Uplink Channel Estimation for Bandlimited MC-DS-CDMA Systems Relying on Long Spreading Codes

Shuai Wang†, Jing-yi Lu†, Jianping An†, and Lajos Hanzo*,
† School of Information Science & Electronics, Beijing Institute of Technology, Beijing, China,
* School of ECS, University of Southampton, SO17 1BJ, United Kingdom.
Tel: +44 23 8059 3125, Fax: +44 23 8059 4508
Email: swang@ecs.soton.ac.uk, espanastella@sina.com, an@bit.edu.cn, lh@ecs.soton.ac.uk
http://www-mobile.ecs.soton.ac.uk

Abstract—This paper considers pilot-based parameter estimation for bandlimited MC-DS-CDMA systems relying on long spreading codes. Three different schemes are proposed and compared. The two so-called unstructured algorithms, namely the Least Squares Estimator (LS-E) and the Least Absolute Shrinkage and Selection Operator (LASSO-E) first estimate the composite channel impulse response, and then extract the propagation delay, amplitude and phase. By contrast, the third algorithm namely the Structured LS Search Estimator (SLSS-E) exploits the a priori knowledge of the chip waveform and directly estimates the channel parameters. Parallel interference cancelation (PIC) is incorporated in the SLSS-E for the sake of mitigating the effect of multiple access interference and hence to further improve the performance. The complexity of PIC assisted LS-E and LS-E only increases linearly with the number of users \( K \), with the number of subcarriers \( U \) and with the length of the pilot sequence \( N_p \). Simulation results indicate that the PIC assisted structured estimator outperforms its unstructured counterparts.

I. INTRODUCTION

As a competitive candidate for the physical-layer technique in future wireless networks, Multi-Carrier Direct-Sequence Code Division Multiple Access (MC-DS-CDMA) is proposed for example for the evolution of the High Speed Package Access (HSPA) system, where the detrimental effect of the uplink Multiple Access Interference (MAI) may be mitigated by Multiple User Detection (MUD) [1]. The performance of MUD rests a lot with channel estimation. However, open literatures on channel estimation for MC-DS-CDMA systems often term as being unstructured [7], in contrast to the unstructured LS and LASSO based estimators of [6, 14]. Therefore, we may characterize the fading channel of the \( u \)-th user as

\[
\begin{align*}
\end{align*}
\]

In the above expression, \( T_b \) is the duration of symbol interval, and \( b_k(q) \in (-1, +1) \) is the \( q \)-th bipolar information symbol transmitted by the \( k \)-th user. Furthermore, \( A_k \) and \( \bar{\tau}_u \) represent the amplitude and propagation delay of the \( k \)-th user’s signal, while \( \phi^{(u)}(t) \) is the zero-mean complex-valued white Gaussian noise with Power Spectral Density (PSD) equal to \( 2N_0 \). Since each subband signal is assumed to experience time-invariant block fading during a packet’s transmission, we may characterize the fading channel of the \( k \)-th user on the \( u \)-th subcarrier by its impulse response of

\[
\begin{align*}
\end{align*}
\]

We consider the asynchronous uplink of a bandlimited long-code aided MC-DS-CDMA system that supports \( K \) active users, all of whom transmit on the same \( U \) subcarriers. We assume that the \( U \) subchannels of each user are sufficiently far apart so that they do not overlap with each other [13]. Then the baseband equivalent of the signal received on the \( u \)-th subcarrier is given by:

\[
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\end{align*}
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\[
\begin{align*}
\end{align*}
\]
both sides of Eq.(1) with $\eta_{\text{brc}}(4T_c - t)$, we arrive at:

$$r^{(u)}(t) = \sum_{k=0}^{K-1} \sum_{q = \infty}^{-\infty} A_k b_k(q) s_k^{(u)}(t - \tau_k - qT_k) * c_k^{(u)}(t) + n^{(u)}(t),$$

where $\eta_{\text{brc}}(t)$ represents a raised cosine chip waveform time-limited to $[0, 8T_c]$ and $s_k^{(u)}(t) = \sum_{n=\infty}^{-\infty} \beta_{k,n}(q) s_{k,n}(q)(t - nT_k)$. Finally, we note that $n^{(u)}(t) = w^{(u)}(t) * h_{\text{brc}}(4T_c - t)$ is a colored Gaussian noise process.

Upon denoting the effective signature waveform by $h_k^{(u)}(t; \tau_k) = A_k s_k^{(u)}(t - \tau_k) * c_k^{(u)}(t)$, Eq.(2) can be rewritten as:

$$r^{(u)}(t) = \sum_{k=0}^{K-1} \sum_{q = \infty}^{-\infty} b_k(q) h_k^{(u)}(t - qT_k, \tau_k) + n^{(u)}(t).$$

(3)

Assuming $0 \leq (\tau_k + T_m) < T_c$, where $T_m$ stands for the maximum delay spread [6], it may be readily seen that $h_k^{(u)}(t - qT_k; \tau_k)$ has a TD support of $[qT_c, q^2T_c + T_z]$. During the q-th symbol interval $T_z \triangleq [qT_c, q^2T_c + T_z]$, the term $g_k^{(u)}(t)$ in Eq.(3) is influenced by at most three consecutive bits, namely by $\{b_k(q - 2), b_k(q - 1), b_k(q)\}$. Given the sampling rate equal to $(M/T_c)$ per sec, the vector $r^{(u)}(t) \in C^{MN \times 1}$ that contains the $MN$ samples of $r^{(u)}(t)$ coming from $T_m$ can be formulated as:

$$r^{(u)}(t) = \sum_{k=0}^{K-1} \sum_{q = \infty}^{-\infty} b_k(q - 2) h_k^{(u)}(t - (q - 2)T_k; \tau_k) + b_k(q - 1) h_k^{(u)}(t - (q - 1)T_k; \tau_k) + b_k(q) h_k^{(u)}(t - qT_k; \tau_k) + n^{(u)}(t),$$

where $h_k^{(u)}(t - 2T_k), h_k^{(u)}(t - T_k), n^{(u)}(t)$ comprise the $MN$ samples of $h_k^{(u)}(t)$ and $h_k^{(u)}(t - 1) = h_{k-1}^{(u)}(t)$. By defining $g_k^{(u)} = C^{MN \times (M + 1)}$ and $\beta_k(q)$ for $0 \leq q < \infty$.

$$g_k^{(u)} = \begin{bmatrix} g_k(q) & \cdots & g_k(0) \\ T_k & \cdots & T_k \end{bmatrix} + \begin{bmatrix} \frac{8M - 1}{M}T_k & \cdots & \frac{8M - 1}{M}T_k \end{bmatrix}^T,$$

(6)

we may arrive at: $h_k^{(u)}(t) = C_{k-2}^{u}(q)g_k^{(u)}h_k^{(u-1)}(t) + g_{k-1}(q)g_k^{(u)}$, where $C_{k-2}^{u}(q), C_{k-1}^{u}(q)$, and $C_k^{(u)}$ are $MN \times (M + 8M - 1)$ dimensional matrices determined by $\beta_k(q)$. Then the following more compact formulations of $r^{(u)}(t)$ may be obtained:

$$r^{(u)}(t) = \sum_{k=0}^{K-1} A_k^{(u)}(g_k^{(u)} + n^{(u)}(q)),$$

where we have $A_k^{(u)} = b_k(q)C_{k-2}^{u}(q) + b_k(q - 1)C_{k-1}^{u}(q) + b_k(q)C_k^{u}(q)$. $A_k^{(u)}(q) = [\{A_k^{(u)}(q)\}, \cdots, A_{k-1}^{(u)}(q)]$. $g_k^{(u)} = [g_k^{(u)}(0), \cdots, g_k^{(u)}(K-1)]^T$.

If the first $N_l$ symbols sent by each user, $\{b_k(q)\}_{q=0}^{K-1}$, are known to the receiver as pilots, then we can stack the data collected within $N_l$ consecutive symbol interval on all $U$ subchannels into a $UN_lMN \times 1$ dimensional vector $r = [r^{(u)}(0)]^T, \cdots, [r^{(u)}(N_l - 1)]^T$, which takes the form:

$$r = \begin{bmatrix} A^{(u)}(t) \cdots A^{(u)}(t) \\ g^{(1)} \cdots g^{(U)} \end{bmatrix} \begin{bmatrix} n^{(u)}(t) \\ g^{(1)} \cdots g^{(U)} \end{bmatrix} = Ag + n,$$

(8)

where $A^{(u)}(t) = [\{A^{(u)}(t)\}], \cdots, [A^{(u)}(t)(N_l - 1)]^T$. $n^{(u)}(t) = [n^{(u)}(t)]^T, \cdots, [n^{(u)}(t)(N_l - 1)]^T$. In some cases we might prefer to decompose $r$ into a sum of components corresponding to different users, like:

$$r = \sum_{k=0}^{K-1} \begin{bmatrix} A_k^{(u)}(t) \\ g_k^{(u)} \end{bmatrix} = \sum_{k=0}^{K-1} A_k^{(u)}g_k + n,$$

(9)

where $A_k^{(u)} = [\{A_k^{(u)}(t)\}], \cdots, [A_k^{(u)}(t)(N_l - 1)]^T$. To handle the colored noise vector $n$ that may affect the estimation quality, we can multiply both sides of Eq.(8) and Eq.(9) with a “whitening matrix” $W \in C^{MN \times N_lMN}$, which leads to:

$$y = Wg + w = \sum_{k=0}^{K-1} F_k g_k + w,$$

(10)

where $y = Wf = W(A, f) = W(A, 1, K)$. $w$ is a complex-valued white Gaussian vector with a zero mean and a covariance matrix of $\sigma^2 I_{MN \times N_lMN}$. Given $K$ is the Cholesky factor of the covariance matrix of $n$, then we have $\sigma^2 = 1$ provided that $W = L^{-1}$.

III. LS BASED CHANNEL ESTIMATION (LS-E)

Given Eq.(10), the overall CCIR $g$ may be directly estimated by invoking the LS estimation procedure of [6]:

$$\hat{g}_L = \arg \min_{g} \|y - Fx\|^2_2 = \frac{F^HH^{-1}F^H}{F^HH^{-1}} y.$$ 

(11)

Upon rearranging the entries in $\hat{g}_L$ according to the relationships specified in Eq.(7)-(9), we can formulate $\hat{g}_L$ as the estimate of the k-th user’s CCIR $g_k$. The next step is to extract the channel parameters $\{\hat{g}_L(q)\}_{q=0}^{K}$ from $\hat{g}_L$. An in-depth description of the extraction algorithm can be found in [6] for a single carrier system and its extension to multi-carrier transmission is quite natural. For conciseness, we leave the details for readers.

IV. LASSO BASED CHANNEL ESTIMATION (LASSO-E)

The question as to why $g$ is a sparse vector has been answered for DS-CDMA in [9], and most of the arguments hold for MC-DS-CDMA as well. One way to exploit the sparsity of $g$ is to estimate $g$ as the solution to the following LASSO problem [9, 10]:

$$\hat{g}_L = \arg \min_{g} \|y - Fx\|^2_2 + \lambda \|x\|_1,$$

(12)

where $\lambda > 0$ and the larger $\lambda$ is, the more entries of $\hat{g}_L$ will become zero. In [9] $\lambda$ was determined empirically in the form of $\lambda = \sqrt{2r \ln |KU(MN + 8M - 1)|}$. Once $\hat{g}_L$ was obtained, the channel parameters of each user can be extracted in the same way as for the LS-E of Section III.

V. PIC-SLSS BASED CHANNEL ESTIMATION

Compared to the unstructured estimators of [6, 9] which rely on the explicit estimation of the CCI, structured estimators directly estimate the channel parameters and generally lead to an improved performance, since the number of unknowns to be estimated is substantially reduced [7]. To facilitate discussion, let us now commence our discussion of structured estimator designed for bandlimited long-code based MC-DS-CDMA with the simplest single-path single-user scenario, and then extend it step-by-step to more relevant scenarios, where both multipath interference and MAI are present.
A. Single-path Single-user Channel: the Simplest Scenario

Without loss of generality, we assume the uplink channel is solely occupied by user $k$. Then Eq.(10) reduces to:

$$ y = F_k g_k + w. $$

(13)

Recalling the single-path assumption of $L = 1$ and invoking Eq.(5) and (6), we have:

$$ g_k^{\text{w}} = \alpha_k^{\text{w}} RHC(\tau_k). $$

where $RHC(\tau_k) = [h_{RC}(\tau_k), \ldots, h_{RC}(\tau_k + (8M - 1)\tau)]$. Hence $g_k$ in Eq.(13) may be rewritten as:

$$ g_k = G(\tau_k) \alpha_k^{\text{w}}. $$

(14)

where $\alpha_k = \{\alpha_k^{(1)}, \ldots, \alpha_k^{(U)}\}^\top$ is a $(U \times 1)$-element vector that contains the complex-valued fading coefficients of $U$ subcarriers, and the matrix $G(\tau_k) \in \mathbb{C}^{(U \times MN-KM-1) \times U}$ is defined as $G(\tau_k) = I_U \otimes h_{RC}(\tau_k)$. Here we use the notations $G(\tau_k)$ and $h_{RC}(\tau_k)$ to emphasize the fact that both $G$ and $h_{RC}$ are functions of $\tau_k$. Upon substituting Eq.(14) into Eq.(13) and letting $F_k G(\tau_k) = S_k(\tau_k)$, we arrive at:

$$ y = S_k(\tau_k) \alpha_k^{\text{w}} + w. $$

(15)

From Eq.(15) we may obtain the Equivalent Log-Likelihood (ELL) function as $f_{\text{ELL}}(y | \tau_k, \alpha_k^{\text{w}}) = -\|y - S_k(\tau_k) \alpha_k^{\text{w}}\|^2$, which implies that the joint Maximum Likelihood Estimation (MLE) of $\tau_k$ and $\alpha_k$ may be formulated as:

$$ \{\hat{\tau}_k, \hat{\alpha}_k\}_{\text{MLE}} = \underset{\{\tau_k, \alpha_k\}}{\text{arg min}} \|y - S_k(\tau_k) \alpha_k\|^2. $$

(16)

To compute the joint MLE by directly solving Eq.(16) is rather challenging. However, if $\tau_k$ is known a priori, the optimal estimate of $\alpha_k$ which satisfies Eq.(16) may be formulated as:

$$ \hat{\alpha}_k^{\text{SLS}}(\tau_k) = S_k^{-1}(\tau_k) y. $$

(17)

where the superscript “SLS” is the abbreviation of Structured LS.

Based on Eq.(17), a grid search scheme may be developed for numerically solving Eq.(16). More particularly, we may hypothesize a specific value of $\tau_k \in [0, T_0)$ and obtain the corresponding $\hat{\alpha}_k(\tau_k)$ from Eq.(17). Once this procedure has been repeated for all possible candidates of $\tau_k$, an approximate joint MLE can be found as the combination of $\{\tau_k, \hat{\alpha}_k(\tau_k)\}$ which minimizes the Cost Function (CF) specified in Eq.(16). In our forthcoming discourse, we will refer to this algorithm as the Structured Least Squares Search Estimator (SLS-E). Note that the above mentioned grid search performed by the SLS-E is one-dimensional, proceeding across the limited range of $\tau_k \in [0, T_0)$ only. As the resolution of the grid search increases, SLS-E asymptotically converges to the MLE. Given the estimated complex channel gain $\hat{\alpha}_k^{\text{SLS}}$, the further extraction of the amplitude and phase estimates becomes trivial.

B. Multopath Single-user Scenario

To accommodate the multipath effect, Eq.(15) should be extended as:

$$ y = \sum_{l=1}^{K} S_k(\tau_k) \alpha^{\text{w}}_{k,l} + w. $$

(18)

The direct application of the SLS-E to dispersive channel entails a multi-dimensional grid search and leads to a complexity exponentially dependent on $L$. Following the approach proposed in [8], we may first apply the single-path SLS-E to Eq.(18) and find the SLS estimates $\{\hat{\alpha}_k^{\text{SLSS}}[1], \ldots, \hat{\alpha}_k^{\text{SLSS}}[L]\}$ corresponding to path 1, i.e., the strongest path. Then the contribution of path 1 to the single-user multipath signal can be regenerated by $S_k(\hat{\alpha}_k^{\text{SLSS}}[1])$ and subtracted from $y$ of Eq.(18). Applying the single-path SLS-E to the resultant residual, we may obtain the SLSS estimation corresponding to the 2nd-strongest path. Obviously, the entire estimation task can be completed after $L$ cycles, and this successive grid search imposes a modest complexity, which increases linearly with $L$.

C. Multiuser Multipath Channel: the Practical Scenario

In an uplink multiuser system, the accuracy of channel estimation is often affected by the MAI. To overcome this difficulty, we now generalize the above-mentioned SLS-E by combining it with Parallel Interference Cancellation (PIC) [11, 12]. This potent combination results in a new iterative estimator, which we refer to as PIC assisted SLS-E (PIC-SLS-E). Given the multipath, multiuser signal of:

$$ y = \sum_{k=1}^{K} \sum_{l=1}^{L} S_k(\tau_k, l) \alpha_k^{\text{w}} + w. $$

(19)

the PIC-SLS-E scheme operates as follows:

Step 1: (Initialization) Set $v = 1$, where $v$ is the index of the PIC stage. For the $k$-th user, the parameter estimate of Stage-$I$ $\{\hat{\tau}_k[1], \hat{\alpha}_k^{\text{w}}[1]|l = 1, \ldots, L\}$ is obtained by applying the corresponding multipath SLS-E to $y$ of Eq.(19). The initial value of the CF should be set to $C_0 = +\infty$.

Step 2: (Reconstruction) Based on the channel estimates obtained in the previous stage (Stage-$v$), the $k$-th user’s contribution to the received signal may be reconstructed by:

$$ \hat{y}_k = \sum_{l=1}^{L} S_k(\hat{\tau}_k[l], \alpha_k^{\text{w}}[l]). $$

(20)

Step 3: (Verification) Calculate the CF value for the previous stage:

$$ C_v = \|y - \sum_{k=1}^{K} \hat{y}_k\|^2 $$

and check the following two conditions: $I$. $C_v < C_{v-1}$ (This is always true for $v = 1$), II. the maximum iteration limit, say $V$, has not been reached, i.e., we still have $v < V$. Proceed to Step 4, if both of the above two conditions are satisfied. Otherwise, the PIC-SLS-E is terminated at Step 3.

Step 4: (Subtraction) For the $k$-th user ($k = 1, \ldots, K$), the refined received signal $\hat{y}_k^{v-1}$ to be used for channel estimation in the $(v+1)$-th stage is constructed as:

$$ \hat{y}_k^{v+1} = y - \sum_{k' = 1, k' \neq k}^{K} \hat{y}_{k'} = y - \sum_{k' = 1}^{K} \hat{y}_{k'} + \hat{y}_k. $$

(21)

Step 5: (Estimation) For the $k$-th user ($k = 1, \ldots, K$), the channel parameter estimates $\{\hat{\tau}_k[1], \hat{\alpha}_k^{\text{w}}[1]|l = 1, \ldots, L\}$ generated during the $(v+1)$-th stage are obtained by applying the multipath SLS-E to $\hat{y}_k^{v+1}$, then we set $v = v + 1$ and return to Step 2.

VI. IMPLEMENTATION ISSUES AND COMPLEXITY ANALYSIS

A. Whitening

With the covariance matrix of the colored noise vector $n$ in Eq.(8) denoted by $\Psi = \text{E}(n(n^H))$, the whitening matrix $W$ has to satisfy:

$$ W \Psi W^H = \sigma^2 I_{MN,MN}, $$

where $\sigma^2$ is the variance of the whitened noise. As $\sigma^2$ may assume arbitrary positive values, $W$ is not unique. Recalling $n = \{n^{(1)}|^T, \ldots, n^{(U)}|^T\}^T$ from Eq.(8), and observing the noise vectors in the $U$ different subchannels are independent of each other, then for any $u_1$ and $u_2 \in \{1, \ldots, U\}$ we have:

$$ \{n^{(u_1)}|^T, n^{(u_2)}|^T\} \in O_{MN,MN}, \quad u_1 \neq u_2, $$

(22)

where the $(i,j)$-th entry of $\Phi \in \mathbb{R}^{MN \times MN}$ equals $2\text{Re}(\text{det}(\Phi^H + |i-j|T/\Psi + I))$ [6]. Upon setting $N_0 = 1$ and obtaining the corresponding whitening matrix for $\Phi$, say $P \in \mathbb{R}^{MN \times MN}$, we may readily verify that $W = \text{diag}[P, \ldots, P] \in \mathbb{C}^{MN \times MN}$ is a whitening matrix for $\Psi$.

The above discussions suggest that the on-line calculation of the whitening matrix may be avoided by pre-computing the real-valued matrix $P$ and storing it in the memory. The resultant memory requirements and the computational complexity associated with $W \hat{y}$ have been summarized in Tab I.
B. LS-E

Following the arguments of [6], the complexity of the LS-E is $O[(K^2U^2)(MN + 8M - 1)]^2$ due to the inversion of $(F^H F)$ in Eq.(11), or at least $O[(K^2U^2)(MN + 8M - 1)]^2$ by Gauss-Seidel iterations. However, this is actually not so much of a problem since $F^1 = (F^H F)^{-1} F^H$ can be computed off-line. To see this, recall from Eq.(10) that we have $F = WA$, where $W$ can be pre-computed as $W$, and $A$ is solely dependent on the spreading codes and the pilot symbols, both of which are known to the receiver. Therefore, the implementation cost associated with the LS-E of Eq.(11) is contributed by the memory space required for storing $F^1$ and the computational complexity of the $y$-residual of $K$. Moreover, a close inspection of $K$-multiplications (RM) and $K$-additions (RA) are provided:

$\hat{y}_k = y_k - \hat{y}_k^u$, 

(24)

where $2UNMNK$ Rs are required for reconstructing all the $K$ users’ signals. As for Step 3, the calculation of $\sigma^2 = \|y - \sum_{k=1}^{K} \hat{y}_k^u \|_2^2$ involves $2UNMN$ RMs and $2UNMN(K + 1) - 1$ RAs. The last constituent part of the PIC stage is Step 4, where we evaluate $y_k^{u+1} = y - \hat{y}_k^u$ for each user. Note that $(y - \sum_{k=1}^{K} \hat{y}_k^u)$ has already been calculated during Step 3, hence Step 4 only entails $2UNMNK$ RAs. Assuming the PIC runs for $V$ stages, we report the complexity of PIC-SLSS-E also in Tab I.

D. LASSO-E

Iterative approaches [10, 15] have been proposed for solving the LASSO based channel estimation problem formulated by Eq.(12). However, to the best of our knowledge, there is no closed-form LASSO solution available, which makes the evaluation of its complexity quite a challenge. Owing to the lack of a closed-form solution, LASSO-E cannot pre-calculate any quantitative for the sake of saving on-line computational complexity, which constitutes an obvious disadvantage in contrast to LS-E and PIC-SLSS-E. Moreover, the whitened noise variance $\sigma^2$ must be estimated in advance to determine $\lambda_c$. In the simulations, we assume $\sigma^2$ to be known and solve Eq.(12) as a quadratic programming problem with the so-called SeDuMi toolbox [16] aided with a Yalmip interface [17].

VII. NUMERICAL EXAMPLES

The key system parameters adopted in our simulations are $K = 5$, $N = 16$, $M = 2$, $N_r = 10$ and $U = 4$. The long spreading codes are generated as random bipolar sequences where every entry has the same probability of assuming $+1$ or $-1$. As in [6], we consider a near-far ratio (NFR) of 10dB, where the power of active users are randomly fluctuated around their average with $\pm 5$dB deviation, and we report the simulation results of the weakest user to characterize the lower bound of the uplink channel estimation quality. The parameter extraction in LS-E and LASSO-E is performed at a resolution of $T_c/10$, while we set $R = 10$ for PIC-SLSS-E for the sake of fair comparison. Unless otherwise stated, $V$ is set to 4, i.e., the PIC-SLSS-E iterates for at most three stages after initialization. All results were averaged over 1000 independent runs.

In Fig.1 we compare the probability of correct acquisition ($P_{ac}$) of the three proposed estimators for transmission both over single- and multi-path channel. The acquisition of a certain path is deemed as correct if the corresponding delay estimation error has an absolute value less than $T_c/2$. We observe that the $P_{ac}$ of the LS-E without employing PIC (hence referred to as “unassisted”) fails to approach 100% even when $E_b/N_0$ is quite high. This is because the unassisted SS-E scheme acts essentially in a decentralized way, i.e., the MAI is not mitigated. Furthermore, although a notable performance discrepancy is observed between $V = 1$ and $V = 2$, increasing $V$ brings little gain when $V \geq 4$. As expected, LASSO-E is superior to LS-E, but both of them are outperformed by PIC-SLSS-E for $V \geq 2$.

In order to evaluate the channel parameter estimation quality, in Fig.2-4 we illustrate the unconditioned Mean Squared Error (MSE) and MSE conditioned on the correct acquisition (ca). As for delay estimation, there is a large disparity between the conditioned and non-conditioned MSEs in each of the three proposed schemes, especially for the MA-based PIC-SLSS-E. In Fig.3, we compare the BER performance of the LASSO-E scheme with and without the proposed PIC enhancement, where the performance gap is significant at high SNR. In Fig.4, we compare the average delay error in the PIC-SLSS-E and LASSO-E, where the results indicate that LASSO-E is superior in terms of delay estimation accuracy.
unconditioned MSE in the low $E_b/N_0$ range, but they tend to coincide with each other in the high $E_b/N_0$ range, as $P_a$ tends to unity. Interestingly, the difference between the conditioned and unconditioned MSE is less evident in amplitude and phase offset estimation. In general, these results demonstrate again that the structured PIC-SLSS-E compares favorably with the two unstructured algorithms.

References


