A Wiener model for memory high power amplifiers using B-spline function approximation

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Modeling of the operation of high power amplifiers (HPA)

- Power-efficient nonlinear HPA transmitters are employed in broadband communication systems.
- Serious nonlinearities potentially leading to the deterioration in system performance
- Predistorter design approaches
    - Look up table (LUT)
    - The polynomials
    - The Wiener model that is a linear dynamical model followed by a nonlinear static functional transformation.
- The B-spline curves and De Boor recursions

A Wiener model for memory high power amplifiers

- HPA as complex valued Wiener system
- The nonlinearity in the Wiener model is mainly dependent on the input signal amplitude
- Two real valued B-spline curves, one for the amplitude distortion and another for the phase shift
- The Gauss-Newton algorithm is applied, which incorporates with the De Boor algorithm, including both the B-spline curve and the first order derivatives recursion
- Offers modeling versatility as well as enables stable and efficient evaluations of functional and derivative values, as required in nonlinear optimization algorithm.

The general complex valued Wiener system

The general complex valued Wiener system consists of a cascade of two subsystems, a linear filter of order n representing the memory effect on the input signal, followed by a nonlinear memoryless function $\Psi(\bullet) : \mathcal{C} \to \mathcal{C}$.

$$\begin{aligned} w(t) &= H(z)y(t) \\ &= y(t) + h_1 y(t-1) ... + h_n y(t-n) \quad (1) \\ d(t) &= \Psi\left(w(t)\right) + \xi(t) \quad (2) \end{aligned}$$

with z transfer function $H(z)$ defined by

$$H(z) = \sum_{i=0}^{n} h_i z^{-i}, \quad h_0 = 1 \quad (3)$$

where $d(t) \in \mathcal{C}$ is the system output and $y(t) \in \mathcal{C}$ is the system input. $\xi(t) \in \mathcal{C}$ is assumed to be a white complex valued noise.

The nonlinearity of the traveling wave tube (TWT) for $\Psi(\bullet)$

For the baseband HPA model, $\Psi(w(t))$ can be specified by a nonlinearity of the traveling wave tube (TWT). The input to the TWT nonlinearity can be expressed as

$$w(t) = |w(t)| \exp(j\angle w(t)) = r(t) \exp(j \cdot \phi(t)) \qquad (4)$$

The output of TWT, $\Psi(w(t))$, is distorted in both the amplitude and the phase, with the distortion dependent mainly on the input signal amplitude, i.e. $r(t)$. So $\Psi(w(t))$ is given by

$$r_\Psi(t) = \begin{cases} \frac{\alpha_1 r(t)}{1+\alpha_2 r^2(t)}, & 0 \le r(t) \le r_{Sat} \\ \Psi_{max}, & r(t) > r_{Sat} \end{cases} \qquad (5)$$

$$\phi_\Psi(t) = \frac{\beta_1 r^2(t)}{1 + \beta_2 r^2(t)}. \qquad (6)$$

Outline

B-spline basis functions and De Boor recursions

Univariate B-spline basis functions are parameterized by the order of a piecewise polynomial of order k, and also by a knot vector specified by $(M+k)$ knot values, $\{R_1, R_2, \cdots, R_{M+k}\}$. M B-spline basis functions and its derivatives can be formed by using the De Boor recursion

$$\mathcal{B}_l^{(0)}(r) = \begin{cases} 1 & \text{if } R_l \le r < R_{l+1} \\ 0 & \text{otherwise} \end{cases} \qquad l = 1, \cdots, (M+k)$$

$$\mathcal{B}_l^{(i)}(r) = \frac{r-R_l}{R_{i+l}-R_l}\mathcal{B}_l^{(i-1)}(r) + \frac{R_{i+l+1}-r}{R_{i+l+1}-R_{l+1}}\mathcal{B}_{l+1}^{(i-1)}(r),$$
$$\text{for } l = 1, \cdots, (M+k-i), \quad i = 1, \cdots, k$$

$$\frac{d}{dr}[\mathcal{B}_l^{(k)}(r)] = \frac{k}{R_{k+l} - R_l}\mathcal{B}_l^{(k-1)}(r) - \frac{k}{R_{k+l+1} - R_{l+1}}\mathcal{B}_{l+1}^{(k-1)}(r)$$
$$l = 1, \cdots, M$$

Two real valued B-spline curves

We model $\Psi(\bullet)$ as two univariate B-spline curves, one for the amplitude and another for the phase shift in the form of

$$r_{\Psi}(t) = \sum_{l=1}^{M} \mathcal{B}_l^{(k)}(r(t))\omega_l \quad (7)$$

$$\phi_{\Psi}(t) = \sum_{l=1}^{M} \mathcal{B}_l^{(k)}(r(t))\vartheta_l \quad (8)$$

where ω_l's and ϑ_l's are weights to be determined. Note that due to the piecewise nature of B-spline functions, there are only $(k+1)$ basis functions with nonzero functional/derivative values at any point r. Hence the computational cost of the De Boor algorithm is determined by the polynomial order k, rather than the number of knots, and this is in the order of $O(k^2)$.

Outline

Cost function

The complex valued B-spline network output is denoted by $\hat{d}(t) = \hat{d}_R(t) + j \cdot \hat{d}_I(t)$ in which

$$\begin{aligned} \hat{d}_R(t) &= r_\Psi(t)\cos[\phi_\Psi(t) + \phi(t)] \\ \hat{d}_I(t) &= r_\Psi(t)\sin[\phi_\Psi(t) + \phi(t)] \end{aligned}$$

Let the error between the Wiener system output $d(t)$ and the B-spline network output $\hat{d}(t)$ be denoted by $e(t) = d(t) - \hat{d}(t) = e_R(t) + j \cdot e_I(t) \in \mathcal{C}$. Our task is to estimate $\mathbf{h}$, $\boldsymbol{\omega}$ and $\boldsymbol{\vartheta}$. This could be achieved by minimizing

$$J = \sum_{t=1}^{K} [e_R(t)]^2 + \sum_{t=1}^{K} [e_I(t)]^2 \tag{9}$$

The Gauss Newton algorithm combined with the De Boor algorithm

- A simple parameter initialization using least squares algorithm has been proposed (see paper)
- With an initial $\boldsymbol{\theta}^{(0)}$, the iteration formula is given by

$$\boldsymbol{\theta}^{(\tau)} = \boldsymbol{\theta}^{(\tau-1)} - \alpha\{[\mathbf{J}^{(\tau)}]^T \mathbf{J}^{(\tau)}\}^{-1}[\mathbf{J}^{(\tau)}]^T \boldsymbol{\epsilon}(\boldsymbol{\theta}^{(\tau-1)}) \quad (10)$$

where $\boldsymbol{\theta} = [\theta_1 \,, \theta_2, \cdots, \theta_{2(d+n)}]^T = [\omega_1, \omega_2, \cdots, \omega_M, \vartheta_1, \vartheta_2, \cdots, \vartheta_M, h_{1,R}, \cdots, h_{n,R}, h_{1,I}, \cdots, h_{n,I}]^T \in \Re^{2(M+n)}$.

- The De Boor algorithm are used to evaluation all entries in $\mathbf{J}$.

A modeling example

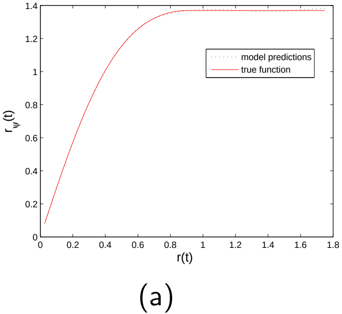
- 2000 training data samples and 500 validations data samples $d(t)$ were generated with $H(z) = 1 + 0.7692z^{-1} + 0.1538z^{-2} + 0.0769z^{-3}$, in which the TWT nonlinearity is used to generate the training data set and specified by $\alpha_1 = 3$, $\alpha_2 = 1.2$, $\beta_1 = \pi/12$, $\beta_2 = 0.25$.
- $y(t)$ was uniformly distributed complex random variable with $y_R(t) \in [-1.5, 1.5]$ and $y_I(t) \in [-1.5, 1.5]$.
- The variances of the additive noise to the system output are set $\sigma^2 = 0.02^2$.
- The polynomial degree of the B-spline basis function was set as three (i.e. $k = 4$, piecewise cubic).

A modeling example

Table: Results of linear subsystem parameter estimation.

	True parameters	Initial estimates	Final estimates
h_1	0.7692	0	$0.7691 - j6 \times 10^{-7}$
h_2	0.1538	0	$0.1537 + j8 \times 10^{-6}$
h_3	0.0769	0	$0.0769 - j2 \times 10^{-5}$

A modeling example



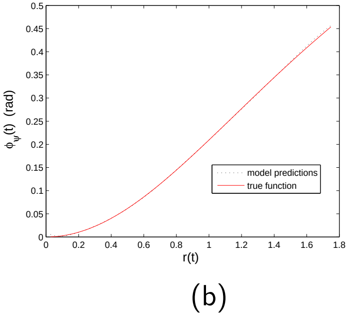


Figure: The TWT nonlinearity modeling results; (a) Amplitude distortion with respect to the amplitude of the input; and (b) Phase shift with respect to the amplitude of the input.

Conclusions

- A new Wiener system modeling approach has been introduced in order to model the memory high power amplifiers in communication systems.
- We model the complex valued nonlinear static function using two real valued B-spline neural networks, one for the amplitude distortion and another for the phase shift, respectively.
- The parameter is estimated using the Gauss-Newton algorithm. The De Boor algorithm, including both the B-spline curve and the first order derivatives recursion, is utilized in the algorithm.
- Future work will be focused on predistorter design based on the proposed model.