Reduced-complexity transmit-beamforming codebook search algorithm

Y.J. Kim, S.H. Won, N.Y. Park and L. Hanzo

A two-stage reduced-complexity index search algorithm is proposed for finding the best vector in the codebook of quantised equal gain transmission based multiple-input multiple-output arrangements. When the number of transmit antennas is more than three, the normalised complexity is halved while maintaining the same symbol error rate as the benchmark.

Introduction: The de facto fourth generation (4G) wireless systems have encompassed codebook (CB) based closed-loop multiple-input multiple-output (MIMOs) [1]. Quantised equal gain transmission (QEGT) imposes less stringent transmit amplifier linearity requirements than other MIMOs. Similar to other CB based MIMO schemes, QEGT requires a substantially increased CB size, as the number of transmit antennas increases. Hence, employing an efficient CB index search algorithm becomes essential. A single-user MIMO-OFDM system relying on transmit beamforming (BF) and using M_t transmit as well as M_r receive antennas is considered. The system model and QEGT CB design are the same as in [2]. This allows the direct contrasting of our efficient search to that in [2]. The matrix of channel impulse responses (CIR) $H \in \mathbb{C}^{M_r \times M_t}$ is assumed to have IID complex-valued Gaussian elements having a unit variance. The CIR is assumed to be perfectly estimated at the downlink receiver. Given the transmitted symbol of rank one $s \in \mathbb{C}$, the received symbol $x \in \mathbb{C}$ is described by

$$x = z^H H w s + z^H n \tag{1}$$

where $z^H \in \mathbb{C}^{1 \times M_r}$ is the combining vector, $\boldsymbol{w} \in \mathbb{C}^{M_r \times 1}$ is the BF vector having $\|\boldsymbol{w}\| = 1$ and $\boldsymbol{n} \in \mathbb{C}^{M_r \times 1}$ represents the complex-valued AWGN having a variance of N_o . When the number of feedback bits is L, the CB size is denoted by $N = 2^L$.

Two-stage index search algorithm: To obtain a fast index search for the QEGT CB, we propose to group the BF vectors in the CB into a given number of categories based on their chordal distance expressed by

$$d(\mathbf{w}_j, \mathbf{w}_k) = \sqrt{1 - |\mathbf{w}_j^H \mathbf{w}_k|^2}$$
(2)

where w_i and w_k are arbitrary BF vectors in the CB. The BF vectors can be grouped by utilising a Euclidean space. This Letter applies Lloyd's algorithm to the BF vectors of the CB to generate their optimal grouping. To illustrate the grouping method, we consider the case of $(M_t, N) = (2, N)$ 64). The first element of the BF vectors corresponding to the first transmit antenna can be forced to be real-valued owing to the rotation-invariant property of BF vectors [3]. Then, the second element of the BF vectors corresponding to the second transmit antenna can be plotted on the complex-valued plane, as seen in Fig. 1, which illustrates eight groups of BF vectors as the result of Lloyd's clustering algorithm. In the notation of $w_{i,j}$ seen in Fig. 1, *i* represents the *i*th element of the BF vector corresponding to the *i*th transmit antenna and *j* denotes the index of the BF vector in the CB. Fig. 1 also magnifies a group of eight BF vectors at the right, where the cluster centroid is indicated by a square mark, while the others are indicated by triangles. In the same way, we can group the BF vectors in the CB, when the number of transmit antennas is higher than or equal to three. Assuming that the QEGT CB elements are uniformly quantised, they can be arranged into P groups, each having Q BF vectors. Therefore the CB size is N = PQ.



Fig. 1 Example of CB index grouping when $(M_t, N) = (2, 64)$

Now we are ready to apply our two-stage index search algorithm. In the first stage, the best of the centroids is selected, which maximises the channel gain of

$$c_{opt} = \arg \max_{c_r \in C_{out}} \|\boldsymbol{H}\boldsymbol{c}_p\| \tag{3}$$

where c_p , p = 1, 2, ..., P are the centroids, while C_{set} is the set of $\{c_1, c_2, ..., c_p\}$. At the second stage, finally, the optimal BF vector is determined within the group of the selected centroid, which maximises the following equation:

$$\boldsymbol{m}_{opt} = \arg \max_{\boldsymbol{m} \in \mathcal{M}} \|\boldsymbol{H}\boldsymbol{m}_q\| \tag{4}$$

where m_q , q = 1, 2, ..., Q, represents the BF vectors of the chosen group and M_{set} is the set of $\{m_1, m_2, ..., m_Q\}$. As a benefit, the entire search complexity is significantly reduced. The problem now becomes how to determine the number of groups P and the number of elements Q in each group, when the CB size, N, is fixed. Based on our investigations not included here, the grouping rule can be applied that the number of groups P should be the same as, or the nearest integer to, the value of N/P when the BF vectors in the CB are distributed almost evenly to each group.

Simulation results: The CB search complexity and the average SER will be characterised for both the conventional and the proposed search. BF arrangements having three-to-five transmit antennas and two receive antennas are considered. QPSK modulation is assumed for the MIMO-OFDM LTE system, the carrier frequency of which is 2 GHz, system bandwidth is 5 MHz, sampling rate is 7.68 MHz, cyclic prefix size is 30, FFT size is 512, and the number of subcarriers occupied is set to 300. The channel model is assumed to be the ITU-R Pedestrian-B model having six paths [4]. It is also assumed that the channel estimation and symbol synchronisation are ideal. There is no spatial correlation amongst the antennas. Table 1 quantifies how much the CB search complexity is reduced in comparison to the conventional search. When the number of groups P is fixed, the number of vectors in each group may vary around the nearest integer of N/P. The number of indices searched, namely $P + \bar{Q}$ (where \bar{Q} is the mean of Q) becomes the lowest among the other grouping configurations, when our grouping rule defined in the preceding Section is applied. The CB search complexity was also evaluated in real time based on a Texas Instruments TMS6713 DSP chip. The normalised complexity of the proposed technique is given by dividing the search complexity of the proposed method by the conventional one, as summarised in the last column of Table 1. As the CB size N increases, the normalised complex gain of the proposed scheme becomes more substantial.

Table 1: Comparisons among number of average candidate weight
vectors under two systems considered when assuming
 $M_r = 1$

M _t	$N = 2^L$	Р	Q	Number of indices searched/number of total indices $\{(P + \bar{Q})/N\}$	Normalised complexity of proposed search for TI DSP
3	16	2	8	10/16 = 0.625	0.628
		4	4	8/16 = 0.500	0.503
		8	2	10/16 = 0.625	0.628
4	16	2	8	10/16 = 0.625	0.628
		4	4	8/16 = 0.500	0.503
		8	2	10/16 = 0.625	0.628
	64	4	16	20/64 = 0.313	0.313
		8	8	16/64 = 0.250	0.251
		16	4	20/64 = 0.313	0.313
5	16	2	8	10/16 = 0.625	0.628
		4	4	8/16 = 0.500	0.503
		8	2	10/16 = 0.625	0.628
	64	4	16	20/64 = 0.313	0.313
		8	8	16/64 = 0.250	0.251
		16	4	20/64 = 0.313	0.313

The SER performance of both the proposed schemes as well as of the conventional one is shown in Fig. 2. The number of transmit antennas is three to five and the number of receive antennas is set to two. When the value of *N* is fixed to 16 or 64, there are various grouping configurations. As the number of groups *P* increases, the SER is improved at the cost of an increased CB search. When the two-stage grouping rule is applied, i.e. we have (N, P) = (16, 4) or (64, 8), the performance degradation

becomes less than 0.2 dB, while maintaining the lowest search complexity. Hence, an attractive trade-off was struck between the CB search complexity and SER performance.



Fig. 2 SER performance of various grouping configurations when $(M_t, N) = (3, 16), (4, 16)$ and (5, 64)

Conclusion: A reduced-complexity closed-loop downlink BF scheme has been investigated, which relies on a grouping strategy and imposes only a slight SNR loss, when operating at half the complexity

of the conventional search and relying on three-to-five transmit antennas.

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