Performance Analysis of Multihop Wireless Links Over Generalized-K Fading Channels

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***Abstract*—The performance of multihop links is studied in this contribution by both analysis and simulations when communicating over generalized-K (K_G) fading channels. The performance metrics considered include symbol error rate (SER), outage probability, level crossing rate (LCR), and average outage duration (AOD). First, the expressions for both the SER and outage probability are derived by approximating the probability density function (pdf) of the end-to-end signal-to-noise ratio (SNR) using an equivalent end-to-end pdf. We show that this equivalent end-to-end pdf is accurate for analyzing the outage probability. Then, the second-order statistics of LCR and AOD of multihop links are analyzed. Finally, the performance of multihop links is investigated by either simulations or evaluation of the expressions derived. Our performance results show that the analytical expressions obtained can be well justified by the simulation results. The studies show that the K_G channel model and the expressions derived in this paper are highly efficient for the prediction of the performance metrics and statistics for the design of multihop communication links.**

***Index Terms*—Average outage duration (AOD), generalized-K distribution, level crossing rate (LCR), multihop communications, outage probability, performance analysis, symbol error rate (SER).**

I. Introduction

In wireless communications systems, radio signals experience (large-scale) propagation path loss and shadowing, as well as small-scale fading [1]. Conventionally, the effect of wireless channels on system performance is investigated by separately treating these phenomena, typically assuming two to four orders of power decay for the propagation path loss; lognormal distribution for the shadowing; and Rayleigh, Rician, or Nakagami-m distribution for the small-scale fading [2]. To accurately predict the achievable performance, it is highly desirable that a model can simultaneously cope with all the aforementioned phenomena, including propagation path loss, shadowing, and small-scale fading. Because of this, composite channel models have been formed by combining the distributions for small-scale fading with lognormal distribution for shadowing, yielding Rayleigh-, Rician-, Nakagami-lognormal, etc., composite channel models [2]. However, these composite channel models often result in complicated performance analysis.

In recent years, the generalized-K (K_G) channel model has been proposed [3], which is formed by first approximating the lognormal distribution using the Gamma distribution [2] and then combining it with the Nakagami-m distribution. It has been recognized [3] that the K_G channel model can simultaneously take into account propagation path loss, shadowing, and small-scale fading. It can usually cover more communications scenarios encountered in real mobile wireless systems, compared with the other composite channel models. Furthermore, the K_G channel model often results in closed-form solutions when applied for system performance analysis [4]–[6]. Therefore, in this paper, we are interested in analyzing the performance of multihop links when they are operated in wireless channels characterized by the K_G channel model. The symbol error rate (SER), outage probability, level crossing rate (LCR), and average outage duration (AOD) of the multihop links are analyzed. A range of closed-form expressions is obtained. Furthermore, the performance of the multihop links is investigated by both simulations and evaluation of the formulas derived.

By dividing a long transmission link into multiple more reliable short links supported by relays, multihop transmission has the potential to improve the system's energy efficiency and extend the coverage area. Recently, the performance analysis of relay networks has received much attention. Specifically, in [7], the general framework for studying the outage probability of relay links over Nakagami-m fading channels has been proposed. By assuming fixed relay gain, in [8], Wu *et al.* have investigated the performance of nonregenerative dual-hop links over K_G fading channels. The second-order statistical properties, including both the LCR and AOD, of the regenerative multihop links have been analyzed over general multipath fading channels [9]. Furthermore, in [10], Krantzik and Wolf have studied the LCR and AOD performance of the point-to-point links when communicating over the modified Suzuki fading channels, which are formed by the product of two independent random processes, with their envelope following Rayleigh and lognormal distributions, respectively. However, to the best of our knowledge, the performance of the multihop links aided by regenerative relays for communications over K_G fading channels has not been analyzed in the open literature.

The remainder of this paper is organized as follows: Section II provides a brief overview of the K_G fading channels

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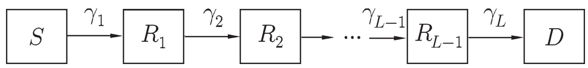

Fig. 1. Schematic of a multihop transmission link, where source S sends messages to destination D through $(L-1)$ relays, and the SNR of the lth hop is denoted by γ_l for $l = 1, 2, \ldots, L$.

and derives the equivalent probability density function (pdf) of the multihop link's end-to-end signal-to-noise ratio (SNR). In Section III, the average SER and outage probability are studied. Section IV addresses the second-order statistics of both the LCR and AOD. Section V provides performance results, and finally, in Section VI, we summarize the conclusions.

II. Statistics of End-to-End Signal-to-Noise Ratio

A. System and Channel Model

We consider an L-hop link as shown in Fig. 1, where source S sends messages to destination D via $(L-1)$ intermediate relays that are expressed as $R_1, R_2, \ldots, R_{L-1}$, respectively, in time-division principles. We assume that the $(L+1)$ nodes are sufficiently separated, so that a node can only receive signals from its adjacent nodes. The relays are regenerative and are operated under the decode-and-forward cooperative strategy. The SNRs of the L hops are denoted by $\gamma_1, \gamma_2, \ldots, \gamma_L$, respectively. This type of communication links exist in wireless ad-hoc networks, wireless sensor networks, etc.

Let us assume that the envelops of the L hops are expressed as $Z_1, Z_2, \ldots, Z_L$. Then, when communicating over the K_G fading channels, these envelops obey the K_G distribution with the pdf [4]

$$f_{Z_l}(z_l) = \frac{4 z_l^{\beta_l}}{\Gamma(m_l)\Gamma(k_l)} \left(\frac{m_l k_l}{\Omega_l}\right)^{(\beta_l+1)/2} K_{\alpha_l}\left[2\sqrt{\frac{m_l k_l}{\Omega_l}} z_l\right]$$
$$z_l \geq 0,\ l = 1, 2, \ldots, L \quad (1)$$

where $\alpha_l = k_l - m_l$, $\beta_l = k_l + m_l - 1$, k_l and m_l are the distribution shaping parameters accounting for the shadowing and Nakagami-m (small-scale) fading associated with the lth hop, $K_{\alpha_l}(\cdot)$ is the second kind modified Bessel function of order α_l, and $\Gamma(\cdot)$ denotes the Gamma function. In (1), $\Omega_l = E\{Z_l^2\}$, where $E\{\cdot\}$ is the expectation operation, represents the local mean determined by propagation path loss [3]. In practice, Ω_l can be obtained by the relatively long-time average over various locations within a certain area to remove the effects of both shadowing and small-scale fading. Specifically, when giving a transmission distance d_l and a propagation path-loss exponent η, we then have $\Omega_l = d_l^{-\eta}$. Therefore, by controlling the values of the shaping parameters k_l and m_l, as well as the value of Ω_l, (1) can be used to approximate the channel distribution in numerous communication environments. Multipath fading, shadowing, the composite of shadowing and multipath fading, additive white Gaussian noise (AWGN) channels, etc., can all be modeled by the distribution of (1) by appropriately setting the parameters of k_l, m_l, and Ω_l [6].

To facilitate the performance analysis, usually, the pdf of the instantaneous SNR per hop of $\gamma_l = Z_l^2 E/N_0$ is required, where E denotes the common energy per symbol, and N_0 is the single-sided power spectral density of AWGN. This pdf can be readily derived from (1) and can be expressed as

$$f_{\gamma_l}(\gamma) = \frac{2\Psi_l^{(\beta_l+1)/2} \gamma^{(\beta_l-1)/2}}{\Gamma(m_l)\Gamma(k_l)} K_{\alpha_l}\left[2\sqrt{\Psi_l \gamma}\right],\ \gamma \geq 0 \quad (2)$$

where $\Psi_l = m_l k_l / \bar{\gamma}_l$, and $\bar{\gamma}_l = \Omega_l E/N_0$.

As shown in Fig. 1, the relay link has the property of regenerative transmission. Hence, if any of the first $(L-1)$ hops suffers from severe fading, resulting in an outage event that its SNR falls below a preset threshold $\gamma_{\rm th}$, the whole transmission from source S to destination D is apparently failed. Otherwise, if the last relay R_{L-1} can successfully decode the messages transmitted by source S, then the statistical property of the SNR observed at destination D is mainly determined by that of the last hop's SNR. Therefore, inspired by the approaches proposed in [11], the cascaded multihop transmissions from source S to destination D, as shown in Fig. 1, can be reduced to an equivalent point-to-point link to simplify the analysis. With the aid of this equivalency, the pdf of the end-to-end SNR of Fig. 1 can be represented as [11]

$$f(\gamma) = A\delta(\gamma) + (1-A) f_{\gamma_L}(\gamma) \quad (3)$$

where A is the probability that outage occurs with the first $(L-1)$ hops, whereas $f_{\gamma_L}(\gamma)$ denotes the pdf of the last hop's SNR. Explicitly, the equivalent pdf of the end-to-end SNR is constituted by two components, i.e., the discrete component of $P(\gamma = 0) = A$ and the continuous K_G pdf scaled by a factor $(1-A)$.

We assume that the channels of the L hops are independent, which is reasonable, provided that any two of the $(L+1)$ nodes in Fig. 1 are separated by a sufficient distance, such as by a few wavelengths. In this case, the SNRs γ_l, $l = 1, \ldots, L$ are also mutually independent and have the pdfs $f_{\gamma_l}(\gamma)$, $l = 1, \ldots, L$, as shown in (2). Note that, since the SNRs γ_l, $l = 1, \ldots, L$ may be associated with different values for parameters k_l, m_l, and Ω_l, the pdfs $f_{\gamma_l}(\gamma)$, $l = 1, \ldots, L$ may be different. Under the aforementioned assumption, when given a preset threshold $\gamma_{\rm th}$, the probability A in (3) can be derived as

$$\begin{aligned} A &= 1 - \Pr\{\gamma_1 > \gamma_{\rm th}, \gamma_2 > \gamma_{\rm th}, \ldots, \gamma_{L-1} > \gamma_{\rm th}\} \\ &= 1 - \prod_{l=1}^{L-1} \Pr\{\gamma_l > \gamma_{\rm th}\} \\ &= 1 - \prod_{l=1}^{L-1} \left[1 - F_{\gamma_l}(\gamma_{\rm th})\right] \end{aligned} \quad (4)$$

where $F_{\gamma_l}(\cdot)$ represents the cumulative distribution function (cdf) of γ_l, which, for noninteger parameter α_l, is given by [6]

$$\begin{aligned} F_{\gamma_l}(\gamma) = {} & \pi \csc(\pi\alpha_l) \\ & \times \left[\frac{(\Psi_l\gamma)^{m_l} {}_1F_2(m_l; 1-\alpha_l, 1+m_l; \Psi_l\gamma)}{\Gamma(k_l)\Gamma(1-\alpha_l)\Gamma(1+m_l)} \right. \\ & \left. - \frac{(\Psi_l\gamma)^{k_l} {}_1F_2(k_l; 1+\alpha_l, 1+k_l; \Psi_l\gamma)}{\Gamma(m_l)\Gamma(1+\alpha_l)\Gamma(1+k_l)}\right], \gamma \geq 0 \end{aligned} \quad (5)$$

where ${}_pF_q(a; b, c; z)$ is the generalized hypergeometric function with integer parameters p and q, as defined in [12]. When parameters m_l and k_l are integers, then the cdf of γ_l can be expressed as [13]

$$F_{\gamma_l}(\gamma) = 1 - \frac{2(\Psi_l\gamma)^{k_l/2}}{\Gamma(k_l)} \times \sum_{q=0}^{m_l-1} \frac{1}{q!}(\Psi_l\gamma)^{q/2} K_{k_l-q}[2\sqrt{\Psi_l\gamma}], \ \gamma > 0. \quad (6)$$

B. Moments of End-to-End SNR

With the equivalent pdf, as shown in (3), the nth moment of the end-to-end SNR $\mu_\gamma(n)$ can be evaluated by the formula

$$\mu_\gamma(n) = \int_0^\infty \gamma^n f(\gamma)\, d\gamma. \quad (7)$$

Upon substituting (3) into (7) and using [12, eq. (6.561/16)], we obtain

$$\mu_\gamma(n) = (1-A)\frac{\Gamma(k_L+n)\Gamma(m_L+n)}{\Gamma(k_L)\Gamma(m_L)\Psi_L^n}. \quad (8)$$

Furthermore, with the aid of (8), the amount of fading (AoF) often used to measure the severity of fading can be computed by the formula [2]

$$\text{AoF} = \frac{\mu_\gamma(2)}{\mu_\gamma^2(1)} - 1. \quad (9)$$

Let us now analyze the SER and outage probability of the general multihop links, as shown in Fig. 1, when they are operated in K_G fading channels.

III. Analysis of Symbol Error Rate and Outage Probability

In this section, we adopt the moment generation function (MGF) approach [2] to analyze the SER and outage performance of the multihop links over K_G fading channels. The MGF $M_\gamma(s)$ of the end-to-end SNR can be expressed as

$$M_\gamma(s) = \int_0^\infty \exp(-\gamma s) f(\gamma)\, d\gamma. \quad (10)$$

By substituting (3) into (10) and using [12, eq. (6.643/3)], it can be shown that the MGF of the end-to-end SNR can be expressed as

$$M_\gamma(s) = A + (1-A)\left(\frac{\Psi_L}{s}\right)^{\frac{\beta_L}{2}} \exp\left(\frac{\Psi_L}{2s}\right) \times W_{-\frac{\beta_L}{2},\frac{\alpha_L}{2}}\left(\frac{\Psi_L}{s}\right) \quad (11)$$

where $W_{v,\mu}(z)$ is the Whittaker's function, as defined in [12].

A. Average SER

For many coherent demodulation schemes, such as M-ary amplitude shift keying, binary phase-shift keying (BPSK), binary frequency-shift keying (BFSK), etc., the SER $P_e(\gamma)$ conditioned on the SNR γ can be expressed in the form as [2], [14]

$$P_e(\gamma) = a \int_0^{\frac{\pi}{2}} \exp\left(-\frac{g\gamma}{\sin^2\theta}\right) d\theta \quad (12)$$

where parameters a and g are determined by the specific modulation scheme employed. For example, for BPSK, we have $a = 1/\pi$ and $g = 1$. The average SER of the multihop links using a specific modulation scheme can be evaluated by integrating $P_e(\gamma)$ with respect to the pdf of γ, i.e.,

$$\bar{P}_e = \int_0^\infty P_e(\gamma) f(\gamma)\, d\gamma. \quad (13)$$

Upon substituting (3) and (12) into the aforementioned equation and exchanging the order of integration, we obtain

$$\bar{P}_e = a\int_0^{\frac{\pi}{2}}\int_0^\infty \exp\left(-\frac{g\gamma}{\sin^2\theta}\right) f(\gamma)\, d\gamma\, d\theta = a\int_0^{\frac{\pi}{2}} M_\gamma\left(\frac{g}{\sin^2\theta}\right) d\theta \quad (14)$$

which relates the average SER to the MGF of (11). Therefore, when applying (11) into (14), the average SER can be expressed as

$$\bar{P}_e = \frac{\pi a A}{2} + a(1-A)\left(\frac{\Psi_L}{g}\right)^{\frac{\beta_L}{2}} \int_0^{\frac{\pi}{2}} (\sin\theta)^{\beta_L} \times \exp\left(\frac{\Psi_L\sin^2\theta}{2g}\right) W_{\frac{-\beta_L}{2},\frac{\alpha_L}{2}}\left(\frac{\Psi_L\sin^2\theta}{g}\right) d\theta. \quad (15)$$

To simplify (15), we first make use of the formula [15, eq. (07.45.26.0005.01)]

$$e^{z/2}W_{v,\mu}(z) = \frac{1}{\Gamma(1/2-\mu-v)\Gamma(\mu-v+1/2)} \times G_{1,2}^{2,1}\left(z \,\middle|\, \begin{matrix} v+1 \\ \mu+1/2, 1/2-\mu \end{matrix}\right) \quad (16)$$

where $G(\cdot)$ is the Meijer's G-function, as defined in [12, eq. (9.301)]. Then, we make the variable transform of

$t = \sin^2\theta$. Finally, after some simplifications with the aid of [12, eq. (7.811.2)], we obtain the average SER

$$\bar{P}_e = \frac{\pi a A}{2} + \frac{a(1-A)\Gamma\left(\frac{1}{2}\right)}{2\Gamma\left(\frac{1-\alpha_L+\beta_L}{2}\right)\Gamma\left(\frac{1+\alpha_L+\beta_L}{2}\right)}\left(\frac{\Psi_L}{g}\right)^{\frac{\beta_L}{2}} \times G_{2,3}^{2,2}\left(\frac{\Psi_L}{g}\,\middle|\,\begin{matrix}(1-\beta_L)/2, (2-\beta_L)/2\\ (1+\alpha_L)/2, (1-\alpha_L)/2, -\beta_L/2\end{matrix}\right). \quad (17)$$

When noncoherent demodulation, such as BFSK, binary differential phase-shift keying, etc., employing square-law detection is considered, the conditional SER can be expressed as $P_{e,non}(\gamma) = C\exp(-D\gamma)$, where C and D are constants determined by the corresponding noncoherent demodulation scheme employed. Therefore, by averaging $P_{e,non}(\gamma)$ using the pdf of (3) and with the aid of [12, eq. (6.643/3)], we can obtain the average SER of the multihop links over K_G fading channels, which is

$$\bar{P}_{e,non} = CA + C(1-A)\left(\frac{\Psi_L}{D}\right)^{\frac{\beta_L}{2}} \times \exp\left(\frac{\Psi_L}{2D}\right) W_{-\frac{\beta_L}{2},\frac{\alpha_L}{2}}\left(\frac{\Psi_L}{D}\right). \quad (18)$$

Let us now analyze the outage probability.

B. Outage Probability

The outage event occurs provided that there is at least a hop having its SNR lower than a threshold γ_{th}. Using the equivalent pdf of (3), we have the outage probability

$$P_{\text{out}} = \int_0^{\gamma_{\text{th}}} f(\gamma)\, d\gamma. \quad (19)$$

By substituting (3) into (19), it can be readily shown that the outage probability can be represented as

$$P_{\text{out}} = A + (1-A)F_{\gamma_L}(\gamma_{\text{th}}). \quad (20)$$

The outage probability can also be directly derived from the multihop links of Fig. 1, which can be expressed as

$$\begin{aligned} P_{\text{out}} &= 1 - P\left[\gamma_1 > \gamma_{\text{th}}, \gamma_2 > \gamma_{\text{th}}, \ldots, \gamma_L > \gamma_{\text{th}}\right] \\ &= 1 - \prod_{l=1}^{L} \Pr\{\gamma_l > \gamma_{\text{th}}\} \\ &= 1 - \prod_{l=1}^{L}\left[1 - F_{\gamma_l}(\gamma_{\text{th}})\right]. \end{aligned} \quad (21)$$

Let $\bar{A} = \prod_{l=1}^{L-1}[1 - F_{\gamma_l}(\gamma_{\text{th}})]$ in the aforementioned equation. According to (4), explicitly, we have $A + \bar{A} = 1$. Upon applying them into (21), we can readily prove that (21) is the same as (20). This implies that the equivalent model associated with the pdf of (3) is accurate in terms of the outage probability.

IV. Level Crossing Rate and Average Outage Duration

In addition to the performance measurements based on the average SER and outage probability, in wireless communications, the LCR and AOD are also very important for system design and optimization. For example, both the LCR and AOD have a strong impact on the selection of packet length, channel-coding schemes, length of interleaver, etc. The reader who is interested in more details about the impact of LCR and AOD on system design and optimization is referred to [9] and [16]–[18], as well as the references therein. Therefore, in this section, we study the LCR and AOD of the multihop links. To the best of the authors' knowledge, the second-order statistics of the LCR and AOD of multihop links operated over K_G fading channels have not been studied in literature. Note that, the LCR is used to quantify how often the fading process crosses a preset threshold, usually in the positive-going direction, whereas the AOD is defined as the average period of time during which the channel quality, such as SNR, is below a predefined threshold [9], [16]–[18].

Here, we first derive the LCR and AOD in the context of the lth hop when it is operated over K_G fading channels. Then, the results are extended to the multihop paradigms with the aid of the approaches proposed in [9]. For the sake of simplicity, the subscript/superscript l is dropped during our analysis of the lth hop, yielding no confusion.

According to [18], the AOD $T(z_{\text{th}})$ in seconds can be approximately expressed as

$$T(z_{\text{th}}) \approx \frac{P_{\text{out}}}{N(z_{\text{th}})} \quad (22)$$

where P_{out} is the outage probability, which has been derived in Section III-B, and $N(z_{\text{th}})$ represents the LCR for a given threshold z_{th}. Therefore, to derive $T(z_{\text{th}})$ of the AOD, we first need to obtain $N(z_{\text{th}})$ of the LCR, which can be evaluated by the integration [19]

$$N(z_{\text{th}}) = \int_0^{\infty} \dot{z} f_{Z\dot{Z}}(z_{\text{th}}, \dot{z})\, d\dot{z} \quad (23)$$

where $f_{Z\dot{Z}}(z,\dot{z})$ represents the joint pdf of the random processes $Z(t)$ and its time derivative $\dot{Z} = dZ(t)/dt$ at time t.

To derive the joint pdf of $f_{Z\dot{Z}}(z,\dot{z})$, we can commence from the composite structure of the K_G distribution [5]

$$f_Z(z) = \int_0^{\infty} f_{Z|Y}(z|y) f_Y(y)\, dy \quad (24)$$

where $f_Z(z)$ is the K_G distribution in the form of (1), $f_{Z|Y}(z|y)$ denotes the pdf of the Nakagami-m distribution conditioned on the local mean $Y = y$, and $f_Y(y)$ represents the Gamma pdf approximating the distribution of the local mean.

In detail, the conditional Nakagami-m and Gamma pdfs for deriving the K_G distribution shown in (1) are given by

$$f_{Z|Y}(z|y) = \frac{2m^m}{\Gamma(m)y^m} z^{2m-1} \exp\left(-\frac{mz^2}{y}\right) \quad (25)$$

$$f_Y(y) = \frac{k^k}{\Gamma(k)\Omega^k} y^{k-1} \exp\left(-\frac{ky}{\Omega}\right). \quad (26)$$

Let us use the variable transform of $S = Z/Y$ in (25). It can be shown that, for a given value of Y, variable S also obeys the Nakagami-m distribution, which is related to the original Nakagami-m pdf through the relationship of

$$f_{Z|Y}(z|y) = \frac{1}{y} f_S\left(\frac{z}{y}\right) \quad (27)$$

where $f_S(\cdot)$ represents the pdf of variable S. Upon applying the preceding relationship into (24), we obtain

$$f_Z(z) = \int_0^{\infty} \frac{1}{y} f_S\left(\frac{z}{y}\right) f_Y(y)\, dy. \quad (28)$$

Finally, according to [10], the joint pdf of $Z(t)$ and $\dot{Z}(t)$ can be expressed in the integral form as

$$f_{Z\dot{Z}}(z, \dot{z}) = \int_0^{\infty} \frac{1}{y^2} \int_{-\infty}^{\infty} f_{S\dot{S}}\left(\frac{z}{y}, \dot{\left(\frac{z}{y}\right)}\right) f_{Y\dot{Y}}(y, \dot{y})\, dy\, d\dot{y} \quad (29)$$

where $f_{S\dot{S}}(\cdot,\cdot)$ represents the joint pdf of the random process $S(t)$ and its derivative $\dot{S}(t)$, and $f_{Y\dot{Y}}(\cdot,\cdot)$ represents the joint pdf of the random process $Y(t)$ and its derivative $\dot{Y}(t)$.

Since the Nakagami-m distributed random process and its time derivative are mutually independent [20], we have $f_{S\dot{S}}(\cdot,\cdot) = f_S(\cdot) f_{\dot{S}}(\cdot)$. Furthermore, according to [20], the samples from the time derivative of a Nakagami-m distributed random process obeys the well-known Gaussian distribution with zero mean and a variance of $\sigma_S^2 = f_m^2 \pi^2 / ym$, where $f_m = f_m^{(l)}$ denotes the maximum Doppler frequency shift in the context of the lth hop. Hence, the joint pdf $f_{S\dot{S}}(z/y, \dot{(z/y)})$ seen in (29) can be obtained as the product of the Nakagami-m distribution of $f_S(z/y)$ and the Gaussian distribution $f_{\dot{S}}(\dot{(z/y)})$, which is expressed as

$$f_{S\dot{S}}\left(\frac{z}{y}, \dot{\left(\frac{z}{y}\right)}\right) = \frac{2m^m}{\Gamma(m)y^{m-1}} z^{2m-1} \exp\left(-\frac{mz^2}{y}\right) \times \frac{1}{\sqrt{2\pi}\sigma_S} \exp\left(-\left(\frac{\dot{z}}{y} - \frac{z\dot{y}}{y^2}\right)^2 \Big/ 2\sigma_S^2\right) \quad (30)$$

where $\dot{(z/y)} = \dot{z}/y - z\dot{y}/y^2$ has been applied.

Unfortunately, a Gamma-distributed random process and its time derivative are not independent. Hence, we cannot derive the pdf $f_{Y\dot{Y}}(y, \dot{y})$ in the same way as for the Nakagami-m distributed random process. However, the joint pdf of $f_{Y\dot{Y}}(y, \dot{y})$ can be obtained from [21], which studies the joint pdf between the random process obeying the generalized Gamma distribution and its time derivative. It can be shown that the generalized Gamma distribution reduces to the Gamma distribution of (26) when the parameter β in [21, eq. (4)] equals one. Therefore, by setting $\beta = 1$ in [21, eq. (15)], we obtain the joint pdf of the Gamma-distributed random process and its time derivative as

$$f_{Y\dot{Y}}(y, \dot{y}) = \frac{k^{k+1/2}}{2\Gamma(k)\Omega^{k+1/2}\sqrt{-\rho''(0)\pi}} y^{k-\frac{3}{2}} \times \exp\left(\frac{k\dot{y}^2}{4\Omega\rho''(0)y} - \frac{ky}{\Omega}\right) \quad (31)$$

where $\rho(\tau)$ is the normalized correlation function of the Gamma-distributed random process, and $\rho''(0)$ is its second-order derivative at $\tau = 0$. For example, for land mobile communication systems, usually, $\rho(\tau) = J_0(2\pi f_m \tau)$, where $J_0(x)$ is the zeroth-order Bessel function of the first kind. Correspondingly, we have $\rho''(0) = -2\pi^2 f_m^2$.

Finally, when substituting the joint pdfs of (30) and (31) into (29) and then using [12, eq. (3.321.4)] to simplify it, we can obtain the joint pdf of the K_G distributed random process $Z(t)$ and its time derivative $\dot{Z}(t)$ as

$$f_{Z,\dot{Z}}(z, \dot{z}) = \frac{2\sqrt{ab}m^m k^k z^{2m-1}}{\sqrt{\pi}\Gamma(m)\Gamma(k)\Omega^k} \int_0^{\infty} \frac{y^{\alpha}}{\sqrt{ay^3 + bz^2 y}} \times \exp\left(-\frac{k}{\Omega}y - \frac{b\dot{z} + mz^2}{y} + \frac{b^2 z^2 \dot{z}^2}{ay^3 + bz^2 y}\right) dy \quad (32)$$

where, by definition, $a = -k/4\Omega\rho''(0)$, and $b = m/2\pi^2 f_m^2$.

Furthermore, let $z = z_{\text{th}}$ in (32). Then, when substituting (32) into (23) and completing the integration with respect to $\dot{z}$, we can obtain the LCR for a single-hop link as

$$N(z_{\text{th}}) = \frac{m^m k^k z_{\text{th}}^{2m-1}}{\sqrt{ab\pi}\Gamma(m)\Gamma(k)\Omega^k} \int_0^{\infty} y^{\alpha-\frac{3}{2}} \times \sqrt{ay^2 + bz_{\text{th}}^2} \exp\left(-\frac{k}{\Omega}y - \frac{mz_{\text{th}}^2}{y}\right) dy \quad (33)$$

which contains just one integration that can be easily evaluated using existing software packages, such as MATLAB, IT++, etc.

When the multihop links are considered, according to [9], the end-to-end LCR over the K_G channels can be evaluated by the formula

$$N_T(z_{\text{th}}) = \sum_{n=1}^{L} N_n(z_{\text{th}}) \prod_{\substack{l=1 \\ l \neq n}}^{L} [1 - F_{Z_l}(z_{\text{th}})] \quad (34)$$

where $N_n(\cdot)$ denotes the nth hop's LCR, which is given by (33) with parameters m, k, Ω, and f_m replaced by m_n, k_n, Ω_n, and $f_m^{(n)}$, respectively. In (34), $F_{Z_l}(\cdot)$ denotes the cdf of Z_l of the lth hop's envelope, which can be derived from (5) and (6) based on the variable transform of $\gamma_l = \bar{\gamma}_l Z_l^2/\Omega_l$. From this variable transform, it can be shown that $F_{Z_l}(z) = F_{\gamma_l}(\bar{\gamma}_l z^2/\Omega_l)$. Hence, for noninteger parameter α_l, from (5), we obtain that

$$F_{Z_l}(z) = \pi \csc(\pi\alpha_l) \times \left[\left(\frac{m_l k_l}{\Omega_l} z^2\right)^{m_l} \frac{{}_1F_2(m_l; 1-\alpha_l, 1+m_l; m_l k_l z^2/\Omega_l)}{\Gamma(k_l)\Gamma(1-\alpha_l)\Gamma(1+m_l)} - \left(\frac{m_l k_l}{\Omega_l} z^2\right)^{k_l} \frac{{}_1F_2(k_l; 1+\alpha_l, 1+k_l; m_l k_l z^2/\Omega_l)}{\Gamma(m_l)\Gamma(1+\alpha_l)\Gamma(1+k_l)} \right]. \quad (35)$$

By contrast, when m_l and k_l are integers, we can have from (6) that

$$F_{Z_l}(z) = 1 - \frac{2}{\Gamma(k_l)} \left(\frac{m_l k_l}{\Omega_l} z^2\right)^{k_l/2} \sum_{q=0}^{m_l-1} \frac{1}{q!} \left(\frac{m_l k_l}{\Omega_l} z^2\right)^{q/2} \times K_{k_l-q}\left[2\sqrt{\frac{m_l k_l}{\Omega_l}} z\right]. \quad (36)$$

Finally, the AOD of multihop links communicating over K_G channels can be obtained by substituting P_{out} of (20) and $N_T(z_{\text{th}})$ of (34) into (22).

Let us now provide the simulation and numerical results to characterize the performance of multihop links in the next section.

V. Performance Results

In this section, the SER, outage probability, AoF, LCR, and AOD performance of the multihop links communicating over K_G fading channels are investigated by both simulation and numerical approaches. In our investigations, we assumed that the channels of the L hops were independent and identically distributed (i.i.d.) K_G fading channels. The impact of propagation path loss, shadowing, and Nakagami-m fading on the achievable performance was all considered. For convenience, the intermediate relays were assumed to be equally located on a line connecting source S and destination D, making all the L hops the same distance. We assumed that the maximum distance from source S to destination D was $L_{\max} = 5$, after being normalized by the distance of one hop. Hence, when given $L \leq L_{\max}$ number of hops, the parameter Ω_l in (1), which accounts for the propagation path loss of the lth hop, was $\Omega_l = \Omega = (L/L_{\max})^{\eta}$, where η represents the global propagation path-loss exponent, which was set to $\eta = 3$ in our simulation and numerical examples. Furthermore, for the sake of fairness of comparison, the transmission power was assumed to be evenly allocated to the L transmitters, making $\bar{\gamma}_l = \bar{\gamma} = \Omega \times \text{SNR}/L$, where SNR is the total SNR without propagation path loss, which was used for depicting the figures. Additionally, when concerning the outage probability, the threshold was set according to $\log_2(1 + \gamma_{\text{th}})/L = R$, with the spectrum efficiency R being set to 0.3 bit/s/Hz.

Figs. 2–4 show the average error performance of the three-hop ($L = 3$) links employing coherent BPSK (see Fig. 2), coherent 4ASK (see Fig. 3), or noncoherent BFSK (see Fig. 4) modulation, when communicating over the K_G fading channels with various values for the shaping parameters k and m, whereas $\Omega_l = \Omega = (3/5)^3$. From the results of Figs. 2–4, we

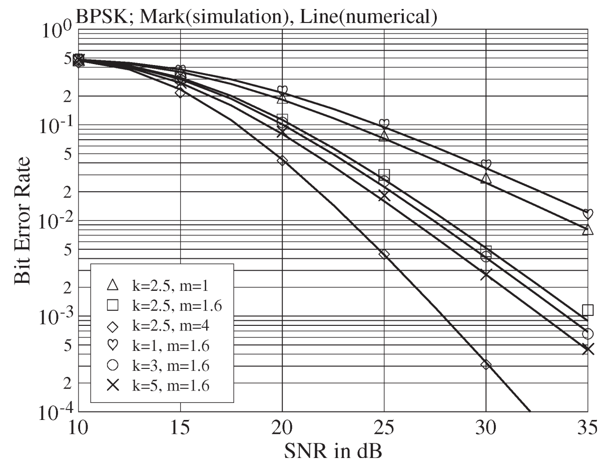

Fig. 2. BER performance of three-hop links using BPSK modulation when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters.

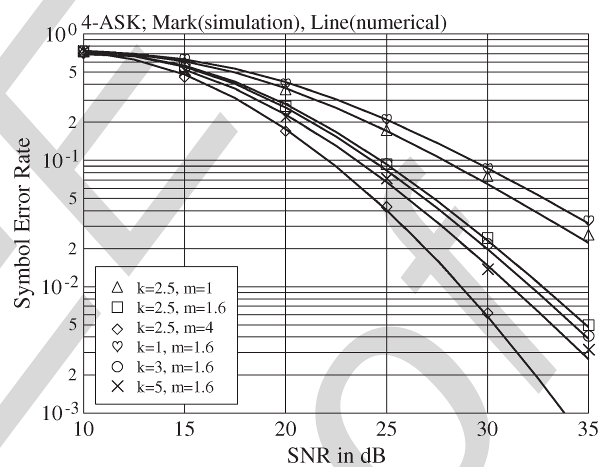

Fig. 3. SER performance of three-hop links using 4ASK modulation when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters.

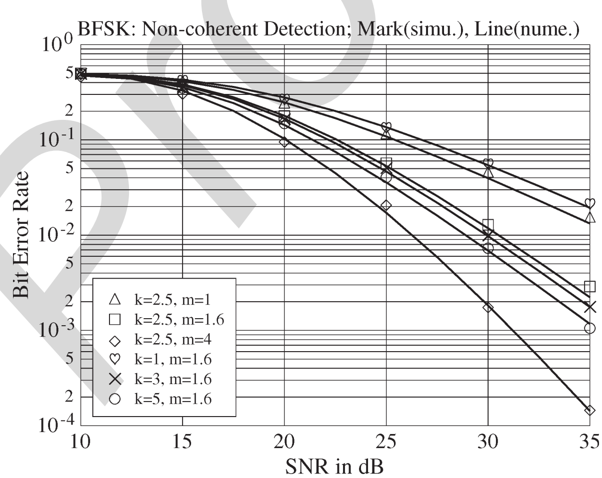

Fig. 4. BER performance of three-hop links using noncoherent BFSK modulation when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters.

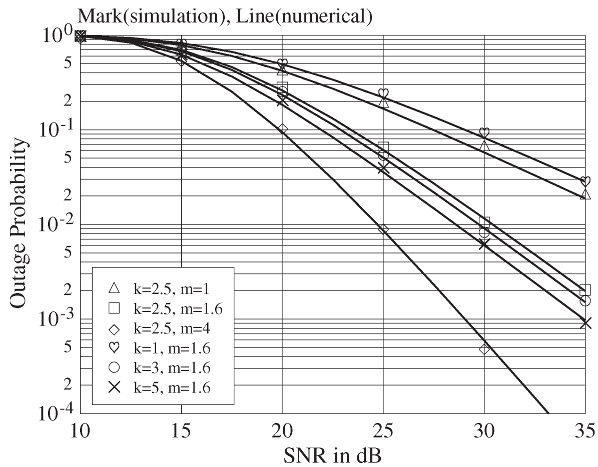

Fig. 5. Outage probability of three-hop links when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters.

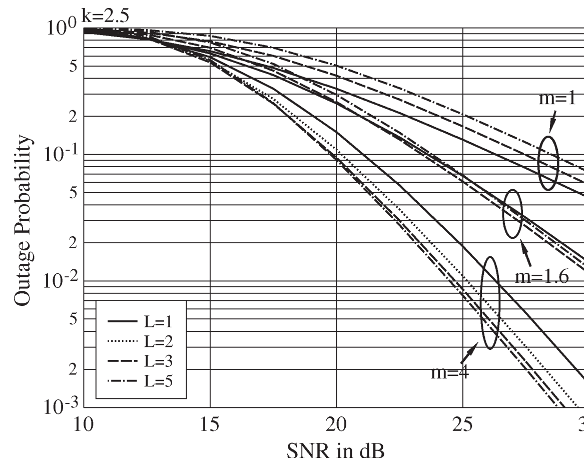

Fig. 7. Outage probability of one-, two-, three-, or five-hop links when communicating over K_G fading channels with $\Omega_l = \Omega = (L/5)^3$.

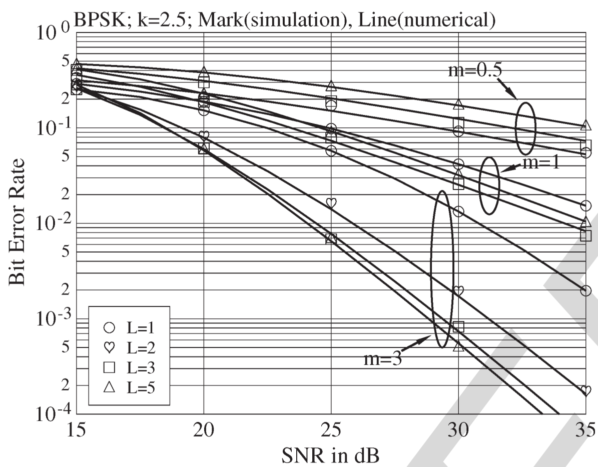

Fig. 6. BER performance of one-, two-, three-, or five-hop links using BPSK modulation when communicating over K_G fading channels with $\Omega_l = \Omega = (L/5)^3$.

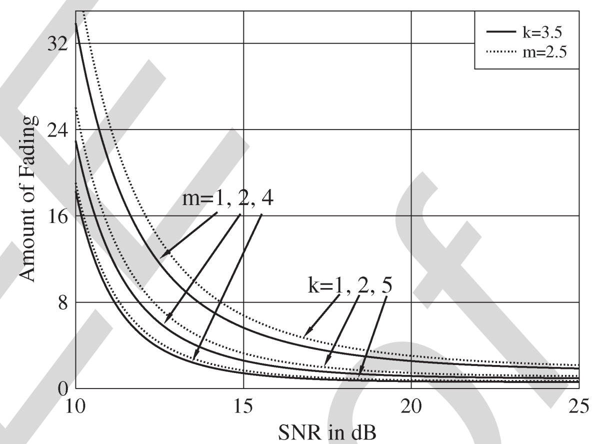

Fig. 8. Amount of fading of three-hop links when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters, as shown in the figure.

can first observe that the numerical results agree well with the simulation results. Therefore, the equivalent point-to-point link with the pdf given by (3) is an effective model for studying the error performance of a corresponding multihop link. Second, given a fixed value of $m = 1.6$ corresponding to a specific small-scale fading, the BER performance of the multihop links improves as the value of k increases, implying that the channel increasingly shadows less. Similarly, for a given value of $k = 2.5$, the BER performance improves as the value of m increases, corresponding to the small-scale fading becoming less severe.

Fig. 5 shows the exact outage probability of the three-hop ($L = 3$) links, when communicating over the K_G fading channels with $\Omega = (3/5)^3$ and various values for the other shaping parameters k and m. Again, when the channel quality improves, i.e., when the value of k and/or m increases, the outage probability at a given SNR decreases. However, it seems that the outage performance is sensitive more to the small-scale fading determined by the value of m than to the shadowing determined by the value of k.

Figs. 6 and 7 investigate the effect of the number of hops on the achievable BER (see Fig. 6) or outage (see Fig. 7) performance. Explicitly, when given the total propagation distance ($L_{\max} = 5$), shadowing ($k = 2.5$), and the small-scale fading of a communication link, there exists an optimum number of hops, which yields the lowest BER or lowest outage probability. Specifically, when the small-scale fading is very severe corresponding to $m = 0.5$ in Fig. 6 and $m = 1$ in Fig. 7, using only one-hop link results in the best performance. When $m = 1$ in Fig. 6 and $m = 1.6$ in Fig. 7, depending on the available SNR, a three- or five-hop link may perform best. Finally, when the channel quality in terms of the small-scale fading is very good, for example, when $m = 3$ in Fig. 6 and $m = 4$ in Fig. 7, generally, the link is beneficial to using as many hops as possible to attain the best BER and outage performance.

The AOF of a three-hop ($L = 3$) link was numerically evaluated based on (9), which is shown in Fig. 8, when various sets of parameters for the K_G fading channels were considered. Explicitly, given $k = 3.5$, the fading becomes severer, as the value of m decreases. Similarly, given $m = 2.5$, the fading becomes more severe as the value of k decreases.

In Fig. 9, we compare the end-to-end LCR of the fading over a three-hop link when the communication channels over the three hops are modeled as the i.i.d. K_G fading channels with various shaping parameters, as shown in the figure. In Fig. 9, the

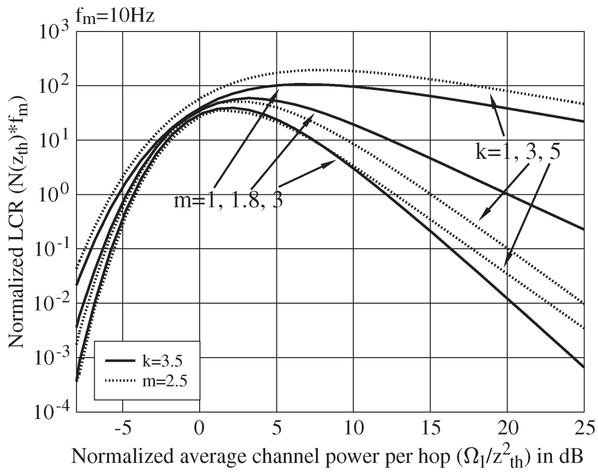

Fig. 9. Level-crossing rate of three-hop links when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters, as shown in the figure.

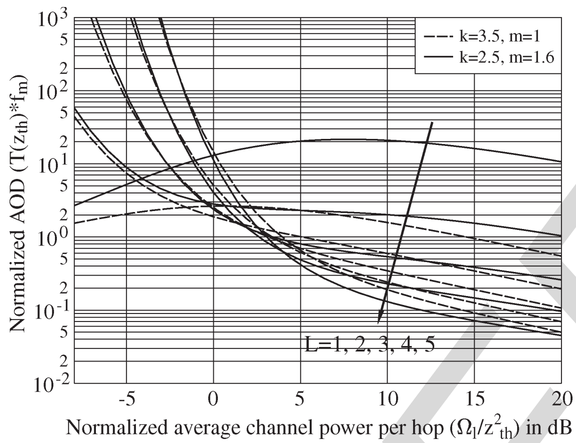

Fig. 10. AOD of multihop links when communicating over K_G fading channels with $\Omega_l = \Omega = (3/5)^3$ and other various shaping parameters, as shown in the figure.

results were numerically evaluated based on (34) when given the maximum Doppler frequency shift $f_m = 10$ Hz for any of the three hops. From the results of Fig. 9, it seems that the LCR of the three-hop link considered is sensitive more to the change of the small-scale fading determined by parameter m than to the varying of the shadowing explained by parameter k. When the average channel power per hop is low, the LCR is low since, in this case, the three-hop channel stays mainly at the "poor" state without many chances going to the "good" state. Similarly, when the average channel power is high, the LCR is also low. This is because, in this case, the three-hop channel stays mainly at the "good" state without much fluctuation. By contrast, as shown in Fig. 9, there is a range for the average channel power, which results in the three-hop channel highly fluctuating, yielding relatively high LCR. Finally, from Fig. 9, we observe that, for a given value of the average channel power, the three-hop channel becomes more fluctuated, as the value of k or m decreases.

Finally, Fig. 10 shows the normalized AOD of the links having different numbers of hops when operated in K_G fading channels with the parameters as shown in the figure. Note that, for Fig. 10, we assumed that all the hops had the same maximum Doppler frequency shift expressed as f_m. In this case, after being normalized by the maximum Doppler frequency shift, the AOD is independent of f_m. As shown in Fig. 10, when the average channel power is low, such as $\Omega_l/z_{\rm th}^2 < -5$ dB, the AOD of the one-hop link may be significantly shorter than the AOD of the multihop links. By contrast, when the average channel power is high, such as $\Omega_l/z_{\rm th}^2 > 5$ dB, the AOD decreases as the number of hops increases. The principles behind the aforementioned observation may be explained as follows: When the average channel power is low, each of the hops is an unreliable channel. Hence, the multihop link has a higher chance to stay at the "poor" state as the number of hops increase. By contrast, when the average channel power is high, due to the fluctuation of the component channels, the multihop channel oscillates the server as the number of hops increases. Correspondingly, the AOD of the multihop link becomes shorter as the number of hops increases.

VI. Conclusion

The SER, outage probability, and second-order statistics, including both LCR and AOD, of multihop links have been investigated, when communicating over the K_G fading channels, which can simultaneously take into account the propagation path loss, shadowing, and small-scale fading. Our studies and performance results show that the K_G channel model and the expressions derived in this contribution are highly efficient for evaluating the SER and outage performance of multihop communication links, as well as for revealing the statistical behavior of multihop channels. Given the distance of a transmission link and the corresponding fading channel determined by the parameters for the propagation path loss, shadowing, and small-scale fading, our studies show that there exists an optimum number of hops for signal delivery, which results in the best SER and outage performance. In general, when the channel is very poor, one-hop direct transmission is the desired signal transmission option. When the quality of the channels improves, the communication link expects more hops to augment the SER and outage performance. If the channels are very reliable, it seems that the best SER and outage performance are attained when the communication link uses as many hops as possible.

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