Entropy production in an energy balance Daisyworld model

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Abstract

Daisyworld is a simple mathematical model of a planetary system that exhibits self-regulation due to the nature of feedback between life and its environment. A two-box Daisyworld is developed that shares a number of features with energy balance climate models. Such climate models have been used to explore the hypothesis that non-equilibrium, dissipative systems such as planetary atmospheres are in a state of maximum entropy production with respect to the latitudinal flux of heat. When values for heat diffusion in the two-box Daisyworld are selected in order to maximize this rate of entropy production, the viability range of the daisies is maximized. Consequently planetary temperature is regulated over the widest possible range of solar forcing.

Introduction

Although not intended as such, Daisyworld can be regarded as an example of an artificial life model. It develops a conceptual framework that explores life and environment as they could be on a planet similar to the Earth. Daisyworld was originally proposed as a mathematical proof of concept for James Lovelock's Gaia Hypothesis: that the Earth and its biota are a self-regulating system that will reduce the effects of otherwise deleterious perturbations (Lovelock, 1995). Since its creation some twenty five years ago (Lovelock, 1983) Daisyworld has been extended and modified in order to address a number of different research questions. See (Wood et al., 2008) for a recent review. The consensus is that it has matured to the extent that it can be considered as a model in its own right rather than as an ancillary component of Gaia theory. A central message from Daisyworld is that when life affects the environment as well as being affected by the environment, self-regulation may emerge.

Modelling of the Earth's climate can take place at various levels of complexity, from zero-dimensional models such as the original Daisyworld to three-dimensional general circulatory models that are very computationally expensive and defy exact analysis. A relatively recent development of a two-box Daisyworld (Harvey, 2004) is conceptually similar to one-dimensional energy balance climate models that have been used to explore the *Maximum Entropy Production Principle* (MEPP) (Lorenz et al., 2001). It is proposed that certain energetically open and driven systems, such as planetary atmospheres, are in states that maximise the rate of entropy production. It is straightforward to introduce thermodynamic constraints into the two-box Daisyworld such that maximum entropy production can be achieved. If the Earth's atmosphere is in a non-equilibrium state that maximises entropy production then it seems reasonable to ask what would the effects on Daisyworld regulation and stability be if it were in a MEPP state?

Daisyworld

Daisyworld is an imaginary grey planet orbiting a star similar to the Sun. It is home to two daisy types: black and white. Albedo is a measure of the reflectivity of an object. In Daisyworld the black daisies have a low albedo (0.25), the grey bare earth intermediate albedo (0.5) and the white daisies a high albedo (0.75). The white daisies, having the highest albedo in the model, reflect more of the short wave energy from the star and so have a lower temperature than either the grey planet or the black daisies. The same applies to the black daisies but in reverse. The black and white daisies share a viability range of temperature. They are only able to grow when the local ambient temperature is within 5-40 degrees Celsius. Within this range growth rates of the daisies vary, with optimum growth being achieved when the temperature is 22.5C. Simulations begin when the star is dim and the temperature of the planet is below 5C. As the star increases in brightness the planetary temperature reaches 5C and black daisies begin to grow. The increase in the number of black daisies is further increased via a feedback loop: more black daisies leads to a lower planetary albedo and so more energy is absorbed that warms the ground that leads to an increase of the growth rate. This feedback loop is regulated by the parabolic growth rate of the daisies. As the temperature increases past 22.5C, the daisy growth rate decreases. At steady state the ambient temperature, growth rate and death rate are at equilibrium. As the energy output of the star continues to increase, coverage in black daisies decreases and white daisies begin to grow. This initiates a feedback loop that is the inverse of the effect of the black daisies and so decreases their temperature. Again, this feedback loop is regulated by the parabolic growth rate of the daisies. Increasing the amount of energy from the star results in a progressive increase in white daisies (and decrease in black daisies) until the maximum coverage of white daisies is reached. Any further increase in energy takes the ambient temperature past the point where growth rates balance death rates and so the coverage of white daisies decreases. This leads to a rapid collapse of white daisies similar in nature to the population explosion of the black daisies. The differential coverage of white and black daisies results in a system that effectively regulates ambient planetary temperature to within the viability range. Whereas the temperature of a bare lifeless planet would increase in an approximately linear fashion with increases in luminosity, when black and white daisies are present, ambient temperature remains within the viability range over a wide range of solar forcing.

The Maximum Entropy Production Principle

The Earth's atmosphere can be viewed as a heat engine: motions are driven by the flow of heat from the hot equator to the cold polar regions. The amount of work that can be done by this flow of heat depends on the temperatures of the reservoirs: the greater the drop in temperature for a given amount of heat flow, the greater the thermodynamic efficiency of the system and so a greater amount of work output and entropy produced. Such heat processes do not produce entropy at an arbitrary rate. Two extreme principles have been formulated to describe their characteristic behaviour. For systems near thermodynamic equilibrium with fixed boundary conditions, (Prigogine, 1962) formulated the principle of minimum entropy production (MinEP) stating that the steady state of the process is associated with a MinEP state. However, many processes do not have fixed boundary conditions and are far from equilibrium. For those processes it has been proposed that they maintain steady states in which the production of entropy is maximized if there are sufficient degrees of freedom associated with the processes - the Maximum Entropy Production Principle (MEPP). It may not be necessary to understand the detailed internal dynamics of such systems in order to make accurate models and predictions. An impressive demonstration of this was with (Paltridge, 1975) who successfully reproduced the Earth's latitudinal temperature profile in a simple energy balance climate model by assuming that the rate of diffusion was that which maximized the rate of entropy production via latitudinal heat transport. However, in the absence of any proposed mechanism or other system that maximized entropy production in this manner, Paltridge's results failed to gain significant traction within the scientific community. This situation has changed somewhat as MEP has been observed in the atmospheres of Mars and Titan (Lorenz et al., 2001) and there have been attempts (Dewar, 2003) and (Martyushev and Seleznev, 2006) to place the MEPP on firm information theoretic foundations. See (Ozawa et al., 2003) for a review.

Two-Box Daisyworld

The two-box Daisyworld was motivated by a desire to produce the simplest implementation of the Daisyworld control system. Whilst the original Daisyworld was itself a simple model of complex real-world phenomena, analysis was not trivial. The two-box model simplifies Daisyworld further by removing space competition dynamics by having each daisy occupy separate 'beds' or boxes and dispensing with a separate birth and death rate and finding steady state daisy coverage with a single linear function. Each grey box is seeded with either black or white daisies. When the seeds are dormant, the boxes have the same temperature. As the seeds germinate and daisies begin to cover each box, a temperature gradient is established. The two-box Daisyworld is represented schematically in Figure 1.

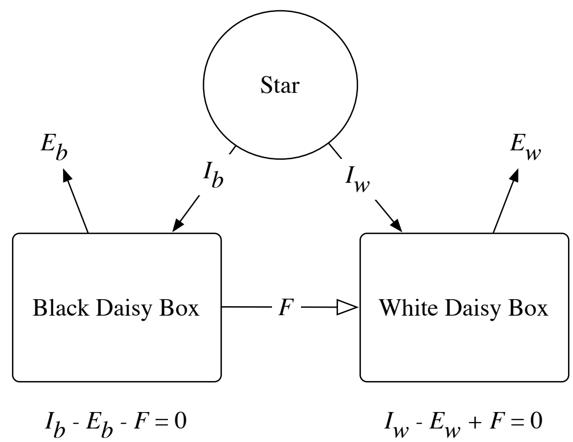

Figure 1: Schematic of Two-Box Daisyworld

The two-box Daisyworld which is conceptually similar to a two-box energy balance climate model (North et al., 1981). Incoming energy in the form of short-wave energy I_i warms each box and is radiated back into space as long-wave emissions E_i with an amount of heat flux F proportional to the temperature gradient between the boxes. Only black daisies are seeded in the black daisy box and only white daisies in the white daisy box. White daisies, being lighter than the black daisies will reflect more energy from the star. Black daisies, being darker, absorb more energy. Hence the black daisies are warmer and the white daisies are cooler than the grey bare earth. The two boxes are coupled via a heat conducting medium which when there is non-zero coverage of daisies allows heat to flow from the black to white daisy box.

Those with a familiarity of climate modelling will immediately see the similarity to a zonal energy balance model. The temperature of each daisy box is found with

$$T_b^4\sigma = I(1 - A_b) - F, \quad (1)$$

$$T_w^4\sigma = I(1 - A_w) + F, \quad (2)$$

where A_i is the albedo of the boxes, σ is the Stefan-Boltzmann constant having a value of 5.67×10^{-8} J $\mathrm{s}^{-1}\mathrm{m}^{-2}\mathrm{K}^{-4}$, I is insolation, the amount of energy received on the surface of the planet from the star in units of $\mathrm{W}^{-1}\mathrm{m}^{-2}$. As in previous studies on Daisyworld, the amount of insolation will be parameterized by a luminosity variable, L. L is a non-dimensional index of stellar brightness. $I = L \times 1000\mathrm{W}^{-1}\mathrm{m}^{-2}$ where $L \in [0, 2]$. We can think of luminosity as a 'dimmer switch' that modulates the brightness of the star and thus the amount of energy received on the surface of the planet. The temperature of the planet is the mean of the two box temperatures, $T_p = 0.5(T_b - T_w)$. The heat flux F between the two boxes is found with

$$F = D(T_b - T_w). \quad (3)$$

The flux of heat is proportional to the temperature difference between the two boxes and a diffusion parameter D normally measured in units of $\mathrm{W}^{-1}\mathrm{m}^{-2}\mathrm{K}^{-1}$. As we will be investigating the comparative effects of entropy production, we will set the surface area between the two boxes to unity, and heat flux and entropy will be measured in non-dimensional or arbitrary units. The diffusion term D will be scaled from 0, which produces no heat flux, to 1 which produces maximal heat flux with the boxes being isothermal. The albedo of each daisy box is found in a similar fashion to the original Daisyworld

$$A_b = AG(1 - \alpha_b) + AB\alpha_b, \quad (4)$$

$$A_w = AG(1 - \alpha_w) + AW\alpha_w. \quad (5)$$

AG, AB, AW are parameters that determine the fixed albedo of bare ground, black daisies and white daisies, and these are set to 0.5, 0.25 and 0.75 respectively. α_i is the proportional daisy coverage from 0 to 1 which is found with equation 6. There is an optimal temperature T_{opt} that produces 100% coverage, with coverage decreasing linearly to zero as temperature decreases or increases away from this value.

$$\alpha_i = Max[1 - 2(|T_{opt} - T_i|)/R, 0], \quad (6)$$

where $T_{opt} = 22.5$ and R is the range of temperature over which non-zero daisy coverage is achieved. It is fixed at 35 thus daisies grow at 5C, have maximal coverage at 22.5C and are back to zero coverage at 40C. It is assumed, as with other studies on Daisyworld, that the rate of change of daisy coverage is sufficiently faster than that of luminosity so as to allow the above equations to be numerically integrated to steady state whilst luminosity is fixed. The implementation details can be found in previous studies (Harvey, 2004), (Dyke and Harvey, 2005) and (Dyke and Harvey, 2006). Essentially, for any fixed luminosity and fixed rate of heat diffusion, the daisy coverage is initialised to 1 (maximum coverage), albedo, temperature, heat flux and then temperature plus heat flux and new coverage values are computed. The current daisy coverage is then adjusted a small amount towards this new coverage. When the percentage change of daisy coverage is 0.001 per iteration of this loop, 200,000 iterations produce changes in coverage, albedo, heat flux and temperature that are no greater than 10^{-22}. Figure 2 shows numerically computed steady state values for daisy coverage and planetary temperature when the diffusion parameter is fixed at 0 and 0.5.

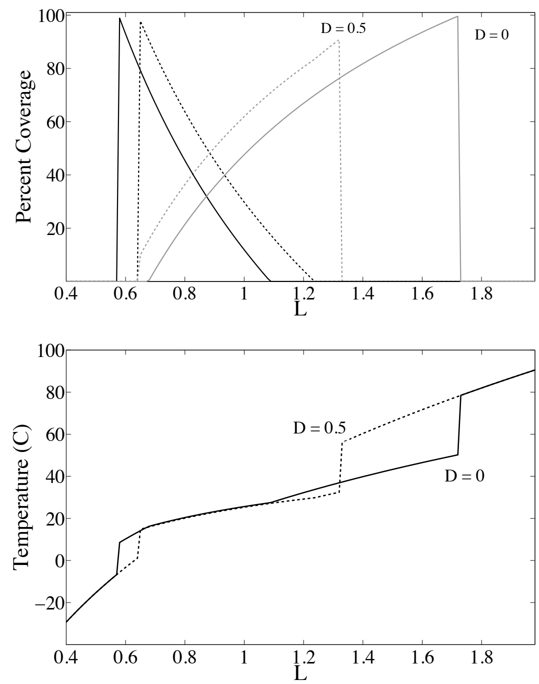

Figure 2: Two-Box Daisyworld $D = 0, D = 0.5$

Numerically computed results when $D = 0$ (solid lines) and $D = 0.5$ (dashed lines). The top plot shows daisy coverage over luminosity. When $D = 0$ the black daisies (black line) grow at lower luminosity and white daisies (grey line) grow at higher luminosity than when diffusion is set at an 'intermediate' value of 0.5. When $D = 0.5$ white daisies grow at lower and black daisies grow at higher luminosities. The bottom plot shows planetary temperature for the same results. The rate of change of planetary temperature when both daisy types are present is less than when only one daisy type is present.

Initializing daisy coverage to 1 for any luminosity removes hysteresis from the system. Hysteresis is recovered if, as in the original Daisyworld model, daisy coverage is allowed to 'evolve' by initializing new luminosity coverage with previous steady state luminosity coverage whilst making changes in luminosity very small.

Entropy production in Daisyworld

Previous studies have investigated entropy production in Daisyworld: (Pujol, 2002) within the original version, (Tonaizzo et al., 2004) with a version that allowed an arbitrary number of daisy types and a two dimensional cellular automata version in (Ackland, 2004).

The first two studies faced certain limitations due to the particular implementation of heat flux in the respective models. However they both concluded that when Daisyworld maximizes the rate of entropy production, there is an increase in the range of luminosity over which daisies grow. Rather than assuming Daisyworld is in a maximizing entropy production state, (Ackland, 2004) tests the contrary hypotheses that Daisyworld self-organises to either maximize entropy production or maximize the total amount of life for any given luminosity ('MaxLife'). It is found that a maximising life not maximising entropy principle is selected. However, due to the modelling assumptions of cellular automata Daisyworlds, computing the rate of entropy production via heat flux is not possible. Consequently it is the rate of biodiversity entropy that (Ackland, 2004) measures. See (Wood et al., 2008) for a more detailed discussion of these studies.

The two box formulation of Daisyworld is similar to energy balance climate models in which there is a flux of heat from warm to cool regions (North et al., 1981). Whereas in these models such heat gradients are produced via different amounts of energy received on the surface of the planet due to different latitudes, in the two-box Daisyworld model the difference in temperature is due to different albedo. However, heat will flow from warm to cool regions irrespective of how such a situation was produced, and the method of calculating entropy production budgets in energy balance box models such as (Lorenz et al., 2001) can be naturally employed in order to calculate entropy production in the two-box model. The rate of entropy production is a function of heat flux over the difference in temperature between the hot and cold boxes.

$$\frac{dS}{dt} = \frac{F}{T_w} - \frac{F}{T_b}. \tag{7}$$

The greatest rate of entropy production will be achieved with the greatest temperature difference and the greatest heat flux between the two daisy boxes. Attempting to increase entropy production by increasing heat flux via increasing diffusion may lead to a decrease in the temperature gradient and thus a decrease in thermal efficiency of the 'heat engine' and so a decrease in the rate of entropy production. Consequently maximizing entropy production is a balancing act with the value of D required to produce maximum rates of entropy production varying with the driving of the system. Figure 3 shows the unimodal function of entropy production over D with daisies present and fixed luminosity.

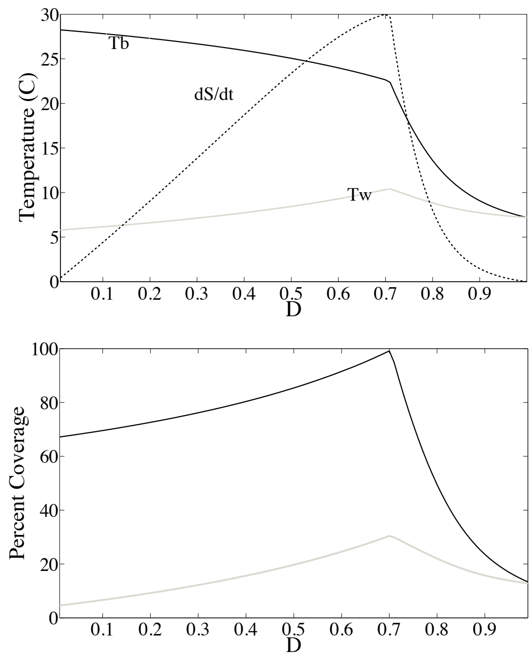

Figure 3: Two Box Daisyworld MEP results

The rate of entropy production and daisy box temperatures are shown in the top plot and black daisy coverage (solid black line) and white daisy coverage (solid grey line) is shown in the bottom plot for various values of heat diffusion where luminosity is fixed at 0.7. The greatest rate of entropy production is produced when $D \approx 0.7$. Increasing diffusion leads to a decrease in the temperature difference between the boxes and entropy production until the daisy boxes are isothermal with no entropy production and steady state coverage being that of a 'grey' daisy type with albedo of 0.5. The coverage of daisies undergoes a sharp decline as increasing heat flux drives the black and white box temperatures away from the optimal temperature of 22.5C. The value of D required to produce maximum entropy production will vary with L.

Maximising entropy production

It has been postulated that certain dissipative systems maximize the rate of entropy production. *How* these systems reach such non-equilibrium stable states is a different topic of enquiry. Here I will assume that the imaginary two-box planetary system, like the Earth, possesses an atmosphere with sufficient degrees of freedom to produce heat fluxes that lead to the maximisation of entropy production. In doing so we can construct a thought experiment analogous to Maxwell's Demon which is used to explore certain aspects of the second law of thermodynamics. Rather than monitoring the velocity of air molecules, our demon monitors the flux of heat and temperature of the daisy boxes. It has a dial at its disposal that modulates the diffusivity of the atmosphere. For any luminosity value, the demon can alter the diffusion and in doing so find the rate of diffusion that maximizes equation 7. The behaviour of the demon can be implemented in a search algorithm that modulates D for any fixed L in order to find the value of D that produces the highest rate of entropy production.

Results

Figure 4 shows the effects of the maximising demon on the two-box Daisyworld. It can be seen that the 'any-daisy' range, the range of luminosity over which any daisy is present, is the same as when D is fixed at zero. It will be shown that this represents the greatest range of daisy growth possible. Planetary temperature is regulated with either daisy type present. The effects of increasing luminosity on planetary temperature are further reduced within the 'both-daisy' range, the range of luminosity over which black and white daisies are present. It will be shown that when the rate of entropy production is maximized the both-daisy range is also maximized and so the range of luminosity that sees the smallest rate of change of planetary temperature is maximized. By altering D to maximize the rate of entropy production we also maximize the any-daisy and both-daisy ranges and thus maximize the range of luminosity over which planetary temperature is regulated at all and regulated most effectively.

It is straightforward to show that maximising entropy production will lead to a maximisation of the any-daisy range. In order for there to be non-zero entropy production, there must be a temperature gradient between the two daisy boxes. Therefore there must be non-zero coverage in either or both boxes. This equates to black daisies growing at the lowest possible luminosity and white daisies growing at the highest possible luminosity. In order for black daisies to grow when the planet is cool, the amount of heat flux must be reduced. Consequently at the limits of the any-daisy range, $D \to 0$. The limits of the any-daisy range are also when the daisies reach maximum coverage. The intuition is that the external forcing drives the daisy coverage higher until the biota can no longer respond at which point any further increases lead to a population collapse. The daisy box temperature that produces maximum coverage is 295 degrees Kelvin. Setting $D = 0$ enables us to find the luminosity for the start and end of the any-daisy range. With the parameter and constant values as detailed in the Two-Box Daisyworld section, computing equations 8 and 9 give the start and end of the any-daisy range to be be 0.5764 and 1.729 respectively (to within 4 significant figures). These are the values returned with numerical results

$$L_{start} = \frac{295.5^4\sigma}{S(1-AB)}, \tag{8}$$

$$L_{end} = \frac{295.5^4\sigma}{S(1-AW)}. \tag{9}$$

Finding the limits of the both-daisy range is not a trivial exercise [1]. At lower luminosities we want to find that combination of luminosity and heat flux that increases the white daisy box to 5C. This will be achieved at lower luminosities with higher heat flux. But if heat flux is too high, the temperature of the black daisy box can be cooled so far as to lead to a collapse in the black daisy population. Similarly we want to find the greatest heat flux that can maintain the black daisy box to within 40C without increasing the temperature of the white daisy box beyond 22.5C and so its collapse. The hypothesis that maximising the both-daisy range is achieved when entropy production is maximized can be checked by computing steady states in which D is adjusted in order to maximize the both-daisy range. These produce values (to 4 significant figures) of 0.6285 for the start and 1.283 for the end of the both-daisy range which match the values for the both-daisy range when D is adjusted to maximize the rate of entropy production. These values are also returned when D is adjusted in order to maximize the total coverage of daisies for any luminosity (a Maxlife scenario). Results are shown in figure 5. An important difference between the MEPP and Maxlife models is that D reaches a maximal value in Maxlife such that the black and white boxes are isothermal and thus entropy production falls to zero; $T_b = T_w = T_{opt}$ and so $dS/dt = 0$.

Discussion

We have seen that when diffusivity is altered in the two-box Daisyworld model in order to maximise the rate of entropy production via latitudinal heat flux, the range of luminosity over which daisies grow is maximised. Care must be exercised when interpreting results from such simple models, especially in the absence of empirical data. The model presented in this paper is based upon established climate models and the observed real world phenomena of entropy maximisation via latitudinal heat flux. However the maximisation of entropy production was *imposed* on the model by a notional demon. This is not necessarily a limitation, but does need to be appreciated. MEPP models can be regarded as 'black box' models in that one need not know the details of the system's dynamics. Within the two-box model we have assumed that there are sufficient degrees of freedom within the processes that determine the diffusivity between the two boxes to afford the system the ability to configure itself into a state that maximizes the rate of entropy production. When the two-box model is in a MEP state with respect to latitudinal heat flux, it is not in a MEP state with respect to

[1] For this author.

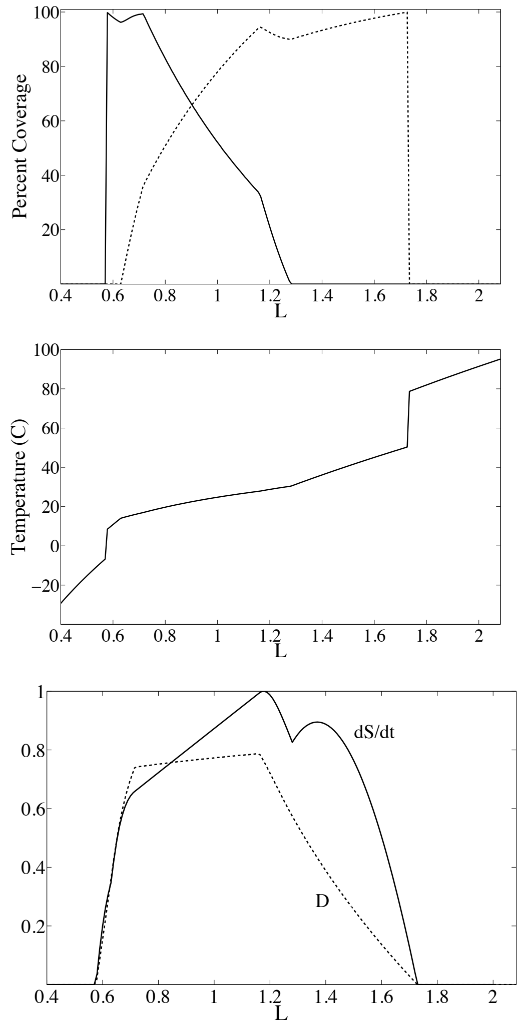

Figure 4: Two-box Daisyworld MEP results

Black (solid line) and white (dashed line) daisy percentage coverage are shown in the top plot. Planet temperature is shown in the middle plot. Normalised entropy production (solid line) and the diffusion parameter D (dashed line) are shown in the bottom plot. When adjusting heat flux to maximize the rate of entropy production, the any-daisy and both-daisy ranges are maximsed. Entropy production is greatest when $L \approx 1.2$.

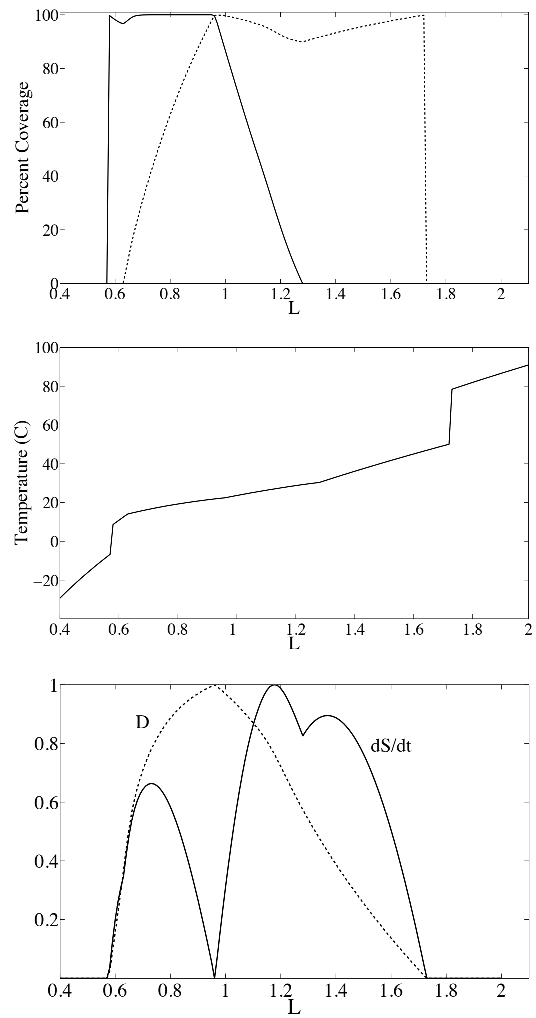

Figure 5: Two-box Daisyworld MaxLife results

Black (solid line) and white (dashed line) daisy percentage coverage are shown in the top plot. Planet temperature is shown in the middle plot. Normalised entropy production (solid line) and the diffusion parameter D (dashed line) are shown in the bottom plot. When adjusting heat flux to maximize the rate of entropy production, the any-daisy and both-daisy ranges are maximised. Diffusion reaches a maximum value and entropy production a minimum value when $L \approx 0.95$.

short wave to long wave radiative balance. This would be achieved by making Daisyworld as dark as possible thus absorbing as much of the star's energy and so converting the maximum amount of short wave radiation to long wave radiation. On Earth, the majority of entropy production is due to short wave radiation from the Sun warming the surface and atmosphere of the planet and then radiating this now long wave energy back into space. The Earth does *not* maximize the rate of this entropy production. This is due to the insufficient degrees of freedom the radiative mechanism possesses with MEP only to be expected in complex, turbulent, dissipative systems such as planetary climates (Ozawa et al., 2003). Arguably the utility of the results presented here will depend on the plausibility and conceptual coherence of the MEPP. If the MEPP is 'real' and applicable to a range of systems then it seems reasonable to investigate its effects on systems such as life-mediated climate models.

The effects of maximising entropy production in the two-box model was to maximize the any-daisy and both-daisy ranges and so maximize the range of solar forcing over which the system is self-regulating. It is tempting to claim that by maximizing entropy production we have maximized self-regulation. However the current results must not be overstated. For example the model's response to stochastic external or internal perturbations was not explored. We assumed that the rate of change of daisies was sufficiently faster than the star to allow the star to remain fixed whilst steady state coverage was found. We also assumed that diffusion was fixed whilst the system moved to steady state. A wide range of different model dynamics could well be produced by relaxing or altering these assumptions. That said, there are immediate intuitive connections between self-regulation and entropy production. The system can only regulate temperature when daisies are present. Latitudinal heat flux entropy can only be produced when daisies are present. If the two-box Daisyworld atmosphere had sufficient degrees of freedom to maximize entropy production, then it would maximize the range of luminosity over which daisies can grow.

Future Work

At a planetary level, it is not immediately obvious how individual organisms would self-organize in such a way as to lead to situations observed in this model. Given the 'choice' of evolutionary or thermodynamic maximizing principles, there seem to be no initial reasons to think that life would adhere to the latter. One very important mechanism absent from the two-box model is evolution. The daisies have fixed responses and effects on environmental variables. Finding real-world organisms in states that appear to maximise the rate of entropy production via metabolic processes may be the result of such states being more efficient than lower entropy producing states. A more efficient organism may be more fitter and so natural selection rather than thermodynamics would be the mechanism that explains how such states arose and persist. However, a recent study has shown how Daisyworld regulation can emerge via evolutionary dynamics with minimal assumptions (McDonald-Gibson et al., 2008). An intriguing next step would be to develop new models that incorporate evolutionary mechanisms into energy balance models in order to assess the relationships between entropy production, self-regulation and evolutionary dynamics. This would combine evolutionary and thermodynamic mechanisms and so build a potentially more complete picture of the Earth's climate.

Conclusion

A simple two-box Daisyworld model has been presented. It has been shown that this model can be regarded as an example of an energy balance climate model. Energy balance models have been used to explore the hypothesis that planetary atmospheres maximise the rate of entropy production via the transport of heat from the hot tropics to the cold poles. In the two-box Daisyworld model, the difference in zonal temperature is produced by a difference in albedo rather than latitude. It was found that when the amount of diffusion was adjusted to maximize the rate of entropy production, the range over which any and both daisies grow was maximised and consequently the viability range of life on the planet was maximised. Maximising the rate of entropy production led to a maximisation of the range of luminosities over which self-regulation is observed. It is speculated that developing new models that incorporate thermodynamic and evolutionary dynamics could produce new results that have direct applicability to the Earth and its climate.

Acknowledgements

The author would like to thank Inman Harvey for suggesting the maximising demon analogy and significantly supporting the development of a number of ideas contained herein and James Lovelock for stimulating discussions on Daisyworld and climate modeling.

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