Theoretical and experimental investigation on the dynamic behaviour of disk blades during sharpening

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Vibrations arising during the sharpening process of disk blades are a crucial problem in paper manufacturing because they can cause a considerable loss of quality and productivity in the paper cutting process. In the technical literature no specific works are reported but similar problems such as those related to grinding and brake disks are dealt with.

In this work the sharpening process of a disk blade has been simulated by means of the explicit finite element code LS-DYNA®. Some phenomenological aspects have been studied highlighting the most important parameters affecting them such as stiffness, contact force and rotational speed. The results of an experimental investigation carried out on a paper cutting machine are also presented and discussed.

Keywords: vibrations, instability, stick-slip, disc blade, sharpening

1. Introduction

Vibrations arising during the sharpening process of disk blades are a crucial problem in paper manufacturing. In particular, since paper rolls are cut using disk blades, defects in the blade edge caused by vibrations, though small, can cause remarkable quality and productivity losses. Common practice has shown that in some cases the disk edge exhibits a thickness periodic irregularity caused by such vibrations. In the technical literature no specific work is reported but similar problems such as those related to grinding [1,2], saw blades, turbine rotors, computer magnetic recording disks [3-6] and brake disks [7,8] are dealt with. The problems mainly concern the response of a rotating disk to an external transverse force.

Lisini and Bartalucci [1,2] studied the grinding process and showed that the regenerative effect of grinding wheels is the predominant effect of the process instability: rotation oscillations of grinding wheel and work-piece feed successive vibrations, that are strictly connected with cutting forces. The problem was studied with a linear model in spite of non-linearity caused by backlash, friction, deformation, wear etc. studying only the onset of instability.

Early work on the vibration of spinning elastic disks dealt with the determination of the natural frequencies and the effect of centrifugal stresses. In general, disk/spindle vibrations have both in-plane and out-of-plane components. The out-of-plane ones primarily result from the deflection of the spinning disks and can be predicted accurately by the classical vibration analysis of rotating disks.

Mote [9] formulated the Green’s function for the transverse response of a uniform, stationary, centrally clamped annular plate and determined the specific response for a harmonic load moving at constant velocity about the plate periphery and for a peripheral load circumnavigating the plate at a speed which is the sum of a constant and a harmonic component. Circular plates containing radially symmetric membrane stresses and thickness were also considered.

Iwan and Moeller [10] used the Galerkin’s method to study the stability of a spinning disk subjected to a spring-mass-dashpot load. Their analysis revealed that the disk system was unstable in a range speed above the critical speeds. When a load system is included, instabilities can occur at speeds other than the classical critical speeds of the disk. It has been shown that a combination of a mass, stiffness and damping in the load system gives rise to several regions of instability not directly related to the classical critical vibration speed of the disk.

More recently, the dynamic behaviour of spinning disks in the storage system of modern computers has been studied. The most difficult problem encountered in the flexible disk drive is the interaction between the spinning disk and the recording head. Many researchers have formulated this problem according to different approaches. Huang and Chiou [5] considered also the radial motion of the head and determined the spinning disk response with a radially moving, harmonic excitation.

Moreover interface friction couples longitudinal and transversal degrees of freedom and makes the system potentially unstable [8]. The motion may not be continuous, but intermittent and proceed by a process of stick-slip.
Stick-slip concerns low velocities and is due to the transition from static to dynamic friction in a range where the friction coefficient decreases with velocity. When damping is insufficient, unstable vibrations occur at the natural frequency of the damped system. Ouyang and others [11] studied the in-plane stick-slip vibration of a flexible disk rotating between elastic sliders, by means of a complex mathematical model. In any case there isn’t yet a unique general theory for the occurrence of friction vibrations, which are still unpredictable because the friction-velocity dependency varies randomly with contamination, surface finish, misalignment of sliding surfaces and other factors.

In this work the disk blade sharpening process is numerically and experimentally examined. The results of an analytical investigation carried out by means of the explicit finite element (FE) code LS-DYNA® are presented and discussed. The influence of the friction decay coefficient and of idle grinding wheels has been studied. Some hypotheses are proposed to explain the phenomenon, among which the effect of stick-slip. Since the analysed disk is rather thick, geometric non-linear behavior, typical of thin rotating disks and amply illustrated by Raman and Mote [12], has not been considered. An experimental investigation carried out on a paper roll cutting machine is also presented highlighting the problems concerning the sharpening of a disk blade.

2. Numerical Analysis

2.1 Model definition

The sharpening process is a quick process that occurs in a very short time. So a transient analysis is doubtlessly necessary. So the process has been simulated by means of the explicit FE code LS-DYNA®. Explicit analysis is quite suitable for dynamic simulations such as impact and crash even if it can become prohibitively expensive for long duration or static analyses. The terms explicit refers to time integration algorithms. In the explicit approach, internal and external forces are summed at each node point, and a nodal acceleration is computed by dividing by the nodal mass. The solution is obtained by integrating acceleration in time. The maximum time step size is limited by the Courant condition, producing an algorithm which typically requires many relatively inexpensive time steps.

The investigation presented in this paper faces a real problem encountered in a paper manufacturing industry. The disk blade sharpening process of paper cutting machine is fundamental because blade edge imperfections must be avoided. So the model definition takes actual paper cutting conditions into account.

Therefore a disk of 0.6 m diameter and tapered thickness from 4.8 mm to 1.5 mm disk has been considered. Shell Hughes-Liu elements with four integration points have been utilized in the FE model. Such elements have been chosen especially because they haven’t hourglass energy problems. A model of a perfectly elastic material with steel properties has been used to represent the disk blade behaviour. A little damping has been added to the system in order to avoid too high oscillations.

In a paper cutting machine the disk blade is usually connected to the shaft by means of a very stiff tight fit. So the circular disk model is free at the outer radius and clamped at the inner radius by a central core, assumed to be rigid. The rigid core is constrained to just rotate around its axis at given angular velocities. Models with one and two grinding wheels have been considered. The grinding wheels, assumed idle, with a 100 mm diameter and a 0.3 kg mass, were modelled as shell rigid bodies. Each idle grinding wheel is pushed against the disk blade by a spring placed in the wheel centre.

![Figure 1 Disk blade model with one and two grinding wheels](image)

The contact parameter definition is the principal problem concerning the sharpening process simulation because it is the interface friction that couples longitudinal and transversal degrees of freedom and makes the system potentially...
unstable. Moreover it’s difficult to find a real abrasive-steel friction coefficient vs. sliding velocity curve. However since the cutting and repulsing forces during the grinding process have about the same size when the cutting ratio is small, the static friction coefficient was assumed equal to 1. The sliding friction coefficient was assumed equal to 0.4, lightly greater than for steel on steel. Moreover the friction curve was assumed to have an exponential shape:

\[ \mu_c = FD + (FS - FD) \cdot e^{-DC|v_{rel}|} \]  

where \( FS \) is the static friction coefficient, \( FD \) is the dynamic friction coefficient, \( DC \) is the exponential decay coefficient and \( v_{rel} \) is the relative velocity of surfaces in contact. For \( DC \), values of 1, 10 and 100 were considered in order to simulate different contact behaviours ranging from a smooth to an irregular friction curve coefficient.

![Figure 2 Friction coefficient curve: \( DC=100, DC=10 \) and \( DC=1 \)](image)

### 2.2 Impact Simulation

First, in order to identify natural frequencies, an impact simulation of one grinding wheel against the non-rotating disk blade was carried out. In figure 3 the nodal displacements power spectral density (PSD) related to a disk blade peripheral node and to a grinding wheel node are reported as representative of the dynamic behaviour of the two components. The disk blade signal spectrum (a) shows a 193 Hz peak and some smaller peaks (70, 93, 105, 117, 125, 138, 155, 220, 288 Hz). They correspond to the system natural frequencies. The grinding wheel signal spectrum (b) shows a pronounced 70 Hz peak and a flattened 193 Hz peak. So it can be deduced that the grinding wheel acts as a filter for higher order harmonics. Then an impact simulation with two grinding wheels was carried out (fig. 3). The disk blade signal spectrum (c) shows outstanding natural frequencies at 120 Hz and 140 Hz and some less significant peaks (70, 93, 200, 220 Hz). The grinding wheel signal spectrum (d) shows a pronounced 70 Hz peak and a flattened 140 Hz peak.

Table 1 reports natural frequencies calculated with the FE code ANSYS®. Grinding wheels were also considered in the model. They were represented by two simple mass-spring sub-systems.

![Table 1](image)

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**Figure 3 PSD of impact simulation for non-rotating disk blade for the systems with one (a, b) and two grinding wheels (c, d): corresponding to a peripheral node of the disc blade (a, c) and to a node of the grinding wheel (b, d)**
Figure 4 Single grinding wheel sharpening: one sided cascade plot for (a) DC=100, (c) DC=10, (e) DC=1 and corresponding two sided plots (b-d-f)

Figure 5 Double grinding wheels sharpening: one sided cascade plot and corresponding two sided plots DC=100
Table 1
Natural frequencies:

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<td></td>
<td>8</td>
<td>402; 426</td>
</tr>
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<td>4</td>
<td>183; 196</td>
<td></td>
<td>9</td>
<td>465; 491</td>
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2.2 Sharpening Simulation

Several test simulations were carried out to build waterfall plots for different decay coefficient values. The angular velocity range from 0 to 47.75 Hz was investigated. Firstly sharpening with a single grinding wheel was simulated. In figure 4 there are one sided and two sided waterfall plots for different cases. Then a $DC=100$ sharpening cascade plot referred to the double grinding wheel configuration was also obtained (see figure 5). In the figures MAX indicates the maximum PSD peak value of the plot. The solid straight lines are drawn as reference for forward and backward travelling waves.

It can be assumed that the sudden approach of the grinding wheels produces an excitation which is mainly noise. So when the disk angular velocity is set to zero the corresponding PSD peaks show just the natural frequencies of the system.

In addition to one-sided cascade plots that are a classical tool to study vibration of rotating bodies, two-sided cascade plots are presented because they permit to distinguish between forward and backward vibrations and it is well known that instabilities are generated by backward travelling waves. Moreover one-sided cascade plots are not very clear and straightforward to interpret. Two-sided cascade plots break up forward and backward vibrations and the graphs become more readable. In the presented plots peaks related to 1X and 2X harmonics are evident. Moreover a band of constant frequency peaks can be observed around 100 Hz corresponding to stationary waves. Comparing the plots with different friction decay coefficients a clear trend is not found. The grinding wheel angular speed was checked in the three cases observing that it took different values so as to reach the same friction force in the sliding contact.

Fig.5 shows waterfall plots corresponding to the simulation with two grinding wheels. The plots are similar to the previously presented ones but they are somewhat more neat.

Figure 6 PSD of the disk edge (above) and of the grinding wheel (below) related to sharpening simulations with a torsional spring added to the axis of the disk model (145 rad/sec angular speed)
In the investigation on the influence of stick-slip on the disk vibrations a torsional spring was also added to the axis of the disk model. From some preliminary trials it can be inferred that decay coefficient dependence is more evident when the torsional spring stiffness \((K_t)\) is smaller (figure 6). Moreover when the peaks of the grinding wheel signal are higher, those of the disc signal are lower as if the former acted as a dynamic damper for the latter.

3. Experimental Investigations and Observations

The experimental verification of the simulated behaviour was carried out on a paper roll cutting machine. The steel disk blade initial diameter is about \(600 \text{ mm}\) and the tapered thickness varies from \(4.8 \text{ mm}\) to \(1.5 \text{ mm}\). The sharpening is achieved by means of two idle, opposed grinding wheels with a \(100 \text{ mm}\) diameter pressed down on the blade edge by means of two springs. The sharpening process is about \(0.5 \text{ sec}\) long and is periodically repeated.

Investigations over a machine in not-working and working conditions were carried out. In the former case the disk blade was excited by an impact hammer or by the grinding wheels suddenly pressing it. In the latter case the working conditions have been simulated. The sampling acquisition rate was fixed at \(5000 \text{ Hz}\) in order to obtain reliable results up to the maximum frequency of the range of interest, \(500 \text{ Hz}\). In the analysis of the recorded signals spectrograms have been used to find the onset moment of particular phenomena while power spectral densities (corresponding to different time intervals) have been plotted to show the principal harmonic components. Moreover the disk blade was rotated manually in order to find possible misalignments and thickness irregularities by means of a displacement transducer.

Firstly impact tests using the impulse force hammer were carried out on the stopped machine in order to find natural frequencies with contacting and not-contacting grinding wheels. PSD graphs showed two predominant peaks (about \(180\) and \(270 \text{ Hz}\)) when the disk edge was constrained by the grinding wheels and three peaks (\(105, 180\) and \(270 \text{ Hz}\)) when the disk was free. Secondly the impact tests by pressing the grinding wheels were carried out in order to be closer to the real working conditions. In figure 7 a spectrogram shows the instants of the grinding wheels pressing (\(1.2 \text{ sec}\)) and moving off (\(4.5 \text{ sec}\)). In figure 8 signal spectra of an accelerometer fixed to the disk blade in two different phases are presented. During the pressing the predominant peaks were \(105, 180\) and \(275 \text{ Hz}\) whereas during the moving off there was a general attenuation and the greater peak was at \(260 \text{ Hz}\).

Then sharpening tests in working conditions were carried out. The disk blade rotational speed was set to \(22.2 \text{ Hz}\) and a very short sharpening (about \(0.3 \text{ sec}\)) was performed. Instead of accelerometers, non-contact displacement transducers were preferred because less sensitive to environmental disturbance. In figure 9 spectra corresponding to different phases show that a \(200 \text{ Hz}\) peak appears during sharpening. The \(22\) and \(44 \text{ Hz}\) peaks, related to the disk blade rotational frequency, are always present.

Consecutive sharpening experiences were carried out in order to measure the grinding wheel angular velocity. The signal trigger, recorded during such tests, showed that the angular velocity quickly reaches a constant value of \(100 \text{ Hz}\). Such value is equal to about five times the disk blade angular velocity. This velocity was reached after a short transient (2-3 sharpening phases) but then kept almost constant. Figure 10 reports the results of intermittent sharpening experiments. The graphs on the left refer to sharpening periods whereas those on the right refer to periods in which the grinding wheels are not in contact. During the sharpening a \(200 \text{ Hz}\) peak is noticed even if its amplitude diminishes in the following phases whereas a \(100 \text{ Hz}\) peak increases. Moving the grinding wheels away from the disk such peaks vanish. Smaller peaks at \(22, 44, 66\) and \(88 \text{ Hz}\), related to the disk blade rotational frequency, and a \(180 \text{ Hz}\) peak during the no sharpening period can also be noticed. Increasing the pre-load of the two grinding wheels the \(200 \text{ Hz}\) peak
increases. On the contrary increasing the pre-load of only one grinding wheel the 200 Hz peak decreases. Lastly intermittent sharpening experiences at a higher temperature (the disk blade shaft was at about 65°C) were carried out. In this case the 180 and 200 Hz peaks disappeared.

![Figure 9 PSD of a short sharpening test](image)

A 200 Hz vibration, that is a multiple of the rotation frequency ($\times 9$), during the sharpening process was noticed. The amplitude of such vibrations diminishes increasing the grinding wheel angular velocity and the disk hub temperature, and is influenced by the grinding wheel pre-load. The following disk blade edge examination showed a nine lobe waviness, that increased in time until the disk blade was spoilt.

The influence of velocity can be related to the variation of friction conditions. The temperature increase concerns the disk inner zone, nearby the bearings, and can produce a tight fit at the disk inner edge. Consequently there is stiffening and an increase of the natural resonant frequencies. Lastly the pre-load of the grinding wheel springs can increase the impact force and so transient oscillations get larger.

The 200 Hz vibration, detected by a ground based observer can be explained referring to a disk-based observer. Such vibration might be related to a natural vibration with three nodal diameters, at 138 Hz.

In fact, $200\text{Hz} - 3\Omega^2 \pi = 134\text{Hz}$, close to 138 Hz.

Moreover the 200 Hz response frequency is about nine times the disk blade rotational frequency and so also the observed nine lobe thickness irregularities could be explained. The fact that the phenomenon is dependent on the wheels-disk sliding velocity suggests that friction conditions and stick-slip could be at the root of the detrimental vibrations.

![Figure 10 PSD of a repeated sharpening test](image)
These observations were partly confirmed by a simulation in which all data of the working conditions of the paper cutting machine were considered. Results of such simulation are reported in figure 11. For a correct comparison between numerical and experimental results it must be considered that first are referred to a local coordinate system whereas the second to a ground-based observer. In the PSD of the simulated disk blade signal a peak around 130-140 Hz appears (corresponding to about 200 Hz for a ground-based observer).

![Figure 11 PSD of the simulation of experimental working conditions: corresponding to a disk blade edge node (a) and to a grinding wheel node (b)](image)

### 4. Conclusions

The disk blade sharpening process has been examined both numerically and experimentally. The results of an analytical investigation carried out by means of the explicit code LS-DYNA® have been presented and discussed. The influence of the friction decay coefficient and of idle grinding wheels has been taken into account. Some hypotheses have been proposed to explain the phenomenon, among which the effect of stick-slip.

Moreover the results of an experimental investigation carried out on a paper roll cutting machine have been presented and the problems concerning the sharpening of a disk blade have been highlighted. A harmful vibration has been observed. This vibration is a multiple of the blade angular velocity and gives rise to a wavy edge thickness.

The model adopted in the simulation is suitably simplified and encouraging results, in agreement with the theoretical formulation, have been obtained. Nevertheless more specific and extensive experimental tests are planned to validate the model in order to obtain more significant results.

### 5. References