COMPARATIVE ANALYSIS OF A–V AND A–T–T_0 CALCULATIONS OF INDUCED CURRENTS IN MULTIPLY CONNECTED REGIONS

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Abstract

The paper offers a comparative analysis of two methods, both using potentials, for electromagnetic field computation in multiply connected regions, including a conventional $\boldsymbol{A}$–V approach and a fairly new and much less popular $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation. The relevant finite element equations are provided. To facilitate comparisons the TEAM Workshop Problem No. 7 has been solved and the results of both methods verified by measurements. Computational times have been considered and the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ approach found to be much more efficient.

1. Introduction

In the analysis and synthesis of electromagnetic fields the formulations relying on potentials are most commonly used. There are three basic possibilities: (a) the Ω–$\boldsymbol{T}$ description, where the magnetic field is described by the scalar potential Ω, while the electric field in terms of the electric vector potential $\boldsymbol{T}$; (b) the probably best known $\boldsymbol{A}$–V formulation, with the vector potential $\boldsymbol{A}$ describing the magnetic field and the electric scalar potential V applied to the electric field; and (c) the $\boldsymbol{A}$–$\boldsymbol{T}$ description utilizing both vector potentials. In the analysis of induced currents in three dimensions the $\boldsymbol{A}$–V approach has dominated, mainly due to the simplicity and universality of the associated finite element (FE) algorithm. Moreover, the procedures for solving simply and multiply connected regions are similar. The disadvantage of the $\boldsymbol{A}$–V formulation arises from inefficient computation, a result of slow convergence of the iterative scheme of solving the finite element equations [1]. On the other hand, the $\boldsymbol{A}$–$\boldsymbol{T}$ and Ω–$\boldsymbol{T}$ methods are ill suited to calculations of induced currents in multiply connected regions [1,4]. The classical $\boldsymbol{T}$ description needs to be supplemented by auxiliary equations in terms of the electric vector potential $\boldsymbol{T}_0$ describing current distribution around the 'holes' of the multiply connected volumes [2,4]. As a result a joint $\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation has emerged [4], slowly gaining popularity.

The authors of this article have been searching for more efficient formulations of the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ description by utilizing edge values of the vector potentials; these have been shown to be competitive with other well established methods [4]. Here we focus on the comparison of performance of our own algorithms based on the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation with typical $\boldsymbol{A}$–V computations employing edge values of the vector potential $\boldsymbol{A}$ and nodal values of the scalar potential V. The TEAM Workshop Problem No.7 [8] (Fig. 1) has been solved to investigate the accuracy and efficiency of calculations. The finite element equations have been set up following [3] and [7] and described using the language of circuit theory. The finite element equations arising from the scalar formulation and nodal elements are equivalent to nodal equations of an edge network constructed from branches associated with element edges; whereas the equations for the vector potentials for the system described by edge elements are represented by loop equations of the facet network made of branches joining element mid-points.

2. FE equations for the A–V formulation

The finite element equations based on the use of the magnetic vector potential $\boldsymbol{A}$ and electric scalar potential V are equivalent to the loop equations of the magnetic facet network (FN) coupled with the nodal equations of the electric edge network (EN) [3]. The coupling between the networks is provided by the sources and the equations may be written as

$$\begin{bmatrix} \boldsymbol{k}_n^T \boldsymbol{G}_g \boldsymbol{k}_n & -\mathrm{p}\boldsymbol{k}_n^T \boldsymbol{G}_g \\ -\boldsymbol{G}_g \boldsymbol{k}_n & \boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e + \mathrm{p}\boldsymbol{G}_g \end{bmatrix} \begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{\phi}_e \end{bmatrix} = \begin{bmatrix} 0 \\ \boldsymbol{\theta}_m \end{bmatrix} \tag{1}$$

where $\boldsymbol{V}$ is the vector of nodal potentials, $\boldsymbol{\phi}_e$ represents the loop fluxes, i.e. edge values of $\boldsymbol{A}$, $\boldsymbol{G}_g$ is the matrix of branch conductances in EN, $\boldsymbol{R}_\mu$ is the matrix of branch reluctances of the FN, $\boldsymbol{k}_n$ is the transposed nodal incidence matrix of EN and $\boldsymbol{k}_e$ is the transposed loop matrix for FN. The symbol $\boldsymbol{\theta}_m$ represents the vector of the external loop *mmf*s.

3. FE equations for the A–T–T_0 formulation

The FE equations for edge elements and vector potentials $\boldsymbol{A}$, $\boldsymbol{T}$ and $\boldsymbol{T}_0$ are equivalent to loop equations of the coupled magnetic and electric facet networks (FNs). In the electric network, distinction has to be made between equations for loops around the element edges and those surrounding a 'hole' in the multiply connected case. The loop currents $\boldsymbol{i}_m$ around the element edges represent the edge values of the potential $\boldsymbol{T}$, while currents $\boldsymbol{i}_o$ embracing the 'hole' correspond to the edge values of the potential $\boldsymbol{T}_0$. The derivation of currents $\boldsymbol{i}_o$ may follow either of two equivalent formulations, where the description of the loops may be linked to: (a) cuts with of loop surfaces with element edges, or (b) cuts of loop edges with element facets [4]. Here, we follow the latter description and the FE equations have been written as

$$\begin{bmatrix} \boldsymbol{k}_e^T \boldsymbol{R}_\rho \boldsymbol{k}_e & \boldsymbol{k}_e^T \boldsymbol{R}_\rho \boldsymbol{z}_f & \mathrm{p}\boldsymbol{k}_e^T \boldsymbol{K} \\ \boldsymbol{z}_f^T \boldsymbol{R}_\rho \boldsymbol{k}_e & \boldsymbol{z}_f^T \boldsymbol{R}_\rho \boldsymbol{z}_f & \mathrm{p}\boldsymbol{z}_f^T \boldsymbol{K} \\ -\boldsymbol{K}^T \boldsymbol{k}_e & -\boldsymbol{K}^T \boldsymbol{z}_f & \boldsymbol{k}_e^T \boldsymbol{R}_\mu \boldsymbol{k}_e \end{bmatrix} \begin{bmatrix} \boldsymbol{i}_m \\ \boldsymbol{i}_o \\ \boldsymbol{\phi}_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \boldsymbol{\theta}_m \end{bmatrix} \tag{2}$$

where $\boldsymbol{R}_\mu$ and $\boldsymbol{R}_\rho$ are the matrices of branch reluctances and resistances of relevant FNs, $\boldsymbol{z}_f$ is the matrix describing the distribution of loops around the 'holes', and $\boldsymbol{K}$ is a matrix transposing the branch values of the FN into the values related to branches of the EN.

It should be noted that the difference between the formulations $\boldsymbol{A}$–V and $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ applies only to the equations describing the flow of induced currents. To find the current distributions a nodal method is used in $\boldsymbol{A}$–V, whereas a loop scheme in $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$.

4. Results and comparison of methods

The TEAM Workshop Problem No.7 [8] (Fig. 1) has been solved using both formulations ($\boldsymbol{A}$–V and $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$) and results compared with measurements published in [5]. Parallelepiped elements have been used resulting in about half a million equations for the edge values of the vector potential $\boldsymbol{A}$, 20 thousand for the scalar potential $\boldsymbol{V}$ and 63 thousand for the edge values of $\boldsymbol{T}$–$\boldsymbol{T}_0$. Complex potentials were used throughout and the equations were solved using the BR-ICCG method [9]. Figures 2 and 3 present the computed and measured selected distributions: (a) of the magnetic flux density B_z along the line A_1–B_1, and (b) of the current density J_y along the line A_2–B_2, for two frequencies (50Hz and 200Hz). The positions of the lines A_1–B_1 and A_2–B_2 are depicted in Fig. 1.

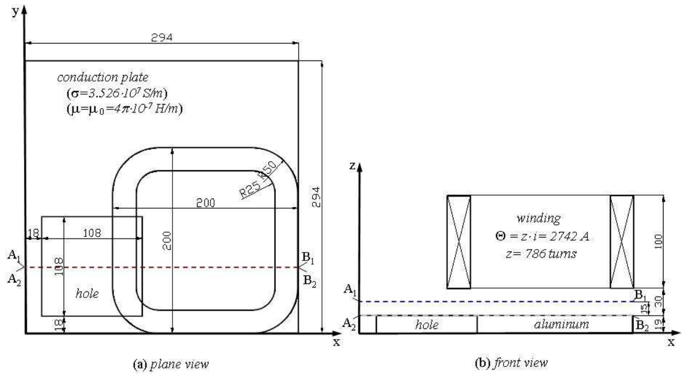

Fig. 1. The TEAM Workshop Problem No. 7 – Asymmetrical Conductor with a Hole [8]

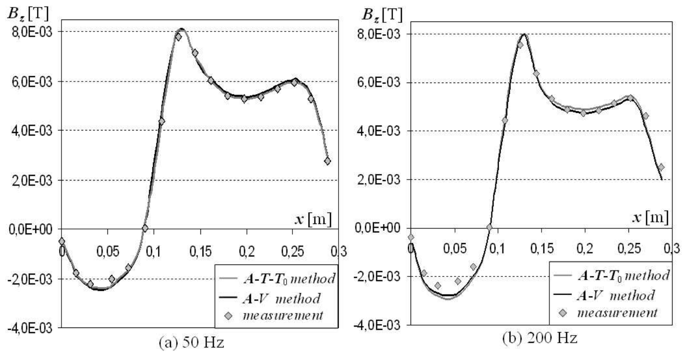

Fig. 2. Magnitude of B_z along the line A_1-B_1 as shown in Fig. 1.

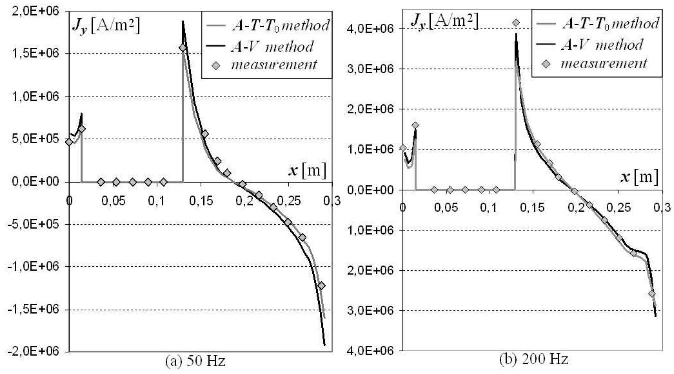

Fig. 3. Magnitude of J_y along the line A_2-B_2 as shown in Fig. 1.

The comparison shows good agreement between the computed results and measurements. A more careful inspection of the results has revealed that the results of the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ approach are slightly more accurate.

The main purpose of this investigation was to assess the efficiency of both algorithms by comparing the computational times required to achieve a prescribed accuracy. Both calculations were performed on a worktop PC with a 2.93 GHz IntelCore Duo processor and 2GB of RAM. In the case of the $\boldsymbol{A}$–V approach the solution was reached after 6 hours 23 minutes and 20 seconds for the case of frequency of 50 Hz, whereas computations using the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ approach converged after only 19 minutes and 52 seconds. Similar time savings were observed at the frequency 200 Hz. This dramatic reduction in the computational effort was possible thanks to the imposition of the ungauged formulation, for both magnetic and electric field loop equations, as first suggested independently by Ren [6].

5. Conclusions

The paper compares and contrasts two methods, the commonly employed $\boldsymbol{A}$–V approach and a new formulation based on the vector potentials $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ adapted by the authors for problems with induced currents in multiply connected regions. Both methods yield similar accuracy of the solution but the $\boldsymbol{A}$–$\boldsymbol{T}$–$\boldsymbol{T}_0$ formulation was found to be computationally much more efficient leading to significant reduction of execution times. It is therefore argued that such formulation is more appropriate for problems in multiply connected regions.

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