GROUP FORMATION AND COMPETITION:
INSTRUMENTAL AND EXPRESSIVE APPROACHES

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Abstract
We construct models of group formation designed to capture some of the key features of political and social competition. The models draw on the ‘citizen candidate’ approach and allow competition to be modelled as either compromise - where all groups influence outcomes; or conflict - where one group wins the right to dictate. We also consider both instrumental and expressive approaches to understanding group formation, respectively separately and then in a setting which encompasses both approaches.

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Keywords: Group Formation, Expressive Behaviour, Conflict, Compromise.

1 Introduction

The major objective of this paper is to contribute to the analysis of political competition. However, our starting point is to note that political competition is almost always best seen as competition between groups, rather than between individuals. These groups might be rival political parties competing within a highly structured constitutional framework that sets out the rules of competition in some detail, and where elections may be thought of as the major battle field on which political competition is played out. Alternatively, the groups might be rival gangs competing in an essentially unstructured environment where the idea of a battle field may be all too literal a description of the mode of competition. Political competition, as we conceive it, therefore covers the full range of activities from electoral competition to political violence. But, wherever in this range any particular case of political competition may lie, we identify the group nature of political competition as an important part of what makes any particular instance of competition political.

In most models of political competition in the rational actor tradition, however, the group nature of political action is glossed over - political rivals may
sometimes be referred to as ‘parties’, but - following Downs (1957) - they are almost always analysed as if they were individuals. In addition, citizens/voters are usually treated as separated from the political parties which compete for their support. However, to varying degrees in different societies, parties are actually the representatives of the distinct groups from which they emerged. We follow Demsetz (1990) in seeing political parties as appealing not just to an ‘external constituency’, but also to an ‘internal constituency’ and in this paper we wish to focus our attention upon the emergence of these internal constituencies. It will not, however, be our purpose in this paper to explicitly model the emergence of political parties. Rather, as a first and fundamental step towards that future task, our purpose is to model the emergence of political groups, some of which might develop into political parties.

In moving away from the narrowly individualistic model of political competition, we should emphasise that we are not abandoning the commitment to methodological individualism. We are merely concerned to push the level of analysis back one step to focus attention on the formation of the relevant political groups - whether parties, factions, gangs or whatever. The basic idea is to think about the emergence of political groups from individual behaviour as a preliminary step to employing those political groups as key features of a variety of models of political competition, although, of course, some discussion of the eventual roles of political groups will often (but not necessarily always) be relevant to the analysis of their formation. One clear aspect of political groups that must be built into the discussion from the outset, is that political groups are by their nature conflictual. We are not dealing with the emergence of consumption clubs (see Buchanan (1965) and Cornes and Sandler (1996)) where, at least in many cases, it is sensible to begin with the case in which clubs, once formed, do not interact with each other. Political groups, as we understand them here, are by their nature rivals in at least some key aspects of their activities.

The paper is organised as follows. In Section 2 we introduce the two key elements that we use to develop the idea of the endogenous formation of political groups. These are the citizen-candidate approach, which has been used to endogenise candidate emergence in elections, and the idea of expressive - as distinct from instrumental - political behaviour. Section 3 will briefly review two extant approaches to social and political groups that lie broadly in the rational actor tradition and that are of direct relevance to this paper. Section 4 will then develop a series of models that attempt to capture important elements of the process of group formation. Finally, section 5 will offer some concluding comments.

2 Key Ideas

As already indicated, the two fundamental ideas that will be key to our analysis are the ideas of endogenous political agents and of expressive political action. Each of these ideas may be thought of as developing in reaction to perceived limitations in the benchmark Public Choice model of political competition. In
the case of the citizen-candidate model of endogenous political agency, the relevant limitation was the ad hoc nature of the specification of candidates in 'standard' models of electoral competition, ad hoc both in the identification of the number of candidates to be considered (with or without any possibility of entry), and in the specification of their motivation. In the case of expressive political behaviour - and particularly expressive voting - the relevant limitation concerned the classic 'paradox of voting' as an example of the free-rider problem that applies in many large-number political situations, where it may be difficult to identify individual incentives to act on purely instrumental grounds.

We will use the citizen-candidate approach and expressive motivation in Section 4 to derive models of the emergence of political groups, but first, we will offer some brief introductory comments on each idea in turn.

2.1 Endogenous Political Agents - Citizen-Candidate Models

In response to the limitations of the standard model associated with the ad hoc and exogenous specification of political candidates, Osborne and Slivinski (1996) and Besley and Coate (1997) introduced a class of models which endogenises candidate emergence. The basic set-up in these models identifies a three-stage game. In stage one, each individual citizen faces a decision of whether to stand as a candidate or not; in stage two, voting takes place given a set of candidates; in stage three, policy is implemented given the results of the election. In both Osborne and Slivinski and Besley and Coate, individuals operate with the same preferences over policy outcomes as candidates as they do as citizen-voters, and the voting rule employed in stage two of the game is simple plurality voting so that the election always produces a single winner, and that winning candidate implements her ideal policy in stage three of the game. Hamlin and Hjortlund (2000) extend the analysis to the case of proportional representation, so allowing compromise between candidates in the third stage of the game and a richer sense of representation within the model. They also discuss the possibility of allowing office rents as well as policy considerations to motivate citizens in their choice of whether or not to stand as candidates.

While these citizen-candidate models clearly make a considerable step forward, they equally clearly retain the commitment to modelling political actors as individuals. It is of the essence of these models that individual citizens make individual decisions to become individual candidates. In order to extend the structure of citizen-candidate models to the study of electoral competition between groups we might think in terms of a four-stage game, where the first stage now identifies the endogenous emergence of political groups or parties; the second stage determines the choice of platform within each party; the third stage includes the election; while the fourth stage implements policy.\footnote{For an analysis which aims to achieve this extra dimension of political party emergence prior to policy emergence see Haan (1999). Note that Haan's model assumes that individuals are instrumentally motivated.}
a model would then be a specific model of political groups in electoral competition, rather than a more general model of political groups in forms of political competition that may range far beyond elections. An alternative line of thought that offers a less restricted account of the nature of the eventual competition between groups, and a more detailed account of the process of group emergence, would reinterpret the basic citizen-candidate model differently. We might now conceive of the first stage of the game as one in which potential group founders or focal points emerge which offer a set of group norms to be followed. The second stage of the game could be conceived as one in which citizens decide which - if any - group to join, and a third stage of the game as the forum within which groups compete and which determines political or social outcomes. We shall explore models of this form below.

2.2 Expressive Political Action

In response to the limitations of the standard model associated with the critique of the instrumentally rational political action in the face of the free rider problem, Brennan and Lomasky (1993) and Brennan and Hamlin (1998) provide an account of expressive voting that motivates political action by reference to direct, expressive benefits. The basic starting point of this line of argument is that in many cases of political action (such as voting in mass elections), individual action may be almost entirely inconsequential, so that any individual who was motivated only by instrumental considerations, and also faced a small cost of action, would not act. The more constructive part of the argument then suggests that once the individual is released from the simple instrumental calculus, she may engage in a more expressive calculus in which rather different considerations will weigh in determining the nature of the political act. Political acts will be opportunities to identify with certain causes, display loyalty to a particular candidate or party, or express some other feeling or prejudice at low cost. The idea of expressive voting, in itself, offers an explanation of certain types of political acts but it does not offer simple or direct normative analysis of those acts, since the link between expressive concerns and interests is by no means direct. At the same time, expressive voting is only one ingredient in any model of political process. The relevance and impact of expressive behaviour will depend greatly on the details of the institutional arrangements, and on other structural aspects of the model. It is important to note that expressive behaviour is no less rational than instrumental behaviour. Formally, this is just to say that both expressive and instrumental motivation can be explained side-by-side within a generalised utility function, with the particular circumstances of a choice situation selecting which type of motivation is the more relevant.

In the standard citizen-candidate models outlined above, all political agents are taken to act instrumentally. One aspect of this assumption is that behaviour in the first stage of the game is motivated by reference to its impact on outcomes.

\footnote{Applications of the idea of expressive voting are provided by Brennan and Hamlin (1999, 2000).}
in the final stage of the game. It is precisely this sort of assumption that is subject to the criticism from expressive behaviour, since the link between individual behaviour and final social outcome is so tenuous as to break any link of this type, once a cost of action (voting) is added and action is made voluntary. The expressive line of argument would have to offer some more proximate and non-contingent motivator of activity in the first stages of the game. We believe that the shift to the focus on political groups offers one such possibility. In particular, we believe that group membership may be motivated by direct, expressive considerations linked to the simple and proximate benefits derived from inclusion in, and identification with, a relevant group; rather than by indirect, instrumental consideration of the policy effects or other social effects that may arise at a later stage of the political process. This is not to deny that instrumental considerations may be important to some political actors at some stages of the game, but merely to argue that the balance between instrumental and expressive considerations as motivators of political action will depend on the structure of the game and the role of the individual within that structure.

One clear possibility raised by the introduction of expressively motivated political activity is that the behaviour in early stages of a political game may be largely - or even wholly - independent of the consequences of that behaviour in later stages. Thus, for example, decisions of whether or not to join a particular political group may be effectively divorced from the exact specification of how that group will operate to determine final political outcomes. The converse is, however, not the case. While individual behaviour may be divorced from its long-range and contingent consequences as a matter of motivation, the long-run outcomes are still consequences of the initial individual behaviour in a causal sense. This opens up an additional line of argument - characteristic of the expressive approach to political action - for identifying political outcomes as the unintended consequences of individual political action, even though the political action was fully rational in the relevant sense.

3 Related Approaches to Social and Political Groups

This paper is clearly linked to spatial theories of voting. However, we focus on a broader class of ‘group activity’ rather than limiting attention to the case of voting. Relevant ‘group activities’ may include obviously collective activities as attending meetings, but may also include more private activities such as following rules of behaviour laid down by the group. More generally, ‘group activities’ will be distinguished by the adoption of a distinct group norm. As already stated, the idea of expressive voting provides an explanation for why people actually vote and likewise, in the present setting, expressive motivation may provide an explanation for the adoption of a group norm, and hence group activity.

In this way, expressive motivation is linked to the classic Olson (1965) ex-
planation of how groups overcome collective action problems. Olson looked to the provision of selective incentives (either negative or positive) as inducements to group action, as any public good incentive is effectively zero. Applications of the Olson approach are myriad (see Sandler (1992) for a survey), for example Tullock (1971) analysed how revolutions might come to be provided.³ In this paper expressive considerations play the role of direct incentives to individuals to engage in group action.

Against this background two recent developments in the discussion of groups are particularly noteworthy. Hardin (1995) departs from the Olson model by emphasising the coordination aspect of the decision to form a group, rather than the free-rider or prisoner’s dilemma aspect. Kuran (1995) emphasises the importance of social psychology and argues that reputational concerns may be sufficient to induce individuals into preference falsification. These approaches to collective action deserve special attention for five main reasons. First, both emphasise that group behaviour is identified by the adoption of group norms. Second, the adoption of group norms provide a source of power to the group, which may lead the group into conflict with other groups. Third, they both provide a theory (in their different ways) of how unforeseen and very possibly unwanted outcomes may emerge. Fourth, the ‘issue space’ upon which the groups gather is interpreted much more broadly than is usual to include religion or ethnicity. Finally, both approaches operate within the broad rational choice tradition. We will discuss these two contributions in turn in slightly more detail.

3.1 Hardin - Coordinating Action

Hardin provides a relatively informal analysis of both the formation of groups and inter-group conflict. He starts from the observation that a primary reason for being in a group is to achieve power. Power comes in two forms: ‘coordination power’ that derives from individuals conforming to some group-specific norm or activity; and ‘exchange power’ which derives from the group’s ability to amass economic resources - either through production or through predation. Coordination power is seen as logically prior to exchange power. These two types of power may be seen as underpinning the Hobbesian social contract. In that case, the whole population co-ordinates to form a social contract whereby some liberty is surrendered to a sovereign. However this provides the basis for secure property rights and the accumulation of economic resources. Here coordination power is seen as a prerequisite to exchange power, where exchange power relates to production which depends on secure property rights and protection from predation.

Hardin extends these basic features to explain how separate and contending groups may form. Coordination power is derived from individuals coordinating around particular behavioural norms which provide the group with its identity. For some members this might be seen as the playing of a simple coordination
game where they face no internal conflict in pursuing these norms. However, others may face a prisoners' dilemma incentive to free-ride on the groups' activities, while enjoying the benefits of membership. This distinction between those facing a coordination game and those facing incentives to defect, separates the dedicated core of the group from the less dedicated fringe. However, the fringe are faced with social pressures from those that have co-ordinated that may prevent them from free-riding, and as such social pressures act as negative selective incentives.

Clearly, for inter-group conflict to arise, separate groups must form. Why do we not see the whole population conform around a single set of norms?

Why would members of a group wish to be different, to exclude non-members? Often because there might be benefits to membership. Benefits can take at least two quite different forms. First, some subgroup or coalition can benefit its members most quickly by excluding others from access to the limited resources. Here the group is a means to other goods.

Second, there may be straightforward benefits of comfort, familiarity and easy communication in one's group. (1995, p.76.)

The source of political conflict relates to predatory behaviour or 'rent-seeking' in public choice terminology.

In this account, identification with a group matters because it can lead to coordination for great power. That power might then be used more for destruction than for creation just because destruction is easier and more readily focused on specific, extant objectives. (1995, p.9.)

Thus, for political violence to emerge separate groups must form around certain types of norms, and at least one of these groups must then engage in predatory behaviour.

The models we develop below attempt to formalise some aspects of Hardin's account by discussing the potential benefits to be derived from group membership under two distinct headings - one associated with the external role of groups acting in potential conflict with other groups, and the other associated with the internal role of groups as providing its members with a sense of identity or belonging. His emphasis on group norms as the basis of group strength is followed here and the idea of a fringe and core of more or less dedicated members is developed in this model, where the less dedicated engage in less group activity.

3.2 Kuran - Preference Falsification

Kuran begins from the idea that when individuals perform an action they may receive utility from three different sources. He identifies intrinsic utility with the instrumental outcome of an action, reputational utility with the approval or
disapproval one receives from other people, and finally expressive utility with
the utility one receives from being true to one's self regardless of the actual
outcome. One result of the interaction between these three sources of utility is
that if the pressure of reputational utility is strong, individuals may conform
to certain types of group norms, even if they would privately prefer different
types of behaviour. Kuran calls this type of behaviour preference falsification,
since it amounts to hiding one's 'true' preferences out of a desire to fit into a
reference group.

Kuran uses his theory of preference falsification as a basis for discussing how
particular norms become embedded within a society and how they can change
rapidly through the mechanism of a reputational cascade. At this stage we will
not discuss this more dynamic part of his analysis. The key point for now is
that social pressures may exact what one says and what one does - that groups
imply their own norms of activity and may demand compliance to at least some
extent.

Kuran's theory is most powerful in providing a microfoundation for adhering
to group norms. However, while Kuran explains group solidarity, he fails
to explain either group identity or voluntary membership. Group solidarity is
achieved through the application of social pressure, but the preference falsification that follows does not provide a sense of individuals identifying with the
group. Equally, Kuran takes membership of a group as his starting point with
no option of either joining or leaving the group. While this is appropriate for
his discussion of ethnic groups, it is clearly not satisfactory more generally.

While the models to be developed below are similar to Kuran's in some ways
- Kuran distinguishes between instrumental/intrinsic utility and expressive utility (although our sense of expressive utility is not exactly that of Kuran), and in the
following models we depict individuals pursuing norms that would not be their
personal preference - there are also clear differences in that we focus explicitly
on the membership decision, and therefore on the potential trade-off between
costs and benefits of membership. Membership of a group may help to achieve
particular instrumental benefits, and may also help to forge a sense of identity;
but there may be costs of membership in terms of compliance to norms that are
not ideal from the individual's perspective.

4 The Models
As already indicated, the basic structure of the citizen candidate model will
provide our point of reference. However, there will be several departures from
the basic version of the citizen-candidate model. These may be introduced via
two distinctions: between instrumental and expressive behaviour on the one
hand, and between conflict and compromise on the other. As already outlined,
the basic point underlying the instrumental/expressive distinction is that an

\footnote{For example they may state political preferences simply to conform, such as support of
communist dictatorships (Kuran 1989, 1991 and 1995), or conform to norms of ethnicity
(Kuran 1998).}
essentially instrumental model of individual behaviour will always tie individual behaviour at the early stages of the model - the stage of group formation - to the eventual outcome of the competition between groups. In this way, instrumental actors may be said to 'see through' the structure of the model and map their own actions at each stage to eventual outcomes. The expressive line of argument, by contrast, denies this link and looks for more proximate explanations of political action. While, up to now, we have stressed the relative signiﬁcance of expressive motivation as opposed to instrumental motivation in political settings, we will initially present models where we assume that only one type of motivation is present. Models 1 and 2 will be purely instrumental and model 3 will be purely expressive. This allows us to make the distinction between them as sharp as possible. Once the nature of these extreme case models has been clariﬁed, model 4 will attempt to combine both instrumental and expressive elements.

Before turning in detail to the models, we shall ﬁrst discuss the underlying game structure of these models. As already mentioned, the citizen candidate approach adopted a three-stage game, which is solved by backwards induction to provide subgame perfect equilibria. For illustration we shall take the plurality rule game (as used by Besley and Coate and Osborne and Slivinski) as our reference point and consider each stage in turn.5

In the nal stage of the game, the only credible policy for the winner of the election is to implement her preferred policy, (where in the case of an electoral tie a winner is determined by the toss of a coin). This is apparent to all citizens in stage two of the game, and here a voting equilibrium will exist, such that - given the set of candidates, and the policies that each would implement if elected - each citizen’s vote is a best response to the votes of all other citizens. Finally, in the rst stage of the game, given the anticipated voting equilibrium and impact on nal policy outcome, each individual citizen will decide whether to stand as a candidate or not subject to some entry cost, so that an equilibrium of the entry game is identiﬁed.6 Hamlin and Hjortland follow the same formal structure, but instead of plurality rule, the policy in stage three is determined by proportional representation, which substantially affects the equilibria of the game.

The models to be presented here also maintain the same formal structure and general solution method. However, just as Hamlin and Hjortland changed one aspect of the Besley and Coate model to consider proportional representation, the models presented here will change other aspects (often radically) within the three stages of the game. However, the common outline of all of the models to be discussed may be sketched as follows: in stage one individual decide whether or not to provide a focal point around which a group may emerge, subject to an entry cost. Those that decide to act in this way will be referred to as the founders of groups and may be thought of as identifying a particular norm. In stage two, individuals who did not themselves found groups either attach

5 See Besley and Coate for a formal description of the game.
6 See Besley and Coate on the existence of an equilibrium. There will always be a mixed strategy equilibrium at the entry stage. However, the focus of their paper, Hamlin and Hjortland's and of this paper will be on pure strategy equilibria.
themselves to a group or choose to be independent. Finally, in stage three, the groups interact in some way to determine a social outcome that affects all individuals.

The instrumental/expressive distinction and the conflict/compromise distinction may now be located within this general outline. In the purely instrumental models the payoff to individuals will be specified solely in terms of the final social outcome determined by the interaction between groups; it is in this sense that individuals are modelled as seeing through the entire game to the final social outcome which will dictate their choices over whether to found a group, join a group or be independent. In the expressive model, by contrast, individuals base their group attachment decisions upon factors such as the size of groups they would prefer to be members of, and their location within those groups, without reference to the final social outcomes that may emerge from the interaction between groups. Note, however, that while social outcomes play no motivational role \textit{ex ante} in the expressive model, they will have an effect upon utility \textit{ex post}. The conflict/compromise distinction relates to the form of the eventual interaction between groups. Conflict identifies a relatively sharp form of competition, while compromise identifies a more moderate form of rivalry. More specifically, conflict situations will be ones in which groups face each other in a winner-take-all contest; so that the idea of conflict might be associated with simple first-past-the-post electoral rules or with recent development in political economy that stress the violent element in the emergence of social institutions (see Usher (1992) and Gar..nkel and Skaperdas (1996)). By contrast, compromise situations will be ones in which groups interact in such a way that the final social outcome can be defined in terms of a weighted average of the positions that would be chosen by each group if it had monopoly power; so that the idea of compromise might be associated with electoral rules supporting proportional representation and coalition building, or with ideas of a social contract. Since the contrast between conflict and compromise relates to the form of the eventual interaction between groups, it is clear that it will be significant as a motivation for individual action only in models of group formation that emphasise instrumental behaviour. Models 1 and 2 below will examine this distinction by viewing compromise and conflict respectively.

Formally, models 1 and 2 below are closely related to the models of Hamlin and Hjortlund and Besley and Coate respectively. This is because, as we have just said, the contrast between compromise and conflict relates quite closely to the distinction between proportional representation and plurality voting. In one sense, then, the discussion of models 1 and 2 may be seen as a reinterpretation of the formal structure of the Hamlin and Hjortlund and Besley and Coate models into the context of group formation, and an opportunity to establish ideas and appropriate notation for models 3 and 4 to follow. However, there is one further novelty of the sequence of models to be discussed that requires some further introduction - the activity rate of an individual member of a group. An individual's activity rate - or the total activity within a particular group - bears rather different interpretation in each of the models to be presented, and also provides a clear departure from the models of Hamlin and Hjortlund or Besley...
and Coate, since there is no analogue of the activity rate in a voting model. The broad idea of the activity rate is that it stands as a measure of the degree of involvement of the individual in the group. This might be interpreted as the level of identification that the individual feels for the group, or an index of the personal cost of membership, for example. We will comment more specifically on the interpretation of individual activity rates in each of the following models.

4.1 Model 1 - Instrumental Social Compromise

We begin with the case in which all agents are purely instrumental in their approach to decision making, and where the eventual form of competition between groups involves compromise. The issue space is taken to be one dimensional and the ideal points of individuals are assumed to be uniformly distributed on the interval $[0,1]$. As already mentioned, in stage one of the process, individuals decide whether or not to establish a group (become a group founder). If a group is formed, that group takes on the ideal point of its founder as its collective position. This is revealed by the group pursuing a set of behavioural norms that are uniquely associated with the preferences over social outcomes that characterise the founder. Under instrumental calculation, an individual will decide to found a group if the benefit in terms of the expected impact on the eventual social outcome outweighs the cost of group formation. This cost will be taken to be a fixed transaction cost, $c$, associated with establishing one's position as a group founder. The form of the payoff to individual $j$ who founds a group is:

$$U_j = \left( \sum_{i} x_{ji} - c \right) P$$

where $x_{ji}$ is $j$'s ideal point and $P$ is the eventual social outcome. We now need to explain how the outcome $P$ is determined.

In stage two, all individuals who have decided not to found groups will affiliate themselves with the group that is closest to them. This statement may be broken into two parts. First, that all individuals will join a group, so that there will be no independents or 'outsiders'. For the moment, we simply assume that for all individuals, the marginal benefit of group activity/membership is (weakly) greater than marginal cost over the relevant range and that there are no transaction or fixed costs to group membership. Second that each individual will join the group whose founder is closest to her, this can be demonstrated given the other assumptions of the model.

We now turn to the individual's activity within a group. In a fully specified model, this activity rate would itself be endogenous, however this would introduce additional complexity which we do not wish to explore in this paper. The idea that we wish to capture is that group members face alternative activities - activity within the group must be balanced against private activities outside of the group - so that the level of group activity can be expected to vary across group members. We will assume that the further away an individual is from the group's ideal position, the less will be that individual member's activity within the group - so that activists are identified as those close to the founder, while
more peripheral members will be less active in support of the group’s position. While we assume that there is no transaction or fixed cost of group membership, there is an implicit cost in terms of activity. This cost could be seen as the opportunity cost of time spent on group activity, and will be higher for individuals located further from the founder, thus lowering their activity rate. To simplify matters, we will assume that the activity rate of individual member \( i \), \( y_i \), is given by:

\[
y_i = 1 - \frac{1}{|x_j - x_i|}
\]

(2)

where \( x_j \) is the position of group founder \( j \), and \( x_i \) is the position of the individual member \( i \). For a particular group, with a founder at \( x_j \) we can then calculate the total group activity rate, \( Y_j \), by summing activity rates for all individual members or, in the continuous case that we study here, taking the relevant integral over all members of the group:

\[
Y_j = \int y_i \, dx
\]

(3)

In this particular version of the model, then, the activity rate is to be interpreted at the individual level as a measure of the strength of an individual’s contribution to the group, and at the aggregate level as a measure of the impact or weight that the group will exert in influencing social outcomes.

In stage three, the final social compromise emerges. We define this outcome as the weighted mean of the ideal positions of the groups, where the relevant weight for each group is related to that group’s activity level:

\[
P = \frac{\sum_j v_j x_j}{\sum_j v_j}
\]

(4)

where \( v_j \) is a normalized measure of the aggregate activity level of the group based at \( x_j \):

\[
v_j = \frac{Y_j}{\sum_j Y_j}
\]

(5)

A Nash equilibrium in pure strategies exists if:

\[
jP_i - x_j > 0 \quad \text{for all group founders } j, \text{ and}
\]

\[
jP_i - x_j < 0
\]

(6)

(7)

\footnote{As stated, we assume for the meantime that there is an instrumental benefit to group activity, which is positive enough to outweigh the marginal cost of activity over some range. However, it is just this sort of assumption that leaves models in political settings which are built on instrumental motivation open to attack.}
for all non-founders \(i\), where \(P_j\) is the outcome from \(j\) forming a group and \(P_{ij}\) is the outcome if \(j\) decides not to form a group. The following results all follow from these equilibrium conditions. Clearly the number of groups that form in equilibrium is endogenous. To rule out the possibility of no group emerging, we assume that \(U = z\) for all individuals when there are no groups, where \(z > c\), so that each individual would prefer to found a group rather than accept an equilibrium in which no groups emerge. It should also be clear that however many groups emerge, they must be distinct in the sense that no equilibria can exist in which more than one group founder occupies any given position (this mirrors Hamlin and Hjortlund’s lemma 1). To see this, simply notice that if two individuals were to found groups at the same position, the payoff for one of them would be greater if she disbanded her own group, with all members joining the other group. The second group would have no impact on social outcomes, but would incur the fixed cost. We will proceed by investigating equilibria in which there is just one group, before discussing two-group and n-group equilibria.

4.1.1 One-Group Equilibrium

To establish an equilibrium with the single group based at \(a\) we must demonstrate that no individual will wish to found a rival group. To prevent group formation at, say, \(b\) we require that:

\[
\frac{b \cdot p_j}{Y_j} \cdot X + \frac{a \cdot p_j}{Y_j} \cdot X \quad a \cdot i \cdot c < 0
\]

(8)

or

\[
\frac{aY_a + bY_b}{Y_a + Y_b} \quad a < c
\]

(9)

for all \(b > a\) and:

\[
\frac{a \cdot p_j}{Y_j} \cdot X + \frac{b \cdot p_j}{Y_j} \cdot X \quad c < 0
\]

(10)

or

\[
\frac{aY_a + bY_b}{Y_a + Y_b} < c
\]

(11)

for all \(b < a\).

To see how any position may support a one-group equilibrium, one may begin by considering the value \(c\) must take to allow the extreme positions of 0 or 1 to be one-group equilibria. Here our model departs significantly from
the model in Hamlin and Hjortlund since, in Hamlin and Hjortlund's model all votes count equally in support of the chosen candidate, whereas in our model, members contribute differentially to their groups depending on their activity rates. So, while, in Hamlin and Hjortlund's model, it is clear that the potential candidate with the greatest incentive to enter against a candidate located at 0 is an individual located at 1 (and vice versa), the analogous statement is not true in our model. This is because, ceteris paribus, a group founder at an extreme point generates less group activity than a group founder at an interior position.

In order to find which potential group founder would have the greatest incentive to form a group in the presence of a group located at 0, we substitute \( a = 0 \) in the left hand side of equation 9 to obtain:

\[
\frac{bY_b}{Y_a + Y_b}
\]

(12)

Plugging in the values for \( Y_a \) and \( Y_b \) and maximising with respect to \( b \) gives an optimum value of \( b \) of approximately 0.935, with a maximised value of the expression of 0.5052. Therefore, an individual located at 0.935 has the greatest incentive to form a group in the presence of a group located at 0, and this incentive would be sufficient to overcome any fixed cost less than 0.5052: So, if \( c > 0.5052 \) is certainly a sufficient condition for one-group equilibria. With \( c \) above this critical level, a group located at any position will constitute a one-group equilibrium. As \( c \) falls below the critical value the range of possible one-group equilibria tightens as described by equations 9 and 11 until the point where these two constraints intersect at \( c' = 0.1359 \). At this value of \( c \), 0.5 is the only location that could support one-group equilibrium.

We can summarize the conditions for one-group equilibria as follows:

\[ Y_a = \int_{0}^{b} (1 + ax) \, dx = \frac{1}{2}b^2 + \frac{1}{8}b^2 \]

\[ Y_b = \int_{b}^{\infty} (1 + bx) \, dx = \frac{1}{2}b^2 + \frac{5}{8}b^2 + \frac{1}{2} \]

After some manipulation, we obtain

\[ \frac{bY_b}{Y_a + Y_b} = \frac{\frac{1}{2}b^2 + \frac{5}{8}b^2 + \frac{4}{2}}{b^2 + \frac{1}{2}} \]

which yields:

\[ \frac{1}{2}b^2 + \frac{5}{8}b^2 + \frac{4}{2} \]

Maximising with respect to \( b \) gives an optimum value of \( b \) of approximately 0.935, with a maximised value of the expression of 0.5052.
i If $c > 0.5052$ any position can support a one-group equilibria.

ii If $0.1359 < c < 0.5052$, then point $a$ can support a one-group equilibrium if it satisfies both equations 9 and 11, which gives

$$b_i \frac{c(Y_a + Y_b)}{Y_b} < a < \frac{c(Y_a + Y_b)}{Y_b} + b$$  \hspace{1cm} (13)

for all $b$

A key point to note here is that one individual will become a group founder, but will attract no group activity. In an instrumental model, individuals are motivated to act in order to achieve instrumental gains. In a one-group equilibrium however, group activity will bring about no change in social outcomes and therefore the incentive to join a group is gone. As such, the outcome would be determined by one founder, who will receive no group affiliation. This seems strange when we think of dictatorships, which receive group-oriented behaviour. However, this is precisely the point. Group participation in situations where only one group exists is difficult if not impossible to explain using an instrumental model. Participation suggests that other factors are at work, as we shall explore later.

4.1.2 Two-Group Equilibrium

Clearly, two-group equilibrium requires both that two founders are willing to found groups in each others presence, and that no other founder would emerge. For two founders to be willing to run against each other requires essentially the reverse of the conditions described in the last subsection. So, for a potential founder located at $a < b$ to emerge alongside $b$, it must be that the case that:

$$b_i \frac{\mu aY_a}{Y_a + Y_b} + \frac{bY_b}{Y_a + Y_b} = c$$  \hspace{1cm} (14)

which reduces to

$$\frac{Y_a (b_j a)}{Y_a + Y_b} = c$$  \hspace{1cm} (15)

and for $b > a$ to be willing to form a group in the presence of $a$, it must be that:

$$\mu \frac{bY_b}{Y_a + Y_b} + \frac{aY_a}{Y_a + Y_b} = j \quad a, c$$  \hspace{1cm} (16)

which reduces to:

$$\frac{Y_b (b_j a)}{Y_a + Y_b} = c$$  \hspace{1cm} (17)

15
Note that as \( c \) falls from approximately 0.5052 (so that equilibria with more than one group become feasible) the co-existing groups may be closer together.

The next step is to demonstrate the conditions under which no third group will wish to enter. Hamlin and Hjortlund's lemma 2 eliminates the possibility of intermediate candidates in their election model, but this lemma does not carry over to the present case. Again, the explanation lies in the variable activity rate in this model. If all activity rates were constant, it should be clear that an intermediate group could have no impact on the eventual social outcome, given the weighted average nature of the compromise outcome. This is essentially the analogue of Hamlin and Hjortlund's lemma 2. However, with variable activity rates it should be equally clear that this argument fails. A new intermediate group would generate more activity out of its members than would have been generated by their membership of the existing groups, and this opens up the possibility of intermediate groups.

However, given the assumption we have made for activity rates, the possibility is rather remote. That is, the \( c \) has to be rather low before any intermediate group is sustainable in equilibrium. We can use the case in which the two established groups, \( a \) and \( b \), are located at 0 and 1 respectively to illustrate this, by observing that if \( c \) is sufficient to prevent the formation of an intermediate group in this case, then it will certainly be sufficient to prevent intermediate entry in all two-group equilibria. In this case a potential group founder, located at \( d \), would face the following incentive to form a group:

\[
\frac{d^2}{2} - \frac{d Y_d + Y_b}{Y_a + Y_b + Y_d} d^2 - \frac{d Y_d + Y_d}{Y_a + Y_b + Y_d} d^2 - \frac{d Y_d + Y_b}{Y_a + Y_b + Y_d} d^2 - \frac{d Y_d + Y_d}{Y_a + Y_b + Y_d} d^2
\]

Making the relevant substitutions, it is straightforward, if tedious, to check that the individuals facing the maximal incentive to form a group are located at approximately 0.8 and 0.2. The intuition here is clear enough, in the absence of the third group the compromise outcome will be 0.5. A new group can be worthwhile to its founder only if she has an ideal point significantly different from 0.5. However, if the third group's ideal point is too close to either 0 or 1, the new group will have little impact on the compromise outcome, since they would effectively replace one of the existing groups. The trade off between these two forces identifies the location of the individual with the maximal incentive. However, even when the incentive is maximised, it is rather small. The impact on the final compromise outcome is to shift it by only approximately 0.01445. Therefore, if \( c > 0.01445 \) there can be no formation of an intermediate group, and therefore no equilibria with more than two groups.

Of course, for a two-group equilibrium, we require not only that the two groups will form in each other's presence, and that no intermediate group will form, but also that there will be no incentive for a further group to form outside of the range \( a; b \). This will be assured if \( a \) (the left most group) is such as to preclude further group formation on the left, and \( b \) (the right most group) is such as to preclude further group formation on the right. Call the potential leftist group \( l \), and the potential rightist group \( r \), then the condition to prevent the formation of \( l \) is the natural extension of equation 11:
where $P_{ab}$ is the compromise outcome when $a$ and $b$ are the only groups in place. Similarly, the condition to prevent the formation of $r$ is:

$$\frac{aY_a + bY_b + lY_r}{Y_a + Y_b + Y_r} < c$$

(20)

To summarise the conditions for two-group equilibria:

i If $c > 0.5052$ no two group equilibrium is possible.

ii If $0.01445 < c < 0.5052$, groups located at $a$ and $b$ (where $a < b$) will form a two-group equilibrium provided that conditions 15 and 17 are satis…ed. This requires that:

$$\frac{c(Y_a + Y_b)}{Y_b} \leq \frac{\mu}{b}$$

and:

$$\frac{c(Y_a + Y_b)}{Y_a} + a \leq \frac{\mu}{a}$$

(21)

and that conditions 19 and 20 are satis..ed.

iii If $0 < c < 0.01445$ we require the additional condition that no intermediate group will emerge. This requires that:

$$\frac{aY_a + dY_d + bY_b}{Y_a + Y_d + Y_b} < c$$

(23)

for $d < \frac{a+b}{2}$ and:

$$\frac{aY_a + dY_d + bY_b}{Y_a + Y_d + Y_b} < c$$

(24)

for $d > \frac{a+b}{2}$.

The interpretation of the conditions in (ii) and (iii) are relatively straight-forward. Condition 21 and 22 ensures that $a$ and $b$ will form in each others presence, while conditions 19 and 20 ensure that there will be no outside groups formed. Conditions 23 and 24 ensure no intermediate entry when costs are low enough to allow this to happen. In particular, while for any value of $c$ in the identi..ed range, there will always exist a two-group equilibrium (many such equilibria, in general) it is not the case that one can select an arbitrary group founder $a$ and guarantee that there will be a second group founder $b$, such
that \((a; b)\) forms a two-group equilibrium. A key point to note now is that all individuals may be seen as having an incentive to engage in group activity, as their action will have some bearing on the overall social outcome.\(^9\)\(^10\).

4.1.3 Discussion of model 1

This model of instrumental social compromise builds quite directly on the Hamlin and Hjortlund model of proportional representation. The mechanism of compromise between groups is essentially similar to the mechanism of proportional representation in the voting context analysed by Hamlin and Hjortlund. The major difference lies in the idea of variable activity rates, which makes good sense in the context of the operation of political groups, but has no counterpart in a model of voting where all votes are of equal weight. This difference gives us a more realistic sense of a political/social group to which members attach themselves to varying degrees.

There are three further points to note. First, we have assumed no fixed cost of group membership, only a cost of founding a group.\(^11\) Second, note that in this model the group founders do not have to be representative of the group in the sense that they do not necessarily occupy a position at the centre of their group. Indeed, it can be the case in a two group equilibrium, that the founders of the groups are seen as extremists within their groups. Finally, we assume individuals believe that they will have an effect upon social outcomes. If they were not to believe this, they would not engage in group activity, since they incur a positive marginal cost in group action. Indeed, a feature of one-group equilibrium in the case discussed above is that it would attract no group activity.

4.2 Model 2 - Instrumental Social Conflict

If the model of instrumental social compromise is built on Hamlin and Hjortlund's study of proportional representation, the model of instrumental social conflict to be considered in this section relates most directly to Besley and Coate's analysis of the simple plurality voting rule, since both social conflict and the plurality rule are defined in terms of a winner-take-all competition between groups/candidates. Aside from the re-interpretation of the model, the key differences between the model to be developed and that of Besley and Coate concerns the explicit modelling of a cost of conflict.

We assume that this cost is borne by all individuals who engage in group activity and not just the founder. Furthermore, we shall make the (perhaps...
rather strong) assumption that the cost falls on all group members in the same way regardless of their own group activity level, and that the group with the most members wins the ultimate contest. With this in mind, activity rates play no driving role in the model of conflict.

We shall assume that the cost of conflict for a member \( i \) is given by

\[
\mu n_0 + \bar{\mu} \frac{n_o}{n_i}
\]

where \( \bar{\mu} \) is a weight and \( n_i \) is the number of individuals active within the group of which \( i \) is a member and \( n_o \) is the number of individuals in groups of which \( i \) is not a member.

The payoff functions (1) and (2) are now rewritten as:

\[
j_P i x_i j - j_P j x_j j + \mu n_o \frac{n_o}{n_i} > 0
\]

for all founders \( j \)

\[
j_P i x_i j - j_P i x_j j + \mu n_o \frac{n_o}{n_i} < 0
\]

for all non-founders \( i \)

We must also pay attention to the payoff functions of all non-founders in stage two of the game. They now incur a conflict cost if they are group members. Thus, they will only join groups if the instrumental benefit of shifting the social outcome outweighs this conflict cost. This is formalised as follows

\[
j_P d x_d i - j_P i x_d i + \mu n_o \frac{n_o}{n_d} > 0
\]

for all members \( d \) and

\[
j_P k x_k j - j_P j x_k j + \mu n_o \frac{n_o}{n_k} < 0
\]

for all independents \( k \).

4.2.1 One-group equilibrium

Two conditions are required for a one-group equilibrium to exist:

i. If no individual stands as a founder, the payoff to each individual is \( z \), where \( z > c \). As before, this simply assures us that at least one individual will found a group.

ii. No second individual wishes to enter as a founder. Consider the possibility of an individual located at \( b \) entering against \( a \) such that the group founded at \( b \) would win the eventual conflict with certainty. In this case:

\[
j_b x_a j < c
\]

is sufficient to ensure that \( b \) does not enter. Note that if \( c > 1 \), any position can constitute a one group equilibrium, and that as \( c \) approaches zero,
the set of possible one-group equilibria shrinks until 0.5 is left as the only potential one-group equilibrium. Note that in this case \( \bar{n}_b / n_a = 0 \), which simply means that there is no actual conflict in a one-group equilibrium. In addition, since there is no individual, instrumental incentive to join the single group, we will see no participation in group activity, so that all except the founder will be independents. This is then a limiting - almost degenerate - case in which the relatively high cost of acting as a founder and the potential costs of conflict between groups yield an outcome in which there are no effective groups at all.

4.2.2 Two-group equilibrium

The conditions for a two-group equilibrium are as follows:

i. For any positive cost of entry an individual will not wish to enter if she expects to lose. Therefore, only individuals who expect to obtain the same level of membership will compete against each other. This requires that they occupy symmetric positions around the median.

\[ \frac{a+b}{2} = 0.5 \]

ii. For \( b \) and \( a \) to run against each other requires

\[ \frac{1}{2} j_{bi} a_j , c + \bar{\mu} \left( \frac{n_b}{n_a} \right) \]

where \( \bar{n}_b / n_a = 1 \), we get;

\[ j_{bi} a_j , 2(c + \bar{\mu}) \]

The result must be a tie for no individual would found a group knowing he is going to lose. That is, his payoff in terms of effecting the social outcome would be zero, but he would still incur entry and conflict costs. Thus he would be better off not entering.

iii. No other individual wishes to enter against \( a \) or \( b \). Exactly which kind of entrants will be deterred depends on whether we assume ‘honest’ attachment (that is one joins the group founded closest to you) or ‘strategic attachment’ (a possible Nash equilibrium strategy in the attachment stage of the game, would be to join a group founder not located closest to you, in order to prevent one’s least preferred founder from winning). Allowing for strategic attachment expands the set of possible two-group equilibria, to any set of symmetric individuals, whereas, assuming honest attachment would prevent extremists from being a two-group equilibria.

For two group equilibria to exist, it must be the case that \( \bar{n} \leq 0.5 \) and that as \( j_{bi} a_j \) become closer together, the value of \( \bar{n} \) must fall to allow for two-group equilibria to continue to exist. This follows Besley and Coate in that we are simply adding \( \bar{n} \) to \( c \) in the decision process for potential group founders.
Where this model differs is in the behaviour of non-founders. In Besley and Coate all individuals who are not indifferent between candidates vote as it is costless. This no longer holds here. Only individuals $d$ for whom

$$jd_i \leq 0.5(jd_i aj_i + jd_i aj) , \quad \text{if (32)}$$

or

$$jd_i aj_i, \quad \text{if (33)}$$

when $d$ is closer to $b$ and

$$jd_i bj_i, \quad \text{if (34)}$$

when $d$ is closer to a will participate in group activity.

Note that for individual $0.5$ the payoffs for group activity is $0$. Therefore if $\bar{r} > 0$ individual $0.5$ will not participate. This is unsurprising because he is indifferent between the two groups and this is no different to the Besley and Coate scenario. However, where the result does differ here is that individuals who do have a preference over one group founder than another will not participate if $2^\bar{r} > jd_i aj_i jd_i bj$. Therefore if $\bar{r}$ is high few individuals will engage in social conflict and extremists are the most likely combatants in two group equilibria. In fact if $\bar{r} = 0.5$, and $c = 0$, $1$ and $0$ will sustain a two-group equilibrium, but no other individual will wish to engage in group activity. As $\bar{r}$ falls the range of two group equilibria expands (assuming $c$ is held constant at a low value) as does the number of individuals prepared to engage in group activity. For all individuals to engage in group activity requires that $\bar{r} = 0$, but this in fact means there is no conflict. Thus it is not possible to have group conflict with all individuals participating.

4.2.3 Discussion of Model 2

By shifting the emphasis to conflict rather than entry costs, we draw attention to the cost-benefit analysis facing potential members of groups (followers rather than leaders) as well as the founders. As is the nature of plurality rule models with complete information, individuals may be pivotal in determining the overall outcome, and this provides the claimed instrumental incentive to participate. The assumptions used here greatly simplify the analysis of conflict, for instance, assuming that larger groups automatically win ignores a host of interesting issues in the modelling of conflict (see Hirshleifer (1988) and (1995)). Further, we have not discussed how group activity actually translates into group conflict, and whether individuals with different roles within a group may incur different costs of conflict. For now though, this simplified analysis serves our present purposes. It demonstrates that, as conflict costs increase, group participation diminishes until $\bar{r} > 0.5$ where no group activity will exist at all. Further, where individuals do participate they have calculated that the instrumental benefits

\[12\] These issues will be a central concern of later work.
are greater than the costs. Note again, as in the case of the compromise model, that the group founders do not have to be representative of their groups. These observations provide a useful benchmark for the expressive model that we shall turn to now.

4.3 Model 3 - Expressive Motivation

A purely expressive model is sharply different from its instrumental counterpart because we no longer need to discuss actual social outcomes in analysing initial decisions to join a group. Behaviour is assumed to be unrelated to overall social outcomes as individuals believe that the effect of their actions on overall outcomes are negligible. An individual’s motivation will be assumed to be determined by two factors: utility gained from the size of the group; and the individual’s location within the group relative to the founder. Clearly, actual utility ex post will be affected by...al social outcomes, but these could be viewed as externalities over which the individual has no effective influence ex ante. Turning attention to preferences for group size captures the sense that individuals are motivated to expressing themselves within a ‘group frame of reference’.

The “utility function” for an individual who is a member of group is given by

\[ U_{ij} = f(n_j; x_{ij} - x_{ij}) \]  

(35)

If an individual stands as founder his utility will be given by

\[ U_{ii} = f(n_i; 0) - c \]  

(36)

and if the individual chooses to be independent of all group adherence (where I stands for independent), his utility will be

\[ U_{II} = f(1; 0) \]  

(37)

where \( U_{ii} \) may be positive, negative or zero depending on an individuals ideal group size. \( U_{ij} < 0 \) reflects the idea that individuals become worse off as the distance between themselves and the group founder increases. If we were to ignore the issue of distance from the founder, then this model would in effect become a simple club good model. The idea here is similar; one may wish to...

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13 In this sense, when costs of conflict are high, we should expect to see very little group activism. However, just as we do see group participation even where only one group exists, we often see high levels of group activity where the conflict costs are substantial. Again, this points to a model less tied to eventual social outcomes.

14 In addition to the work by Hardin and Kuran discussed earlier, this section relates to a paper by Van Winden (1999) calling for a group frame of reference in political economics. It also relates to a recent paper by Akerlof and Kranton (2000) which provides a model for how group identity affects decision-making. Finally, the purely expressive model presented here has strong similarities with the sorting/segregation models in Schelling (1978).

15 Note again that we use the term utility function loosely here as ex post utility will be affected by social outcomes.
be in groups simply for reasons of camaraderie and communication. When the
group becomes large, benefits such as these may be diminished as congestion
sets in. We assume (for ease) that all individuals have the same preferences over
the size of the group, and differ only in their locations.

In the analysis to follow, to clarify matters, we will assume an additively
separable functional form as follows. In the case where individuals join a group
with founder \( j \) their utility is:

\[
U_{ij} = \beta(g(n_j)) + \phi(l(j)) \cdot \xi_{ij}
\]  

(38)

As indicated we assume that the effect of changes in \( n_j \) upon utility may be
either positive or negative, depending upon the size of group individuals prefer
to be in. However, greater distance from the group founder unambiguously
makes the individual worse off. \( \beta \) and \( \phi \) are weights upon these factors. We will
use this form of the utility function to derive conditions concerning equilibria.

If an individual stands a group founder, his utility will be:

\[
U_{ii} = \beta(g(n_i)) + \phi(l(0)) \cdot c
\]  

(39)

and if he chooses to be independent his utility will be:

\[
U_{ij} = \beta(g(1)) + \phi(l(0))
\]  

(40)

We will assume that \( n \) must be greater than one to be defined as a group.

The structure of the game to be played here is that in stage one group
founders enter, in stage two individuals who did not found groups decide whether
to attach themselves to groups or decide whether to be independent. In stage
three, the outcomes in terms of group size and location within groups are given.
This set-up for the expressive game would require much more specification to
allow for the kind of numerical solutions which we derived in the instrumental
models. However, this simple approach allows to set out clear conditions for
equilibria. We provide these conditions for one and two-group equilibria and
then provide a fairly informal discussion as to how these equilibria might come
to exist.

4.3.1 One-Group Equilibrium

We assume that if no groups form the value of being independent is inferior
to being a group founder (even if no other individuals join one's group),
this is given by

\[
U_{ij} \leq c \geq U_{ij}(0)
\]  

(41)

where \( U_{ij}(0) \) refers to the utility that individual \( j \) would receive from
being independent when no groups have formed. This ensures that one
individual will enter as a group founder.
To ensure that no other individual wishes to enter against individual \( j \), we require that for all other individuals either: they prefer to join group \( j \) or they prefer to be independent, rather than enter as founder. In the first case this means that for all individuals \( i \);

\[
U_{ii} + c < U_{ij}
\]

which becomes;

\[
\otimes(g(n_i) - g(n_j)) + \theta(l(0) - l(x_i - x_j)) < c
\]

The case of independence means that

\[
U_{ii} + c < U_{ii}
\]

which becomes

\[
\otimes(g(n_i) - g(1)) < c
\]

How might one-group equilibria emerge? Note that three factors determine the equilibria in this model, the preference for the size of the group one would like to be in, preferences for location within a group and the cost of entry. To keep matters tractable, we shall start by ignoring the effects of entry costs (by assuming they are constant at some arbitrary low level) and we shall assume that \( \theta = 0 \), thus preferences are only for group size. Let us also assume, for simplicity, that individuals wish to be in a group with the whole population. It should be clear here, that a one-group equilibrium will emerge and further, the founder may be located at any point on the distribution.\(^{16}\) Indeed, even if we assume that preferences identify ideal groups smaller than the whole population, but still very large, it is still highly likely that there will be a one-group equilibrium.

Now let us assume that \( \theta > 0 \), so that individuals care about their location within groups. How might this affect one-group equilibria? Once again ignoring the effects of entry costs and still assuming that the preference is for very large groups we should expect the set of potential one-group equilibria to be reduced, so that only more central individuals could emerge as founders in one-group equilibrium. In this case, extremists will not be able to found groups unopposed, as individuals at the opposite extreme will also wish to found groups.

Finally, let us consider the effect of differing cost levels. The effect of high costs are twofold, first, they make the emergence of one-group equilibria more likely, for instance even if preferences are for small groups joined with a deep concern for one's location, if costs are set at an extremely high level, only one individual will found a group in equilibrium. Second, a related point, is that the effect of high entry costs is to allow for a wider set of potential points for one-group equilibria. Obviously, if the costs of entry are low and the preference is for small groups then one-group equilibria are not likely to be possible.

\(^{16}\) Individuals are assumed to want to be in a group with everybody else, but since we assumed \( \theta = 0 \), they will not care who the founder of that group will be.
In summary, then, one-group equilibrium will be associated with (a) preferences which identify large ideal group size, (b) preferences that attach relatively little weight to location within group and (c) relatively high costs of entry as a group founder.

4.3.2 Two-Group Equilibrium

This requires that two individuals are prepared to enter as founders against each other and that no other individual wishes to form a group.

A two group equilibria founded by \(i\) and \(j\) requires that for \(i\)

\[
U_{ii} - c < U_{ij}
\]  
(46)

\[
U_{ii} - c < U_{il}
\]  
(47)

These conditions are clearly replicated for \(j\):

If we take individual \(i\) and plug in the functional form, conditions 46 and 47 become;

\[
\delta (g(n_i) - g(n_j)) + \circ (l_i | j | i | x_i j | x_i j) , c
\]  
(48)

\[
\delta (g(n_i) - g(1)) , c
\]  
(49)

For all other individuals \(k\) it is required that either

iii

\[
U_{kk} - c < U_{ki}
\]  
(50)

or

\[
U_{kk} - c < U_{kj}
\]  
(51)

or

\[
U_{kk} - c < U_{kl}
\]  
(52)

Constructing n-group equilibria would require going through this process for situations where there are more than two groups.

How might two-group equilibria emerge. Similar observations to those made for one-group equilibria may be made. First, if the preference is for very large groups, there must be a concern with location to enough of an extent to allow for two individuals to enter at stage one. Second, if the preference is for moderately sized groups, then the concern with location can be reduced to some extent and
two groups may still emerge. However, some concern over location would be required for segregating equilibria to emerge.\(^{17}\) Third, costs of entry cannot be so high as to prevent a second group emerging. Finally, if costs are low and the preference is to be in small groups, two-group equilibria are unlikely to be possible.

4.3.3 Discussion of Model 3

In this section we have set out the conditions for group formation in one or two-group equilibria in a model containing a sharply different source of motivation to that of the previous instrumental models. Individuals now form and join groups on the basis of a preference for interaction within groups divorced from the eventual social outcomes which the interaction between groups brings about. We mentioned earlier that while social outcomes play no role \emph{ex ante} in individual decision-making, they do carry consequences \emph{ex post}. In this case individuals may be engaging in group activity under expressive motivation which they would not have engaged in at all under purely instrumental motivation.

4.4 Model 4 - Instrumental/Expressive Motivation

We now wish to construct a model which incorporates both instrumental and expressive motivation. The instrumental motivation contained in the models for social compromise and conflict constructed earlier is generated by a shift in the social outcome towards one's preferred outcome. The expressive motivation lies in one's preferences over the size of the group one belongs to, and one's location within that group.

We can formalise this as follows, the utility function for an individual \(i\) who is a member of group \(j\) is given by

\[
U_{ij} = f \left( n_j; jx_i; x_{ij}; jP_{ij} - x_{ij} \right) - \frac{\mu n_o}{n_i} \tag{53}
\]

where \(n_j\) is the size of group \(j\), \(jx_i\) is \(i\)'s location within group \(j\) and \(jP_{ij} - x_{ij}\) is the difference between the social outcome and the individual's preferred social outcome, given that he has chosen to be a member of group \(j\). Obviously utility is declining in \(jP_{ij} - x_{ij}\) and this reflects the instrumental element to group involvement. \(- \frac{\mu n_o}{n_i}\) reflects potential conflict costs and clearly this term is not relevant in models of compromise.

If \(i\) stands as a founder, his utility function is given by

\[
U_{ii} = f \left( n_i; 0; jP_{ii} - x_{ij} \right) - \frac{\mu n_o}{n_i} \tag{54}
\]

\(^{17}\) A segregating equilibrium can be defined as containing groups whereby if \(x_i\) and \(x_{i+r}\) are the boundary members of a group, all individual \(x\) such that \(x_i < x < x_{i+r}\) are also members of the group.
Finally if \( i \) opts to be independent, his utility function is given by

\[
U_{iI} = f(1; j|P_{iI} \ i \ x_{ij})
\]  

(55)

If we continue to work with the more specific utility function used earlier, the conditions 53, 54 and 55 become

\[
U_{ij} = g(n) + h(\lambda P_{ij} \ i \ x_{ij}) \ i - \frac{\mu}{n} \frac{\eta}{\eta_i}
\]  

(56)

where \( \pm \) is a weight upon the instrumental factor.

\[
U_{ii} = g(0) + h(\lambda P_{ii} \ i \ x_{ij}) \ i - \frac{\mu}{n} \frac{\eta}{\eta_i}
\]  

(57)

\[
U_{iI} = g(1) + h(\lambda P_{iI} \ i \ x_{ij})
\]  

(58)

4.4.1 Discussion of mixed model

As before, the mixed motivation approach relates to two possible settings: social compromise and social conflict. We could go through the rather tedious task of setting out the algebra for one and two-group equilibria to exist in each of these environments, but instead we shall provide an informal discussion of how expressive concerns may impact on the instrumental equilibria already identified, and in so doing, pick out what we consider to be the most striking features of combining the two motivations.

Consider first, the setting of social compromise (model 1). A number of features were associated with the equilibria derived there. In the case of one-group equilibria, we found that costs of entry had to be above a certain level for equilibria to exist, and that the higher the costs, the larger the set of potential equilibria. In addition, there would be no incentive for group participation (recall that since no instrumental incentive to action would be on offer for group action, a positive marginal cost would rule out the possibility of group action). The addition of expressive motivation modifies this conclusion in two main ways. First, the factor of cost is less relevant if individuals wish to be in very large groups (and do not have too deep a concern about their within-group location). In addition, participation will generally arise, individuals may now obtain direct expressive group benefits, rather than indirect instrumental benefits. This is similar to the idea that an expressive model of voting will account for participation even when the candidates on offer are identical.

In two-group equilibria, under purely instrumental motivations costs were required to be below a certain level, there was no requirement that founders should be symmetric around the median position and participation was now possible (assuming that positive marginal instrumental returns to group action outweighed positive marginal costs). The addition of the expressive line of
reasoning to some extent augments these conclusions, such that it produces additional benefits to participation and concerns regarding one's location within a group are similar to concerns regarding one's location with respect to the social outcome. However, a difference is that expressive concerns may lead some individuals to choose to join a group where the founder is located further from them. This is possible if a preference to be in a large group plays a major role in the individual's utility function, and this may be so even when preferences for group size are the same across all individuals. In the instrumental two-group equilibrium, individuals always join the group located closest to them.

Let us now consider the other possible setting of social conflict with both instrumental and expressive concerns. Similar to the compromise setting, one-group equilibria in the conflict setting may arise due to high costs of entry and expected conflict. If these costs were lowered, the set of potential one-group equilibria shrinks until 0.5 would be the only remaining point as costs approach zero. For the same reasons as for compromise, there would be no group participation. Adding expressive concerns here has the same effect as it did for one-group equilibrium in the compromise setting. Significantly, it provides a rationale for group participation, and a desire to be in large groups would add to the instrumental reasons for a one-group equilibrium emerging.

Under social conflict, costs were required to be below a certain level for two-group equilibrium to emerge. The group founders had to be symmetric around the median, and the level of participation depended upon the conflict cost. The introduction of an expressive component significantly alters this picture. A radical example of how equilibria may be altered, is that a (certainly) losing group may form in equilibrium, whereas under purely instrumental motivation, all groups must have a positive probability of winning. For a losing group to emerge in equilibrium, the expressive return would be required to outweigh the expected conflict cost. This is simply to say that the desire to be in a group of like-minded individuals may be strong enough to outweigh the associated costs of conflict even when it is acknowledged that there is no prospect of winning in the competition for social outcomes.

5 Conclusion

This paper has attempted to reinterpret and extend the model of political competition to provide for a focus on political groups rather than individuals, and to forms of competition between groups more general than elections. To this end we have adopted and adapted the citizen candidate approach to provide the formal basis for endogenising group emergence, and studied both instrumental and expressive motivations and two forms of competition between groups.

We believe that the inclusion of expressive motivation allows us to understand examples of group behaviour that would be difficult to rationalise using only instrumental motivation. For instance, one significant advantage of the

\[ 18 \text{We do, however, accept that the model of conflict used in this paper is highly stylised.} \]
mixed-motive model outlined here is that it offers an explanation of the existence of political groups that seem to have little or no realistic chance of affecting social outcomes. Examples might include small minority parties in situations where the electoral system offers such parties no real prospect of power or influence; or small revolutionary groups that have little or no prospect of success. The explanation lies not in attributing such groups false beliefs about their own instrumental significance, but in understanding the expressive value of group membership as a potentially powerful influence on behaviour.

The models derived here could be viewed as a starting point for a sequence of studies. We have provided an account of the formation of competing political groups. Given that we model group members as possessing both instrumental and expressive concerns, a next step will be to ask how political organisations emerge out of these basic groups, and how groups provide incentives for group members to supply leadership or organisational services that may improve the prospects of the group. We would also be interested in how political representatives with the support of their group may choose violence rather than ‘normal politics’ as a means to achieve group goals - which might be thought of in terms of a choice between forms of competition. In addition, normative issues are raised by the models outlined here, since the outcomes that emerge may be inefficient in the standard sense. This then leads to a consideration of whether groups (or founders) can reach constitutional agreements that may serve to improve the outcomes that emerge. The role of expressive rather than instrumental motivations in providing a different perspective on constitutional issues is one that has yet to be fully explored.

References


