Numerical Analysis and Optimization of an Electromagnetic Micro-Generator for Vibration Energy Harvesting Applications

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This work investigates the behavior of an electromagnetic micro-generator for power generation utilizing stray vibrations. The analysis is conducted through finite element simulations. The fundamental equations describing the performance of such devices are derived and the maximum power generation condition is discussed for the case of a cored winding structure.

***Index Terms*—Vibration energy harvesting, energy conversion, magnetic circuit, self-contained electromagnetic generator.**

I. Introduction

In the is last decade, the development of miniature sensors for various monitoring applications, such as structural health monitoring [1], condition-based maintenance [2], etc., has seen a tremendous progress. In many situations it is more favorable for the sensors to be embedded within the structure being monitored with no physical connection to the outside world. One of the main problems with such an arrangement is that the sensing device then needs its own power supply for continuing operation. Most of the sensors use batteries, while the more advanced ones have the power transferred wirelessly to them. Batteries have limited energy capacity, whereas wireless power transfer is restricted by distance. A possible solution for this sort of applications is the emerging energy harvesting technology [3] whereby a miniature self-contained power generator is integrated with the sensing device.

Many environments, such as aircrafts, bridges, buildings, etc., where sensors are to be used, often experience ambient vibrations. The mechanical energy freely available in these stray vibrations is not commonly utilized, but it is in principle possible to convert harvested mechanical energy into electrical energy to power up sensing devices for prolonged periods of time. Such conversion of energy could be accomplished through electromagnetic induction, by using a piezoelectric material or via an electrostatic generator [3], [4]. Electromagnetic (EM) induction based devices have received particular attention due to the highest power density achievable among the three mentioned possibilities. Several research programs have been undertaken and reported in literature ([5], [6]) on the design of efficient EM generators. However, there appears to be a lack of thorough design considerations of the magnetic circuit aspects of such miniature electromagnetic generators.

In this paper, a micro electromagnetic induction based device for vibration energy harvesting (VEH) is studied and its magnetic circuit optimized. A design reported in [5] was taken as the starting point and various 2D and 3D finite element (FE) simulations have been undertaken. The performance of the generator has been analyzed for various operating conditions, like changing the frequency and acceleration levels of the ambient vibrations. Another significant contribution of the paper is the investigation conducted to study the implications of different aspects of its magnetic circuit with respect to the total electrical power extracted. The findings are based entirely on finite element simulations and a series of numerical experiments and design studies.

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II. Generator Model

The micro electromagnetic (EM) generator, used in this research study, has been designed to harvest energy at frequencies between 30 and 70 Hz with acceleration levels between 0.2 and 1.2 m/s^2. These operating conditions are typical for vibration levels found in industrial equipment such as a compressor or a refrigeration unit [4]. The EM generator consists of two pairs of magnets arranged as shown in Fig. 1 and a coil wound around a steel core placed in the gap between the magnets. In this structure it is assumed that the magnets are fixed and the coil moves in response to the external continuous vibration.

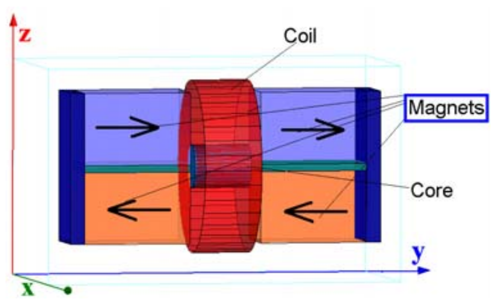

Fig. 1. VEH generator structure.

The coil is supported by a spring mechanism (a cantilever beam in this case [5]) that is connected to the vibrating part of the equipment and moves along the *z*-axis as illustrated in Fig. 1. A reverse arrangement is also possible – with moving magnets and a fixed coil; however, from the magnetic analysis point of view, this aspect of the arrangement does not matter. The choice of the moving part is primarily guided by the mass of the moving component as available power varies linearly with mass in such mass-spring systems. Therefore the heavier the moving part the more electrical power available for extraction [7]. According to [6] the average power dissipated is:

$$P_{av} = \frac{m\xi_t Y^2 \left(\frac{\omega}{\omega_n}\right)^3 \omega}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left[2\xi_t\left(\frac{\omega}{\omega_n}\right)\right]^2} \quad (1)$$

where m is the mass of the moving part, Y and ω are the amplitude and angular frequency of the harmonic movement, respectively, ω_n is the resonant angular frequency of the mass-spring oscillator, $\xi_t = c_t/2m\omega_n$ is the total damping ratio, and c_t is the total damping factor. Equation (1) assumes that the generator is driven by a harmonic motion, hence under steady-state conditions; P_{av} is equal to the kinetic energy supplied per second from the existing vibrations. The maximum power dissipation occurs at resonance [7], which is given as

$$P_{av} = \frac{mY^2\omega_n^3}{4\xi_t} \quad (2)$$

The total energy dissipated within this system is represented by the damping coefficient, c_t. Although (2) implies that power varies linearly with mass, increases with the cube of the frequency and with the square of the amplitude of motion, designing such small inertial generators cannot based on this equation alone. Due to the size of these devices, the mass displacement is limited; the amount of mass is restricted too. It has been shown in [6] that the amount of electrical power that can be extracted from such a device may be expressed as

$$P_{\max} = \frac{mY^2\omega_n^3}{16\xi_p}\left(\frac{R_{load}}{R_{load}+R_{coil}}\right) \quad (3)$$

where ξ_p is the open circuit damping and R_{load} and R_{coil} are the load resistance and coil resistance, respectively. One disadvantage of such an EM generator is that an air-cored coil is normally used, thus resulting in a weak magnetic flux linkage between the moving coil and the fixed magnets [5], [6], [8]. To overcome this drawback, this paper presents a numerical study of the effects of the magnetic core on the maximum extractable power of the EM generator used in vibration energy harvesting (VEH) applications. The relevant dimensions of the micro generator are summarized in Table I.

TABLE I. MICRO-GENERATOR DIMENSIONS

Moving mass	m = 0.66 g
Size of magnets (NdFeB34)	1x1.5x1 mm
Cu coil size - inner, outer radius and number of turns	0.3 mm, 1.2 mm, 600 turns
Silicon cantilever beam size – length, width, thickness	L = 9 mm, w = 4 mm, t = 50e-3 mm
Distance between magnets Air gap	D = 1 mm ag = 0.1 mm

III. SIMULATION METHODOLOGY

In this study, several 2D and 3D FE models of the micro generator have been setup and solved using two commercially available software packages: Magnet from Infolytica (2D) and Maxwell from Ansoft (3D). Both FE solvers have the capability to simulate electro-mechanically coupled problems using their transient solvers; however, the time to get the solution for more than several periods is quite long, measured in hours, hence a faster model was developed and used to analyze and optimize the dynamics of the micro-generator. This model was setup in Simulink and solves the following equation of motion:

$$m\frac{d^2z}{dt^2} + c_t\frac{dz}{dt} + kz(t) + F_{mn}\left[z(t)\right] = F_d(t) \quad (4)$$

where c_t is the total damping coefficient, k is the spring constant, F_{mn} is the magnetic force acting on the winding core and F_d is the external driving force due to the harmonic vibrations. The spring constant k of the silicon cantilever beam considered in the simulation set-up is based on

$$k = \frac{E \cdot w \cdot t^3}{4 \cdot L^3} \quad (5)$$

where E is the Young's modulus (E_{Si} = 130 GPa), w is the width of the beam, t its thickness and L its length, respectively. The damping coefficient, c_t, was assumed initially to be very small and its effect ignored in the motion equation. The magnetic force, F_{mn}, on the iron core was calculated for different positions (in z-axis direction as seen in Fig. 1) using a 2D FEM magnetostatic model (see Fig. 2). Referring to Fig. 2, the iron-cored coil was moved between −3.5 mm to −0.4 mm and 0.4 mm to 3.5 mm in steps of 0.1 mm and from −0.4mm to 0.4mm in steps of 0.01 mm. For an air core case, this magnetic force is zero.

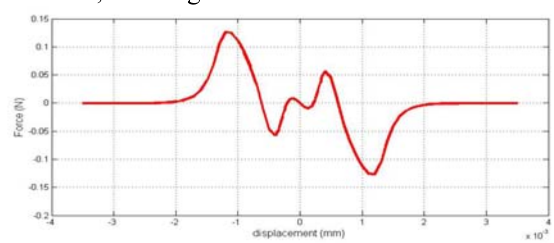

Fig. 2. Magnetic force variation with position (z in mm).

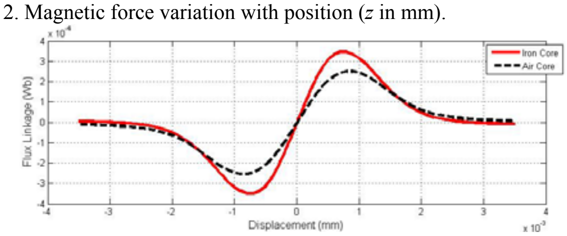

Fig. 3. Flux linkage variation with position (z in mm).

The iron-cored coil flux linkage variation with its position was also computed using the FE model, and its profile is shown in Fig. 3. It can be seen that the maximum flux linkage of the EM generator with an iron-cored coil is 3.5 $\times 10^{-4}$ Wb, which is about 40 % larger than the EM generator with a simple air cored coil of 2.5 $\times 10^{-4}$ Wb. This is an encouraging result which promises higher power density to be available when an iron-cored coil is used. Moreover, the relevant data (force and flux linkage) from the FEM magnetostatic model was imported into a dynamic model built in Simulink and a time dependent solution was obtained. To verify the validity of the Simulink model, a simulation using an external driving force, F_d = 0.019sin(2π*50t), with the spring constant of the cantilever beam calculated using (5) and the numerical data from Table I, was performed. A 2D FEM transient model with the same parameters of the motion was also created and the results compared against the Simulink solution. A very good agreement was observed as shown in Fig. 4.

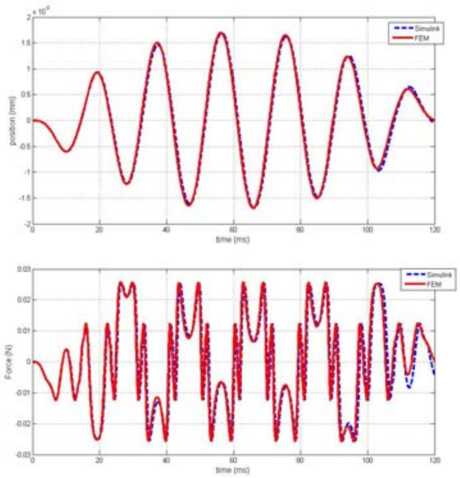

Fig. 4. Position and magnetic force variation with time.

By using the flux linkage values obtained from the 2D FEM magnetostatic model, the induced voltage in the coil can be computed in the Simulink model and the results are very similar to those obtained from the 2D FEM transient model (Fig. 5). Using this approach means significant savings in computing times. For example, a full transient 2D FEM solution took more than two and half hours to simulate 120ms with a 0.1ms time step, whereas the Simulink model took less than 2s for the same time interval and same time step.

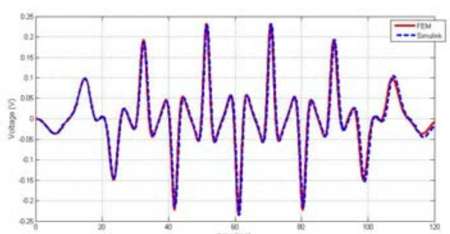

Fig. 5. Open circuit induced voltage variation with time.

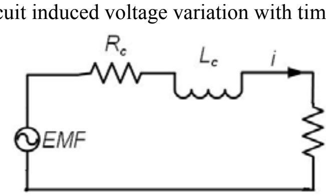

Fig. 6. Equivalent circuit of the loaded micro-generator.

The Simulink model is also capable of simulating the behavior of a loaded micro-generator. When loaded with a resistance, the equivalent circuit of the generator becomes a simple voltage source behind the impedance of the coil, to which terminals the load resistance is connected (Fig. 6). The resistance and inductance of the coil were computed from a magnetostatic FEM model where the coil was excited with a 1A current. The resistance of the coil was R_c = 17.32Ω and the inductance of the iron-cored coil was L_c = 0.268mH.

The time dependent current in the circuit is calculated using the following equation

$$L_c \frac{di}{dt} + \left(R_c + R_l\right) \cdot i(t) = EMF(t) \tag{6}$$

The power dissipated in this circuit is seen by the spring mechanism of the micro-generator as damping illustrated by (4). Hence the power dissipated in the electrical circuit should be equal to the power dissipated in the damping mechanism when the open circuit damping is zero as shown in (7) below

$$\frac{1}{T}\int_0^T i^2(t) \cdot (R_c + R_l) dt = \frac{1}{T}\int_0^T b_e \frac{dz}{dt} \cdot \frac{dz}{dt} dt \tag{7}$$

$$b_e = \frac{i^2 (R_c + R_l)}{\left(\frac{dz}{dt}\right)^2} \tag{8}$$

Therefore, in the case of a loaded generator, $c_t = b_m + b_e$, where b_m is the damping coefficient due to mechanical effects (friction, windage, etc.) and b_e is the damping coefficient due the electrical power that is dissipated in the two resistances of the circuit. The variation of b_e for an air-cored coil is shown in Fig. 7. For this calculation the frequency of the external driving force was the same as the oscillating frequency of the spring mass system $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ and b_m= 0.2 was assumed.

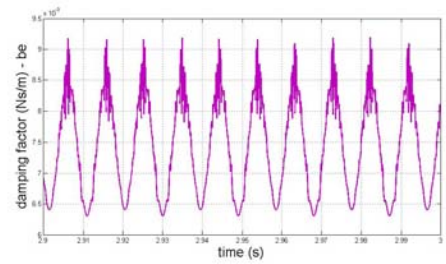

Fig. 7. Electrical damping factor (b_e) variation.

IV. Results and Analysis

The EM generator with an iron-cored coil was simulated and the results are analyzed in this section. The magnetic forces and flux linkages used to generate the results which follow were obtained from the 2D FEM models, but the Simulink model is capable of using 3D FEM results as well.

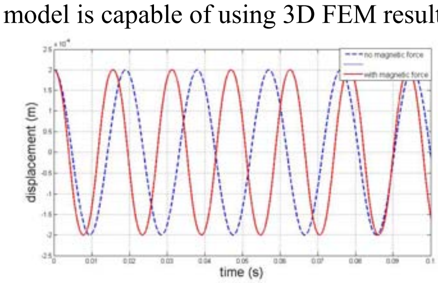

Fig. 8. Motion of the mass-spring system with and without magnetic force.

There are several differences in the behavior of the micro-generator that became apparent when an iron core was used. The increase in flux density is accompanied by a magnetic force that acts on the core that is experienced by the generator like a cogging force. As a result the natural resonance of the mass-spring system increases as this force effectively stiffens the spring. The motion of the mass-spring system with and without the iron core is illustrated in Fig. 8.

Another effect of the magnetic force is that it may limit the amplitude of the movement through which the generator can be used effectively. The force of the spring, $F = -k*z$, has to be larger than the magnetic force throughout the displacement of the moving part of the generator. If this condition is not met, there can be situations when the moving part is stuck in a position where the magnetic force is larger than the spring force and it will oscillate with very small amplitude around that position. For example, in the structure studied here this phenomena was observed and it is illustrated as the 'Magnetic force 1' in Fig. 9.

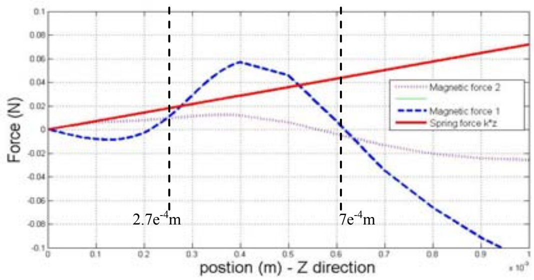

Fig. 9. Spring force and magnetic force along the *z*-direction.

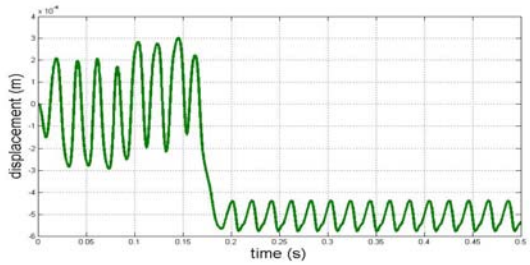

Fig. 10. Moving part motion when amplitude larger than 0.27 mm.

It can be noticed from Fig. 9 that for amplitudes of motion less than 0.27 mm, the micro-generator will function as expected. However, when the amplitude of the EM generator motion is larger than 0.27 mm and smaller than 0.7 mm, the generator will get stuck and it will only oscillate with very small amplitude around that position as can be seen in Fig. 10. To get around such situations, the iron core and the magnetic circuit of the micro-generator should be designed in such a way that the spring force is always larger than the magnetic force, as exemplified by the 'Magnetic force 2' in Fig. 9. The limitation is that the flux density in the core of the coil of the EM generator is likely to be reduced, and consequently its power density. Alternatively, the beam of the mass-spring system can be made shorter and thinner, according to (5), which will increase *k*, but this approach may be limited by the mechanical properties of the beam material.

Several simulations under realistic conditions were run to understand better the micro-generator behavior when an iron core is used. The mechanical damping coefficient, b_m, was set to 0.2 and the generator loaded with a resistive load that was varied from 1Ω to 150Ω. The generator was driven by an external force, $F_d = 0.019\sin(2\pi*50t)$. The simulation results of the power generated by the EM generator with an iron core and with an air core versus different load resistance values are presented in Fig. 11.

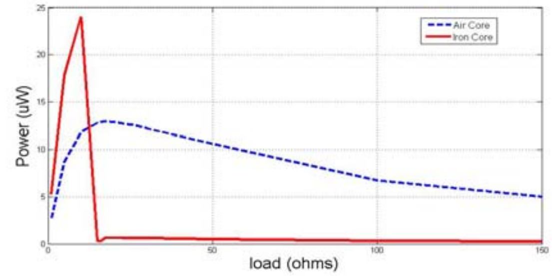

Fig. 11. Extracted electrical power for different load resistance.

With reference to Fig.11, it is noted that for low resistance loads the iron-cored generator exhibits superior performance to its air counterpart. For load resistances larger than 15Ω, however, the power of the iron-cored generator drops sharply. This behavior can be explained by the variation of the total damping coefficient as given by (8), which is related to the load and the current flowing through it. While the load resistance is low and the current relatively large, the total damping coefficient is large enough to limit the amplitude of the motion. Once the current drops too much, it will reduce the electrical part of the damping coefficient and it will allow a larger amplitude of motion. This may push the generator into the situation described in Fig. 10, in which it will oscillate around a different position with very small amplitude. This will result in the drop of power observed in Fig. 11.

V. Conclusions

The effects of an iron-cored winding in a micro inertial electromagnetic generator have been investigated through FEM modeling. The numerical results suggest that the magnetic force between the magnets of the generator and the magnetic core of the coil has a significant effect on the behavior and maximum power capabilities of such devices.

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