we may obtain a closed-form expression of TPA matrix **P** by minimizing the BER in (15).

C. Application to Uncoded QAM

When uncoded M-ary QAM is used for THP transmission, the BER for the K-th sub-channel can be tightly approximated as an exponential function of ρ_k [14], [16]

$$P_b(\rho_k) = \frac{1}{5} \exp\left(-\frac{3\rho_k}{2(M-1)}\right) . \tag{20}$$

The average uncoded BER can be expressed as

$$\overline{P}_b = \frac{1}{K} \sum_{k=1}^{K} \frac{1}{5} \exp\left(-\frac{3\rho_k}{2(M-1)}\right) = \sum_{k=1}^{K} \alpha \exp(-\beta_k P_k) , \tag{21}$$

where $\alpha = \frac{1}{5K}$ and $\beta_k = \frac{3|s_{kk}|^2}{2(M-1)\sigma_n^2}$.

Taking (21) into (18) and solving equations (19), the closed-form solution of the transmit power is

$$P_k = \left(\frac{\ln \frac{\alpha\beta_k}{\gamma_k} - \ln\lambda}{\beta_k}\right)^+ , k=1,...,K , \tag{22}$$

where $(x)^+ \triangleq \max(x,0)$, the Lagrange multiplier λ is chosen to satisfy the constraint (17), i.e.,

$$\lambda = \exp\left(\frac{\sum_{k=1}^{K} \frac{\gamma_k}{\beta_k} \ln \frac{\alpha\beta_k}{\gamma_k} - K}{\sum_{k=1}^{K} \frac{\gamma_k}{\beta_k}}\right) . \tag{23}$$

Applying (23) to (22), the closed-form expression of TPA matrix is obtained. It should be noted that the TPA scheme in (22) is more reasonable for we considered the precoding loss factor γ_k and the results are shown in the next section.

IV. SIMULATION RESULTS

Computer simulations are performed to evaluate the uncoded BER performance over signal to noise ratio (SNR). Two kinds of modulation, square 4QAM and square 16QAM, with Gray encoding are employed [18] during the simulation. We assume $N_T = N_R = K = 4$ i.e., the channel matrix **H** is square 4×4, quasi static and assumed to be perfectly known at the transmitter.

Fig.3 shows the performance comparisons using square 4QAM modulation for MIMO-THP systems. There are three kinds of curves, which are THP without TPA, conventional approximate minimum SER (AMSER)-TPA [15] and proposed AMBER-TPA (22), employed to THP system under ZF and MMSE case, respectively. It is clearly that both TPA schemes achieve better performance than systems without TPA in all cases. More importantly, because we considered the precoding loss factor, proposed AMBER-TPA also achieves better performance than conventional AMSER-TPA, Under ZF cases, we observe that at BER of 10^{-2}, proposed AMBER-TPA offers 3.7 and 0.8 dB SNR gains over THP without TPA and conventional AMSER-TPA schemes, respectively. When MMSE case is employed at BER of 10^{-3}, proposed AMBER-TPA offers nearly 7.4 dB SNR gains over THP without TPA. More importantly, It should be noted that at BER of 10^{-4}, proposed AMBER-TPA achieves nearly 2.0 dB SNR gains than conventional AMSER-TPA schemes.

Fig.4 shows the performance comparisons using square 16QAM modulation for MIMO-THP systems. There are also six curves the same as Fig. 3. Noteworthy, both under ZF case and MMSE case, the curves of proposed AMBER-TPA and AMSER-TPA are almost indistinguishable. This indicates that when the modulation size is moderate, the precoding loss effect of TPA is negligible. Because the AMSER-TPA scheme in [15] needs iterative procedure, and yet the proposed AMBER-TPA can get a closed-form solution, complexity of the latter algorithm is lower. Therefore, we focus on the comparisons between proposed AMBER-TPA and THP without TPA. At BER of 10^{-2}, the ZF-THP with AMBER-TPA gives approximately 2.4 dB gains over ZF-THP without TPA, and the MMSE-THP with AMBER-TPA gives approximately 2.1dB gains over MMSE-THP without TPA.

V. CONCLUSIONS

An AMBER-TPA algorithm for MIMO-THP system is proposed under flat fading scenarios. This scheme is achieved by minimizing the average uncoded BER under sum-power constraint and considering the precoding loss. The comparisons show that the proposed AMBER-TPA method can effectively improve the BER performance of MIMO-THP systems without TPA. Furthermore, it is not only more efficient calculation, but also achieves better performance than AMSER-TPA method in [15]. Finally, above results are derived under perfect conditions, while in fact it is difficult to obtain ideal information. An important topic for our future work is to apply the algorithm to more realistic situations, such as extended to multiuser case or coded MIMO system and analysis of the penalty for using imperfect or outdated channel information.
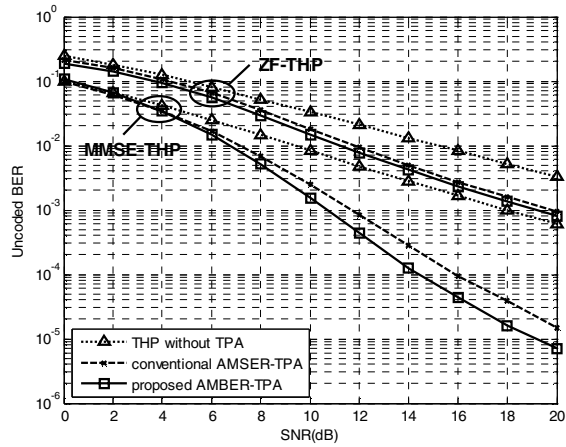
Figure 3. Uncoded BER performance comparisons of proposed AMBER-TPA, conventional AMSER-TPA and without TPA in MIMO-ZF/MMSE-THP for square 4QAM ($N_T = N_R = 4$).
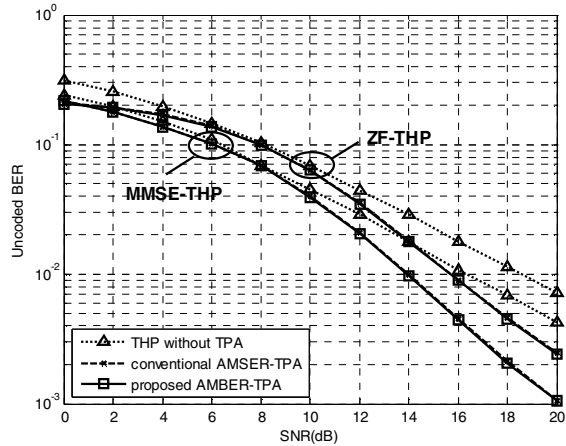

Figure 4. Uncoded BER performance comparisons of proposed AMBER-TPA, conventional AMSER-TPA and without TPA in MIMO-ZF/MMSE-THP for square 16QAM ($N_T = N_R = 4$).

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