The Daisystat: A model to explore multidimensional homeostasis

James Dyke
Max Planck Institute for Biogeochemistry, 07745 Jena, Germany
jdyke@bgc-jena.mpg.de

Abstract
The Homeostat was a physical device that demonstrated Ashby’s notion of ‘ultrastability’. The components interact in such a way as to maintain sets of essential variables to within critical ranges in the face of an externally imposed regime of perturbations. The Daisystat model is presented that bears a number of similarities to Ashby’s Homeostat but which can also be considered as a higher dimensional version of the Watson & Lovelock Daisyworld model that sought to explain how homeostasis operating at the planetary scale may arise in the absence of foresight or planning. The Daisystat model features a population of diverse individuals that affect and are affected by the environment in different ways. The Daisystat model extends Daisyworld in that homeostasis is observed with systems comprised of four environmental variables and beyond. It is shown that the behaviour of the population is analogous to the ‘uniselector’ in the Homeostat in that rapid changes in the population allows the system to ‘search’ for stable states. This allows the system to find and recover homeostatic states in the face of externally applied perturbations. It is proposed that the Daisystat may afford insights into the evolution of increasingly complex systems such as the Earth system.

Introduction
This paper introduces a new model that demonstrates homeostasis in the face of external perturbations: the Daisystat. The Daisystat is a hybrid of ‘Daisyworld’ and ‘Homeostat’ as it shares salient features with both models. The Daisyworld model (Lovelock (1983); Watson and Lovelock (1983)) was initially intended as a cybernetic proof of concept for planetary homeostasis as formulated in Gaia Theory which proposed that the Earth system (where ‘Earth system’ is defined as the Earth’s atmosphere, oceans, cryosphere, lithosphere and biota) was a homeostatic entity that maintained conditions to within the range that allowed widespread life (Lovelock, 1979). The Homeostat was a physical device that exhibited ultrastability - the ability to respond to a particular regime of perturbations in such ways as to maintain certain essential variables to within essential ranges (Ashby, 1960). While the spatial and temporal scales of Daisyworld and the Homeostat are very different (Daisyworld considers self-regulation at a planetary scale over aeons whereas the Homeostat was built from four decommissioned Royal Air Force bomb aiming devices and operated at millisecond speed) both systems exhibit very similar behaviour that can be observed in the Daisystat.

In the following sections, the Homeostat and Daisyworld models will be described. The Daisystat is then presented and two sets of results shown. The first set shows how a single-environmental-variable-Daisystat responds to a progressive driving perturbation, the second set shows how a four environmental variable Daisystat responds to instantaneous shocking perturbations. The establishment and maintenance of homeostasis in both cases is given in terms of ‘rein control’. It will be shown that the behaviour of the population is analogous to the behaviour of the electromechanical Homeostat in that the volume of possible connections between elements of the system is ‘searched’ until new feedback values are found that produce homeostatic states. Such a process is the result of natural selection operating on a population of diverse individuals. No notions of higher level selection, altruism or kin selection are required to explain the homeostatic behaviour of the system. The ‘law of requisite variety’ (Ashby, 1956) is seen operating in the Daisystat in that there are lower bounds for the amount of genetic and phenotypic diversity in the population in order for homeostasis to be established and maintained. It is proposed that the Daisystat can be used as a tool to explore the evolution and emergence of real world complex systems such as the Earth system.

The Homeostat
The Homeostat was an electromechanical device designed and constructed by W. R. Ashby. The Homeostat consisted of four units. Each unit produced an output that was fed into the inputs of the other units and back to itself via a recurrent connection. Fig. 1 shows a schematic of the Homeostat units and their connections. The inputs into the $i$th unit, $I_i$, are the sum of the outputs of the other units multiplied by a set of input weights:

$$I_i = \sum_{j=1}^{j=4} O_j \omega_{ji}$$

(1)
where $\omega_{j,i}$ is the weight for the connection from the $j$th unit to the $i$th unit. A weight can either increase or decrease a connection input. Each unit has a target value, $T$. The unit’s output, $O$, is the difference between the input and target value: $O = T - I$. This represents the first level of homeostatic control in the Homeostat. The second level of control is derived from the establishment of essential ranges for the output of the units. If the output of a unit moves outside of the essential range, then a uniselector component randomly generates connection weights for that unit until the unit output moves back within the essential range. For example, if the essential range is $[-0.5, 0.5]$ and $O = 0.6$ then the uniselector would generate new weights for all connections into that unit until the output moves back within the essential range. The Homeostat demonstrated ultrastability that was a consequence of Ashby’s law of requisite variety. In order for the Homeostat to maintain stable states in the face of perturbations, it must be able to reconfigure itself in at least as many ways as these perturbations demand. Consequently, the volume of possible connection weight values must encompass all possible values that would be required to produce stable states.

Homeostat simulations start by having the uniselectors for each unit create random weights. This produces initially chaotic behaviour whereby one unit drives another unit out of its essential range which responds with new uniselector values which may drive another unit out of its essential range and so on. Given sufficient iterations of the uniselector process, a set of weights will be generated that proves to be stable in that the outputs of all units remain within their essential ranges. An example Homeostat simulation is shown in Fig. 2. The Homeostat finds a stable state and is then perturbed when Time = 200 by decreasing the output of one unit by 1. This leads to all units moving out of their essential range and a period of uniselector activity that creates new random weights which produces a new attractor which the system relaxes towards.

Daisyworld

While Daisyworld is a simple model of a planetary system, it is more complicated than the Homeostat with a number of different feedback mechanisms that feature non-linear functions. However, at its heart it is similar in that two units in the form of two species or type of plants (commonly referred to as ‘daisies’) exert unidirectional effects on a regulated variable in the form of planetary temperature. These effects stem from the different albedo of the daisies. Albedo is a measure of the reflectivity of an object. Black daisies have lower albedo than white daisies. Changing the relative proportion of black and white daisies will affect the planetary albedo and so the global temperature. The black and white daisies share the same parabolic growth response to temperature. Both grow at maximum rates when their local temperatures are 22.5°Celsius with growth progressively decreasing, until it is zero when the temperature is 5°C or 40°Celsius.

Daisyworld simulations consist of seeding a grey planet that has an intermediate albedo of 0.5 with black and white daisy seeds. This planet orbits a star much like the sun which over geological time scales increases in luminosity or brightness. On a lifeless planet, as the star increases in luminosity, the temperature increases approximately linearly (the actual temperature response being a quartic function of luminosity). The situation is markedly different when black and white daisies are present in that the temperature rapidly moves towards the maximum growth rate temperature and then stays within the range that the daisies are able to grow over as luminosity increases. This demonstrates how planetary regulation may emerge as a consequence of biological activity that is not the result of intentional design and
in ways compatible with natural selection. Fig. 3 shows planetary temperature being regulated when both daisies are present and Fig. 4 show how this regulation is the result of the change in the proportional coverage of the black and white daisies.

Figure 3: Temperature as a function of luminosity on Daisyworld. The dashed line represent temperature on a planet with no daisies. This increases approximately linearly with increasing luminosity. The solid black line shows planetary temperature with black and white daisies present. This increases suddenly, after which it is maintained within the growing range of the daisies for a range of luminosity values. There is a sudden increase in planetary temperature that corresponds to the collapse of the daisy populations.

Figure 4: Coverage of black (plotted with solid line) and white daisies (plotted with dashed line) as a function of luminosity on Daisyworld. There is a sudden increase then progressive decline in black daisies that is mirrored by the coverage of the white daisies.

**The Daisystat Model**

While the original Daisyworld demonstrated that planetary homeostasis was at least conceivable, it was subject to a number of quite limiting assumptions. Some of these have been addressed in the literature. See Wood et al. (2008) for a review. The Daisystat is intended to address one of these more important limitations that was succinctly identified by J. Kirchner:

“Daisyworld is a one-feedback model; there is only one environmental variable and it is regulated by extremely strong feedback with the simplest possible biosphere. Such a simple model necessarily exhibits simple behaviour. By contrast, on the real Earth many different environmental variables are coupled simultaneously, through many different feedback relationships, with a highly complex biosphere composed of organisms with diverse (and often incompatible) environmental requirements. Such a complex system can exhibit many kinds of behaviour that a simple Daisyworld model cannot.” Kirchner (2003)

Daisystat features a number of environmental variables that are regulated so that they remain within essential ranges as a consequence of the effects of a diverse population of individuals that respond to selection pressure in ways that means they only ever ‘seek’ to increase their own abundance with no selection for their effects on the environmental variables. Daisystat can be understood as a development of an individual-based Daisyworld model first proposed in McDonald-Gibson (2006) and then analysed and extended in: Dyke et al. (2007); McDonald-Gibson et al. (2008); Dyke (2009). There are three important differences between the Daisystat and these previous models. Firstly, as already stated, Daisystat features multiple environmental variables. Secondly, mutation is not currently modelled in the Daisystat so there is no change in the total amount of genetic information in the population over time. Finally, there is no single carrying capacity for the population. Previous Daisyworld studies typically assumed that all individuals within a population will be limited to a shared carrying capacity amount. Consequently the rate of change of all individuals is a function of the frequency of all other individuals. In Daisystat this assumption is relaxed in that all individuals have separate carrying capacities. The interaction between two individuals is then mediated only via their dependence on shared environmental variables. A population of $K$ individuals are affected by and in turn affect their environment. In all results shown, unless otherwise specified, $K = 100$. The individuals may represent individual organisms, populations, species or guilds etc. All individuals experience the same environmental conditions in that the environment is homogenous so that there are no local conditions or micro-climates. The effect that any individual has on the environment lead to changes in the environment that all individuals experience in the same way. It is assumed that an individual’s effect on this homogenous environment diffuses instantaneously. The term ‘environmental resource’ is used to denote those aspects or elements of the environment that affect individuals and in turn are affected by individu-
als. It is important to note that such environmental resources do not produce monotonically increasing fitness in individuals. It is possible to ‘have too much of a good thing’ so an increasing environmental resource can lead to a decrease in the fitness of an individual. This will be expanded on below. The change over time of the $i$th environmental resource, $R_i$, is given by:

$$\frac{dR_i}{dt} = \alpha I_i + \beta O_i$$

(2)

where $I_i$ is the external perturbing input that is being applied to the $i$th resource and $O_i$ is the population’s effect on the resource which is the sum of the individual’s effects:

$$O_i = \sum_{j=1}^{K_j} E_{i,j}$$

(3)

The effect, $E_{i,j}$, that the $j$th individual has on the $i$th resource varies over the range [-1,1] and is given with:

$$E_{i,j} = A_j \epsilon_{i,j}$$

(4)

where $\epsilon_{i,j}$ is the phenotypic effect which is multiplied by the abundance, $A$, of the $j$th individual, where abundance could be interpreted as numbers of individuals, total biomass, frequency in the population, proportional coverage etc. $\alpha$ and $\beta$ are parameters that determine the relative strengths of the perturbing input and population output. For all the results shown $\alpha = \beta = 1$. There is no momentum in environmental resources, consequently their rate of change will be zero when $\alpha I = -\beta O$. The abundance of the $j$th individual changes over time with:

$$\frac{dA_j}{dt} = A_j (k_j - A_j) F_j - A_j \gamma$$

(5)

where $k_j$ is the carrying capacity of the $j$th individual. This equation is essentially identical to that used in Watson and Lovelock (1983) and gives logistic growth towards the carrying capacity, $k$. In all results shown all $k$ values are set to unity. Therefore, the range of possible abundance values is $[0, 1 - \gamma]$, where $\gamma$ is a fixed death rate and for all results shown is fixed at 0.1. $F_j$ is the ‘fitness’ function for the $j$th individual and is the sum of the fitness function responses for each environmental resource:

$$F_j = \sum_{i=1}^{R_{max}} F_{i,j}$$

(6)

where $R_{max}$ is the number of environmental resources and $F_{i,j}$ is a normal distribution response that determines the $j$th individual’s response to the $i$th environmental resource:

$$F_{i,j} = e^{-((R_{i,j} - T_{i,j})^2)/2\sigma^2}$$

(7)

where $T_{i,j}$ is the ‘target’ $i$th resource value for the $j$th individual in that this is the resource values that gives the maximum fitness of unity. This is analogous to the growth response to temperature in Daisyworld. As the resource increases/decreases from this target value, fitness decreases at a rate determined by the variance, $\sigma^2$. For all results shown, $\sigma^2$ is set to unity.

Simulations consist of initialising a population of individuals with random $\epsilon$ and $T$ values. The method used is to represent each individual as a two loci genome where each locus has a floating point number over the range [0,1]. These values are mapped to the ranges of [0,100] and [-1,1] for the phenotypic traits of $T$ and $\epsilon$ respectively. Resource values are initialised at some value over the range [0,100]. The change over time in resources and abundances of individuals are then numerically integrated.

**Results**

Two sets of results are presented. The first set demonstrates Daisystat’s ability to perform Daisyworld-type regulation; a system consisting of a single environmental resource is stabilised at a series of particular values in the presence of a perturbing driving input that would in the absence of the effects of the individuals increase the resource. The second set demonstrates Daisystat’s ability to perform Homeostat-type regulation or higher dimensional Daisyworld-type regulation; a system consisting of four environmental resources is subjected to a shock which the population responds to with a period of rapid change until a new stable state is achieved.

**Daisyworld-type regulation**

Fig. 5 and Fig. 6 show changes in resource and abundances over time for a system that consists of a single resource when $dI/dt = 3/\tau$, where $\tau = 2000$ is the number of units of time simulated. These results show the resource being maintained at a number of values during a simulation. Decreasing the rate of change of the perturbing input will typically lead to homeostatic states in which the resource is held at one value for the duration of the simulation. The perturbing input progressively seeks to drive the resource higher and higher. Fig. 7 shows that the population responds to this driving so as to produce a counteracting force so that there is no change in the resource: $I = -O$. This regulation proves to be robust to a wide range of parameter values. $K$ can be decreased to approximately 20 and its only upper limit is computational resources for numerically integrating the equations (maximum $K$ value simulated is 10,000). The width of the fitness functions which is determined by $\sigma^2$ can be decreased or increased by a magnitude with no significant effects. The rate of change of the perturbing input, $dI/dt$ cannot be set arbitrarily high. In the original Daisyworld study it was assumed that the rate of change of the luminosity of the star was sufficiently slow and the change in the population was sufficiently fast so as to keep the luminosity value fixed while the population was integrated to steady state. The Daisystat can significantly relax this assumption, however there must be sufficient time for the population to respond to perturbations by changing the abundances of individuals.
It is important to note that what value the resource remains fixed at is not prescribed in the model. Moreover there appears to be no initial reason why the resource should remain fixed at any level. Natural selection can be seen operating on the population via the different target values that each individual has. Individuals with target values nearer to the current resource level would increase in abundance and their effects on the resource would increase. Such effects range over [-1,1] and are an incidental ‘by-product’ of the individual in that there is no selection pressure for these effects. As there is selection pressure for an individual’s response to the environment but no selection pressure for an individual’s effect on the environment, it may appear strange that the population responds to changes in perturbations that affect the environment by changing the effects they have on the environment while keeping their responses fixed. The explanation for this behaviour can be given in terms of ‘rein control’.

**Figure 5:** Daisystat with a single environmental resource. The resource is plotted with a solid line. The approximated resource value in the absence of any individuals is plotted with dashed line. This increases as the perturbing input is increased over time whereas the simulation with individuals present shows that the resource initially increases with increasing perturbations but then remains approximately fixed when it enters the range of values that produce non-zero fitness. There are three periods of relatively rapid change in the resource with homeostasis being recovered after the first two periods.

**Rein control**

The term rein control was coined by M. Clynes in Clynes (1969) within a discussion of unidirectional communication and control in biological organisms. Saunders et al. (1998) and Saunders et al. (2000) developed the notion into a mathematical description of regulatory systems that are comprised of separate ‘reins’ that can only pull a controlled variable in one direction. The notion of rein control has been previously applied to the analysis of Daisyworld-type models: Harvey (2004), Dyke and Harvey (2006), Dyke et al. (2007), McDonald-Gibson et al. (2008), Wood et al. (2008), Dyke (2009). The Daisystat extends the rein control notion in that homeostatic states feature diverse populations that are not necessarily dominated by two individuals/types/species. Fig. 8 shows the establishment of a rein control stable state. Two sub-populations can be seen in that a group of individuals that have \(T\) values lower than the current \(R\) value will collectively have an increasing effect on \(R\), while a group of individuals that have \(T\) values higher then the current \(R\) value will collectively have a decreasing effect on \(R\). The

**Figure 6:** Abundance of individuals changing over time. The change in abundance is analogous to the change in the coverage of black and white daisies in Daisyworld. As the perturbing input seeks to drive the resource higher, the population responds by altering the proportion of increasing and decreasing effect individuals.

**Figure 7:** Population output changing over time. The effect that the population has on the resource is plotted with a solid line. The driving perturbing force is plotted with a dashed line. The increasing perturbing input produces an equal magnitude, but opposite sign response from the population. At Time \(\approx 800\) and \(900\) there are rapid changes in the population output before it is recovered so that \(I = -O\) again.
sum of the individual’s effect will equal that of the perturbing input, \( I \). As \( I \) changes, the abundance of individuals and the net effect of the two sub-populations changes so that \( I = -O \) and so \( R \) remains fixed.

**Homeostat-type regulation**

The Daisystat exhibits Homeostat-type behaviour in response to sudden perturbations. A Daisystat that was comprised of 4 environmental resources was allowed to relax to a stable state in the absence of any perturbations (\( I = 0 \)). This was then subjected to a ‘shock’ in that one resource value was instantaneously increased by 5 units. This lead to a rapid change in the values of all other resource values as the abundance and so population output on the resources varied rapidly as shown in Fig. 9 and Fig. 10. The change in the abundances continued until a new stable state was found.

**Discussion**

Daisystat displays the ability to resist external driving perturbations much the same way as the original Daisyworld model. An important difference from the original Daisyworld model is that the effects the individuals have on their environment and how they are affected by their environment are not prescribed. Consequently, homeostasis may be established anywhere over the range \([0,100]\). The explanation of homeostasis was given in terms of the rein control effects of a population. This also produced uniselector-type behaviour in that if a resource is driven outside of the range of the individuals that are currently regulating it, a sequence of events leads to all resource values being similarly driven and large changes in the population. Such changes continue until a new set of population responses and effects emerges that produce stability. The change in the abundances of individuals in the population can be described in terms of selection pressure, however there is no meaningful selection pressure for a population’s effect on its resources. The homeostatic behaviour of the Daisystat is not a result of higher level selection.
Increasing the number of environmental resources demonstrated that the rein control system will operate in higher dimensions, an observation first made in Saunders et al. (2000). Regulation operating at planetary scales would be a very high dimensional system with a wide range of time and spatial scales. Daisystat can be considered as a first step in exploring higher dimensional regulation that emerges via population dynamics. In the Homeostat, as the number of units increases and so the size of the matrix of weights increases, the probability of randomly generating weight values that will produce a stable system decreases. Such observations resonate with the long-lasting debate surrounding Gaian regulation, that as there is only a single Earth, planetary homeostasis could not have evolved. While population dynamics may provide a possible account for a biological unisselector that can establish and recover stable states, it cannot explain how high dimensional systems could emerge. If we simplify the Daisystat into a network topology of feedback from and to environmental resources, then making the network more complex by increasing resources leads to the probability of it being stable reducing much in the same way as formulated in May (1972). However, the Earth system did not suddenly come into being 4.5 billion years as it is today. The hypothesis is that an effectively intractable problem in the form of determining a set of feedback values that will lead to stability for a high dimensional system can be made tractable by ‘growing’ such a system from initially low dimensions. In more concrete terms, this could involve incrementally adding new environmental resources to currently stable Daisystat systems. This may be seen as the emergence of new ‘guilds’ of organisms that both exploit and affect aspects of the environment that was either previously separated from the biota or did not even exist. Such an account has been proposed for the increase in complexity for the Earth system (Lenton et al., 2004).

Limitations and future work

The Daisystat is a very simple model intended as an ‘opaque thought experiment’ (Di Paolo et al., 2000) much in the same spirit as the original ‘parable’ of Daisyworld. Assumptions concerning population dynamics were very basic. It is important to note they resulted in no individual completely dying and being removed from the population. The number of individuals remained constant. Consequently biodiversity remained constant (if biodiversity is calculated as simply the number of existing species). However the abundances may be so small (approximately $10^{-5}$) that their effects on the resource values can be safely ignored. Moreover, many Daisyworld studies including the original Watson & Lovelock model assumed a constant supply of either daisy ‘seeds’ or floor for the coverage of daisies. However, allowing species to go extinct in Daisystat could lead to a significant decrease in homeostatic behaviour due to the absence of the ‘required’ rein control species for a particular state of the system. Changing the total number of species via extinction in the absence of mutation and so creation of new species can be seen as reducing the Daisystat’s amount of Ashbyian variety. The connection between Ashby’s law of requisite variety and biodiversity can be expressed as the greater the variety of the system (species in Daisystat) the greater the system’s ability to reduce variety in the environment via regulation. There is significant scope to explore the relationship between biodiversity and stability in the Daisystat and how it changes as the dimensions of the environment changes.

A major assumption of the model is that all possible genomes are specified at the start of a simulation. There is no mutation of the alleles that determines an individual’s effect on the environment and how it is affected by the environment. Introducing mutation would allow a range of evolutionary mechanisms to be explored and is a planned item for future work. The current approach of randomly initialising a population of individuals is consistent with the notion that ‘everything is everywhere, but the environment selects’ (see O’Malley (2007) for a historical review) which would support the assumption that it may be sufficient to generate sufficiently diverse simulated populations and then allow environmental conditions to select those individuals that will survive and perish.

No significant assessment of altering the rates at which individuals respond to and affect resources has been undertaken. This corresponds to $\alpha = \beta = 1$ in equation 2. These values can be seen as analogous to the ‘viscosity’ term in models of the Homeostat that modulates the rate of change of a unit’s effect on the other units. There is much scope to explore the parameter space of different rates of change in Daisystat.

All the results presented featured Daisystats that were completely connected; all individuals were affected by and in turn affected all resources. Initial experiments that relaxed this assumption lead to more complex behaviour. For example when the connections were made more sparse, stable states that featured oscillations and limit cycles were observed. Exploring the effects of changing the density of connections in Daisystat represents a fertile area of future research.

Conclusion

A homeostatic model, the Daisystat, has been presented. This shares certain features and behaviour of the Daisyworld and Homeostat models. The Daisystat proved to be robust to two types of perturbation: instantaneous changes in one of the environmental resource values (analogous to one element in the Homeostat being subject to a sudden jolt); progressive driving of environmental resources (analogous to increasing luminosity in Daisyworld). This has demonstrated that Daisyworld-type homeostasis can be observed under minimal assumptions and with numerous environmen-
tal resources being subject to regulation (the original Daisy-world featured a single environmental resource in the form of planetary temperature). This has also demonstrated that a population of diverse individuals can perform the same function as a Homeostat uniselector by generating rapid changes in the feedback operating between the resources until new stable states are found. A plan of future research was outlined that would investigate the ability to incrementally increase the complexity of homeostatic systems and so provide a conceptual framework in order to understand how real world complex systems such as the Earth system have evolved from simpler states.

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