Asymptotic Spectral-Efficiency of MIMO-CDMA Systems with Arbitrary Spatial Correlation

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***Abstract*— In this contribution, we analyze the asymptotic spectral-efficiency (ASE) of multiuser MIMO-CDMA systems, when assuming communications over flat fading channels with arbitrary spatial correlation. Our analysis is built on the operator-valued free probability theory, which is applied to obtain the limit distribution of the correlation matrix's eigenvalues, as the MIMO-CDMA systems' size tends to infinity. The spectral-efficiency (SE) performance of the MIMO-CDMA systems is investigated via both analysis and simulations. Our simulation and numerical results show that the ASE is capable of providing a good measure of the SE achieved by the corresponding realistic MIMO-CDMA systems.**

I. Introduction

In this paper, we study the asymptotic spectral-efficiency (ASE) of uplink multiuser MIMO-CDMA systems, where *asymptotic* means that both the number of users, denoted K, supported by a MIMO-CDMA system and the spreading factor, denoted by N, tend to infinity, while their ratio $\beta = K/N$ is fixed. In the considered MIMO-CDMA systems, the number of transmit antennas of a mobile terminal (MT) and the number of receive antennas at the base-station (BS) are denoted by N_t and N_r, respectively, which are assumed finite. We assume that channels from different MTs to the BS experience independent Rayleigh fading, since different MTs in general distribute geographically at different locations. However, for a given MT, we assume that the signals from different transmit antennas to different receive antennas may be arbitrarily correlated in the spatial domain. In this paper, the ASE of the MIMO-CDMA systems is analyzed with the aid of random matrix theory and operator-valued free probability theory [1–4]. A range of closed-form formulas are derived and an algorithm is proposed for supporting the ASE computation. Furthermore, the spectral-efficiency (SE) performance of some MIMO-CDMA systems is investigated based on evaluation of our derived formulas as well as by simulations. Our studies show that the analytical ASE obtained in this paper is capable of providing good approximation for the SE achieved by the uplink MIMO-CDMA systems with realistic assumptions for supporting a fixed number of users and using a fixed spreading factor.

In wireless communications, the SE performance of MIMO systems with or without spatial correlation as well as that of CDMA systems supported by various multiuser detection schemes have been investigated based on random matrix and free probability theories, as seen, e.g., the references [1–11] as well as the references there in. Specifically, in [2, 5], the ASE performance of multiuser MIMO-CDMA systems has been studied, when assuming that multiple antennas are employed at the BS-receiver and single antenna is employed by every MT-transmitter. By contrast, in [6, 7], the ASE of MIMO-CDMA systems has been considered, when multiple transmit and multiple receive antennas are employed. Explicitly, in this type of MIMO-CDMA systems, data transmitted by different antennas of the same MT may be spread either by the same spreading code for all the transmit antennas or by different spreading codes respectively for different transmit antennas. For convenience, these two schemes are respectively referred to as the *same code assignment scheme* and *different code assignment scheme*. When comparing these two schemes, in general, the same code assignment scheme is capable of achieving better SE performance, when the system load $\beta = K/N$ is high, while the different code assignment scheme achieve better SE performance, when system load is low [6]. In [2, 5–7], no spatial correlation among the transmit/receive antennas was assumed. However, as we know, spatial correlation may significantly degrade the capacity of MIMO systems. For this sake, in [8], spatial correlation and line-of-sight components have been invoked and, furthermore, a generalized resource pooling scheme has been proposed. However, in order to derive the ASE, the approach in [8] requires to obtain first the limit of the received amplitudes' joint distribution and, then, to solve a matrix differential equation. Additionally, the asymptotic outage region in MIMO-CDMA systems has been studied in [9], under the assumption that the two transmit antennas are without correlation.

The so-called operator-valued free probability theory is a more general version of the free probability theory [1, 10]. It allows us to deal with the very general scenarios where arbitrary correlation exists. The landmark work of applying the operator-valued free probability theory in wireless communications is [10], which has analyzed the asymptotic capacity of MIMO systems, either when communicating over multipath fading channels without spatial correlation, or when communicating over flat fading channels with non-separable spatial correlation. Based on the approaches and results provided in [10], [11] has also studied the asymptotic capacity of the MIMO orthogonal frequency-division multiplex (MIMO-OFDM) systems, when without considering spatial correlation. However, to the best of authors' knowledge, no other applications in the wireless communications have so far invoked the operator-valued free probability theory. In this contribution, we make use of the operator-valued free probability theory for analyzing the ASE of MIMO-CDMA systems. As shown in Section III, the channel matrix in the considered MIMO-CDMA systems can be expressed as a block matrix, whose entries are arbitrary correlated due to the spatial correlation. The asymptotic eigenvalue distribution (AED) of this type of channel matrices cannot be directly obtained by the free probability theory, but can be efficiently derived with the aid of the operator-valued free probability theory.

Throughout the paper, there are some not well-known notations used, which are listed below for convenience.

- $\mathbb{C}$, $\mathbb{C}^+$: Complex field and complex upper half-plane;
- $M_d(\mathbb{C})$: Set of $(d \times d)$ matrices with their entries from $\mathbb{C}$;
- tr_d: Normalized trace of $M_d(\mathbb{C})$, $\mathrm{tr}_d(\boldsymbol{X}) = \mathrm{Tr}(\boldsymbol{X})/d$;
- $\Im(x)$: Imaginary part of x.

II. System Model and Main Assumptions

A single-cell uplink MIMO-CDMA system with its schematic block diagram as shown in Fig. 1 is considered. It supports K number of uplink MTs, each of which employs N_t transmit antennas. The BS has N_r receive antennas. The complex channel gain between the n-th receive antenna and the m-th transmit antenna of the k-th MT is denoted by $a_k^{n,m}$ for $m = 1, \ldots, N_t$; $n = 1, \ldots, N_r$; $k = 1, \ldots, K$. Although CDMA channels typically experience frequency-selective fading, in this paper, we, however as done in [2, 5–9], assume flat fading for the sake of simplicity to make the analysis manageable. In this paper, we assume that all user signals are synchronously received and that power-control is employed to make the average power received from each of the K MTs the same. Furthermore, we assume that each of the K MTs is assigned one spreading code of N-length, which is used for spreading by all the transmit antennas of the MT. Therefore,

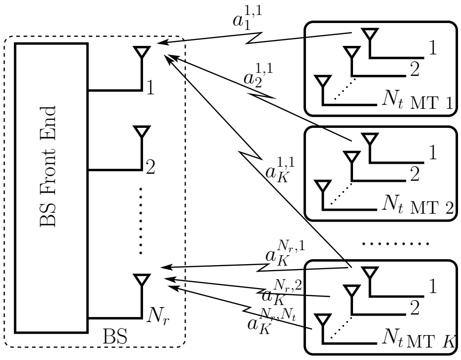

Fig. 1. Schematic block diagram for uplink MIMO-CDMA systems.

our spreading codes are assigned based on the same code assignment scheme, as mentioned in Section I. Note that, the reason for us to use this code assignment is that a full-multiplexing MIMO transmission scheme is assumed, which is capable of distinguishing the symbols transmitted by a MT based on, e.g., the BLAST coding and decoding principles.

At the BS, each of the N_r receive antennas uses a chip-waveform matched-filter (MF) to generate observation samples for detection. Based on the above assumptions, we can readily show that the discrete-time signals received by the n-th, $n = 1, \ldots, N_r$, receive antenna at the BS can be expressed as

$$\begin{aligned} \boldsymbol{y}_n &= \sum_{m=1}^{N_t}\sum_{k=1}^{K} a_k^{n,m}\boldsymbol{s}_k x_{k,m} + \boldsymbol{n}_n = \sum_{m=1}^{N_t} \boldsymbol{S}\boldsymbol{A}_{n,m}\boldsymbol{x}_m + \boldsymbol{n}_n \\ &= \left[\boldsymbol{S}\boldsymbol{A}_{n,1}, \boldsymbol{S}\boldsymbol{A}_{n,2}, \cdots, \boldsymbol{S}\boldsymbol{A}_{n,N_t}\right]\boldsymbol{x} + \boldsymbol{n}_n \end{aligned} \tag{1}$$

where, by definition, we have

$$\begin{aligned} &\boldsymbol{A}_{n,m} = \text{diag}\left\{a_1^{n,m}, a_2^{n,m}, \cdots, a_K^{n,m}\right\} \\ &\boldsymbol{S} = \left[\boldsymbol{s}_1, \boldsymbol{s}_2, \cdots, \boldsymbol{s}_K\right], \ \boldsymbol{x}_m = \left[x_{1,m}, \cdots, x_{K,m}\right]^T \\ &\boldsymbol{x} = \left[\boldsymbol{x}_1^T, \cdots, \boldsymbol{x}_{N_t}^T\right]^T, \ \boldsymbol{n}_n = \left[n_0, n_1, \cdots, n_{N-1}\right]^T \end{aligned} \tag{2}$$

In (1) and (2), $\boldsymbol{s}_k$ is an N-length column vector denoting the k-th MT's spreading code, $x_{k,m}$ is the symbol transmitted by the m-th transmit antenna of the k-th MT and $\boldsymbol{n}_n$ is the corresponding Gaussian noise vector. When the observation samples received by all the N_r receive antennas are collected into $\boldsymbol{y} = [\boldsymbol{y}_1^T, \cdots, \boldsymbol{y}_{N_r}^T]^T$, we can represent it as

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n} \tag{3}$$

where $\boldsymbol{H}$ is a $(NN_r \times KN_t)$ block matrix given by

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{S}\boldsymbol{A}_{1,1} & \boldsymbol{S}\boldsymbol{A}_{1,2} & \cdots & \boldsymbol{S}\boldsymbol{A}_{1,N_t} \\ \boldsymbol{S}\boldsymbol{A}_{2,1} & \boldsymbol{S}\boldsymbol{A}_{2,2} & \cdots & \boldsymbol{S}\boldsymbol{A}_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{S}\boldsymbol{A}_{N_r,1} & \boldsymbol{S}\boldsymbol{A}_{N_r,2} & \cdots & \boldsymbol{S}\boldsymbol{A}_{N_r,N_t} \end{pmatrix} \tag{4}$$

while the N_rN-length complex Gaussian noise vector $\boldsymbol{n} = \left[\boldsymbol{n}_1^T, \boldsymbol{n}_2^T, \cdots, \boldsymbol{n}_{N_r}^T\right]^T$ has zero mean and a covariance matrix of $\sigma^2\boldsymbol{I}_{NN_r}$, where σ^2 denotes the Gaussian noise's variance.

Let the autocorrelation matrix of $\boldsymbol{H}$ be expressed by $\boldsymbol{R}_{\boldsymbol{H}}$, which is a nonnegative definite matrix. In order to derive the ASE of the MIMO-CDMA systems, the distribution of the eigenvalues of $\boldsymbol{R}_{\boldsymbol{H}}$ is required to be obtained first. Our forthcoming analysis is based on the assumptions summarized as follows.

A1. Random spreading codes are employed, whose elements are independently identically distributed (iid) circularly symmetric complex random variables with zero mean and a variance of $1/N$. Hence, they satisfy $\|\boldsymbol{s}_k\| = 1$, $k = 1, \cdots, K$.

A2. Symbols transmitted by any MTs are iid Gaussian variables with zero mean and a common variance of $1/N_t$. Correspondingly, the signal-to-noise ratio (SNR) in terms of a MT is $1/\sigma^2$.

A3. Signals transmitted by MTs experience flat Rayleigh fading. Since uplink communications are considered and MTs are in general geographically at different locations, we hence assume that the channels with respect to different MTs experience independent fading. In mathematics, this implies that $E[a_k^{n,m}(a_{\bar{k}}^{\bar{n},\bar{m}})^*] = 0$, provided that $k \neq \bar{k}$.

A4. For the k-th MT, its channel gains $a_k^{n,m}, \ n = 1, \cdots, N_r; \ m = 1, \cdots, N_t$, can be arbitrarily correlated in the spatial domain. Furthermore, owing to the variance of $a_k^{n,m}$ being normalized to one, the correlation coefficient can be expressed as

$$\begin{aligned} \rho_k(n, m; \bar{n}, \bar{m}) &= \mathbb{E}[a_k^{n,m}(a_k^{\bar{n},\bar{m}})^*] \\ &\equiv \begin{cases} = 1, & \text{if } n = \bar{n}, m = \bar{m} \\ < 1, & \text{otherwise} \end{cases} \end{aligned} \tag{5}$$

A5. Different MTs have the same spatial correlation characteristics, meaning that $\rho_1(n, m; \bar{n}, \bar{m}) = \rho_2(n, m; \bar{n}, \bar{m}) = \cdots = \rho_K(n, m; \bar{n}, \bar{m}) = \rho(n, m; \bar{n}, \bar{m})$, when the values for $m, \ n, \ \bar{m}$ and $\bar{n}$ are the same.

Based on above assumptions, the correlation coefficient between $a_k^{n,m}$ and $a_{\bar{k}}^{\bar{n},\bar{m}}$ can be expressed in a compact form as

$$\begin{aligned} &\rho_{k\bar{k}}(n, m; \bar{n}, \bar{m}) = \mathbb{E}[a_k^{n,m}(a_{\bar{k}}^{\bar{n},\bar{m}})^*] = \delta_{k\bar{k}} \cdot \rho(n, m; \bar{n}, \bar{m}) \\ &k, \bar{k} = 1, \ldots, K; \ n, \bar{n} = 1, \cdots, N_r; \ m, \bar{m} = 1, \cdots, N_t \end{aligned} \tag{6}$$

where δ_{ij} is the Dirac-delta function, defined as $\delta_{ij} = 1$, when $i = j$, and $\delta_{ij} = 0$, otherwise. Let us below analyze the SE of MIMO-CDMA systems based on the above assumptions.

III. Asymptotic Spectral-Efficiency

When assuming that the BS employs the ideal knowledge about $\boldsymbol{H}$, while the MTs know only the distribution information of the fading channels, the optimum detection can be carried out at the BS. In this case, the ergodic SE of the MIMO-CDMA systems normalized by the spreading factor can be formulated as [3, 4, 12]

$$C_{Erg} = \frac{1}{N}\mathbb{E}\left[\log_2\det\left(\boldsymbol{I}_{KN_t} + \frac{\boldsymbol{R}_{\boldsymbol{H}}}{N_t\sigma^2}\right)\right] \tag{7}$$

where the average is taken with respect to both the random spreading codes and the random fading channels. When ASE is concerned by letting $K, N \to \infty$ and $K/N = \beta$, we have

$$\begin{aligned} C_{Asy} &= \lim_{\substack{N\to\infty \\ K\to\infty}} \frac{1}{N}\log_2\left[\det\left(\boldsymbol{I}_{N_tK} + \frac{\boldsymbol{R}_{\boldsymbol{H}}}{N_t\sigma^2}\right)\right] \\ &= \lim_{\substack{N\to\infty \\ K\to\infty}} \frac{N_tK}{N}\left[\frac{1}{N_tK}\sum_{i=1}^{N_tK}\log_2\left(1 + \frac{\lambda_{\boldsymbol{R}_{\boldsymbol{H}}}(i)}{N_t\sigma^2}\right)\right] \\ &= N_t\beta\int_0^\infty \log_2\left(1 + \frac{x}{N_t\sigma^2}\right) f_{\boldsymbol{R}_{\boldsymbol{H}}}(x)dx \end{aligned} \tag{8}$$

where $\lambda_{\boldsymbol{R}_{\boldsymbol{H}}}(i)$ represents the ith eigenvalue of the $(N_tK \times N_tK)$ matrix $\boldsymbol{R}_{\boldsymbol{H}}$, while $f_{\boldsymbol{R}_{\boldsymbol{H}}}(x)$ is the AED of matrix $\boldsymbol{R}_{\boldsymbol{H}}$, which needs to be derived first in order to obtain the ASE formula for the MIMO-CDMA systems.

From (4), we can see that the channel matrix $\boldsymbol{H}$ of the uplink MIMO-CDMA systems can be decomposed into

$$\boldsymbol{H} = \boldsymbol{C}\boldsymbol{A}_K \tag{9}$$

where

$$\boldsymbol{A}_K = \begin{pmatrix} \boldsymbol{A}_{1,1} & \boldsymbol{A}_{1,2} & \cdots & \boldsymbol{A}_{1,N_t} \\ \boldsymbol{A}_{2,1} & \boldsymbol{A}_{2,2} & \cdots & \boldsymbol{A}_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{A}_{N_r,1} & \boldsymbol{A}_{N_r,2} & \cdots & \boldsymbol{A}_{N_r,N_t} \end{pmatrix} \tag{10}$$

is an $(N_r K \times N_t K)$ block matrix and

$$\boldsymbol{C} = \begin{pmatrix} \boldsymbol{S} & & & \\ & \boldsymbol{S} & & \\ & & \ddots & \\ & & & \boldsymbol{S} \end{pmatrix} = \boldsymbol{I}_{N_r} \otimes \boldsymbol{S} \tag{11}$$

is an $(N_r N \times N_r K)$ matrix. In (11) $\otimes$ denotes the Kronecker product operation. Therefore, with the aid of (9), we can rewrite the autocorrelation matrix of $\boldsymbol{H}$ as

$$\boldsymbol{R}_{\boldsymbol{H}} = \boldsymbol{H}^H \boldsymbol{H} = \boldsymbol{A}_K^H \boldsymbol{C}^H \boldsymbol{C} \boldsymbol{A}_K \tag{12}$$

According to the matrix theory [4], the $(N_t K \times N_t K)$ matrix $\boldsymbol{R}_{\boldsymbol{H}}$ and the $(N_r K \times N_r K)$ matrix $\bar{\boldsymbol{R}}_{\boldsymbol{H}} = \boldsymbol{A}_K \boldsymbol{A}_K^H \boldsymbol{C}^H \boldsymbol{C}$ have the same non-zero eigenvalues. Furthermore, the relationship between the AEDs of $\boldsymbol{R}_{\boldsymbol{H}}$ and $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$ is formulated by [4]

$$\eta_{\boldsymbol{R}_{\boldsymbol{H}}}(\gamma) = 1 - \frac{N_r}{N_t} + \frac{N_r}{N_t}\eta_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(\gamma) \tag{13}$$

here $\eta_{\boldsymbol{R}_{\boldsymbol{H}}}(\gamma)$ and $\eta_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(\gamma)$ are, respectively, the η-transform [4] of $f_{\boldsymbol{R}_{\boldsymbol{H}}}(x)$ and $f_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(x)$ of the AEDs of matrices $\boldsymbol{R}_{\boldsymbol{H}}$ and $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$, which are given by [4]

$$\eta_X(\gamma) = \int_0^\infty \left(\frac{1}{1+\gamma x}\right) f_X(x)dx \tag{14}$$

where X is for $\boldsymbol{R}_{\boldsymbol{H}}$ or $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$. Therefore, from above we are implied that the AED of $\boldsymbol{R}_{\boldsymbol{H}}$ can be found through deriving first the AED of $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$, which is now considered below.

A. Asymptotic Eigenvalue Distribution of $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$

Let $\boldsymbol{R}_{\boldsymbol{A}_K} = \boldsymbol{A}_K \boldsymbol{A}_K^H$ and $\boldsymbol{R}_{\boldsymbol{C}} = \boldsymbol{C}^H \boldsymbol{C}$. Then, we have $\bar{\boldsymbol{R}}_{\boldsymbol{H}} = \boldsymbol{R}_{\boldsymbol{A}_K}\boldsymbol{R}_{\boldsymbol{C}}$. In order to make it possible to derive the AED of $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$, we assume (or approximate) that $\boldsymbol{R}_{\boldsymbol{A}_K}$ and $\boldsymbol{R}_{\boldsymbol{C}}$ are asymptotically free. This assumption is reasonable, since, according to [4, 13], two independent unitarily invariant matrices are asymptotically free. In our case, first, $\boldsymbol{R}_{\boldsymbol{A}_K}$ and $\boldsymbol{R}_{\boldsymbol{C}}$ are independent. Second, as pointed out in [14], unitary invariance is a property employed by a set of random matrices with the set size being much larger than the iid Gaussian ensemble. Hence, we can be confident that $\boldsymbol{R}_{\boldsymbol{A}_K}$ and $\boldsymbol{R}_{\boldsymbol{C}}$ are asymptotically free, which is also verified by our results shown in Section IV. Consequently, with the aid of Theorem 2.68 in [4], the η-transform of the AED of $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$ can be represented as

$$\eta_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(\gamma) = \eta_{\boldsymbol{R}_{\boldsymbol{A}_K}}\left(\frac{\gamma}{S_{\boldsymbol{R}_{\boldsymbol{C}}}(\eta_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(\gamma) - 1)}\right) \tag{15}$$

where $S_{\boldsymbol{R}_{\boldsymbol{C}}}(x)$ denotes the S-transform of the AED of $\boldsymbol{R}_{\boldsymbol{C}}$, which is defined [4] by the η-transform as $S_{\boldsymbol{R}_{\boldsymbol{C}}}(x) = -\frac{x+1}{x}\eta^{-1}_{\boldsymbol{R}_{\boldsymbol{C}}}(1+x)$.

From (11) we have $\boldsymbol{C} = \boldsymbol{I}_{N_r} \otimes \boldsymbol{S}$. Hence, the AED of $\boldsymbol{R}_{\boldsymbol{C}}$ is the same as that of $\boldsymbol{S}^H\boldsymbol{S}$. Therefore, using the Marčenko-Pastur (M-P) law [4] for the AED of $\boldsymbol{S}^H\boldsymbol{S}$, we can obtain the S-transform of the AED of $\boldsymbol{R}_{\boldsymbol{C}}$, which is [4]

$$S_{\boldsymbol{R}_{\boldsymbol{C}}}(x) = \frac{1}{1+\beta x} \tag{16}$$

where, as defined previously, $\beta = K/N$ is the system load factor of the MIMO-CDMA systems. Finally, applying $\eta(\gamma) = m(-\gamma^{-1})/\gamma$ of the relationship between the Stieltjes transform[1] and the η-transform [4] as well as (16) to (15), we can obtain the Stieltjes transform of $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$, given by

$$\begin{aligned} m_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(z) &= -\frac{1}{z}\eta_{\boldsymbol{R}_{\boldsymbol{A}_K}}\left(-\frac{1}{z}\left[1-\beta-\beta z m_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(z)\right]\right) \\ &= \int_0^\infty \frac{f_{\boldsymbol{R}_{\boldsymbol{A}_K}}(x)}{[1-\beta-\beta z m_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(z)]x - z}dx \end{aligned} \tag{17}$$

[1]Let X be a real-valued random variable with distribution $F_X(\cdot)$. Then, the Stieltjes transform is defined as [4] $m_X(z) = \int_{-\infty}^{\infty} dF_X(z)/(z-x)dz$.

where $f_{\boldsymbol{R}_{\boldsymbol{A}_K}}(x)$ is the AED of $\boldsymbol{R}_{\boldsymbol{A}_K}$ and $z = -1/\gamma$.

Finally, given the Stieltjes transform of $m_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(z)$, the AED of $\boldsymbol{R}_{\bar{\boldsymbol{H}}}$ can now be obtained by its inversion formula [4], yielding

$$f_{\boldsymbol{R}_{\bar{\boldsymbol{H}}}}(x) = \frac{1}{\pi}\lim_{y\to 0^+}\Im(m_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(x+\sqrt{-1}y)) \tag{18}$$

where $\Im(z)$ means the imaginary part of the complex number z.

As shown in (17), we need first to obtain the AED of $\boldsymbol{R}_{\boldsymbol{A}_K}$, i.e. $f_{\boldsymbol{R}_{\boldsymbol{A}_K}}(x)$, before finding the AED of $\boldsymbol{R}_{\bar{\boldsymbol{H}}}$, which is considered in the next subsection with the aid of the principles of *operator-valued free probability.*

B. Asymptotic Eigenvalue Distribution of $\boldsymbol{R}_{\boldsymbol{A}_K} = \boldsymbol{A}_K\boldsymbol{A}_K^H$

As shown in (10), $\boldsymbol{A}_K$ is a block matrix, where each of the $N_r \times N_t$ blocks is an $(K \times K)$ diagonal matrix. Due to the spatial correlation, entries in $\boldsymbol{A}_K$ may be correlated and the covariance is given by (6). To the authors' best knowledge, the accurate AED of $\boldsymbol{R}_{\boldsymbol{A}_K}$ has not been found and it seems extremely hard to derive this accurate AED. For this sake, we propose an approximation approach, which approximates the AED of $\boldsymbol{R}_{\boldsymbol{A}_K}$ by the AED of $\boldsymbol{B}_K\boldsymbol{B}_K^H$, where $\boldsymbol{B}_K$ is defined as

$$\boldsymbol{B}_K = \begin{pmatrix} \boldsymbol{B}_{1,1} & \boldsymbol{B}_{1,2} & \cdots & \boldsymbol{B}_{1,N_t} \\ \boldsymbol{B}_{2,1} & \boldsymbol{B}_{2,2} & \cdots & \boldsymbol{B}_{2,N_t} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{B}_{N_r,1} & \boldsymbol{B}_{N_r,2} & \cdots & \boldsymbol{B}_{N_r,N_t} \end{pmatrix} \tag{19}$$

where each block $\boldsymbol{B}_{n,m} = \left\{b_{i,j}^{n,m}\right\}_{i,j=1}^K$ is an $(K \times K)$ Gaussian matrix [4]. The entries of $\boldsymbol{B}_K$ are circularly symmetric complex Gaussian variables, with zero mean and covariances

$$\mathbb{E}\left[b_{i,j}^{n,m}(b_{\bar{i},\bar{j}}^{\bar{n},\bar{m}})^*\right] = \frac{1}{K}\delta_{i,\bar{i}}\delta_{j,\bar{j}} \cdot \rho(n,m;\bar{n},\bar{m}) \tag{20}$$

where $\rho(n,m;\bar{n},\bar{m})$ has been defined in (5). As our simulation results in Section IV show, the above approximation is feasible and also very accurate. The reason is that it can be proofed[2] that the AEDs of both $\boldsymbol{B}_K\boldsymbol{B}_K^H$ and $\boldsymbol{R}_{\boldsymbol{A}_K}$ are defined in the positive real axis and that they have the same first and second moments. Furthermore, their third moments are also very close to each other.

According to [10], the AED of $\boldsymbol{B}_K\boldsymbol{B}_K^H$ can be derived based on the operator-valued free probability theory. The results have been stated in a theorem in [10], which is repeated here for convenience.

Theorem 1: [10] Considering a block matrix $\boldsymbol{B}_K$ as defined in (19), for $K \to \infty$, the $(KN_r \times KN_r)$ matrix $\boldsymbol{B}_K\boldsymbol{B}_K^H/(N_r+N_t)$ has almost surely a limiting eigenvalue distribution, whose Cauchy transform $G(z)$ is determined by

$$G(z) = \mathrm{tr}_{N_r}(\mathcal{G}(z)) \tag{21}$$

where $\mathcal{G}(z)$ is an $M_{N_r}(\mathbb{C})$-valued analytic function on the upper complex half-plane, which is uniquely determined by the facts that

$$\lim_{|z|\to\infty,\Im(z)>0} z\mathcal{G}(z) = \boldsymbol{I}_{N_r} \tag{22}$$

and that it satisfies for all z in the upper complex half-plane the matrix equation

$$z\mathcal{G}(z) = \boldsymbol{I}_{N_r} + \psi_1\left((\boldsymbol{I}_{N_t} - \psi_2(\mathcal{G}(z)))^{-1}\right)\mathcal{G}(z) \tag{23}$$

where

$$\psi_1 : M_{N_t}(\mathbb{C}) \to M_{N_r}(\mathbb{C}) \quad \text{and} \quad \psi_2 : M_{N_r}(\mathbb{C}) \to M_{N_t}(\mathbb{C})$$

and the covariance mappings are given by

$$[\psi_1(\boldsymbol{Q})]_{i,j} := \frac{1}{N_r+N_t}\sum_{k,l=1}^{N_t}\rho(i,k;j,l)[\boldsymbol{Q}]_{k,l} \tag{24}$$

$$[\psi_2(\boldsymbol{Q})]_{k,l} := \frac{1}{N_r+N_t}\sum_{i,j=1}^{N_r}\rho(i,k;j,l)[\boldsymbol{Q}]_{j,i} \tag{25}$$

[2]The detailed proof is removed from this paper due to the space limit.

where $\boldsymbol{Q}$ represents a matrix considered.

Note that, in Theorem 1, after multiplied it by -1, the Cauchy transform is the same as the Stieltjes transform [4]. Consequently, the AED of $\boldsymbol{B}_K\boldsymbol{B}_K^H/(N_r+N_t)$ can be obtained from $G(z)$ by carrying out the inverse Stieltjes transform, as shown in (18). Finally, the AED of $\boldsymbol{R}_{\boldsymbol{A}_K}$ can be approximated by the AED of $\boldsymbol{B}_K\boldsymbol{B}_K^H$, which can be directly derived from the AED of $\boldsymbol{B}_K\boldsymbol{B}_K^H/(N_r+N_t)$, since N_r and N_t are constants.

C. Solutions to Equations (17) and (23)

From the previous analysis in Sections III-A and III-B, we can see that, in order to derive the AED of $\bar{\boldsymbol{R}}_{\boldsymbol{H}}$, Eqs. (17) and (23) are required to be solved first. Due to the facts that (17) is an integral equation and (23) is a matrix equation, it is very hard to solve these transcendental equations directly. In this paper, with the aid of the fixed point theorem and the principles of contraction mapping [4], we propose an iterative method to derive the solutions of these two equation.

From Eq. (23), a mapping used in our iterative algorithm is expressed as

$$\mathcal{G}\mapsto\mathcal{F}_z(\mathcal{G}):=\left[z\boldsymbol{I}_{N_r}-\psi_1\left((\boldsymbol{I}_{N_t}-\psi_2(\mathcal{G}))^{-1}\right)\right]^{-1} \tag{26}$$

where we have used $\mathcal{G}(z)=\mathcal{G}$ for simplicity. In [15], it has been proofed that the mapping $\mathcal{F}_z(\mathcal{G})$ is a contraction mapping. Furthermore, in order to speed up the convergence, in [15], an improved iteration structure has been proposed, which can be expressed as

$$\mathcal{G}\mapsto\mathcal{W}_z(\mathcal{G}):=\frac{1}{2}\mathcal{G}+\frac{1}{2}\mathcal{F}_z(\mathcal{G}) \tag{27}$$

In the context of Eq. (17), the mapping used in our iterative algorithm can be expressed as

$$m\mapsto\mathcal{T}_z(m):=\int_0^\infty\frac{f_{\boldsymbol{R}_{\boldsymbol{A}_K}}(x)}{[1-\beta-\beta zm]x-z}dx \tag{28}$$

where we have set $m=m_{\bar{\boldsymbol{R}}_{\boldsymbol{H}}}(z)$, again, for the simplicity of description. Furthermore, it can be shown that $\mathcal{T}_z$ is a contraction mapping.

With the aid of the above mappings, the iterative algorithm for solving Eqs. (17) and (23) can be summarized as follow:

1) For a given argument z, setting an initial value. Specifically, for Eq. (27), it can be $\mathcal{G}_0=\boldsymbol{I}_{N_r}$, while for Eq. (28), it can be $m_0=1+\sqrt{-1}\cdot 1$.
2) For $n=1,2,\ldots$, carrying out the operations:

$$\mathcal{G}_{n+1}\leftarrow\mathcal{W}_z(\mathcal{G}_n),\ \ m_{n+1}\leftarrow\mathcal{T}_z(m_n) \tag{29}$$

until the solutions converge or the maximum number of iterations is reached, yielding the solutions to Eqs. (17) and (23).

IV. Performance Results

In this section, we provide some SE results evaluated from our analysis as well as the corresponding SE results obtained by simulations to show the SE performance of the MIMO-CDMA systems as well as to illustrate how close the analytical results agree with the simulation results. For the sake of illustrating the effect of the correlation among the transmit antennas and of that among the receive antennas, in the examples considered, we assume that the overall spatial correlation is separable in terms of the transmit and receive antennas, which can be expressed as $\rho(n,m;\bar{n},\bar{m})=\rho_r^{|n-\bar{n}|}\rho_t^{|m-\bar{m}|}$. We, however, note that, our analytical results obtained in this paper are suitable for the MIMO systems where the transmit and receive antennas have arbitrary spatial correlation, no matter whether this spatial correlation is separable or non-separable.

Fig.2 shows the ASE and the SE obtained by simulations for the specific MIMO-CDMA systems considered, when various receive antenna correlation is assumed. The parameters configurating the MIMO-CDMA systems are shown associated with Fig.2. When comparing the simulated SE with the corresponding ASE results, we can

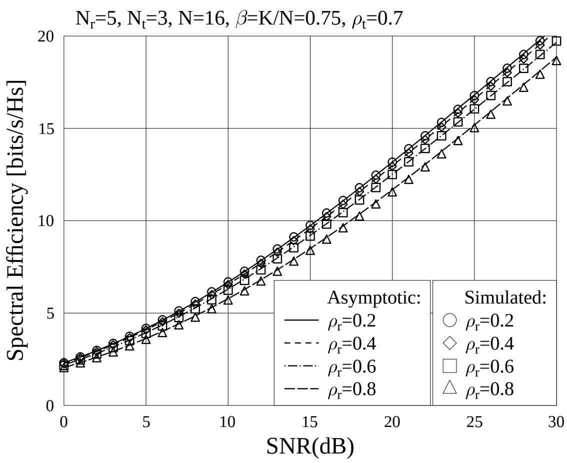

Fig. 2. SE versus SNR performance of MIMO-CDMA systems, when the receive antennas have different correlation.

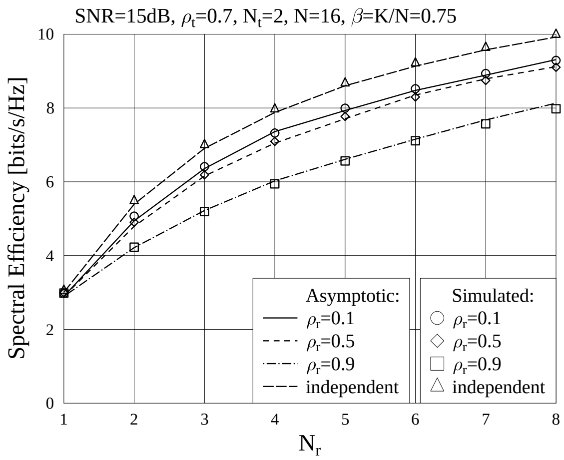

Fig. 3. SE versus the number of receive antennas for the MIMO-CDMA systems, when the receive antennas have different correlation.

observe that they are in good accordance. From this we are implied that, even for the MIMO-CDMA systems of moderate size (here $N=16,\ K=12,\ N_r=5,\ N_t=3$), their SE can be closely estimated by the corresponding ASE. From Fig.2, we can observe that the receive antenna correlation has slight impact on the achievable SE of the MIMO-CDMA systems. Specifically, there is about 2.5dB loss, when ρ_r changes from 0.2 to 0.8. The reason behind is twofold. First, the degrees-of-freedom (DoF) in MIMO-CDMA systems is constituted by both the DoF in the spatial domain and the DoF provided by the spreading codes. Second, the correlation model considered for this figure is exponential type, which results in that the spatial correlation reduces quickly as the spacing between two receive antennas increases.

Fig.3 compares the achievable SE versus the number of receive antennas, when different receive antenna correlation is considered. Again, from Fig.3 we can see that the simulated SE for a specific MIMO-CDMA system is close to its corresponding ASE. As expected, the achievable SE of the MIMO-CDMA systems increases as N_r increases, while decreases as ρ_r increases. Furthermore, the SE corresponding to a relatively small ρ_r value increases faster than that corresponding to a relatively large ρ_r value, when N_r is less than 4. As seen in Fig.3, the MIMO-CDMA systems with $\rho_r=0.1$ is capable of achieving the SE of $8\mathrm{bits/s/Hz}$ by employing $N_r=5$ receive antennas, while the MIMO-CDMA systems with $\rho_r=0.95$ requires $N_r=8$ receive antennas to attain the same SE.

In Fig.4, we compare the achievable SE of the MIMO-CDMA systems employing different number of transmit antennas. As observed in Figs.2 and 3, the results in Fig.4 again show that the ASE can provide

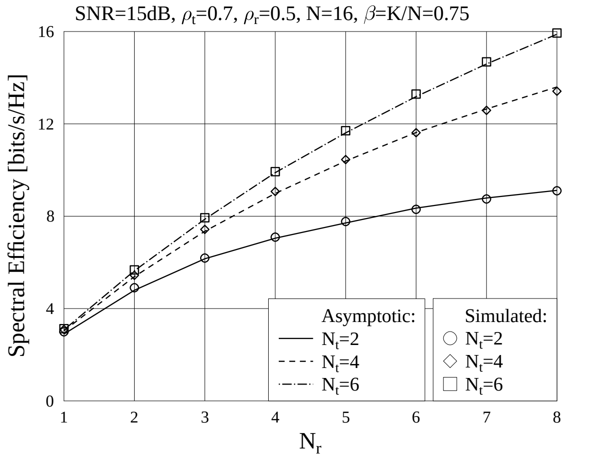

Fig. 4. SE versus the number of receive antennas for the MIMO-CDMA systems employing different number of transmit antennas.

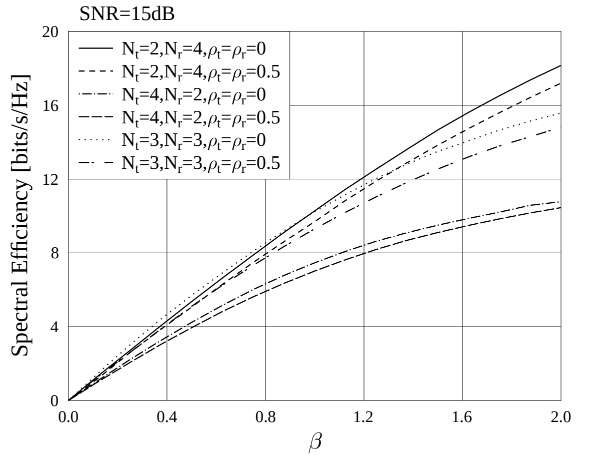

Fig. 6. ASE versus system load β performance of the MIMO-CDMA systems when given $N_r + N_t = 6$.

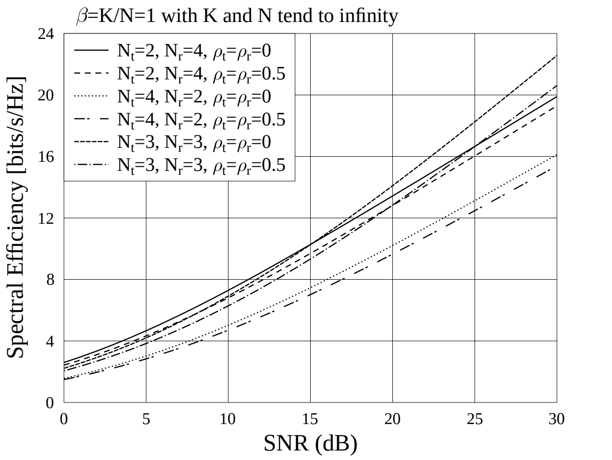

Fig. 5. ASE versus SNR performance of the MIMO-CDMA systems when given $N_r + N_t = 6$.

a good measure of SE achieved by a specific MIMO-CDMA system. Additionally, from Fig.4 we can see that the SE of the MIMO-CDMA systems improves, when the number of transmit antennas increases. However, the SE improvement becomes less and less significant as the number of transmit antennas becomes more and more.

Figs.5 and 6 show the impact of SNR and system load β, respectively, on the ASE of the MIMO-CDMA systems, when the total number of transmit and receive antennas is $N_r + N_t = 6$. As shown in these two figures, the MIMO-CDMA system employing $N_r = 2$ receive antennas and $N_t = 4$ transmit antennas per MT attains the lowest SE. From Fig.5, we can observe that, at a given SNR value, there may exist an optimal combination of the number of transmit and receive antennas, which yields the highest ASE. The results of Fig.6 show that the ASE increases, as the system load increases. Again, at a given system load, there may exist an optimal combination of the number of transmit and receive antennas, which results in possibly the highest ASE.

V. CONCLUSIONS

With the aid of the random matrix theory and the operator-valued free probability theory, we have investigated the ASE of the MIMO-CDMA systems, whose transmit/receive antennas exist arbitrary correlation. A range of formulas have been derived and an algorithm supporting the ASE computation has been proposed. Furthermore, the SE performance of the MIMO-CDMA systems has been investigated either numerically or by simulations, when various scenarios are considered. It can be shown that the corresponding ASE can usually provide a very close approximation for the SE achieved by a realistic MIMO-CDMA system, which has fixed values for the number of transmit antenna, number of receive antennas, spreading factor and the number of MTs supported.

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