Transport properties and current flow patterns in homogeneous strongly anisotropic materials

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Abstract
Purpose – The purpose of this paper is to investigate and explain the unexpected current flow patterns and twisting equipotential surfaces observed in strongly anisotropic materials.
Design/methodology/approach – Potential distributions and current flow paths in highly anisotropic composite materials were studied via numerical simulation and experimentally. Simplified composite panels with two plyes were analysed using a finite-element model; the predictions were then confirmed experimentally.
Findings – The unexpected twisting equipotential surfaces and current flow patterns were found to be consistent with minimising of Joule heat release in the material. Numerical modelling suggests that the twisted profiles of the potential are highly sensitive to the anisotropic electrical conductivity.
Originality/value – This paper discusses the reverse current flows witnessed in a two-layer anisotropic system. Such behaviour has never been predicted or observed experimentally before. The reported results will be of interest to anyone who is considering using anisotropic materials such as carbon fibre composites which might experience applied potential difference, such as lightning strikes.

Keywords Physical properties of materials, Composite materials, Electric current

Paper type Research paper

1. Introduction
Carbon fibre composites (CFCs) consist of two components with very different properties. These components are combined together to obtain a material suitable for a wide range of applications (Lee, 1990). Normally, CFCs contain several laminates arranged in plain layers. Each laminate has continuous carbon fibres aligned along one direction. The large difference in conductivity of epoxy and carbon fibre results in highly anisotropic characteristics of individual laminates (Knibbs and Morris, 1974; Rolfes and Hammerschmidt, 1995). Assembled into the panel, the laminates provide complex conductive paths for electric current and heat flows within the layered structure. There has been very little work into investigating the transport properties of commercial CFCs panels (Kulkarni and Brady, 1997) with most studies focused on unidirectional laminates (Rolfes and Hammerschmidt, 1995; Kulkarni and Brady, 1997; Greenwood et al., 1975; Todoroki et al., 2002). Previous studies looked at averaged transport properties, whereas in this paper we predict the potential distribution and electrical current flow in CFC panels using a homogenous anisotropic material model (the results can also be used to analyse heat flow patterns).

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2. Problem formulation

2.1 Material properties and layout
As a first approximation, it is assumed that the CFC's conductivity is represented by a tensor with three constant different electrical conductivities $\sigma_{11}, \sigma_{22}$ and $\sigma_{33}$, with the principle direction “1” coinciding with the direction of carbon fibres, and the directions “2” and “3” assumed normal to the fibre direction. It has been observed that $\sigma_{11}$ is from two to five orders of magnitude larger than the conductivities $\sigma_{22}$ and $\sigma_{33}$ (Knibbs and Morris, 1974; Rolfes and Hammerschmidt, 1995; Greenwood et al., 1975; Todoroki et al., 2002). For our modelling purposes, the CFC material is assumed to have $\sigma_{11} = 23,726 \text{ S/m}, \sigma_{22} = 4 \text{ S/m}$ and $\sigma_{33} = 4 \text{ S/m}$ (Lago et al., 2006; Uhlig, 1998). The model is designed using two square layers of CFC laminates stacked on top of each other. The analysis is done for the panel made of two laminates with the principle direction “1” rotated by 90° in the bottom layer, see Figure 1. Two circular electrodes are placed above and below the centre of the CFC stack, with a potential difference of 4.5 V applied across the electrodes. The dimensions and overall diagram of the model are shown in Figure 1.

2.2 Governing equations and boundary conditions
The solution for the steady-state current flow problem in terms of the electric current density $\mathbf{J}$ and the electric field $\mathbf{E}$ is given by equation (1), which in turn together with Ohm’s law results in formulation (2) for the electric potential $\varphi$:

$$\nabla \cdot \mathbf{J} = 0, \quad \mathbf{E} = -\nabla \varphi, \quad \mathbf{J} = \sigma \mathbf{E}$$

$$\nabla \cdot \sigma \nabla \varphi = 0$$

(2)

The application of the Dirichlet’s principle to equations (1) and (2) is equivalent to seeking the minimum of the Joule heat problem:

$$S \rightarrow \min \left( \iiint_{\text{volume}} \mathbf{J} \cdot \mathbf{E} dv \right)$$

(3)

The external boundary conditions are provided by electrical insulation on all surfaces except where the electrodes are placed. The potential for the top electrode is assumed to be $\varphi_{\text{top}} = 4.5 \text{ V}$, and the bottom electrode is grounded.

![Figure 1. Schematic diagram of the CFC stack model](image)
Numerical predictions were obtained from a commercial finite-element software COMSOL (2010). Three-dimensional cubic shape elements were used with 110 nodes uniformly spaced along the in-plane directions and 26 nodes through the thickness. It was found that smooth equipotential surfaces could be obtained by Lagrange linear interpolation functions. Owing to a complex shape of the equipotential surfaces, the application of quadratic or higher order interpolation functions results in a wobbling of the surfaces and a drop in the model accuracy.

3. Numerical predictions
The equipotential surfaces within $\pm 1$ per cent of the mid potential $0.5 \varphi_{\text{top}}$ have a twist, as the potential effectively flips over as shown in Figure 2. At first glance, this is very unusual as normally the potential is expected to follow a gradual and uniform change throughout the entire volume resulting in the mid equipotential surface being flat. However, this twist in the potential could be explained by the current flow paths inside the CFC panel. We expect the current to take the path of least resistance. In an isotropic material, this would mean travelling directly through the material between the electrodes. The strong anisotropy of the CFC panel, however, creates a different current flow.

Locally, the current tends to travel mainly along the local principal axis “1” with high $\sigma_{11}$, rather than straight down through the material with low $\sigma_{33}$. The diagram in Figure 3 shows the simplified current flow pattern, while Figure 4 shows the model predictions for the real pattern. Taking the current starting point to be the top electrode,

![Figure 2. Zoomed in image of the equipotential surfaces inside the panel](image)

**Note:** The twisting of the $0.5 \varphi_{\text{top}}$ equipotential surface and a small potential drop in the twisting region

![Figure 3. A simplified current flow pattern is shown with the arrows](image)
the current initially flows along the principal axis “1” in the top layer. As it travels along the top layer, it begins also to penetrate through the thickness of the panel towards the bottom layer. In the bottom layer, the direction with high $\sigma_{11}$ is rotated by 90° in the plane, and so the current prefers to flow in this direction as it reaches the bottom layer, Figure 3. After having been distributed over a large surface area, the current starts to flow back up into the top layer, towards the corner far from the electrode as in Figures 3 and 4. When it re-enters the top layer, it again travels along the top layer’s high-conductive direction and at some point begins to penetrate back down through the panel into the bottom layer. It can now flow along the high-conductive axis to reach the bottom grounded electrode. In summary, the flow path is like travelling around all four sides of the square panel. The interesting observation is that the current travels a much longer distance than expected and also passes through the low-conductivity directions three times.

The rationale behind these somewhat unusual current flows may be better explained by the minimum energy formulation (action integral) equation (3), rather than from equations (1) to (2), as current density $J$ is minimised throughout the sample by following the high-conductivity directions. Such a distribution abnormally reduces the current density without experiencing significant resistance. The only problem is to connect the currents in the top and the bottom layers, the latter being rotated by 90°. This happens through the twisting in the potential which encounters very small variation along the sides and does not contribute to the Joule heat (3) since the field $E$ is kept at a minimum value!

4. Experimental verification
The numerical results were verified against the experiment by measuring the electrical potential down the edge of the sample using a needle and a digital voltmeter.
The potential difference is then compared against the corresponding predictions from the numerical model. Owing to the symmetry of this problem only a quarter of the panel needs to be considered (as shown in Figures 2-4).

4.1 Experimental method
The experiment was designed to mimic that of the numerical model, discussed above, and is shown in Figure 5. A two-layer CFC panel, with the same geometry as the numerical model was obtained for this experiment. The two circular electrodes were made from aluminium and attached to the sample with silver paste. The paste also acted to improve the electrical contact between the CFC and the electrodes. The needle was connected to a digital voltmeter and then placed touching the side of the CFC at \((x, z) = (0, 0)\). The needle is then moved down the edge of the sample from \(z = 0\) mm to \(z = 5\) mm, with the potential difference recorded as the needle is moved. Once the needle has reached \(z = 5\) mm, i.e. the bottom of the CFC, the needle is moved along the \(x\) axis to a new location and the measurements are repeated as before.

4.2 Experimental results
The experimental results are shown in Figure 6. One can see that in the top laminate the potential difference remains constant, which is consistent with predictions shown in Figure 2. In each vertical slice between \(0\) mm \(< x < 45\) mm, the potential difference begins to significantly decrease in the bottom laminate. As measurements are taken further along the edge of the sample \((x > 45\) mm), the behaviour of the potential changes. At this point, the potential in the bottom laminate stops decreasing and begins to increase instead, Figure 6(b).

5. Comparison of numerical predictions with experimental results
The numerical predictions and the experimental results are compared in Figure 7. The graphs show the potential difference along the vertical slice down the edge of the sample for \(x = 0\) mm and \(x = 70\) mm. It can be seen that the experimental results

Note: The needle which is connected to a digital voltmeter for measurements is shown by a red line which is terminated with a diamond.
portray the same potential difference trends as those predicted by the numerical model. This implies that the twisting potential and the upward current flow patterns occur in the experiment. It also supports the real physical nature of the predicted phenomena.

5.1 Parametric study of varying the conductivity

The electrical conductivity values for the CFC sample used in the modelling have not been measured. Instead, the average values were obtained from literature (Lago et al., 2006; Uhlig, 1998). Therefore, a parametric study has been conducted to see how varying the conductivities ($\sigma_{11}$, $\sigma_{22}$ and $\sigma_{33}$) affects the numerical predictions. At this stage, no attempt was made to solve the inverse problem, i.e. to determine the materials conductivities from the potential measurements. The effect of varying the through-thickness conductivity ($\sigma_{33}$) can be seen in Figure 7 for $x = 0$ mm and $x = 70$ mm. As the through-thickness conductivity is reduced to $\sigma_{33} = 1$ S/m, the numerical predictions more accurately resemble the experimental results. The reference input values for $\sigma_{11}$, $\sigma_{22}$ and $\sigma_{33}$ do not reproduce the twisting point of the potential at $x \approx 45$ mm. Varying the $\sigma_{33}$ value seems to have limited the effect on the location at which the potential twists over.

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Figure 7.
Potential difference predicted by the numerical model and the experimental results for (a) $x = 0$ mm and (b) $x = 70$ mm

Note: $\sigma_{33}$ was varied whilst the remaining conductivities were kept constant: $\sigma_{11} = 23,726$ S/m and $\sigma_{22} = 4$ S/m
In an attempt to represent the location of the twisting potential, the in-plane conductivity perpendicular to the fibre ($\sigma_{22}$) is varied. It was noted that $\sigma_{22}$ has a negligible effect on the potential difference away from potential twisting location, i.e. around $x = 0$ mm and $x = 70$ mm. Instead $\sigma_{22}$ has a strong dependence on the location at which the twisting potential occurs. This can be seen in Figure 8, as by decreasing the $\sigma_{22}$ the decreasing potential diminishes at $x = 15$ mm. Experimentally, a noticeable drop in the potential is observed at this location. Notice that if $\sigma_{22} = 1$ S/m, the predicted potential has twisted over at this location. Therefore, a much better agreement with the experimental results can be achieved by a significant increase in $\sigma_{22}$.

The sensitivity of the model to variations of the conductivity in the direction of the fibre ($\sigma_{11}$) was also considered. This does not show any sizeable variation in potential difference profiles.

6. Conclusions
The equipotential surfaces in anisotropic materials have a twisted profile with the twist occurring approximately within ±1 per cent of the mid potential. This conclusion has been obtained from a numerical model and confirmed by experiments. Such potential profiles result in an upward current flow in the corners of the anisotropic material.

A special experiment has been designed to verify the numerical predictions. The model developed uses the electric conductivity as an input parameter. Accurate conductivity values are not available so averaged literature values were used instead. The direct comparison with the experimental results indicates a qualitative agreement. It is possible to conclude that the experimental results indeed demonstrate the same trends as predicted by the numerical model, and the unexpected potential profiles initially observed in numerical model are a real physical phenomenon.

It has been shown that the predictions are sensitive to the electrical conductivity. Comprehensive material characterisation is therefore extremely important for accurate estimates of the potential distribution in composites.
The current flow path has a simple physical explanation. Current seeks to minimise the Joule heat released via reducing the current density component in the direction of low conductivity, rather than just taking the shortest path between the electrodes. This may appear strange as the current travels back and forth through the thickness of the panel three times rather than just once. However, this has been found to be consistent with the shape of measured equipotential surfaces.

References

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