Redundant Residue Number System Based Multicarrier DS-CDMA for Dynamic Multiple-Access in Cognitive Radios

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***Abstract*—Redundant residue number system (RRNS)-based multicarrier DS-CDMA (MC/DS-CDMA) is proposed for dynamic multiple-access (DMA) in cognitive radios (CRs). The proposed RRNS-based MC/DS-CDMA DMA has the merits of low-complexity for implementation, high-flexibility for reconfiguration and spectrum handoff, robustness to spectrum varying, and fault-tolerance to errors. Specifically, in our RRNS-based MC/DS-CDMA DMA system, RRNS-based orthogonal modulation aided by MC/DS-CDMA is employed for information transmission. At the receiver, signals are detected subcarrier-by-subcarrier independently based on suboptimum MMSE interference cancellation (SMMSE-IC). In performance study, we model the arrival process of primary users (PUs) in primary radios (PRs) as a Poisson process. Both the bit error rate (BER) performance and throughput performance are investigated. Our studies and performance results show that the RRNS-based MC/DS-CDMA constitutes one of the highly promising DMA schemes for application in CRs. It is capable of achieving a substantial throughput with required quality for the CR systems, while without degrading the quality-of-services (QoS) of the PR systems.**

I. Introduction

The CRs equipped with flexible software defined architectures, which aim at intelligent wireless communications, demand high-efficiency, high-flexibility and also high-reliability DMA schemes. These DMA schemes should be suitable for online reconfigurations and robust to spectrum varying as well as to the side-effects resulted from spectrum varying. In this paper, we propose and investigate a so-called RRNS-based MC/DS-CDMA DMA scheme, which is shown to employ the above-mentioned properties. In our investigations, we assume that the PUs and cognitive radio users (CRUs) are operated in the interweave paradigm [1], where the CRUs can only communicate opportunistically on the frequency bands not occupied by the PUs. Once there are PUs appearing on some of the frequency bands that the CRUs communicate on, the corresponding CRUs must vacate these frequency bands and move to some other available. This process is referred to as spectrum handoff. Explicitly, the spectrum handoff requires some time for the CRUs to obtain the knowledge of the new spectrum sources, including available frequency bands, channel state information (CSI), etc. Furthermore, the CR system also needs time to coordinate the CRUs after the handoff. In this paper, our studies take into account of the effects due to spectrum handoff.

In this paper, the RNS/RRNS techniques are introduced for further enhancing the CR system's flexibility, robustness and fault-tolerance, in addition to the MC/DS-CDMA, which has been recognized to have high-flexibility and high number of degrees-of-freedom for reconfigurations [5–7]. Due to their inherent characteristics, RNS/RRNS have been widely applied for design of high-speed special-purpose digital hardware, such as very large scale integrated circuits (VLSIC) [8–10]. Specifically, RNS/RRNS-based arithmetics exhibit modular structures that lead naturally to parallelism in digital hardware. Comparing with the conventional weighted number systems, such as the binary weighted number system representation, the RNS/RRNS have two attractive inherent features [8, 10]. First, they belong to the carry-free arithmetic. Second, the residues with respect to different moduli have no ordered significance. The first property implies that the operations related to the different residues are mutually independent. The second property explains that, in a RRNS, any erroneous residues can be discarded without affecting recovery of the information, provided that a sufficient dynamic range remains in the reduced RRNS system in order to unambiguously represent the information. Due to the above-mentioned inherent properties, the RNS/RRNS techniques may offer a variety of new approaches to the realization of digital signal processing algorithms, such as digital modulation/demodulation, fault-tolerant design of arithmetic units, error-detection and error-correction codes [11, 12], as well as novel digital transmission schemes [11].

In this paper, the performance of the RRNS-based MC/DS-CDMA DMA systems is investigated, when assuming communication over Rayleigh fading channels. Both the BER performance and throughput performance are considered. Our studies show that the RRNS-based MC/DS-CDMA constitutes one of the versatile multiple access schemes that are suitable for CRs. It employs the properties well satisfying the requirements of CRs. Furthermore, as our performance results show, it is capable of achieving a substantial throughput for the CR systems without making trade-off of the QoS of the PR systems.

II. System Modeling

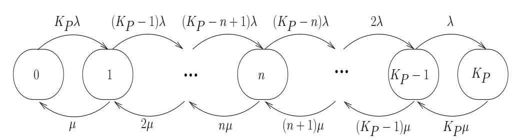

Fig. 1. State transition diagram for modeling the arrival process of PUs.

In order to carry out the related investigation, in this paper, we assume that the whole frequency band accessible by the CRUs is divided into C subbands supported by the corresponding subcarriers. The C subbands are allocated to support maximal K_P PUs. Once a PU becomes active, it is assumed to occupy C_P subbands, where we assume that $K_pC_P \leq C$. Furthermore, we assume that the PUs activate according to the $M/M/K_P/K_P/K_P$ queuing model [13], as depicted in Fig. 1. In this $M/M/K_P/K_P/K_P$ queuing model, the number of active PUs follows a Poisson distribution associated with an arrival rate λ, the service time obeys a negative exponential distribution with mean $1/\mu$, and the number of servers, system capacity as well as the total number of customers are the same value of K_P [13, 14].

Based on the above assumptions for the PRs, we can know that the frequency bands possible for the CR system are dynamic. The number of subbands available for the CRUs is given by

$$\bar{C} = C - nC_P,\ n = 0, 1, \ldots, K_P \tag{1}$$

when there are n active PUs, where n may take any value in $\{0, 1, \ldots, K_p\}$.

Let us assume that the channels allocated to an active PU are uniformly and randomly chosen from the unused channels. In this case, the subbands available for the CRUs also constitute a Poisson process with its parameters that can be readily derived from the corresponding PUs' Poisson process [14]. We assume that the CRUs have the same priority to access the $\bar{C}$ subbands based on the RRNS-based MC/DS-CDMA DMA scheme that will be detailed in our forthcoming discourse. Furthermore, for convenience, when the CR system is capable of accessing all the $\bar{C}$ subbands provided by the above $M/M/K_P/K_P/K_P$ queuing model without other limitations, we say that the CR system is operated under the *ideal* mode. However, when operated under the interweave paradigm [1], it is the usual case that other constraints will be imposed on the CR systems accessing the frequency spectrum. For example, a CR must immediately clear the being used subbands, once it realizes that the subbands are occupied by PUs. Correspondingly, we say that the CR systems are operated under the *constraint* mode.

In this paper, our proposed RRNS-based MC/DS-CDMA DMA system is operated under the constraint mode by taking into account the effect of spectrum handoff, which is hence specifically referred to as the *handoff mode*. Under the handoff mode, let us assume that there are n active PUs at t. After a time duration of Δt, some of the $(K_P - n)$ inactive PUs may activate to communicate. The probability that i out of the $(K_P - n)$ inactive PUs become active within Δt is given by [14]

$$P'_i = \frac{[\lambda(K_P - n)\Delta t]^i}{i!} e^{-\lambda(K_P - n)\Delta t}, \; i = 0, 1, \ldots, K_P - n \quad (2)$$

Once the CRUs realize that there are ‘new’ PUs appearing on some of the subbands they occupy, the CRUs immediately terminate their communications on these subbands. Simultaneously, after a time duration of Δt, some active PUs may complete their communications and become idle. The probability that there are k active PUs become inactive within Δt can be expressed as [14]

$$P''_k = n\mu\Delta t e^{-n\mu(k\Delta t)}, \; k = 0, 1, \ldots. \quad (3)$$

In this case, there will be some new subbands available for the CRUs. However, the CR system has to sense the spectrum first and may also need to estimate the channels of the corresponding subbands found, in order to access them. This process also takes some time. Therefore, the spectrum handoff may have significant impact on the achievable performance of the CR systems, if the DMA scheme is not designed appropriately. Hence, in order to avoid communication interrupt, an efficient DMA scheme should offer sufficient time for spectrum handoff and can tolerate some loss of signals. Our RRNS-based MC/DS-CDMA DMA system takes into account of the spectrum handoff. With the aid of the RRNS techniques, it is robust to the spectrum varying and can implement seamless spectrum handoff, as demonstrated in the next section.

III. RRNS-Assisted MC/DS-CDMA Dynamic Multiple-Access

A. RNS and RRNS: A Brief Overview

A RNS is defined [8] by a set of, say s, positive integers denoted by $m_1, m_2, \ldots, m_s$, which are referred to as information moduli. All the moduli are pairwise relative primes. In this RNS, any integer $0 \leq X < M \leq \prod_{i=1}^{s} m_i$ can be uniquely and unambiguously represented by a residue sequence $(r_1, r_2, ..., r_s)$, where $r_i = X \mod m_i$ represents the residue of X with respect to m_i, $0 \leq r_i < m_i$. Reversely, in this RNS, given an s-tuple residue sequence $(r_1, r_2, ..., r_s)$ with $0 \leq r_i < m_i$, $i = 1, ..., s$, according to the Chinese reminder theorem (CRT) [15], there exists one and only one integer $0 \leq X < M$ satisfying $r_i = X \mod m_i$. In other words, if $(r_1, r_2, ..., r_s)$ conveys a message X, it can be recovered uniquely and unambiguously with the aid of, for example, the Chinese reminder theorem.

For the sake of incorporating error control or making a RNS-based system fault-tolerant, redundant moduli can be appended to the information moduli, forming the so-called RRNS [8, 11, 15–17]. Specifically, $(q - s)$ moduli $m_{s+1}, m_{s+2}, ..., m_q$, referred to as redundant moduli, can be added to the previously introduced RNS, in order to form a RRNS with q positive, pairwise relative prime integers. Usually, it is required that $\min\{m_{s+1}, m_{s+2}, ..., m_q\} > \max\{m_1, m_2, \ldots, m_s\}$. Let the product of $m_{s+1}, m_{s+2}, ..., m_q$ be denoted by M_R. Now in the RRNS, an integer X in the range $[0, M)$ can be represented by a q-tuple residue sequence $(r_1, r_2, ..., r_q)$ with respect to the q moduli of the RRNS. At the receiver, based on $(r_1, r_2, ..., r_q)$, the RRNS decoding processing, such as erasure-only decoding, error-correction-only decoding or error-and-erasure decoding [16, 17], may be implemented to recover the message X transmitted.

In this paper, our RRNS-based MC/DS-CDMA DMA system is designed based on the principles of RRNS, which is now described as follows.

B. Transmitter

The transmitter schematic block diagram for the kth CRU is shown in Fig. 2. The binary bits to be transmitted are first transformed to a residue sequence $(r_1^{(k)}, r_2^{(k)}, ..., r_q^{(k)})$. Then, the q residues are mapped into q number of orthogonal sequences based on the M-ary

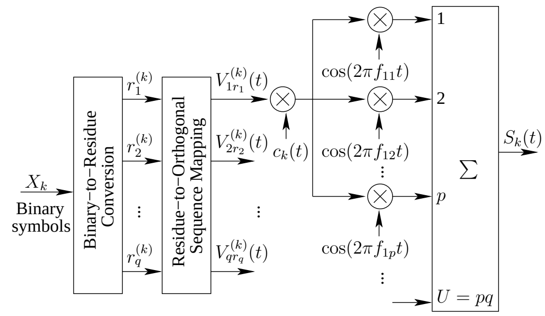

Fig. 2. Transmitter schematic block diagram of the kth CRU in the RRNS-based MC/DS-CDMA DMA system.

orthogonal-shift keying (MOSK) principles [18]. In this paper, Walsh-Hadamard codes are assumed, and every CRU is assigned the same set of orthogonal sequences consisting of $\max\{m_1, m_2, \ldots, m_q\}$ Walsh-Hadamard codes of length N_s, arranged as

$$\left\{ \begin{array}{l} V_{10}(t), V_{11}(t), ..., V_{1(m_1-1)}(t); \\ V_{20}(t), V_{21}(t), ..., V_{2(m_2-1)}(t); \\ \quad \cdots\cdots \\ V_{q0}(t), V_{q1}(t), ..., V_{q(m_q-1)}(t) \end{array} \right\} \quad (4)$$

where the subset $\{V_{i0}(t), V_{i1}(t), ..., V_{i(m_i-1)}(t)\}$, $i = 1, \ldots, q$, is for transmission of the residue $r_i^{(k)}$, $k = 1, \ldots, K$, by taking one out of the m_i orthogonal codes according to the value of $r_i^{(k)}$, $k = 1, \ldots, K$. Note that, due to the orthogonality of the subcarriers, some of the orthogonal codes in the different rows of (4) can be the same. Let $N_s = T_s/T_r$, where T_s denotes the symbol interval and T_r is the duration of the orthogonal codes’ waveforms, referred to as *fraction* for convenience. The orthogonal code waveform can be expressed as $V_{ir_i}^{(k)}(t) = \sum_{n=0}^{N_s-1} V_{ir_i}^{(k)}[n] P_{T_r}(t - nT_r)$, where $V_{ir_i}^{(k)}[n] = \pm 1$, $P_{T_r}(t)$ represents the rectangular waveform of duration T_r. As shown in Fig. 2, after the mapping from residues to orthogonal sequences, each of the q orthogonal sequences is spread by a user specific signature sequence. Furthermore, as seen in Fig. 2. each of the q spreading sequences is transmitted on p subcarriers, in order to achieve a pth order frequency diversity. Therefore, the RRNS-based MC/DS-CDMA DMA system requires a total $U = pq$ number of subcarriers. Specifically, according to Fig. 2, the transmitted signal by CRU k can be expressed as

$$S_k(t) = \sum_{i=1}^{q} \sum_{l=1}^{p} \sqrt{\frac{2P}{U}} V_{ir_i}^{(k)}(t) c_k(t) \cos(2\pi f_{il} t + \phi_{il}^{(k)}), \quad (5)$$

where P is the transmission power of the U number of subcarriers, $c_k(t)$ represents the spreading code assigned to the kth CRU, which can be expressed as $c_k(t) = \sum_{j=0}^{\infty} c_j^{(k)} \psi(t - jT_c)$, where $c_j^{(k)}$ assumes value of $+1$ or -1, T_c is the chip-duration, while $\psi(t)$ is the chip-waveform of the spreading sequences and normalized to $\int_0^{T_c} \psi^2(t) dt = T_c$. Furthermore, we define $N_e = T_r/T_c$, which is the number of chips per fraction. Finally, in (5), $\phi_{il}^{(k)}$ represents the initial phase corresponding to the subcarrier f_{il}.

C. Receiver

We assume that there are K CRUs communicating with a CR base-station (CRBS). Furthermore, we assume synchronous transmission and ideal power-control among the CRUs. Then, the received baseband equivalent signal at the CRBS can be expressed as

$$R(t) = \sum_{k=1}^{K} \sum_{i=1}^{q} \sum_{l=1}^{p} \sqrt{\frac{2P}{U}} h_{il}^{(k)} V_{ir_i}^{(k)}(t) c_k(t) \exp(j 2\pi f_{il} t) + n(t) \quad (6)$$

where $h_{il}^{(k)}$ represents the channel gain with respect to the ilth subcarrier of the kth CRU, $h_{il}^{(k)}$ obeys the complex Gaussian distribution with zero mean and a variance 0.5 per dimension. In (6), $n(t)$ denotes the complex baseband equivalent Gaussian noise, which has zero mean and a single-sided power-spectrum density (PSD) of N_0 per dimension.

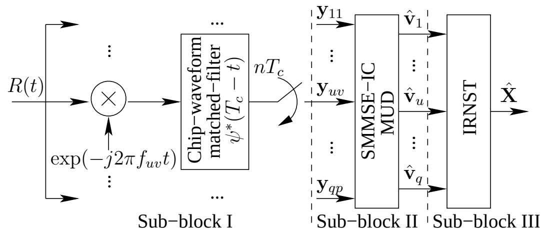

Fig. 3. Receiver schematic block diagram for the RRNS-based MC/DS-CDMA DMA system.

The receiver schematic block diagram for the RRNS-based MC/DS-CDMA DMA is shown as Fig. 3. The receiver can be divided into three sub-blocks. In Sub-block I, the multicarrier demodulation is first executed. We assume that the subcarrier signals are configured so that the subcarrier signals are orthogonal with each other at chip-level [19], which does not yield inter-carrier interference (ICI). Then, a chip waveform matched-filter (MF) with its time domain impulse response $\psi^*(T_c - t)$ is employed by each of the U subcarrier branches. As shown in Fig. 3, the chip waveform MFs' outputs are sampled at the chip-rate in order to provide the discrete observations for detection. It can be shown that the nth observation with respect to the n_sth fraction of $V_{ur_u}^{(k)}(t)$ and the uvth subcarrier can be expressed as

$$\begin{aligned} y_{uv,n}[n_s] = &\frac{1}{\sqrt{2PN_sN_eT_c}}\int_{n_sT_r+nT_c}^{n_sT_r+(n+1)T_c} R(t)\psi^*(t)\\ &\times \exp(-j2\pi f_{uv}t)dt,\ \ u=1,2,...,q;\ v=1,2,...,p;\\ &n_s = 0,1,\ldots,N_s-1;\ n=0,1,...,N_e-1; \end{aligned} \quad (7)$$

where the factor of $1/(\sqrt{2PN_sN_e}T_c)$ is for normalization. When substituting (6) into the above equation, we obtain

$$y_{uv,n}[n_s] = \sum_{k=1}^{K}\frac{1}{\sqrt{N_sN_eU}}h_{uv}^{(k)}c_{n_sN_s+n}^{(k)}V_{ur_u}^{(k)}[n_s] + N_{uv,n}[n_s]$$

where $N_{uv,n}[n_s]$ represents the noise sample, which is Gaussian distributed with zero mean and a variance of $\sigma^2/2 = 1/(2\gamma_s)$ per dimension, where $\gamma_s = E_s/N_0$ denotes the signal-to-noise ratio (SNR) per symbol and $E_s = PT_s$ denotes the energy per symbol. Furthermore, let $B = \log_2 M$ be the number of bits per RRNS symbol. Then, we have $\gamma_s = B\gamma_b$ with γ_b representing the SNR per bit.

As shown in Fig. 3, after the processing in Sub-block I, the observations are sent to the sub-block II, where multiuser detection (MUD) is carried out to suppress the multiuser interference (MUI). Finally, after the inverse RNS transform (IRNST) by sub-block III, the information transmitted by a CRU is recovered. Let us consider the operations of sub-blocks II and III of Fig. 3 in the following sections.

IV. Detection in RRNS-Assisted MC/DS-CDMA DMA Systems

A. SMMSE-IC

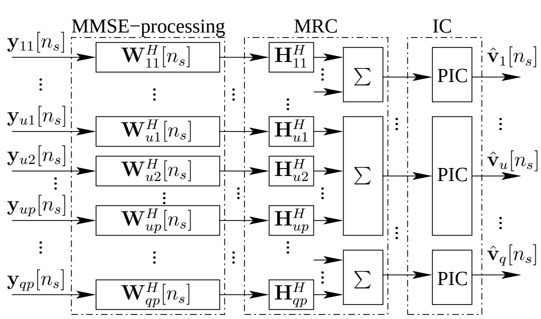

Fig. 4. Schematic block diagram illustrating the SMMSE-IC.

The SMMSE-IC MUD [6, 7] for the RRNS-based MC/DS-CDMA DMA system is shown in Fig. 4. In our SMMSE-IC, the linear MUDs are first implemented in the context of the U subcarriers separately and independently. Then, the subcarrier signals conveying the same residue are coherently combined in order to form a decision variable. Finally, parallel interference cancellation (PIC) is operated to reduce the remaining MUI and enhance the error performance.

Let us define

$$\begin{aligned} \boldsymbol{v}_u[n_s] \ &= \frac{1}{\sqrt{N_s}}\left[V_{ur_u}^{(1)}[n_s], V_{ur_u}^{(2)}[n_s], \cdots, V_{ur_u}^{(K)}[n_s]\right]^T,\\ \boldsymbol{y}_{uv}[n_s] &= [y_{uv,0}[n_s], y_{uv,1}[n_s], \cdots, y_{uv,N_e-1}[n_s]]^T,\\ \boldsymbol{n}_{uv}[n_s] &= [N_{uv,0}[n_s], N_{uv,1}[n_s], \cdots, N_{uv,N_e-1}[n_s]]^T,\\ \boldsymbol{c}_k[n_s] &= \frac{1}{\sqrt{N_e}}[c_{n_sN_s}^{(k)}, c_{n_sN_s+1}^{(k)}, \cdots, c_{n_sN_s+N_e-1}^{(k)}]^T,\\ \boldsymbol{C}[n_s] \ &= [\boldsymbol{c}_1[n_s], \boldsymbol{c}_2[n_s], \cdots, \boldsymbol{c}_K[n_s]],\\ \boldsymbol{H}_{uv} &= \frac{1}{\sqrt{U}}\text{diag}\{h_{uv}^{(1)}, h_{uv}^{(2)}, \cdots, h_{uv}^{(K)}\}. \end{aligned} \quad (8)$$

Then, it can be shown that we have

$$\boldsymbol{y}_{uv}[n_s] = \boldsymbol{C}[n_s]\boldsymbol{H}_{uv}\boldsymbol{v}_u[n_s] + \boldsymbol{n}_{uv}[n_s]. \quad (9)$$

In the context of the SMMSE-IC, when the MMSE-MUD is applied to each of the U subcarriers and after the maximal ratio combining (MRC) of the subcarrier signals conveying the same residue, the decision variable vector for $\boldsymbol{v}_u[n_s]$ can be expressed as

$$\hat{\boldsymbol{v}}_u[n_s] = \frac{1}{p}\sum_{v=1}^{p}\boldsymbol{W}_{uv}^H[n_s]\boldsymbol{y}_{uv}[n_s], \quad (10)$$

where $\boldsymbol{W}_{uv}[n_s]$ is the weight matrix for detection of the n_s fraction of the orthogonal code conveyed by the uvth subcarrier, $\boldsymbol{W}_{uv}[n_s]$ in MMSE sense can be expressed as

$$\boldsymbol{W}_{uv}[n_s] = \boldsymbol{R}_{\boldsymbol{y}_{uv}}^{-1}[n_s]\boldsymbol{R}_{\boldsymbol{y}_{uv}\boldsymbol{v}_u}[n_s], \quad (11)$$

where

$$\begin{aligned} \boldsymbol{R}_{\boldsymbol{y}_{uv}} &= E\left[\boldsymbol{y}_{uv}[n_s]\boldsymbol{y}_{uv}^H[n_s]\right]\\ &= \frac{1}{N_s}\boldsymbol{C}[n_s]\boldsymbol{H}_{uv}\boldsymbol{H}_{uv}^H\boldsymbol{C}^T[n_s] + \sigma^2\boldsymbol{I}_{N_e},\\ \boldsymbol{R}_{\boldsymbol{y}_{uv}\boldsymbol{v}_u} &= E\left[\boldsymbol{y}_{uv}[n_s]\boldsymbol{v}_u^H[n_s]\right] = \frac{1}{N_s}\boldsymbol{C}[n_s]\boldsymbol{H}_{uv}. \end{aligned} \quad (12)$$

Upon substituting (12) into (11), we obtain

$$\boldsymbol{W}_{uv}[n_s] = \left(\boldsymbol{C}[n_s]\boldsymbol{H}_{uv}\boldsymbol{H}_{uv}^H\boldsymbol{C}^T[n_s] + N_s\sigma^2\boldsymbol{I}_{N_e}\right)^{-1}\boldsymbol{C}[n_s]\boldsymbol{H}_{uv}.$$

In (12), we can see that the autocorrelation matrices $\boldsymbol{R}_{\boldsymbol{y}_{uv}}$ are channel dependent and time-variant. Hence, they may result in high-complexity, since they need to be updated once the channels change. Therefore, in our SMMSE-IC, we use the long-term average to find time-invariant autocorrelation matrices $\boldsymbol{R}_{\boldsymbol{y}_{uv}}$, which can be expressed as

$$\boldsymbol{R}_{\boldsymbol{y}_{uv}} = \frac{\Omega}{UN_s}\boldsymbol{C}[n_s]\boldsymbol{C}^T[n_s] + \sigma^2\boldsymbol{I}_{N_e} \quad (13)$$

where $\Omega = E[|h_{uv}^{(k)}|^2]$. Consequently, we can have

$$\boldsymbol{W}_{uv}[n_s] = U\left(\Omega\boldsymbol{C}[n_s]\boldsymbol{C}^T[n_s] + N_sU\sigma^2\boldsymbol{I}_{N_e}\right)^{-1}\boldsymbol{C}[n_s]\boldsymbol{H}_{uv}, \quad (14)$$

where the matrix to be inverted is time-invariant. Finally, when we use the *matrix inverse lemma* [7] in (14) and submit the result into (10), we can express the decision variable vector as

$$\begin{aligned} \hat{\boldsymbol{v}}_u[n_s] = &\frac{U}{p}\sum_{v=1}^{p}\boldsymbol{H}_{uv}^H(\Omega\boldsymbol{C}^T[n_s]\boldsymbol{C}[n_s] + N_sU\sigma^2\boldsymbol{I}_K)^{-1}\\ &\times\boldsymbol{C}^T[n_s]\boldsymbol{y}_{uv}[n_s]. \end{aligned} \quad (15)$$

As Fig. 4 shows, following the MMSE-MUD, a stage of PIC is carried out in order to enhance the error performance of the RRNS-based MC/DS-CDMA system. The operational principles of PIC can be found in [7], which are not repeated here.

Finally, as shown in Fig. 3, the decision variables $\{\hat{\boldsymbol{v}}_u[n_s]\}$ are input to sub-block III, where they are further processed in order to recover the information transmitted by the different CRUs, as considered below.

B. Orthogonal Demodulation and Inverse RNS Transform

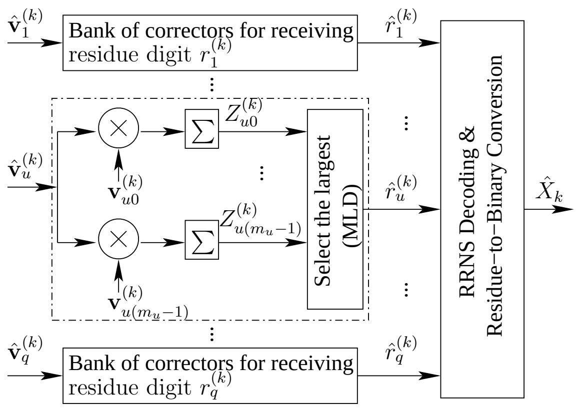

Fig. 5. Schematic block diagram for Sub-block III of Fig. 3, which implements orthogonal demodulation and IRNST.

The coherent demodulation and IRNST for the RRNS-based MC/DS-CDMA system is shown in Fig. 5. It first consists of q banks of correlators operated in the principles of MOSK demodulation [18]. Each bank is for recovering one of the q residues. Specifically, for the uth residue of the kth CRU, let, after the SMMSE-IC, the N_s estimates to the N_s fractions of the transmitted orthogonal code be expressed as

$$\hat{\boldsymbol{v}}_{r_u}^{(k)} = \left[\hat{V}_{ur_u}^{(k)}[0], \hat{V}_{ur_u}^{(k)}[1], \cdots, \hat{V}_{ur_u}^{(k)}[N_s-1]\right]^T,\ k = 1, \ldots, K \tag{16}$$

Then, with the aid of the set of m_u orthogonal codes for the uth residue, m_u decision variables can be formed as

$$Z_{um}^{(k)} = \Re\left\{\sum_{n_s=0}^{N_s-1} \hat{V}_{kr_u}^{(k)}[n_s] V_{um}[n_s]\right\},\ m = 0, 1, m_u - 1. \tag{17}$$

The maximum of the set $\left\{Z_{u0}^{(k)}, Z_{u1}^{(k)}, \cdots, Z_{u(m_u-1)}^{(k)}\right\}$ is selected and its index is then mapped to an integer $\hat{r}_u^{(k)} \in [0, m_u)$, which represents the estimate to $r_u^{(k)}$ of the uth residue transmitted by CRU k.

Finally, let the estimates to the residues transmitted by the kth CRU be collected to a vector

$$\hat{\boldsymbol{r}}^{(k)} = \left[\hat{r}_1^{(k)}, \hat{r}_2^{(k)}, \cdots, \hat{r}_q^{(k)}\right]^T,\ k = 1, 2, \ldots, K \tag{18}$$

Based on $\hat{\boldsymbol{r}}^{(k)}$, the estimate to the message X_k transmitted by the kth CRU can be recovered by the IRNST operation, for example, with the aid of the CRT [15]. Furthermore, when the RRNS is considered, the ratio statistic test (RST) assisted erasure-only decoding, error-correction-only decoding or the RST-assisted error-and-erasure decoding [16, 17] can be employed to decode $\hat{\boldsymbol{r}}^{(k)}$ and recover the message X_k transmitted.

V. Performance Results

In our simulation, we considered a RRNS with $s = 3$ information moduli and maximum $(q-s) = 4$ redundant moduli. The information moduli were $m_1 = 3, m_2 = 5, m_3 = 7$, while the redundant moduli were $m_4 = 8, m_5 = 11, m_6 = 13, m_7 = 17$. Correspondingly, we set $M = 64 \leq \Pi_{i=1}^3 m_i = 105$ and had $B = \log_2 M = 6$ bits binary per symbol. The number of subcarriers for operation of the CR system was assumed to be $U = \bar{C}$. The parameters for the PR system were $C = 56, K_P = 7, C_P = 8$. For simulation of the BER, we assumed that $s \leq \bar{C} \leq C$, implying that the CR system was always active. By contrast, for simulation of the throughput of the RRNS-based MC/DS-CDMA DMA systems, we assumed that $0 \leq \bar{C} \leq C$. For the handoff mode, we set $\Delta t = 1$ for generating new activate PUs. Furthermore, given the total number of subcarriers available for the CR systems, we considered two approaches, namely the *Redundancy First* and *Diversity First*, to allocate them. In the context of the *Redundancy First* approach, the subcarriers are first allocated to make the RRNS-based MC/DS-CDMA DMA system have the required redundancy, and then allocated for achieving the frequency diversity as high as possible. By contrast, for the *Diversity First* approach, achieving diversity has the priority over redundancy, when allocating new subcarriers.

Note that, in the following figures, the lower-bound represents the BER of the corresponding RRNS-based MC/DS-CDMA DMA system supporting single-user, when all the $C = 56$ subcarriers are operated by the CR system. The BER corresponding to the 'single-user' case represents the BER of the RRNS-based MC/DS-CDMA DMA system supporting single-user.

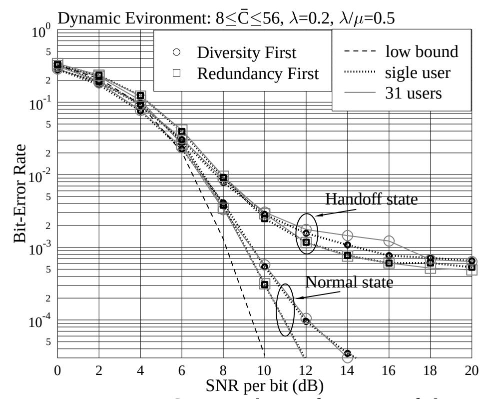

Fig. 6. BER versus average SNR per bit performance of the RRNS-based MC/DS-CDMA DMA systems, when communicating over iid Rayleigh fading channels.

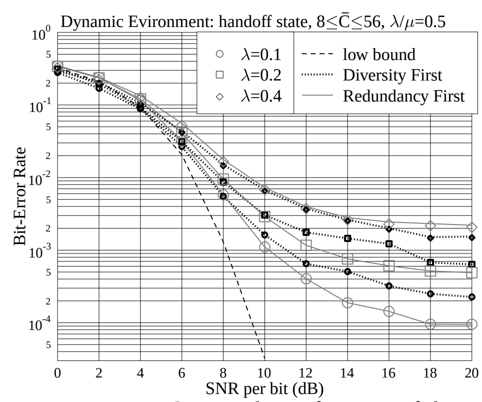

Fig. 7. BER versus average SNR per bit performance of the RRNS-based MC/DS-CDMA DMA systems in handoff mode, when communicating over iid Rayleigh fading channels.

Fig. 6 depicts the BER performance of the RRNS-based MC/DS-CDMA DMA systems communicating over iid Rayleigh fading channels. From the results of Fig. 6, we can observe that the full-load RRNS-based MC/DS-CDMA DMA system is capable of achieving near single-user BER performance. The BER performance operated under the ideal mode is much better than that operated under the handoff mode. Fig. 7 shows the BER performance of the RRNS-based MC/DS-CDMA DMA systems operated in the handoff mode and supporting $K = 31$ CRUs, when communicating over iid Rayleigh fading channels. The results of Fig. 7 demonstrate that the BER performance of the RRNS-based MC/DS-CDMA DMA system becomes worse, as the arrival rate of the PUs increases. This is because, as the arrival rate of the PUs increases, more signals in the RRNS-based MC/DS-CDMA DMA system will be lost due to the handoff mode. Furthermore, from Figs. 6 and 7, we observe that, when $\lambda = 0.1$ and 0.2, the BER performance of the RRNS-based MC/DS-CDMA DMA system with Redundancy First is better than that of the system with Diversity First. However, when $\lambda = 0.4$, the BER performance of the Redundancy First case is slightly worse than that of the Diversity First. Finally, when operated in the handoff mode, the BER curves appear in error floors.

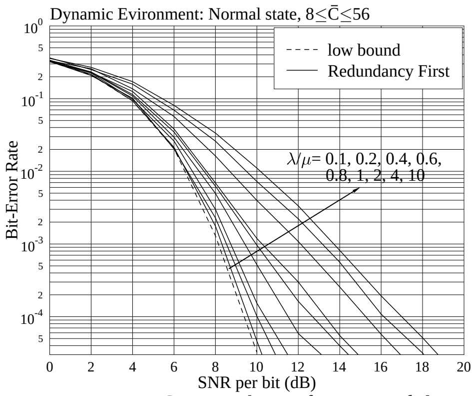

Fig. 8. BER versus average SNR per bit performance of the RRNS-based MC/DS-CDMA DMA systems operated in ideal mode, when communicating over iid Rayleigh fading channels.

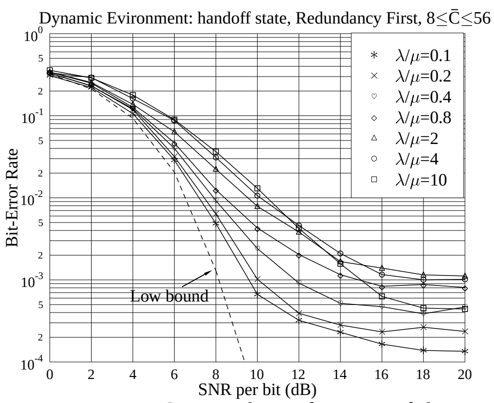

Fig. 9. BER versus average SNR per bit performance of the RRNS-based MC/DS-CDMA DMA systems operated in handoff mode, when communicating over iid Rayleigh fading channels.

Figs. 8 and 9 illustrate the BER performance of the RRNS-based MC/DS-CDMA DMA systems supporting $K = 31$ CRUs and operated in the ideal mode (Fig. 8) and handoff mode (Fig. 9). We can observe from these figures that, for both the ideal mode and handoff mode, the BER performance in general becomes worse, when the value of λ/μ increases. However, as seen in Fig. 9, when the value of $\lambda/\mu = 10$, which is very big, the corresponding BER performance might be better than that of the cases with a relative low λ/μ value, such as $\lambda/\mu = 2, 4$, when the SNR per bit is sufficiently high.

Finally, in Fig. 10, we illustrate the throughput achievable by the RRNS-based MC/DS-CDMA DMA systems operated in both the ideal mode and handoff mode. From this figure we can see that, with the aid of the RRNS-based MC/DS-CDMA DMA, a CR system is capable of attaining a substantial throughput of required quality, without affecting the communications of PR systems.

In conclusions, a RRNS-based MC/DS-CDMA DMA scheme has been proposed for dynamically accessing the frequency spectrums available for CR communications. Our analysis and performance results show that the proposed RRNS-based MC/DS-CDMA DMA constitutes a highly promising DMA scheme for application in CRs. It is capable of accommodating a substantial number of CRUs with a substantial throughput in dynamic spectrum environments, without degrading the QoS of the PR systems. Owing to the employment of RRNS, MC DS-CDMA, SMMSE-IC, etc., techniques, the proposed RRNS-based MC/DS-CDMA DMA scheme has the characteristics of, such as, low-complexity, high-flexibility, high-robustness to spectrum varying, etc. Furthermore, it facilitates seamless and smooth handoff between different frequency bands.

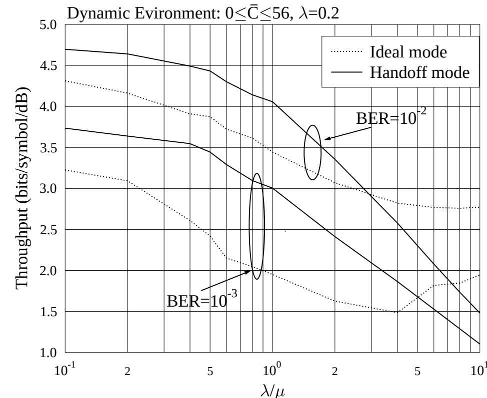

Fig. 10. Throughput achievable by the RRNS-based MC/DS-CDMA DMA systems.

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