Parametric geometry modeling of the carotid artery bifurcation; the need for new flow metrics.

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INTRODUCTION:

Although well established correlations now exist between regions of reversed flow and sites of arterial disease [1], little evidence is available for methodologies that systematically compare how geometrical differences affect such relationships. Drawing on techniques developed in aerodynamic design, a parametric tuning-fork geometry model of the carotid artery bifurcation - based on the glass models described in [2] - is here employed to investigate the impact of bulb size and the angle of the internal carotid artery (ICA) on time-averaged shear stress. Since we seek to directly compare the effect of varying these geometric parameters, we demonstrate the need for a new metric, the integral of negative mean shear stress (INMSS), to highlight the stronger correlation between the extent of reversed flow and bulb width relative to the angle of the ICA.

METHODS:

In order to systematically simulate the flow through a range of different bifurcations a software system is employed that automatically generates new geometries from a baseline shape, constructs a suitable mesh and then solves the unsteady Navier-Stokes equations [3]. The power of the parametric computer aided design (CAD) model lies in the ability to generate alternative geometries by simply adjusting the values of appropriate parameters of a baseline shape (in this article, the angle of the ICA and the width of the sinus bulb are used). The baseline shape is based on the mean values in [4]. Although relatively sophisticated methods are available for populating the (design) space defined by the ranges of the (design) parameters, it is often useful to initially consider geometries at suitable upper and lower limits of these ranges. Here, we construct nine geometries produced by the permutations of small, intermediate and large ICA angles with small, intermediate and large sinus bulbs. The ICA angles are 9.80, 25.4 and 41.0 degrees; the intermediate angle is that of the baseline geometry and the other two are three standard deviations apart and symmetrical about the mean. The bulb widths are less straightforward to define since the bulb shape is actually manipulated by the control points of Bezier curves (such parameterizations are particularly useful because they facilitate shape control with a small number of parameters). In this case, the parameter used to vary the bulb shape is the x-coordinate of the control point of the outer ICA edge and its range of movement is set-up to produce an appropriate set of bulb shapes.

A mean pulse for the human carotid artery is used [5] with a time-step of 0.0001s and the time-averaged shear stress, $\bar{\tau}$, is stored for each boundary surface. Meshes of between 25000 and 30000 cells are employed in each geometry. These spatial and temporal resolutions are consistent those used in other sources and validated in previous mesh and time dependency studies [3].

RESULTS:

Figure 1 depicts the variation in $\bar{\tau}$ for a range of ICA angles with a large sinus bulb. We observe that, as the ICA angle increases, the peak value of $\bar{\tau}$ at the

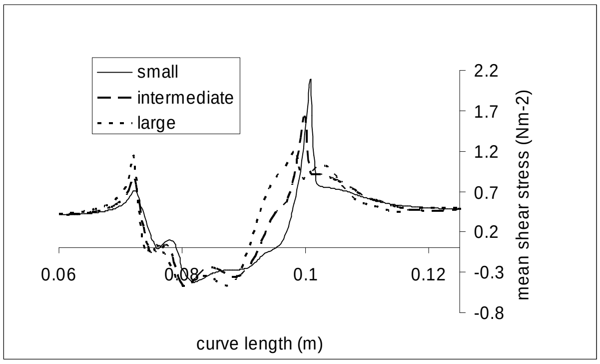

Figure 1: Variation of time-averaged shear stress on the outer ICA wall for a range of ICA angles at large bulb width.

entrance to the bulb increases, the gradient just downstream steepens leading to an earlier onset of the region of negative $\bar{\tau}$ and, at the end of the bulb, $\bar{\tau}$ returns to positive values further upstream. A similar variation exists for small and intermediate bulb widths. Additionally, for intermediate and large sinus bulbs, larger ICA angles produce a weaker peak in $\bar{\tau}$ at the downstream end of the bulb. Visual inspection of Figure 1 reveals that relatively small changes occur for all geometries with respect to both the minimum value of $\bar{\tau}$, $\bar{\tau}_{min}$, and to the variation in area below the x-axis at fixed bulb size. However, significantly larger variations in area are produced by increasing the bulb size at constant ICA angle. This is depicted in Figure 2 where, again, we observe little variation in $\bar{\tau}_{min}$.

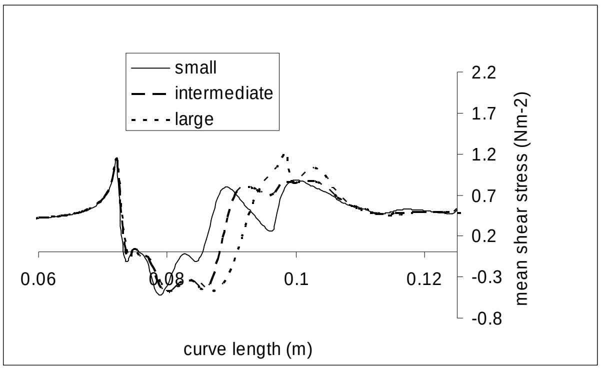

Figure 2: Variation of time-averaged shear stress on the outer ICA wall for a range of bulb widths at large ICA angle.

The persistently large negative shear in relatively large bulbs results from the strong vortex produced during systole that fills the ICA as evidenced in Figure 3.

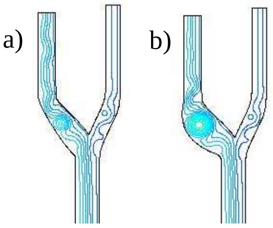

Figure 3: Streamlines of the flow at mid-points in the cycle for geometries with large ICA angle and (a) with small and (b) large sinus bulbs.

Since we are here concerned with the regions of negative $\bar{\tau}$, denoted by $\bar{\tau}_-$, and since we seek to identify suitable metrics (or objective functions) that can be used to quantifiably compare the impact different geometries have on the flow, the following metrics are evaluated for each geometry:

(i) the minimum value of $\bar{\tau}$,

$$m_1 = \bar{\tau}_{\min}$$

(ii) a length integral of $\bar{\tau}_-$

$$m_2 = \int \left|\bar{\tau}_-\right| dl_-$$

Figures 4 and 5 depict, respectively, the variations of m_1 and m_2 on the outer ICA wall for a range of bulb widths and ICA angles. The contours are based upon regressed surface fits to the data produced by the method of kriging [6] Both parameters are non-dimensionalised by the limits of their respective ranges. *A priori*, we may expect that a stronger region of recirculating flow and, hence larger (absolute) values of the metrics m_1 and m_2, will be produced by a large sinus bulb combined with a large ICA angle (relative to a small bulb and small angle). However, this relationship is only evident for m_2 (c.f. Figure 5) whilst m_1 is minimised at a low ICA angle and bulb width (c.f. Figure 4). (N.B. the integral of the region exposed to negative mean shear stress is moreorless independent of the ICA angle).

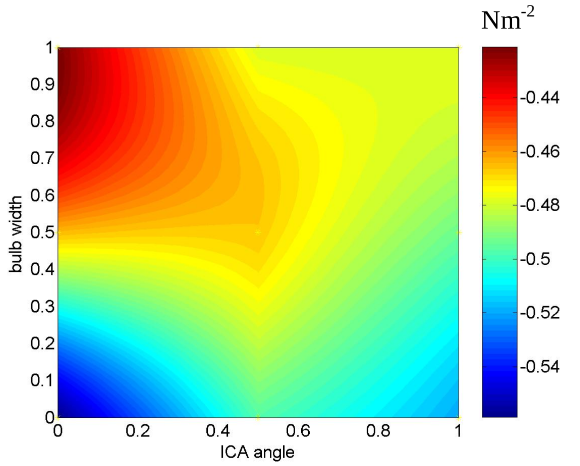

Figure 4: Variation of the minimum value of time-averaged shear stress on the outer ICA wall for a range of bulb widths and ICA angles.

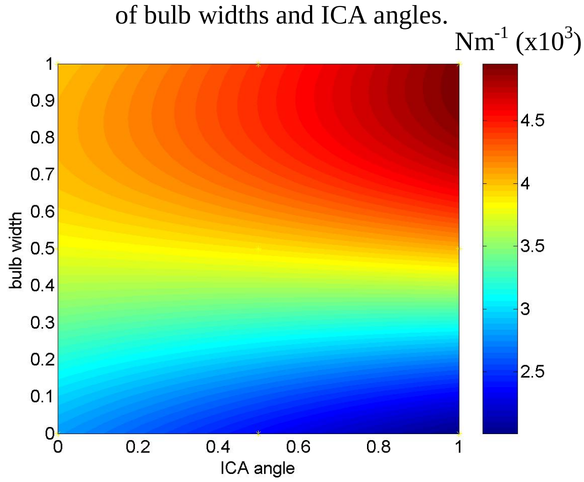

Figure 5: Variation of the length integral of time-averaged shear stress on the outer ICA wall for a range of bulb widths and ICA angles.

DISCUSSION:

The above results suggest that the metric, m_2, is the preferred quantity to use when comparing the impact of geometric changes on the flow of blood in the carotid artery bifurcation. Unlike $\bar{\tau}_{\min}$ it confirms the expected result of stronger recirculation for both larger ICA angles and sinus bulb widths. Additionally, it reveals a stronger dependence on bulb width particularly at larger ICA angles. Interestingly, at small bulb widths, m_2 decreases with ICA angle. Work is now underway to explore these relationships for a three-dimensional extension of the two-dimensional parametric geometric model considered here.

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