Transport properties and current flow patterns in homogeneous strongly anisotropic materials

Richard D. Chippendale, Igor O. Golosnoy,
Paul L. Lewin and Jan K. Sykulski

School of Electronics and Computer Science, University of Southampton, Southampton, UK

Abstract

Purpose – The purpose of this paper is to investigate and explain the unexpected current flow patterns and twisting equipotential surfaces observed in strongly anisotropic materials.

Design/methodology/approach – Potential distributions and current flow paths in highly anisotropic composite materials were studied via numerical simulation and experimentally. Simplified composite panels with two plyes were analysed using a finite-element model; the predictions were then confirmed experimentally.

Findings – The unexpected twisting equipotential surfaces and current flow patterns were found to be consistent with minimising of Joule heat release in the material. Numerical modelling suggests that the twisted profiles of the potential are highly sensitive to the anisotropic electrical conductivity.

Originality/value – This paper discusses the reverse current flows witnessed in a two-layer anisotropic system. Such behaviour has never been predicted or observed experimentally before. The reported results will be of interest to anyone who is considering using anisotropic materials such as carbon fibre composites which might experience applied potential difference, such as lightning strikes.

Keywords Physical properties of materials, Composite materials, Electric current

Paper type Research paper

1. Introduction

Carbon fibre composites (CFCs) consist of two components with very different properties. These components are combined together to obtain a material suitable for a wide range of applications (Lee, 1990). Normally, CFCs contain several laminates arranged in plain layers. Each laminate has continuous carbon fibres aligned along one direction. The large difference in conductivity of epoxy and carbon fibre results in highly anisotropic characteristics of individual laminates (Knibbs and Morris, 1974; Rolfes and Hammerschmidt, 1995). Assembled into the panel, the laminates provide complex conductive paths for electric current and heat flows within the layered structure. There has been very little work into investigating the transport properties of commercial CFCs panels (Kulkarni and Brady, 1997) with most studies focused on unidirectional laminates (Rolfes and Hammerschmidt, 1995; Kulkarni and Brady, 1997; Greenwood *et al.*, 1975; Todoroki *et al.*, 2002). Previous studies looked at averaged transport properties, whereas in this paper we predict the potential distribution and electrical current flow in CFC panels using a homogenous anisotropic material model (the results can also be used to analyse heat flow patterns).

2. Problem formulation

2.1 Material properties and layout

As a first approximation, it is assumed that the CFC's conductivity is represented by a tensor with three constant different electrical conductivities σ_{11}, σ_{22} and σ_{33}, with the principle direction "1" coinciding with the direction of carbon fibres, and the directions "2" and "3" assumed normal to the fibre direction. It has been observed that σ_{11} is from two to five orders of magnitude larger than the conductivities σ_{22} and σ_{33} (Knibbs and Morris, 1974; Rolfes and Hammerschmidt, 1995; Greenwood *et al.*, 1975; Todoroki *et al.*, 2002). For our modelling purposes, the CFC material is assumed to have $\sigma_{11} = 23,726\,\mathrm{S/m}$, $\sigma_{22} = 4\,\mathrm{S/m}$ and $\sigma_{33} = 4\,\mathrm{S/m}$ (Lago *et al.*, 2006; Uhlig, 1998). The model is designed using two square layers of CFC laminates stacked on top of each other. The analysis is done for the panel made of two laminates with the principle direction "1" rotated by 90° in the bottom layer, see Figure 1. Two circular electrodes are placed above and below the centre of the CFC stack, with a potential difference of 4.5 V applied across the electrodes. The dimensions and overall diagram of the model are shown in Figure 1.

2.2 Governing equations and boundary conditions

The solution for the steady-state current flow problem in terms of the electric current density **J** and the electric field **E** is given by equation (1), which in turn – together with Ohm's law – results in formulation (2) for the electric potential φ:

$$\nabla \cdot \mathbf{J} = 0, \quad \mathbf{E} = -\nabla\varphi, \quad \mathbf{J} = \sigma\mathbf{E} \tag{1}$$

$$\nabla \cdot \sigma\nabla\varphi = 0 \tag{2}$$

The application of the Dirichlet's principle to equations (1) and (2) is equivalent to seeking the minimum of the Joule heat problem:

$$S \rightarrow \min \left(\iiint_{volume} \mathbf{J} \cdot \mathbf{E}\,dv \right) \tag{3}$$

The external boundary conditions are provided by electrical insulation on all surfaces except where the electrodes are placed. The potential for the top electrode is assumed to be $\varphi_{top} = 4.5\,\mathrm{V}$, and the bottom electrode is grounded.
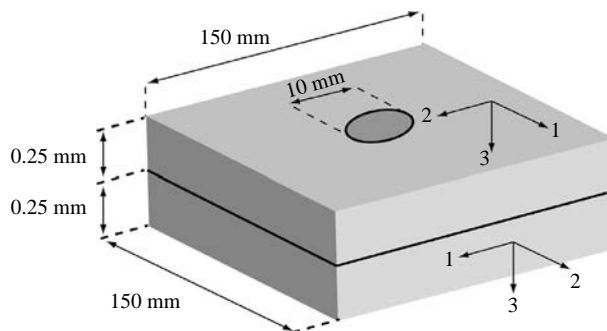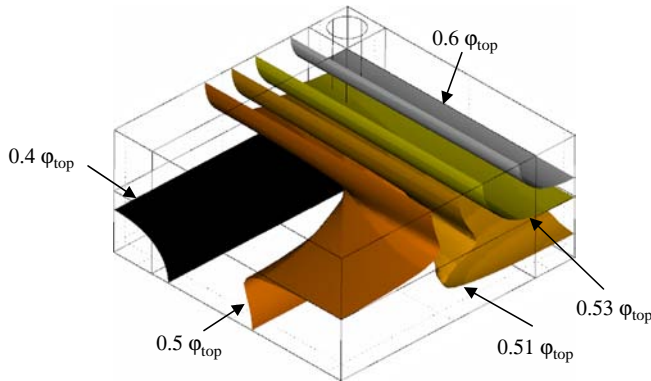

Figure 1.
Schematic diagram
of the CFC stack model

Numerical predictions were obtained from a commercial finite-element software COMSOL (2010). Three-dimensional cubic shape elements were used with 110 nodes uniformly spaced along the in-plane directions and 26 nodes through the thickness. It was found that smooth equipotential surfaces could be obtained by Lagrange linear interpolation functions. Owing to a complex shape of the equipotential surfaces, the application of quadratic or higher order interpolation functions results in a wobbling of the surfaces and a drop in the model accuracy.

3. Numerical predictions

The equipotential surfaces within ± 1 per cent of the mid potential $0.5\,\varphi_{top}$ have a twist, as the potential effectively flips over as shown in Figure 2. At first glance, this is very unusual as normally the potential is expected to follow a gradual and uniform change throughout the entire volume resulting in the mid equipotential surface being flat. However, this twist in the potential could be explained by the current flow paths inside the CFC panel. We expect the current to take the path of least resistance. In an isotropic material, this would mean travelling directly through the material between the electrodes. The strong anisotropy of the CFC panel, however, creates a different current flow.

Locally, the current tends to travel mainly along the local principal axis "1" with high σ_{11}, rather than straight down through the material with low σ_{33}. The diagram in Figure 3 shows the simplified current flow pattern, while Figure 4 shows the model predictions for the real pattern. Taking the current starting point to be the top electrode,

Note: The twisting of the $0.5\,\varphi_{top}$ equipotential surface and a small potential drop in the twisting region

Figure 2.
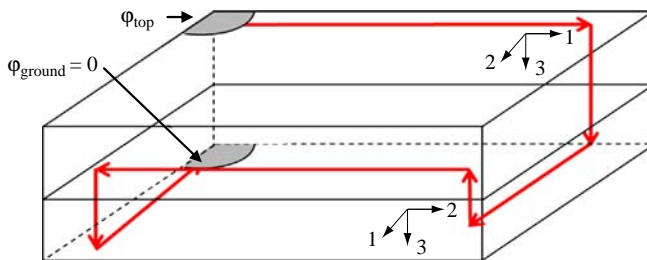Zoomed in image of the equipotential surfaces inside the panel

Figure 3.
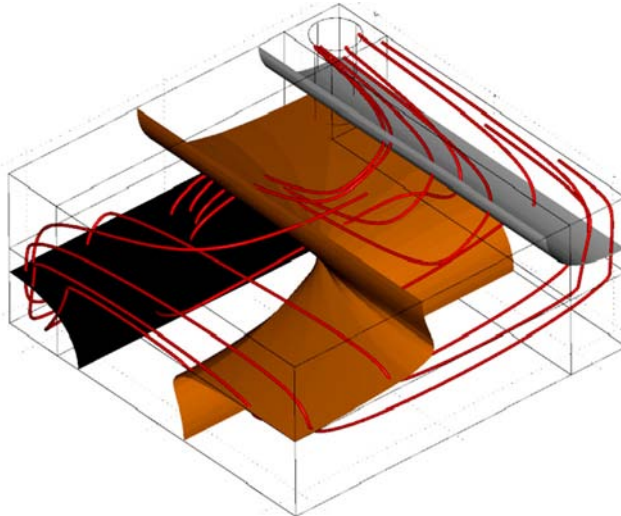A simplified current flow pattern is shown with the arrows

Note: The current predominately flows in the direction of fibres, i.e.
local "1" direction with high conductivity

the current initially flows along the principal axis "1" in the top layer. As it travels along
the top layer, it begins also to penetrate through the thickness of the panel towards the
bottom layer. In the bottom layer, the direction with high σ_{11} is rotated by 90° in the
plane, and so the current prefers to flow in this direction as it reaches the bottom layer,
Figure 3. After having been distributed over a large surface area, the current starts
to flow back up into to the top layer, towards the corner far from the electrode as in
Figures 3 and 4. When it re-enters the top layer, it again travels along the top layer's
high-conductive direction and at some point begins to penetrate back down through the
panel into the bottom layer. It can now flow along the high-conductive axis to reach the
bottom grounded electrode. In summary, the flow path is like travelling around all four
sides of the square panel. The interesting observation is that the current travels a much
longer distance than expected and also passes through the low-conductivity directions
three times.

The rationale behind these somewhat unusual current flows may be better
explained by the minimum energy formulation (action integral) equation (3), rather
than from equations (1) to (2), as current density **J** is minimised throughout the sample
by following the high-conductivity directions. Such a distribution abnormally reduces
the current density without experiencing significant resistance. The only problem is to
connect the currents in the top and the bottom layers, the latter being rotated by 90°.
This happens through the twisting in the potential which encounters very small
variation along the sides and does not contribute to the Joule heat (3) since the field **E** is
kept at a minimum value!

4. Experimental verification
The numerical results were verified against the experiment by measuring the
electrical potential down the edge of the sample using a needle and a digital voltmeter.

The potential difference is then compared against the corresponding predictions from the numerical model. Owing to the symmetry of this problem only a quarter of the panel needs to be considered (as shown in Figures 2-4).

4.1 Experimental method
The experiment was designed to mimic that of the numerical model, discussed above, and is shown in Figure 5. A two-layer CFC panel, with the same geometry as the numerical model was obtained for this experiment. The two circular electrodes were made from aluminium and attached to the sample with silver paste. The paste also acted to improve the electrical contact between the CFC and the electrodes. The needle was connected to a digital voltmeter and then placed touching the side of the CFC at $(x, z) = (0, 0)$. The needle is then moved down the edge of the sample from $z = 0$ mm to $z = 5$ mm, with the potential difference recorded as the needle is moved. Once the needle has reached $z = 5$ mm, i.e. the bottom of the CFC, the needle is moved along the x axis to a new location and the measurements are repeated as before.

4.2 Experimental results
The experimental results are shown in Figure 6. One can see that in the top laminate the potential difference remains constant, which is consistent with predictions shown in Figure 2. In each vertical slice between 0 mm $< x < 45$ mm, the potential difference begins to significantly decrease in the bottom laminate. As measurements are taken further along the edge of the sample ($x > 45$ mm), the behaviour of the potential changes. At this point, the potential in the bottom laminate stops decreasing and begins to increase instead, Figure 6(b).

## 5. Comparision of numerical predictions with experimental results
The numerical predictions and the experimental results are compared in Figure 7. The graphs show the potential difference along the vertical slice down the edge of the sample for $x = 0$ mm and $x = 70$ mm. It can be seen that the experimental results
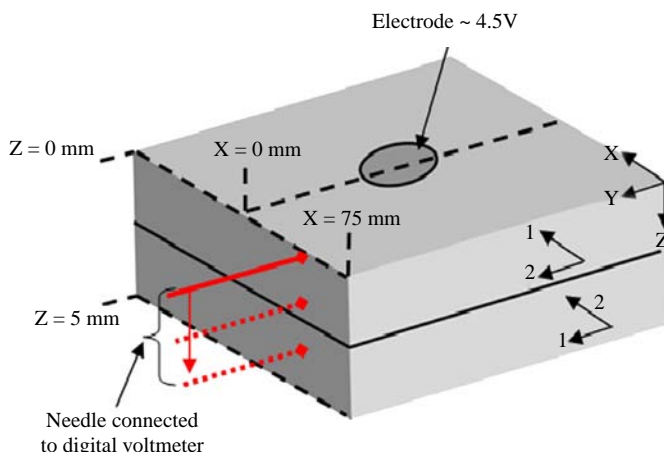
Note: The needle which is connected to a digital voltmeter for measurements is shown by a red line which is terminated with a diamond

Figure 5.
Schematic diagram of the
experimental setup

Figure 6.
Experimental results for
the potential difference
down the edge of the panel
(z) for different locations
along the edge (x)
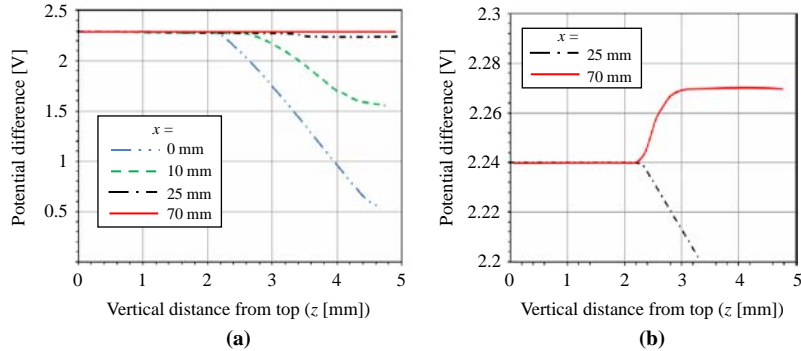
Notes: (a) For $x = 0$ mm, $x = 10$ mm, $x = 25$mm and $x = 70$ mm; (b) zoomed in view of results for $x = 25$ mm and $x = 70$ mm

portray the same potential difference trends as those predicted by the numerical model. This implies that the twisting potential and the upward current flow patterns occur in the experiment. It also supports the real physical nature of the predicted phenomena.

5.1 Parametric study of varying the conductivity

The electrical conductivity values for the CFC sample used in the modelling have not been measured. Instead, the average values were obtained from literature (Lago *et al.*, 2006; Uhlig, 1998). Therefore, a parametric study has been conducted to see how varying the conductivities (σ_{11}, σ_{22} and σ_{33}) affects the numerical predictions. At this stage, no attempt was made to solve the inverse problem, i.e. to determine the materials conductivities from the potential measurements. The effect of varying the through-thickness conductivity (σ_{33}) can be seen in Figure 7 for $x = 0$ mm and $x = 70$ mm. As the through-thickness conductivity is reduced to $\sigma_{33} = 1$ S/m, the numerical predictions more accurately resemble the experimental results. The reference input values for σ_{11}, σ_{22} and σ_{33} do not reproduce the twisting point of the potential at $x \approx 45$ mm. Varying the σ_{33} value seems to have limited the effect on the location at which the potential twists over.
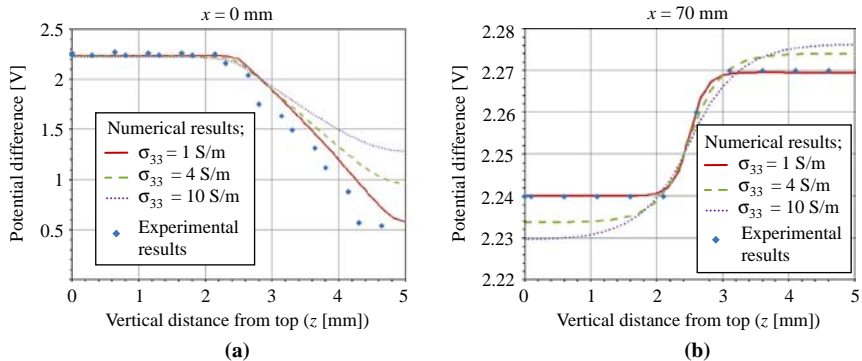

Figure 7.
Potential difference
predicted by the numerical
model and the
experimental results for
(a) $x = 0$ mm and
(b) $x = 70$ mm

Note: σ_{33} was varied whilst the remaining conductivities were kept constant: $\sigma_{11} = 23,726$ S/m and $\sigma_{22} = 4$ S/m

In an attempt to represent the location of the twisting potential, the in-plane conductivity perpendicular to the fibre (σ_{22}) is varied. It was noted that σ_{22} has a negligible effect on the potential difference away from potential twisting location, i.e. around $x = 0$ mm and $x = 70$ mm. Instead σ_{22} has a strong dependence on the location at which the twisting potential occurs. This can be seen in Figure 8, as by decreasing the σ_{22} the decreasing potential diminishes at $x = 15$ mm. Experimentally, a noticeable drop in the potential is observed at this location. Notice that if $\sigma_{22} = 1$ S/m, the predicted potential has twisted over at this location. Therefore, a much better agreement with the experimental results can be achieved by a significant increase in σ_{22}.

The sensitivity of the model to variations of the conductivity in the direction of the fibre (σ_{11}) was also considered. This does not show any sizeable variation in potential difference profiles.

6. Conclusions
The equipotential surfaces in anisotropic materials have a twisted profile with the twist occurring approximately within ± 1 per cent of the mid potential. This conclusion has been obtained from a numerical model and confirmed by experiments. Such potential profiles result in an upward current flow in the corners of the anisotropic material.

A special experiment has been designed to verify the numerical predictions. The model developed uses the electric conductivity as an input parameter. Accurate conductivity values are not available so averaged literature values were used instead. The direct comparison with the experimental results indicates a qualitative agreement. It is possible to conclude that the experimental results indeed demonstrate the same trends as predicted by the numerical model, and the unexpected potential profiles initially observed in numerical model are a real physical phenomenon.

It has been shown that the predictions are sensitive to the electrical conductivity. Comprehensive material characterisation is therefore extremely important for accurate estimates of the potential distribution in composites.
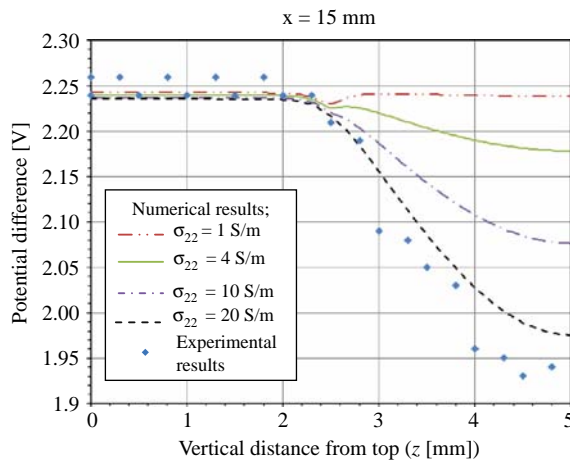
Note: σ_{22} was varied whilst the remaining conductivities were kept constant: $\sigma_{11} = 23,726$ S/m and $\sigma_{33} = 1$ S/m

Figure 8.
Potential difference predicted by the numerical model with the experimental results for $x = 15$ mm

The current flow path has a simple physical explanation. Current seeks to minimise the Joule heat released via reducing the current density component in the direction of low conductivity, rather than just taking the shortest path between the electrodes. This may appear strange as the current travels back and forth through the thickness of the panel three times rather than just once. However, this has been found to be consistent with the shape of measured equipotential surfaces.

References

COMSOL (2010), *COMSOL Multiphysics® 2010 Version 3.5a*, COMSOL AB, Stockholm.

Greenwood, J.H., Lebedat, S. and Bernasconit, J. (1975), "The anisotropic electrical resistivity of a carbon fibre reinforced plastic disc and its use as a transducer", *J. Phys. E: Sci. Instr.*, Vol. 8, pp. 369-70.

Knibbs, R.H. and Morris, J.B. (1974), "The effects of fibre orientation on the physical properties of composites", *Composites*, Vol. 5, pp. 209-18.

Kulkarni, M.R. and Brady, R.P. (1997), "A model of global thermal conductivity in laminated carbon/carbon composites", *Comp. Sci. Techn.*, Vol. 57, pp. 277-85.

Lago, F., Gonzalez, J.J., Freton, P., Uhlig, F., Lucius, N. and Piau, G.P. (2006), "A numerical modelling of an electric arc and its interaction with the anode: Part III. Application to the interaction of a lightning strike and an aircraft in flight", *Journal of Physics D-Applied Physics*, Vol. 39 No. 10, pp. 2294-310.

Lee, S.M. (1990), *International Encyclopedia of Composites*, VCH, New York, NY.

Rolfes, R. and Hammerschmidt, U. (1995), "Transverse thermal conductivity of CFRP laminates: a numerical and experimental validation of approximation formulae", *Comp. Sci. Techn.*, Vol. 54, pp. 45-54.

Todoroki, A., Tanaka, M. and Shimamura, Y. (2002), "Measurement of orthotropic electric conductance of CFRP laminates and analysis of the effect on delamination monitoring with an electric resistance change method", *Comp. Sci. Techn.*, Vol. 62, pp. 619-28.

Uhlig, F. (1998), *Contribution À L'Étude Des Effets Directs Du Foudroiement Sur Les MatÉriaux Structuraux Constituant Un AÉronef*, UniversitÉ De Paris, Paris.

About the authors

Richard D. Chippendale received his MPhys (Hons.) in Physics from the University of Southampton, Southampton, UK, in 2009. He joined the School of Electronics of Computer Science as part of the Electrical Power Engineering group where he is currently pursuing his PhD degree. His research interests include finite-element analysis modelling techniques and thermal degradation of carbon fibre composites. Richard D. Chippendale is the corresponding author and can be contacted at: rc805@ecs.soton.ac.uk

Igor O. Golosnoy received his MSc degree in Applied Mathematics and Physics from Moscow Institute of Physics and Technology, Russia in 1992 and the PhD in Mathematics and Physics from Institute for Mathematical Modelling, Moscow, Russia. He is currently a Lecturer at the Electrical Power Engineering Group, School of Electronics and Computer Science, University of Southampton. His research interests include numerical modelling of various coupled electrical, thermal and mechanical phenomena with emphasis on free and moving boundaries problems.

Paul L. Lewin received the BSc (Hons.) and PhD degrees in Electrical Engineering from the University of Southampton, Southampton, UK, in 1986 and 1994, respectively. He joined the academic staff of the University of Southampton in 1989 and is now the Professor of Electrical Power Engineering in the School of Electronics of Computer Science. His research interests include condition monitoring of high-voltage systems, surface charge measurement, high-voltage

insulation/dielectric materials and applied signal processing. Since 1996, he has published more than 270 refereed conference and journal papers in these research areas. He is the Director of the Tony Davies High Voltage Laboratory at the University of Southampton. Professor Lewin is a Chartered Engineer and a Fellow of the Institution of Electrical Engineers, UK.

Jan K. Sykulski is Professor of Applied Electromagnetics in the School of Electronics and Computer Science at the University of Southampton. His research interests are in the areas of power applications of high-temperature superconductivity, modelling of superconducting materials, advances in simulation of coupled field systems (electromagnetic, circuits and motion), development of fundamental methods of computational electromagnetics and new concepts in design and optimisation of electromechanical devices. He has over 320 publications listed on the official database of the university. He is a Founding Member and former Chair of the Professional Network Electromagnetics of IET, a Founding Secretary/Treasurer/Editor of International Compumag Society, a Visiting Professor at universities in Canada, France, Italy, Poland and China and a member of International Steering Committees of major international conferences, including COMPUMAG, IEEE CEFC, IEE CEM, EMF, ISEF, EPNC, ICEF and others. He is Fellow of IEEE, IET, Institute of Physics and British Computer Society and has an honorary title of Professor awarded by the President of Poland.