Iterative Soft-Detection of Space-Time-Frequency Shift Keying

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Abstract—Inspired by the concept of Space-Time Shift Keying (STSK), the further evolved philosophy of Space-Time-Frequency Shift Keying (STFSK) was proposed for Multiple-Input-Multiple-Output (MIMO) wireless communications, where a beneficial diversity gain may be gleaned from three different domains, namely the space-, time- and frequency-domain. In this paper we proposed soft-detected STFSK in order to conceive its iterative decoding aided version combined with channel codes. Our results showed that the STFSK soft demodulator, which iteratively exchanges extrinsic information with channel codes, may decrease the required transmit power by approximately 3 dB at the exchanges.
Fig. 1. The transceiver block diagram of STFSK scheme.

Q pre-defined dispersion matrices $A_q \in \mathbb{C}^{M \times T}$ ($q = 1, 2, \ldots, Q$). The second stream is mapped to $s_l(i)$ symbols ($l = 1, 2, \ldots, L$) by a conventional modulation scheme, such as L-PSK or L-QAM, while the third one is mapped to the FSK symbol $r(i)$, which is represented by $r(i) = \cos(2\pi f_s t + \varphi_i)$, where $f_s$ ($r = 1, 2, \ldots, K$) is the frequency associated with the $i^{th}$ transmitted FSK symbol and $\varphi_i$ is the random phase during the $i^{th}$ symbol interval. Then, the resultant modulated streams are multiplied in order to create the space-time-frequency block $S(i) \in \mathbb{C}^{M \times T}$, which conveys a total of $\log_2(QLK)$ source bits, yielding

$$S(i) = r(i)s(i)A(i),$$

where each symbol $s(i)$ is a function of time during the period $T_s$ of each time slot.

As in [13], the space-time-frequency signal $\bar{Y}$ received at the destination may be presented as

$$\bar{Y}(i) = \{ \bar{Y}_1(i), \bar{Y}_2(i), \ldots, \bar{Y}_K(i) \},$$

where we have

$$\bar{Y}_k(i) = \sum_{j=0}^{M-1} T(i-j)\chi(K(i) + \bar{V}(i-j) : : \text{at transmit freq.},$$

$$\bar{V}(i) : : \text{otherwise}$$

with the variables formulated as

$$Y = \text{vec}(Y) \in \mathbb{C}^{NT \times 1},$$

$$H = I \otimes H \in \mathbb{C}^{NT \times MT},$$

$$\bar{V} = \text{vec}(\bar{V}) \in \mathbb{C}^{NT \times 1},$$

$$\chi = [\text{vec}(A_1) \ldots \text{vec}(A_Q)] \in \mathbb{C}^{MT \times Q},$$

where $I$ is the $(T \times T)$-element identity matrix and $\otimes$ is the Kronecker product. Furthermore, $K(i) \in \mathbb{C}^{Q \times 1}$ is the equivalent transmitted signal vector, being expressed as

$$K(i) = [0, \ldots, 0, r(i)s(i), 0, \ldots, 0]^T,$$

where $^T$ indicates the matrix transpose operation.

At the receiver, we employ an FSK demodulator consisting of a bank of $K$ parallel square-law detectors [14] in order to detect the activated frequencies of the SFSK symbols. Then the maximum likelihood (ML) detector [15] is employed to search for an appropriate pair of the $l^{th}$ ($l = 1, \ldots, L$) PSK/QAM symbol and the $q^{th}$ ($q = 1, \ldots, Q$) dispersion matrix. More particularly, the estimate $(\hat{q}, \hat{l})$ is given by minimizing the following metric

$$\begin{align*}
(q, l) &= \arg\min_{q,l} \| \bar{Y}(i) - \hat{H}(i)\chi(K_{q, l}(i)) \\
&\quad - \sum_{j=0}^{M-1} H(i-j)\chi(K_{q, l}(i-j))\|^2 \\
&= \arg\min_{q,l} \| \bar{Y}(i) - r(i)s_l(i)(\hat{H}(i)\chi) \\
&\quad - \sum_{j=0}^{M-1} H(i-j)\chi(K_{q, l}(i-j))\|^2,
\end{align*}$$

where $s_l(i)$ is the $l^{th}$ symbol in the $L$-point constellation at the $i^{th}$ block index and the signal vector $K_{q, l,k}$ ($1 \leq q \leq Q, 1 \leq l \leq L, 1 \leq k \leq K$) is presented by

$$K_{q, l,k}(i) = [0, \ldots, 0, r(i)s_l(i), 0, \ldots, 0]^T.$$  \hspace{1cm} (12)

Furthermore, $\sum_{l=1}^{L-1} \hat{H}(i-j)\chi(K_{q, l}(i-j))$ represents the delayed paths of the dispersive channel, which is omitted in flat-fading environments, while $(\hat{H}(i)\chi)_q$ denotes the $q^{th}$ column of the matrix $H(i)\chi$.

Additionally, in order to maintain a unity transmission power for a STSK symbol duration, each of the $Q$ dispersion matrices has to obey the power constraint of [11]

$$tr[A_q^HA_q] = T \quad (q = 1, \ldots, Q),$$

where $tr[\cdot]$ indicates the trace of a matrix, while the superscript $^\dagger$ denotes the complex conjugate transpose operation.

When $K = 1$, the STFSK becomes the STSK scheme [11] while it becomes the SFSK [12], when $L = 1$.

### III. SOFT STFSK DEMAPPER

Let us now detail the soft demapper designed for our STFSK schemes. Based on the equivalent system model of Eq. (3) derived for our STFSK scheme, the conditional probability $p(\bar{Y} | K_{q, l, k})$ is obtained as

$$p(\bar{Y} | K_{q, l, k}) = \frac{1}{(\pi N_0)^{NT}} \exp \left( -\frac{\| \bar{Y} - \bar{H} \bar{X} K_{q, l, k} \|^2}{N_0} \right).$$

Note that the equivalent received signals $\bar{Y}$ carry $B = \log_2(KLQ)$ channel-coded binary bits $b = [b_1, b_2, \ldots, b_B]$, where the resultant extrinsic LLR value of bit $b_k$ for $k = 1, \ldots, B$ may be expressed as [16]

$$L_a(b_k) = \ln \frac{\sum_{K_{q, l, k} \in Z^+_k} p(\bar{Y} | K_{q, l, k}) \cdot e^{\sum_{j \neq k} b_j L_a(b_j)}}{\sum_{K_{q, l, k} \in Z^+_k} p(\bar{Y} | K_{q, l, k}) \cdot e^{\sum_{j \neq k} b_j L_a(b_j)}}$$

where $Z^+_k$ and $Z^0_k$ represent the sub-space of the legitimate equivalent signals $Z$, satisfying $Z^+_k = \{ K_{q, l, k} \in Z : b_k = 1 \}$ and $Z^0_k = \{ K_{q, l, k} \in Z : b_k = 0 \}$, respectively, while $L_a(b_k)$ represents the a priori information expressed in terms of the LLRs of the corresponding bits. Furthermore, Eq. (15) is readily simplified by the max-log approximation [17], yielding:

$$L_a(b_k) = \max_{K_{q, l, k} \in Z^+_k} \left[ -\frac{\| \bar{Y} - \bar{H} \bar{X} K_{q, l, k} \|^2}{N_0} + \sum_{j \neq k} b_j L_a(b_j) \right]$$

$$- \max_{K_{q, l, k} \in Z^0_k} \left[ -\frac{\| \bar{Y} - \bar{H} \bar{X} K_{q, l, k} \|^2}{N_0} + \sum_{j \neq k} b_j L_a(b_j) \right].$$

### EXIT Chart Analysis

The EXIT charts, proposed by S. ten Brink [16], constitute useful tools designed for the analysis of iterative decoding schemes. This tool allows designers to graphically explore the characteristics of a demodulator/decoder based on the soft-input soft-output decisions, which are exchanged between the decoder components. The chart describes the dependence of the
extrinsic information on the a-priori information, which is typically quantified by the mutual information of the Log-Likelihood Ratios (LLR).

In EXIT chart analysis, the mutual information between the a-priori LLRs, $L_A$, or extrinsic LLRs, $L_E$, and the corresponding bits $S$ may be computed as \[ I(L_i; S) = \frac{1}{2} \sum_{x=-1}^{1} \int_{-\infty}^{\infty} p_L(x|s) \log \frac{p_L(x|s)}{p_L(x)} \, dx \quad (i = A, E) \] (17)
with \[ p_L(x) = \frac{1}{2} p_L(x|s = +1) + p_L(x|s = -1), \] (18)
where $p_L(x|s)$ is the probability of the a-priori or extrinsic information conditioned on $s = \pm 1$. According to the amount of mutual information of $L_A$ and $L_E$, the two EXIT functions (or EXIT curves), i.e. the inner and outer EXIT functions, may be drawn. The convergence characteristics of the iterative receiver may now be predicted by examining the relationship between the two curves of the EXIT chart.

The EXIT functions of the various STFSK schemes characterized in Table II are shown in Fig. 2, leading to the following observations:
- Increasing the number of frequencies, $K$, may increase the extrinsic information at the cost of extending the bandwidth used. This fact may be inferred by comparing Schemes 1,2,3 and 4, where we have $K = 16, 8, 4$ and 2, respectively.
- Increasing the number of dispersion matrices, $Q$, reduces the extrinsic information, when the same number of frequencies, $K$, is employed, which may be observed by comparing Schemes 3, 6 and 8.

IV. DETECTION COMPLEXITY

Let us quantify the computational complexity imposed by the ML hard-detection and the soft-detection of STFSK schemes, which is given by the number of real-valued multiplications and real-valued additions. We make the following assumptions:
- Each complex-valued addition is equivalent to two real-valued additions.
- Each complex-valued multiplication is equivalent to four real-valued multiplications plus two real-valued addition.
- Each square of absolute value calculation carried out for a complex number is equivalent to two real-valued multiplication and one real-valued addition.

A. Hard Demodulator

1) STFSK: First, we consider the complexity of the FSK detector. If the square-law FSK detector is employed as mentioned in Sec. II, then the number of multiplications, $C^M$, and the number of additions, $C^A$, may be obtained as
\[ C^M = 2K, \quad (19) \]
\[ C^A = K. \quad (20) \]

In order to evaluate the complexity of the STSK detector, Eq. (10) can be utilized, where the product of $\chi K$ involves the multiplication of two matrices, where one has a dimension of $(MT \times Q)$, which the other has a dimension of $(Q \times 1)$. However, the vector $K$ has only a single non-zero element. Therefore, instead of carrying out the matrix multiplication, the encoder may select the elements of $\chi$ at the specific positions corresponding to the non-zero element in $K$. This reduces the decoding complexity imposed. As a result, the number of multiplications and the number of additions required for executing the decision matrix are equal to
\[ C^M = (4MT + 4MTNT + 2NT)QL, \quad (21) \]
\[ C^A = [2MT + 2(2MT - 1)NT + 2JNT + 2NT - 1]QL. \quad (22) \]

Note that $J$ denotes the number of taps describing our fading model, where the decoder has to eliminate the interfering signals dispersed from the previous symbol intervals. For a flat-fading channel, we have $J = 1$.

Since $K$ host both the FSK symbol and the PSK/QAM symbol, the encoder requires additional multiplications and additions, which are quantified as
\[ C^M = 4QL, \quad (23) \]
\[ C^A = 2QL. \quad (24) \]

Each STFSK block consists of $\log_2(K \cdot Q \cdot L)$ bits. Hence, by employing Eqs. (19-24), the average number of multiplications, $\bar{C}^M$, and the average number of additions, $\bar{C}^A$, required for each bit of a STFSK scheme is given by
\[ \bar{C}^M = \frac{2K + (4MT + 4MTNT + 2NT)QL + 4QL}{\log_2(KQL)}, \quad (25) \]
\[ \bar{C}^A = \frac{K + [2MT + 2(2MT + J)NT - 1]QL + 2QL}{\log_2(KQL)}. \quad (26) \]

2) STSK: STSK is a special case of STFSK, where $K = 1$, no FSK detector is necessary. Furthermore, the vector $K$ now only contains PSK/QAM symbols. Hence, the average number of multiplications and additions for each bit of a STSK scheme may be reduced to
\[ \bar{C}^M = \frac{(4MT + 4MTNT + 2NT)QL}{\log_2(QL)}, \quad (27) \]
\[ \bar{C}^A = \frac{[2MT + 2(2MT + J)NT - 1]QL}{\log_2(QL)}. \quad (28) \]

3) SFSK: Similarly, SFSK is another special case of STFSK, where we have $L = 1$ and the vector $K$ now only hosts FSK symbols. Therefore, the average number of multiplications and additions for each bit of a SFSK scheme may be reduced to
\[ \bar{C}^M = \frac{2K + (4MT + 4MTNT + 2NT)Q}{\log_2(KQ)}, \quad (29) \]
\[ \bar{C}^A = \frac{K + [2MT + 2(2MT + J)NT - 1]Q}{\log_2(KQ)}. \quad (30) \]

4) LDC: LDC is a special case of STSK, where we have $L = 1$. Thus, the vector $K$ is redundant in the decision metric of Eq. (10). Hence, the average number of multiplications and additions for each bit of a LDC scheme may be simplified to
\[ \bar{C}^M = \frac{(4MTNT + 2NT)QL}{\log_2(QL)}, \quad (31) \]
\[ \bar{C}^A = \frac{[2(2MT + J)NT - 1]QL}{\log_2(QL)}. \quad (32) \]
B. Soft Demodulator

1) STFSK: In a soft STFSK demodulator each computation of Eq. (16) consists of two evaluations of Eq. (10) plus $\log_2(QLK)$ multiplications and $\log_2(QLK)$ additions rewired for adding the a-priori information. However, the searching space may be halved. Therefore, the average number of multiplications and additions becomes equivalent to

$$
\bar{C}^+ = \frac{[4MT + 4MTNT + 2NT + 4 + \log_2(QLK)]QLK}{\log_2(QLK)}
$$

2) STSK: Similar to the hard-decision STSK demodulator, the soft-decision STSK demodulator only has to process PSK/QAM symbols. Hence, the average number of multiplications and additions for each bit of a STSK scheme may be reduced to

$$
\bar{C}^+ = \frac{[2MT + 2(2MT + J)NT + \log_2(QLK)]}{\log_2(QLK)}.
$$

3) SFSK: Similarly, the average number of multiplications and additions required for each bit of a soft-decision SFSK scheme, where $L = 1$, may be reduced to

$$
\bar{C}^+ = \frac{[4MT + 4MTNT + 2NT + \log_2(2QLK)]QL}{\log_2(2QL)}.
$$

4) LDC: Finally, the average number of multiplications and additions associated with each bit of a soft-decision LDC scheme becomes

$$
\bar{C}^+ = \frac{[2(2MT + J)NT + \log_2(2QL)]}{\log_2(2QL)}.
$$

The complexity results of the STFSK, STSK and SFSK schemes summarized in Table I are shown in Fig. 3, where the continuous and the dashed lines portray the detection complexity of the hard- and soft-demapper, respectively. As seen in the figure, the complexity of the hard-decision STFSK scheme is comparable to that of the SFSK schemes in terms of the number of multiplications and additions. By contrast, the hard-decision STSK schemes have doubled the complexity of the hard-decision STFSK and SFSK arrangements, when considering the same normalized throughput, while the number of multiplications and additions required by the hard-decision LDC demappers is 50% higher than those of the hard-decision STFSK schemes. Observe furthermore from Fig. 3 that the complexity of the soft-decision STSK, SFSK and STFSK demappers is comparable, while they are slightly more complex than the soft-decision LDC demapper. Furthermore, we found that in case of the SFSK and STFSK schemes the soft demappers doubled the complexity of the hard demappers, whilst in case of the STSK and LDC schemes the complexity of the soft- and hard-demappers remain comparable.

Furthermore, the complexity of the hard- and the soft-decision STFSK schemes, where the block size of 6 bits per symbol is employed, is summarized in Table II. The following observations may be made:

- For the hard-decision, the complexity of decoder is reduced, when the number of frequencies, $K$, increases.
- For a given value of $K$ and for a given value of the product $QL$, all possible combinations of $Q$ and $L$ exhibit the same decoding complexity when hard-decision is employed.
- In case of hard-decision, the complexity of a STFSK scheme increases upon increasing the normalized throughput.
- The complexity of a soft-decision STFSK demodulator depends only on the product of $(Q \times L \times K)$, rather than on each individual component $Q$, $L$ and $K$.

V. PERFORMANCE RESULTS

In this section we consider the achievable iterative detection aided performance, when the soft STFSK demapper iteratively exchanges extrinsic information with the Recursive-Systematic-Convolutional (RSC) decoder. The RSC codec employs a half-rate constraint-length-3 code having the generator polynomial of $1 + 2z^{-1}$. The STFSK scheme is equipped with four transmit and a receive antenna, transmitting two symbols in four time slots ($(M/N/T/Q = 1/1/4/2)$ and assisted by QPSK and BFSK. Observe from Fig. 4 that the achievable performance is significantly improved, when the number of iterations between the soft demapper and the RSC decoder was increased. Quantitatively, an $E_b/N_0$ improvement of 3 dB was achieved at the BER of $10^{-4}$, when the number of iterations was increased from one to five.
The results of Fig. 4 are further supplemented by the EXIT charts. As seen in Fig. 5, at the $E_b/N_0$ value of -3 dB the intersection of the two EXIT curves is at the point of (0.75, 0.55) and up to this point a gradually narrowing tunnel exists between the two EXIT curves. Hence, at this value of $E_b/N_0$ only an insignificant improvement is obtained upon increasing the number of decoding iterations between the STFSK demodulator and the RSC decoder. By contrast, the tunnel is more widely open in case of $E_b/N_0 = -1$ dB in Fig. 5 and the extrinsic information gleaned increases significantly, when the number of iterations increases from one to three. This explains why the attainable BER performance improves rapidly for the first three iterations, while the BER improvement significantly reduces for the fourth and fifth iteration.

Fig. 6 characterizes the performance of all the ten STFSK schemes of Table II. The performance confirms the EXIT-chart based prediction of Fig. 2. More particularly, the results may be divided into four groups. The first group, including Schemes \{1,2,3\}, exhibit the best performance, achieving a BER of $10^{-4}$ at an $E_b/N_0$ value around 0 dB. The EXIT functions of the demappers in these three schemes are shown at top of Fig. 2, where a significant advantage is shown in comparison to the remaining schemes. The second group consisting Schemes \{4,5,6,7\} achieved the same BER, namely $10^{-3}$, at the $E_b/N_0$ of about 2.5 dB. The EXIT functions of these schemes remained significantly below those of the first group. The third group contained Scheme 8 and Scheme 9, whose EXIT functions remained further below those of Schemes \{4,5,6,7\} in Fig. 2. They achieved the BER of $10^{-4}$ at the $E_b/N_0$ value around 5.5 dB. Finally, Scheme 10, which has the demapper EXIT function at the bottom of Fig. 2, acquired the same BER at the $E_b/N_0$ value around 9.5 dB.

![EXIT Charts of the RSC coded STFSK using Iterative Decoding at $E_b/N_0 = -3$ dB (left) and -1 dB (right).](image1)

![The performance of the soft-decision RSC-coded STFSK schemes of Table II, where the EXIT functions of STFSK demodulators at $E_b/N_0 = 0$ dB is shown in Fig. 2.](image2)

**VI. CONCLUSIONS**

In this paper we proposed the soft-modulation aided iterative decoding of STFSK. Our results demonstrated that upon carrying out iterative information exchange between the STFSK soft-demodulator and the RSC channel decoder, the system achieved a 3 dB power gain at the $E_b/N_0$ value of $10^{-5}$ compared to using its counterpart dispensing with iterative decoding. Also, we quantified the detection complexity of both the hard- and soft-detection STFSK demodulators.

**REFERENCES**


