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Credibility of Optimal Monetary Delegation: Do We Really Need Prohibitive Reappointment Costs?

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Abstract

The paper examines the current debate on the real effectiveness of delegation in overcoming the problem of time inconsistency that a- icts discretionary monetary policy. An important contribution by Jensen has shown that, when the government is unable to credibly carry out optimal policy and delegates monetary policy to a central banker with an announced incentive scheme, optimal policy can be credible only if reappointment costs are prohibitive. This finding is questioned in the present analysis. In particular we show that, when delegation is not considered as an alternative, but rather as supplementary, to reputation and is conducive to reputation building for the central banker, the circumstances under which optimal delegation can be credible need not be so extreme. This different result is based on the constraint that the central banker's reputation for low inflation imposes on the government's temptation to deviate from its announcements and on the role played by incentive schemes in strengthening the central banker's reputation.

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1 Introduction

Rogoff's (1985) article is regarded as the first example of strategic delegation in monetary policy. The proposal to change monetary institutions to deal with the issue of time inconsistency and the alleged inflationary bias is the core idea behind the approach of monetary delegation within the literature started by Kydland and Prescott (1977) and Barro and Gordon (1983a). Recently this approach has been extended by the work of Persson and Tabellini (1993), Walsh (1995), and Svensson (1997) among others with the introduction of incentive schemes or policy targets in order to completely remove the inflationary bias.[1] Despite these positive achievements the literature on monetary delegation has been recently criticised by McCallum (1995, 1997), who argues that as there is always the temptation for the government to renege on the chosen monetary institution. Hence the institutional remedies proposed by this approach do not fix the problem of time inconsistency but merely relocate it.[2,3]

Persson and Tabellini, amongst others, have replied to this criticism

[1] Persson and Tabellini (1997) and Walsh (1998) are splendid examples of the most recent available reviews on the literature on monetary delegation, with insightful discussions on the issue of the credibility of optimal monetary delegation.

[2] Posen (1995) also criticises the delegation approach and provides some empirical findings that, in contrast to the previous empirical evidence, suggest that the relationship between central bank independence and disinflationary credibility is not supported. He concludes that central bank independence alone is not sufficient. What is also needed is the presence of a coalition in society committed to protecting the central bank independence as necessary for achieving low inflation. In particular he concludes that, as the financial community is the critical constituency that influences the central bank, the outcomes of monetary policy will predominantly reflect that of the monetary policy desired by the financial community. However, no formal analysis is presented and as shown by Alesina (1995) his empirical findings are controversial.

[3] McCallum refers his criticism only to the Walsh contracting approach. Also Obstfeld and Rogoff (1996) make the same criticism referring to the Walsh solution. There is the view that the Rogoff's approach is immune to McCallum's criticism. On that regard Alesina (1995, p.289) wrote: 'it is institutionally harder to dismiss a "conservative" central banker than it is for the policymaker simply to renege on a policy announcement made without the independent conservative agent'. However in his argument there is the implicit assumption that the presence of reappointment costs will deter the government from over-ruling the banker or sacking him and appointing a less conservative one. If, following Jensen (1997), we interprete the principal-agent approach as a complex institutional arrangement based on a structure of incentives costly to change, then Alesina's argument should hold also for the Walsh approach.

(1997, p.32) by observing that: "in the model that dominates the literature, what is needed is a high cost for changing the institution within the time horizon of existing nominal contracts" and "the cost of suddenly changing the institution could also be a loss of reputation".

This premise, which has been implicitly assumed in the standard literature on delegation, has been recently challenged by Jensen (1997). He explicitly introduces the delegation stage in the government policy choice and adds a quadratic cost for reappointments. In the static one-shot game version higher costs of reappointment reduce the inflationary bias but never remove it. An exception is represented only by the extreme and unrealistic case where the weight on reappointment costs in the government's loss function is infinite. In this situation optimal monetary delegation is not subject to a credibility problem, but all that matters in the loss function are reappointment costs. Moreover if the game is repeated over an infinite horizon, along the lines of Barro and Gordon (1983b), the presence of reappointment costs worsens the credibility of optimal monetary policy under delegation compared to the case when monetary policy is conducted directly by the government.

These results imply a negative view of the contracting solution and in general of the monetary delegation approach. Jensen concludes (quite drastically) by suggesting that too much emphasis has been given to the approach of monetary delegation and that research should focus on other directions, in particular on the relationships between time inconsistency and structural policies.

Al-Nowaihi and Levine (1996) and Herrendorf (1998) provide some opposing results based on the assumption of transparency of the delegation process. They show that if the action of the policymaker cannot be perfectly monitored, but the conditions under which monetary policy is delegated are publicly observed, relocation of the time inconsistency problem can allow the government to commit credibly to the announced institutional design. However, Jensen (1997, p.915 note 8) argues that in the case of imperfect monitoring of the policymaker's action delegation improves the credibility of monetary policy merely because delegation modifications are defined to be transparent and the issue of secrecy is ruled out of the analysis by definition.[4]

[4]Obstfeld and Rogoff (1996) make a similar point in criticising the transparency of the contracting approach. They argue that it is always possible that ex-post the government offers the central banker incentives, explicitly or secretly, that can compensate him for the loss from inflating.

Thus it is not a result of delegation per se.

In the present analysis, using the same model as Jensen, we will raise some objections to his analysis and show formally when the presence of relatively high but not necessarily prohibitive costs of reappointment may ensure that optimal monetary delegation is credible. Section 2 exposes the model. Section 3 recalls briefly the main results of Jensen's analysis. Section 4 contains the results of our analysis and compares them with Jensen's results. Section 5 concludes.

2 The model

Apart from few minor changes in the notation and definitions the model is the same as that used by Jensen. Therefore we will summarise only briefly the key expressions and refer to the original version of the model for a more detailed discussion. Our main purpose here is not to criticise the model and assumptions used by Jensen but instead the analysis developed.[5]

The supply function is given by the standard expectations-augmented Phillips curve

$$y_t = \alpha \left(\pi_t - \pi_t^e \right), \tag{1}$$

where for simplicity the natural level of output is normalised to zero; π_t,π_t^e are the actual and expected inflation rate respectively.

The government's loss function is expressed by

$$L_t = \left[\pi_t^2 + \lambda \left(y_t - \overline{y} \right)^2 + \varphi \left(f_t - f_t^a \right)^2 \right], \tag{2}$$

where deviations of output and inflation from the socially optimal targets are relatively weighted with $\lambda > 0$. As usual in the time inconsistency literature the output target is assumed to be greater than the natural level, $\overline{y} > 0$.

[5] Jensen himself addresses a series of weakness in his assumptions, among which the most relevant is perhaps the issue of wage contracts of longer duration than the frequency with which a new incentive scheme can be chosen. >From a more game-theoretic point of view the model used by Jensen suffers from the criticism generally made to reputational models with trigger strategies; see for example Backus and Driffill (1985) who first pointed out the problems inherent in the game-theoretic framework used by Barro and Gordon (1985b), on which Jensen's analysis is based. On the contrary the models of Herrendorf (1998) and al-Nowaihi and Levine (1996), that also examine the credibility of optimal monetary delegation, use a more satisfactory game-theoretic framework for modelling reputation.

But in contrast with the previous literature on monetary delegation, there is a new additional cost on the reappointment of the central banker expressed by the difference between the announced incentive scheme with the penalty f_t^a and the realised one. In particular if $f_t \neq f_t^a$ we will say that the central banker has been reappointed, which in the present framework will happen at some cost to the government. The parameter φ reflects the distaste for reappointment costs relative to the other costs in the loss function. When monetary policy is delegated we have $\varphi > 0$, otherwise φ is equal to zero. As observed by Jensen, if f_t and f_t^a are understood not simply as the contract proposed to the central banker but as referring to a complex system of monetary regulations, it seems natural to assume that a small change is less costly than a bigger one. This may justify the use of a quadratic cost of reappointments with an assigned weight φ.[6]

Monetary policy is delegated by the government to a central banker whose loss function is the following

$$L_t^b = \left[\pi_t^2 + \lambda \left(y_t - \overline{y}\right)^2 + 2 f_t \pi_t\right]. \tag{3}$$

Here the central banker is fined with the penalty $2f_t$ for inflation rates greater than zero. As we will see later on, the optimal incentive scheme that allows the government to eliminate the inflation bias is $f_t = \lambda \alpha \overline{y}$.

In each period the timing of moves is the following. In stage zero, the government delegates monetary policy to a central banker and announces an incentive scheme f_t^a. In stage 1, the private sector forms expectations about inflation and sets wages. In stage 2, the government sets actual conditions f_t for monetary policy. Finally, in stage 3, the CB sets actual inflation.

In the discretionary regime the central banker minimises the discounted value of his loss function, $\sum_{\tau=t}^{\infty} \beta^{\tau-t} L_t^b$, subject to (1) by taking inflation expectations and actual conditions for monetary policy f_t as given. The parameter β is the discount factor of the central banker. It is assumed that the central banker and the government have the same discount factor. However unlike Jensen, where $0 < \beta < 1$, we assume that the discount factor of the central banker is defined in the range $.5 < \beta < 1$. This weak hypothesis

[6] An alternative way of modelling reappointment costs would be to assume that if the government reneges on his announcement it will also incur a fixed cost and that this fixed component is relatively more important than that dependent on the size of the modifications of the given institutional arrangement. This idea is captured, for example, in the work of Lohmann (1992).

has been used, for example, also by Barro (1986) and al-Nowaihi and Levine (1996).

From the first order condition we obtain the central banker's reaction function

$$\pi_t = \frac{\lambda\alpha^2\pi_t^e + \lambda\alpha\overline{y} - f_t}{\Lambda}, \tag{4}$$

where $\Lambda = (\lambda\alpha^2 + 1)$.When choosing actual monetary conditions for monetary policy, the government must take its prior announcements and inflation expectations as given but incorporates the behaviour of the central banker in it's decision problem. Thus it minimises the discounted value of its loss function, $\sum_{\tau-t}^{\infty}\beta^{\tau-t}L_t$, with respect to f_t subject to (1) and (4). The minimisation yields the following optimal incentive scheme

$$f_t = \frac{\varphi\Lambda}{1+\varphi\Lambda}f_t^a, \tag{5}$$

where f_t^a is the announcement chosen by government. As observed by Jensen the assumption of prohibitive costs of reappointment when monetary policy is delegated by the government to a central banker eliminates by definition the issue of the credibility of optimal monetary delegation. From (5) one can see that announcements will always be fulfilled if the government's only concern is reappointment costs, i.e. when $\varphi \to +\infty$.

The private sector's inflation expectations are obtained by substituting (5) into (4). After taking expectations we get

$$\pi_t^{e,NCD} = \lambda\alpha\overline{y} - \frac{\varphi\Lambda}{1+\varphi\Lambda}f_t^a. \tag{6}$$

Finally the government chooses the optimal announcement. When making this choice the government internalises the effects of its decision on the central banker's behaviour, on its own behaviour when choosing actual monetary conditions, and on the private sector's expectations, Minimising the government's loss function with respect to f_t^a subject to (4), (5) and (6) yields

$$f_t^{a,NCD} = \frac{\Lambda\left(1+\varphi\Lambda\right)\lambda\alpha\overline{y}}{1+\varphi\Lambda^2}. \tag{7}$$

Expression (7) implies that

$$f_t^{NCD} = \frac{\varphi \Lambda^2 \lambda \alpha \overline{y}}{1 + \varphi \Lambda^2}. \tag{8}$$

Here we can observe that if φ tends to infinite we have $f_t^{a,NCD} = \lambda \alpha \overline{y}$ and $f_t^{NCD} = f_t^{a,NCD}$. Thus, if reappointment costs are prohibitive, in the static one-shot game version of Jensen's model optimal monetary delegation is not subject to a credibility problem. However, the more realistic case is when these costs are not all that matters in the government's loss function, or in other words when in expression (2) the weight φ is not infinite.

The equilibrium inflation rate will be under the discretionary regime with delegation

$$\pi_t^{NCD} = \frac{\lambda \alpha \overline{y}}{1 + \varphi \Lambda^2}. \tag{9}$$

If the government does not delegate monetary policy, i.e. $\varphi = 0$, it is straightforward to show that if the government behaves in a discretionary manner the equilibrium inflation rate would be $\pi_t^{NC} = \lambda \alpha \overline{y}$. From expression (9) we can see that delegation reduces the inflation bias but does not remove it. On the contrary if the government could idealistically precommit to an announced policy rule before expectations are formed then the government would not need to delegate monetary policy in order to eliminate the inflation bias and the optimal policy rule, or the precommitment policy rule, would be in this deterministic case to set $\pi_t^{PR} = 0$. Comparing the government's losses under the equilibrium with precommitment and the equilibrium with discretion it is possible to see that in the case of delegation the loss is lower than in the case when the government conducts monetary policy directly and behaves in a discretionary manner, but is greater than in the precommiment equilibrium.

In the subsequent sections we will consider the situation when the policy game is repeated for an infinite number of periods in order to study the precommitment technology where the private sector punishes deviations by a one-period reversion to expectations given by the discretionary solution.

3 Delegation as conducive to reputation building for the government

Let's examine the situation when repeated interactions among the players take place. In particular assume that the game is repeated for an infinite number of periods. In this case Barro and Gordon (1983b) have shown that, if the private sector adopts a punishment strategy triggered by any observed deviation from optimal policy and the government does not discount the future too heavily, it is possible that the future cost for the government of losing its reputation for being committed to zero inflation may more then outweigh the current gain from deviating.

By assuming that the private sector reverts for one period to the discretionary solution whenever a deviation from optimal policy is observed, Jensen has found that the minimal requirement for the patience of the government is given by $\beta \geq \widehat{\beta} \equiv 1/\Lambda$. If β is sufficiently high optimal monetary policy is a perfect Nash equilibrium and therefore it is also credible. Alternatively if the discount factor is not sufficiently high, optimal monetary policy is not credible. In order to achieve the precommitment solution the government might consider delegating monetary policy to a central banker with the optimal incentive scheme $f_t = \lambda\alpha\overline{y}$ and try again to maintain a reputation for low inflation. Also in this case the credibility of optimal monetary delegation, where credibility is understood as the ability to carry out optimal monetary policy, can be studied by examining simple punishment strategies based on a one-period reversion to the discretionary solution.[7] Consider the following strategy combinations:

Government plays:

$$\begin{aligned} f_t &= f_t^a = \lambda\alpha\overline{y} \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ f_t^a &= f_t^{a,NCD} \text{ and } f_t = f_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e. \end{aligned} \tag{10}$$

Private sector plays:

$$\pi_t^e = 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e;$$

[7] As observed by Jensen it is not necessary to analyse explicitly the cases when the announcement of the government is $f_t^a \neq \lambda\alpha\overline{y}$ as we can rule them out through a reversion to the discretionary solution for any value of the discount factor.

$$\pi_t^e = \pi_t^{e,NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e. \tag{11}$$

The expressions of $\pi_t^{e,NCD}$ and f_t^{NCD} are found by substituting $f_t^{a,NCD}$ in expressions (5) and (6) respectively. If there is a deviation from the announced optimal delegation the government minimises the loss function with respect to f_t subject to $\pi_t^e = 0$ and $f_t^{a,DD} = \lambda\alpha\overline{y}$. This yields the following values:

$$\begin{aligned} \pi_t^{DD} &= \frac{\lambda\alpha\overline{y}}{(1+\varphi\Lambda)\Lambda}; \\ f_t^{DD} &= \frac{\varphi\Lambda\lambda\alpha\overline{y}}{1+\varphi\Lambda}. \end{aligned} \tag{12}$$

According to the above strategies the condition of no deviation for the government will be

$$L_t^{PR} - L_t^{DD} \square \beta\left(L_{t+1}^{NCD} - L_{t+1}^{PR}\right), \tag{13}$$

which implies that

$$\beta \geq \widehat{\beta}^D(\varphi) \equiv \frac{1+\varphi\Lambda^2}{\Lambda(1+\varphi\Lambda)}. \tag{14}$$

Now we can compare the condition for the credibility of optimal monetary delegation with the condition for the credibility of optimal monetary policy when monetary policy is conducted directly by the government. With this aim Jensen has proved the following proposition:

Proposition 1 . *For $\beta < \widehat{\beta}$ and all $\varphi > 0$, (i) $\partial\widehat{\beta}^D(\varphi)/\partial\varphi > 0$ and (ii) $\widehat{\beta}^D(\varphi) > \widehat{\beta}$.*

If part (i) of the proposition is true then the credibility of optimal monetary policy will be harder to support the more important reappointment costs are. Moreover as $\lim_{\varphi\to 0}\widehat{\beta}^D(\varphi) = \widehat{\beta}$ and given (i) it follows that $\widehat{\beta}^D(\varphi) > \widehat{\beta}$ for all $\varphi > 0$. Thus the premise made by the standard literature on delegation that it is the presence of reappointment costs that makes delegation to an independent central banker more credible than the conduction of monetary policy itself must be considered false according to Jensen's analysis. The intuition for this result is the following. The punishment subsequent to

a deviation becomes weaker the higher is the weight on reappointment costs. Also the gain from deviating decreases with φ but less than the reduction in the cost deriving from the loss of reputation. The reason is that the reduction of the gain from deviating results from several opposing forces which mitigate the effect of an increase in φ.

4 An alternative view on the process of delegation: delegation as conducive to reputation building for the central banker

Jensen's analysis does not take into account the possibility that an independent and far-sighted central banker might try to establish a reputation for being committed to low inflation and that this possibility might influence the behaviour of the government. Thus the question that we ask here is whether the opportunity for a central banker, institutionally independent, to credibly behave in a committed fashion may also have any effect on the credibility of optimal delegation. In order to answer this question in the present analysis we assume in contrast to Jensen that in the government's delegation problem the central banker's reputation plays a key role.[8]

As we have seen before, in each period when the government chooses the optimal announcement f_t^a it incorporates in its decision process the behaviour of the central banker and the private sector and its behaviour when he chooses actual monetary conditions. Furthermore, when the government chooses f_t it takes into account only the behaviour of the central banker and takes as given private sector's expectations and its announcements. So in both cases the government will also incorporate in its decision problem the possibility that the central banker may be able to maintain a reputation for being committed

[8] A similar approach, where the central banker's reputation plays a key role in the delegation process, has been followed also by Lockwood, Miller and Zhang (1996) in order to extend Rogoff's delegation to a weight conservative central banker to a reputational framework. But they do not analyse the credibility of optimal delegation when the option for the government of reneging on the announcements made is explicitly considered. On the contrary the approach of Jensen in defining the delegation problem faced by the government in terms only of the reputational enforcement of the government itself has been adopted, for example, also by al-Nowaihi and Levine (1996), for examining the renegotiation proveness of the Walsh contract, and Herrendorf (1998), for studying Svensson's inflation targeting regime as a substitute for an explicit precommitment.

to zero inflation.

In the present framework, in each period the timing of moves is the following. In stage 0 the government delegates monetary policy to an independent central banker and announces an incentive structure in order to make him accountable for the outcome of monetary policy. Again, as before, the government repeats in each period the announcement made in the first period of delegation, or period $t = 0$, and has the choice of either sticking to the announcement or deviating from it. The crucial difference is the following. In stage 1, after the announcement of the government and before the private sector's expectations are formed the central banker announces that he will establish a reputation for being committed to a rule for setting inflation independently of the government's behaviour. More exactly the central banker's announcement can be thought of as a costless announcement of an inflation target.[9] The subsequent stages are the same as considered before just shifted one stage forward: in stage 2 expectations are set; in stage 3 actual monetary conditions are chosen by the government; in stage 4 the central banker chooses the inflation rate.

Thus if the central banker announces an inflation target the private sector will expect it to be achieved no matter what the government does - both before and after expectations are set - and whether the central banker is able to establish a reputation for achieving the given inflation target will depend on the usual condition derived from the comparison of the temptation to deviate with the reputational enforcement. In particular if the announced inflation target corresponds to the pre-commitment inflation rate then it must be enforceable simultaneously under both deviation and no deviation of the government from the announced incentive scheme. If this is not true then the central banker will commit to the lowest enforceable inflation rate simultaneously under both deviation and no deviation of the government.

The choice we have made in the analysis of focusing only on the central banker's reputation corresponds first of all to the need to establish clearly

[9] Driffill (1997, 1994) argues that the introduction of inflation targets, in the form of costless announcements, by focusing the attention on a particular reputational solution for inflation may solve the problems of coordination and multiple equilibria that affect reputational models with trigger strategies. This idea is followed by Miller (1997) in extending Rogoff's delegation approach to the reputational framework of Barro and Gordon (1985b). Moreover Miller, considering the same model used in Lockwood, Miller and Zhang (1996), discusses the case when there are observable shocks in the economy and the announced target should have a range around it for stabilisation purposes.

the differences relative to the analysis of Jensen. More importantly, the choice made reflects also the idea that when an independent central banker tries to establish a reputation for low inflation it seems realistic to postulate that the central banker will try to resist any possible influence from the government that would undermine the credibility of his announcements on the inflation target. This implies that the announced inflation target must be sustainable as a reputational equilibrium whether the government deviates or not. Moreover as we will show in our analysis, if this behaviour of the central banker is common knowledge among the players, the central banker has an incentive to behave in this way because then it is possible for him to constrain the government to make credible announcements without relying on its reputation for not reneging on the announced incentive scheme.

Similarly to Jensen's analysis we will rule out, by means of a reversion to the discretionary solution, feasible strategy combinations which include announcements that do not imply the achievement of the precommitment solution.

The delegation problem for the government is the following. Given that the discount factor β is not sufficiently high and that establishing a reputation for low inflation is not an available option, the government decides to delegate monetary policy and must choose an initial announcement f_0^a that minimises its expected losses. Now in order to avoid a reversion to the discretionary solution, triggered by an inflation target announced by the central banker different from zero, the government will consider only announcements $f_t^a > 0$ that imply that the central banker is able to maintain a reputation for being committed to the precommitment inflation rate, independently of whether or not the government deviates from the announced incentive structure. The announcements that satisfy this requirement are optimal for the government and, as we will see later on, another difference relative to Jensen's analysis is that here the number of optimal announcements for the government can be greater than one depending on the assumed value of φ.

So our initial task is to find the set of optimal announcements for the government that ensure the achievement of optimal monetary policy by the central banker. The problem is complicated by the fact that when the central banker commits to the announced inflation target he does not know whether the government will deviate or not. It is possible to show that the key to the solution of the delegation problem for the government is to eliminate the uncertainty about its own behaviour and ensure that the central banker expects that the announcement made by the government will always be ful-

filled. Actually, in the present framework the government has no interest in introducing this uncertainty as it is optimal for it to induce the central banker to announce a zero inflation target and avoid any reversion to the discretionary solution.

Now suppose the central banker has announced an inflation target equal to zero and that this target is sustainable in the case when the government sticks to its announced incentive scheme. Consider a deviation by the government. Due to the presence of reappointment costs a necessary condition for a deviation by the government is that the central banker deviates after a deviation by the government and creates unexpected inflation. Only in this case may the government have an incentive to deviate from the announced institutional arrangement as the gain from surprise inflation can more than compensate the reappointment costs. Assuming again that this is common knowledge among the players, the fact that the central banker deviates after a deviation by the government and creates unexpected inflation implies necessarily that the announced inflation target is not sustainable as a reputational equilibrium. But this contradicts the claim that in order to avoid a reversion to the discretionary solution, triggered by an inflation target announced by the central banker different from zero, the government will consider only announcements that imply that the central banker is able to maintain a reputation for being committed to the precommitment inflation rate.

So it must be the case that, if the announcement of the government is optimal, the necessary condition for a deviation by the government is never satisfied, i.e. if a deviation by the government occurs the central banker never deviates from optimal monetary policy. Notice that in Jensen's analysis the necessary condition for a deviation by the government discussed here is always satisfied as the central banker is assumed to behave only in a discretionary manner. On the contrary, in the present framework the presence of the central banker's reputation combined with the presence of reappointment costs constrains the behaviour of the government and introduces an incentive for the government to always fulfil its announcements and eliminate the uncertainty about its behaviour.

The working of this mechanism will appear more clearly after the formalisation of the necessary conditions for the existence of the reputational equilibrium in which the precommitment inflation rate is sustainable.

4.1 The set of optimal announcements for the government

Before starting with the analysis of the credibility of optimal monetary delegation we need to find the set of optimal announcements for the government. For expositional reasons we separate the set of announcements of the government after which if it does not deviate the central banker is able to sustain zero inflation as a reputational equilibrium, from the set of announcements after which if it deviates the central banker is able to sustain zero inflation as a reputational equilibrium. From the above discussion it is clear that the set of optimal announcements for the government will be the set that is the intersection of the two above distinct sets.

In order that a solution to the delegation problem of the government exists and a reversion to the discretionary solution is avoided, the two set considered must not be mutually exclusive. This latter condition will be examined in the next section. Here we find only the optimal announcements for the government. In the following analysis it is understood that if the discount factor of the government is sufficiently high, i.e. $\beta \geq \widehat{\beta}$, the government does not delegate monetary policy. We will examine the credibility of optimal monetary delegation only in the case when the government is not able to build a reputation for low inflation. Thus here the delegation solution is not alternative to the reputational solution, as claimed in the standard theory of delegation, but rather supplementary.

4.1.1 No deviation by the government

If we assume that the government does not deviate, the announcement $f_t^a > 0$ chosen by the government will be within the set of announcements that imply that the central banker is able to maintain a reputation for being committed to zero inflation. Thus in each period the government will announce $f_t^a = f_0^a \in \Theta$, where f_0^a is the initial announcement and Θ is the set of announcements that ensure that the precommitment solution can be sustained as a reputational equilibrium when the government does not deviate from the announcement made. Consider the following strategy combinations:

Central Banker plays:

$$\begin{aligned} \pi_t &= 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ \pi_t &= \pi_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e. \end{aligned} \tag{15}$$

Private Sector plays:

$$\begin{aligned} \pi_t^e &= 0 \text{ if } \pi_{t-1} = 0; \\ \pi_t^e &= \pi_t^{e,NCD} \text{ if } \pi_{t-1} \neq 0. \end{aligned} \tag{16}$$

Government plays:

$$\begin{aligned} f_t &= f_t^a = f_0^a \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ f_t^a &= f_t^{a,NCD} \text{ and } f_t = f_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e; \end{aligned} \tag{17}$$

where $f_t^{a,NCD}$ and f_t^{NCD} are the same as in (10).

The condition of no deviation from zero inflation for the central banker when the government sticks to the announcement made will be

$$L_t^{b,PR} - L_t^{b,DD} \square \beta \left(L_{t+1}^{b,NCD} - L_{t+1}^{b,PR} \right). \tag{18}$$

The above condition implies that

$$\beta \geq \widehat{\beta}^{b*}(f_0^a; \varphi) \equiv \frac{\left(\lambda\alpha\overline{y} - f_0^a\right)^2 \left(1 + \varphi\Lambda^2\right)^2}{\Lambda\left[\left(\lambda\alpha\overline{y}\right)^2 \left(1 + 2\varphi\Lambda^2\right)\right]}. \tag{19}$$

Considering condition (19) with equality and f_0^a as the unknown term, it is possible to see that there are two values of f_0^a that satisfy this equation:

$$\begin{aligned} \underline{\theta}(\varphi) &= \lambda\alpha\overline{y} \frac{1 + \varphi\Lambda^2 - \sqrt{\beta\Lambda\left(1 + 2\varphi\Lambda^2\right)}}{\left(1 + \varphi\Lambda^2\right)} > 0; \\ \overline{\theta}(\varphi) &= \lambda\alpha\overline{y} \frac{1 + \varphi\Lambda^2 + \sqrt{\beta\Lambda\left(1 + 2\varphi\Lambda^2\right)}}{\left(1 + \varphi\Lambda^2\right)} > 0. \end{aligned} \tag{20}$$

In the appendix it is shown that for the assumed parameter values $\underline{\theta}(\varphi) > 0$. Now, inspection of condition (19) yields the following proposition:

Proposition 2 . *For* $f_0^a = \theta \in \Theta(\varphi)$, *with* $\Theta(\varphi) \equiv \left[\underline{\theta}(\varphi), \overline{\theta}(\varphi)\right]$, *(i)* $\beta \geq \widehat{\beta}^{b*}(f_0^a; \varphi)$; *(ii)* $\Theta(\varphi) \supset \Theta\left(\varphi'\right)$, *with* $\varphi < \varphi'$;*(iii)* $\partial\widehat{\beta}^{b*}(f_0^a; \varphi)/\partial\varphi > 0$.

Part (i) of this proposition says that if the government's announcement is chosen between $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$ the condition (19) is satisfied. According to part (ii) the set $\Theta(\varphi)$, i.e. the set of announcements under which the pre-commitment solution can be sustained as a reputational equilibrium when the government does not deviate, with given φ becomes smaller as reappointment costs become more important. Finally part (iii) says that, for a given government's announcement, increasing the weight φ on reappointment costs makes condition (19) harder to fulfil. Let's proof proposition 2.

PROOF. In the present framework, for given φ, the government's announcement f_0^a is the choice variable in the government's delegation problem. Now considering condition (19) with equality and given φ, we can draw (see Figure 1) the quadratic function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ as a parabola having a global minimum at $f_0^a = \lambda\alpha\overline{y}$ with $\widehat{\beta}^{b*}(\lambda\alpha\overline{y};\varphi) = 0$. The function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ will be equal to β for two values of f_0^a, which correspond to $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$. It is possible to see that the two values $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$ are respectively lower and greater than $\lambda\alpha\overline{y}$ and both tend to $\lambda\alpha\overline{y}$ as $\varphi \to +\infty$.

From the above discussion it follows that $\beta \geq \widehat{\beta}^{b*}(f_0^a;\varphi)$ for all values of f_0^a included between $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$. Thus the set $\Theta(\varphi)$ is defined between these two extreme values. With a minor abuse of notation, in proposition 2 we have assumed that the function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ is defined also for $f_0^a = \lambda\alpha\overline{y}$. Formally it would be more correct to say that this function is not defined in that point as the central banker always chooses zero inflation independently of the value of β.

Part (ii) can be proved using the following first derivative:

$$\frac{\partial I^*(\varphi)}{\partial\varphi} = -\frac{2\varphi\Lambda^4\sqrt{\Lambda\beta}}{(1+\varphi\Lambda^2)^2\sqrt{1+2\varphi\Lambda^2}} < 0; \tag{21}$$

with

$$I^*(\varphi) \equiv \overline{\theta}(\varphi) - \underline{\theta}(\varphi) = \frac{2\sqrt{\beta\Lambda(1+2\varphi\Lambda^2)}}{1+\varphi\Lambda^2}; \tag{22}$$

which shows that for the assumed parameter values the interval $I^*(\varphi)$ is always reduced by higher values of φ. As the limit of $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$ for $\varphi \to +\infty$ is in both cases $\lambda\alpha\overline{y}$, it follows that as φ increases the set $\Theta(\varphi)$ becomes smaller and will shrink to the element $\lambda\alpha\overline{y}$.

We can now turn to part (iii) of proposition 2. Here we can consider the following first derivative

$$\frac{\partial \widehat{\beta}^{b*}(f_0^a;\varphi)}{\partial \varphi} = \frac{2\left(\alpha\lambda y - f_0^a\right)^2\left(1+\varphi\Lambda^2\right)\Lambda^3\varphi}{\left(\lambda\alpha\overline{y}\right)^2\left(1+2\varphi\Lambda^2\right)^2} > 0; \tag{23}$$

which for the assumed parameter values is always positive. Thus for a given announcement f_0^a as φ increases the condition (19) will be harder to fulfil. This result is a consequence of part (ii) of proposition 2. To see this consider Figure 1. If part (ii) holds, then an increase of φ shrinks the parabola constituted by the function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ to the vertical axis passing through $f_0^a = \lambda\alpha\overline{y}$. Thus as shown in Fig.1 with the dotted line, for given f_0^a, the value of the function will be higher. QDE.

4.1.2 Deviation by the government

Consider now the case of a deviation by the government from the announced f_t^a. In stage 1 the central banker does not know whether or not the government will deviate. However the announced inflation target must be enforceable also under a deviation by the government.

As reappointments with $f_t \neq f_t^a$ and $\varphi > 0$ are costly, the government may deviate from the announced incentive structure if and only if the central banker follows the government's deviation and deviates as well. So a necessary condition for a deviation by the government is $f_0^a \notin \Phi$, where Φ is the set of announcements under which the precommitment solution can be sustained as a reputational equilibrium when the government deviates. On the contrary, if the central banker is able to maintain a reputation for low inflation after a deviation by the government, then the government, due to the presence of reappointment costs, has never an incentive to deviate and the announcement $f_0^a = \phi \in \Phi$ will always be fulfilled.

Notice that the fact that the announcement is always fulfilled does not imply that it is also optimal for the government to announce it because, as we have seen previously, it is possible that the central banker might not be able to sustain the precommitment inflation rate as a reputational equilibrium when the government does not deviate.

Now, as implied by the above discussion, the announcements $f_0^a = \phi \in \Phi$ can be derived from the necessary condition for a deviation by the government assuming that the central banker has announced a zero inflation target. In

this case the strategies of the central banker and the private sector are the same as in the previous case, given by (15) and (16). On the contrary, the strategy of the government is now based on the assumption that also the central banker will deviate after its deviations. We have:

Government plays:

$$\begin{aligned} f_t &\neq f_t^a = f_0^a \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ f_t^a &= f_t^{a,NCD} \text{and } f_t = f_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e; \end{aligned} \tag{24}$$

where $f_t^{a,NCD}$ and f_t^{NCD} are the same as in (10). When the government deviates the first time from the announcement it chooses $f_t = f_t^{DD}$, which is the same of expression (12).

Here the minimal condition for the patience of the central banker implies that

$$\beta \geq \widehat{\beta}^{b**}(f_0^a;\varphi) \equiv \frac{\left[\lambda\alpha\overline{y}\left(1+\varphi\Lambda\right) - f_0^a\varphi\Lambda\right]^2\left(1+\varphi\Lambda^2\right)^2}{\left(1+\varphi\Lambda\right)^2\Lambda\left(\lambda\alpha\overline{y}\right)^2\left(1+2\varphi\Lambda^2\right)}. \tag{25}$$

Again it is possible to show that there are two values of f_0^a that satisfy the condition (25) taken with equality. Here we have

$$\begin{aligned} \underline{\phi}(\varphi) &= \lambda\alpha\overline{y}\frac{1+\varphi\Lambda^2 - \sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{\left(1+\varphi\Lambda\right)^{-1}\left(1+\varphi\Lambda^2\right)\varphi\Lambda} > 0; \\ \overline{\phi}(\varphi) &= \lambda\alpha\overline{y}\frac{1+\varphi\Lambda^2 + \sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{\left(1+\varphi\Lambda\right)^{-1}\left(1+\varphi\Lambda^2\right)\varphi\Lambda} > 0. \end{aligned} \tag{26}$$

Again in the appendix it is shown that for the assumed parameter values $\underline{\phi}(\varphi) > 0$. Following the discussion made before, the necessary condition for a deviation of the government is given by $\beta < \widehat{\beta}^{b**}(f_0^a;\varphi)$. On the contrary if (25) holds, the government does never have an incentive to deviate and the announcement made by the government will always be fulfilled. Inspection of condition (25) yields the following proposition:

Proposition 3 . *For* $f_0^a = \phi \in \Phi(\varphi)$, *with* $\Phi(\varphi) \equiv \left[\underline{\phi}(\varphi), \overline{\phi}(\varphi)\right]$, *(i)* $\beta \geq \widehat{\beta}^{b**}(f_0^a;\varphi)$; *(ii)* $\Phi(\varphi) \supset \Phi\left(\varphi'\right)$, *with* $\varphi < \varphi'$;*(iii)* $\partial\widehat{\beta}^{b**}(f_0^a;\varphi)/\partial\varphi \gtreqless 0$.

Part (i) of this proposition says that if f_0^a is chosen between $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$, then for given φ condition (25) is satisfied. According to part (ii) the set Φ, i.e. the set of announcements under which the precommitment solution can be sustained as a reputational equilibrium when the government deviates, becomes smaller as reappointment costs become more important. Finally part (iii) says that the weight φ on reappointment costs has an ambiguous effect on condition (25). Let's proof proposition 3.

PROOF. Again we consider for given φ the government's announcement f_0^a as the choice variable in the government's delegation problem. Taking condition (25) with equality we can draw (see Fig.2) the quadratic function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ as a parabola having a global minimum at $\sigma(\varphi) = \lambda\alpha\overline{y}\left[(1+\varphi\Lambda)/\varphi\Lambda\right]$, with $\widehat{\beta}^{b**}(\sigma;\varphi) = 0$. The function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ will be equal to β for two values of f_0^a which correspond to $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$. The two values $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$ are respectively lower and greater than $\sigma(\varphi)$ and, as well as $\sigma(\varphi)$, both tend to $\lambda\alpha\overline{y}$ as $\varphi \to +\infty$. The above discussion implies that $\beta \geq \widehat{\beta}^{b**}(f_0^a;\varphi)$ for all values of f_0^a included between $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$. Thus the set Φ of announcements is defined between these two extreme values. As before, with a minor abuse of notation, in proposition 3 we have assumed that the function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ is defined also for $f_0^a = \sigma(\varphi)$. However in this case $f_t = f_t^a\left[\varphi\Lambda/(1+\Lambda\varphi)\right] = \lambda\alpha\overline{y}$ and therefore it would be more correct to say that this function is not defined in that point as the central banker always chooses zero inflation independently from the value of β.

Part (ii) can be proved using the following first derivative:

$$\frac{\partial I^{**}(\varphi)}{\partial\varphi} = -\frac{2\sqrt{\beta}\left(1+3\varphi\Lambda^2+3\varphi^2\Lambda^4+\varphi^3\Lambda^5\right)}{\varphi^2\sqrt{\Lambda\left(1+2\varphi\Lambda^2\right)}\left(1+\varphi\Lambda^2\right)^2} < 0; \tag{27}$$

with

$$I^{**}(\varphi) \equiv \overline{\phi}(\varphi) - \underline{\phi}(\varphi) = \frac{2\left(1+\varphi\Lambda\right)\sqrt{\beta\left(1+2\varphi\Lambda^2\right)}}{\varphi\sqrt{\Lambda}\left(1+\varphi\Lambda^2\right)}. \tag{28}$$

The above derivative is always negative for the assumed parameter values. This implies that as φ increases the set Φ becomes smaller and will shrink to the element $\sigma(\varphi)$, i.e. the intermediate element between the lower and upper bound of the set Φ. Thus as φ increases the parabola constituted by the function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ shrinks to the vertical axis passing through $\sigma(\varphi)$.

However, differently from before, as the limit of $\underline{\phi}(\varphi)$, $\overline{\phi}(\varphi)$ and $\sigma(\varphi)$for $\varphi \to +\infty$ in all three cases is $\lambda\alpha\overline{y}$. Thus, as shown in Fig.2 with the dotted line, the parabola shifts also to the left in the cartesian coordinate plane together with the vertical axis passing through $\sigma(\varphi)$.

Now considering part (iii) of proposition 2, we can see from the following first derivative

$$\begin{aligned} & \frac{\partial \widehat{\beta}^{b**}(f_0^a;\varphi)}{\partial\varphi} \\ = \; & -2\frac{f_0^a + f_0^a\left[\varphi\Lambda^2\left(3+\varphi^2\Lambda^3+3\varphi\Lambda^2\right)\right] - \lambda\alpha\overline{y}\left[\varphi\Lambda^2\left(\Lambda+\varphi^2\Lambda^3+2\varphi\Lambda^2\right)\right]}{\left[\lambda\alpha\overline{y}\left(1+\varphi\Lambda\right) - f_0^a\varphi\Lambda\right]^{-1}\left(1+\varphi\Lambda^2\right)^{-1}\left(1+\varphi\Lambda\right)^3\left(\lambda\alpha\overline{y}\right)^2\left(1+2\varphi\Lambda^2\right)^2}, \end{aligned} \tag{29}$$

that a marginal increase in φ will have an ambiguous effect depending on the value of f_0^a. It is possible to show that the derivative becomes negative for

$$\gamma(\varphi) < f_0^a < \sigma(\varphi); \tag{30}$$

with

$$\gamma(\varphi) = \lambda\alpha\overline{y}\frac{\varphi\Lambda^3\left(1+\varphi\Lambda\right)^2}{\left[1+\varphi\Lambda^2\left(3\varphi\Lambda^2+\varphi^2\Lambda^3+3\right)\right]} < \lambda\alpha\overline{y}. \tag{31}$$

For $f_0^a > \sigma(\varphi)$ or $f_0^a < \gamma(\varphi)$the sign of the derivative will be positive. Similarly to the previous proposition, this result is a consequence of part (ii) of proposition 3. The ambiguous sign of the derivative is determined by the fact that now an increase of φ both restricts and shifts to the left in the cartesian coordinate plane the parabola implied by the function $\widehat{\beta}^{b**}(f_0^a;\varphi)$. QDE.

4.2 The analysis of the credibility of optimal monetary delegation

As observed above the government's announcement is optimal if it belongs both to the set of announcements after which, if the government does not deviate, the central banker is able to sustain zero inflation as a reputational equilibrium and to the set of announcements after which, if the government

deviates, the central banker maintains his reputation too. Using propositions 1 and 2, it follows that an announcement ψ that belongs to the set $\Psi(\varphi) \equiv \Theta(\varphi) \cap \Phi(\varphi)$ is optimal. The set of optimal announcements is illustrated in Fig.3, where the two curves intersect each other.

In the present framework examining the credibility of optimal delegation - where credibility is understood as the ability to carry out optimal monetary policy - implies to study the circumstances under which there exists an optimal announcement for the government, i.e. the set Ψ is not empty. It is possible to show that, if the weight on reappointment costs is sufficiently high but not necessarily infinite, there is always at least one optimal announcement $\psi \in \Psi \neq \emptyset$ available for solving the government's delegation problem. To see this we consider the following proposition:

Proposition 4 . *If $\beta \geq \widehat{\beta}$ the government conducts directly monetary policy, whereas if $\beta < \widehat{\beta}$ the government delegates monetary policy to an independent central banker and announces an optimal incentive scheme f_0^a. For $f_0^a = \psi \in \Psi(\varphi) \equiv \Theta(\varphi) \cap \Phi(\varphi)$ and $\varphi \geq \widehat{\varphi}$,(i) $\Psi(\varphi) \neq \emptyset$ and (ii) $0 < \widehat{\varphi} < +\infty$.*

This proposition says that if the weight φ is greater or equal to the threshold value $\widehat{\varphi}$ the set of optimal announcements for the government $\Psi(\varphi)$ is not empty and there exists at least one announcement $\psi \in \Psi(\varphi)$ available at the delegation stage. Moreover, in the most important part of this proposition, it claims that this threshold value is not infinite and therefore reappointment costs need not be prohibitive in order to ensure that delegation credibly delivers the same outcomes of the precommitment equilibrium.

PROOF. Consider first part (i). As we said above there exists an optimal $f_t^a = \psi \in \Psi(\varphi)$ as long as $\Psi(\varphi) \equiv \Theta(\varphi) \cap \Phi(\varphi) \neq \emptyset$. From propositions 2 and 3 it is possible to see that, for $0 < \varphi < +\infty$, $\overline{\phi}(\varphi) > \underline{\theta}(\varphi)$(as $\overline{\phi}(\varphi) > \sigma(\varphi)$ and $\underline{\theta}(\varphi) < \lambda\alpha\overline{y}$). Moreover it is possible to see that

$$\begin{aligned} \overline{\phi}(\varphi) - \overline{\theta}(\varphi) &= \frac{\overline{\phi}(\varphi)}{1+\varphi\Lambda} > 0; \\ \underline{\phi}(\varphi) - \underline{\theta}(\varphi) &= \frac{\underline{\phi}(\varphi)}{1+\varphi\Lambda} > 0. \end{aligned} \tag{32}$$

It follows that $\Psi(\varphi) \equiv \left[\underline{\phi}(\varphi), \overline{\theta}(\varphi)\right]$. Now $\Psi(\varphi) \neq \emptyset$ if and only if

$$\overline{\theta}(\varphi) - \underline{\phi}(\varphi) \geq 0. \tag{33}$$

So we need to find the values of φ that solve this expression. Unfortunately this is a third order polynomial. By using, for instance, Maple© we can get three roots which are very complicated to study. This implies that, considering part (ii), in order to show that $0 < \widehat{\varphi} < +\infty$ we need to perform a very complicated numerical simulation.

Let's follow a simpler route. After some simplifications the weak inequality (33) can be rewritten as

$$\frac{(1+2\varphi\Lambda)\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{(1+\varphi\Lambda^2)} \geq 1. \tag{34}$$

If the government delegates monetary we must have $\beta < 1/\Lambda$. This implies that $\beta\Lambda < 1$. Moreover, the inequality $\beta < 1/\Lambda$ implies also that under delegation we have $1 < \Lambda < 2$, as $.5 < \beta < 1$. Taking the limit for $\varphi \to 0$ we obtain

$$\sqrt{\beta\Lambda} \geq 1; \tag{35}$$

which is never true if $\beta\Lambda < 1$. So the threshold value for φ must be greater than zero. If $\varphi \to +\infty$, the term on the left-hand side of the weak inequality (34) tends to infinity. Moreover the first derivative with respect to φ of this term is

$$\frac{\Lambda\sqrt{\beta\Lambda}\left[2+\varphi\Lambda^2\left(6-\Lambda+2\varphi\Lambda^2\right)\right]}{\left(1+\varphi\Lambda^2\right)^2\sqrt{1+2\varphi\Lambda^2}} > 0; \tag{36}$$

which is always positive for $1 < \Lambda < 2$. Hence, there always exists a value of $\widehat{\varphi}$, such that $0 < \widehat{\varphi} < +\infty$, that satisfies the weak inequality (33). The weak inequality is satisfied also by all $\varphi \geq \widehat{\varphi}$.

In order to have an idea of the range of $\widehat{\varphi}$ it is possible to proceed in the following way. Inspection of (34) yields some useful information. In particular we can see that, if $\Lambda < 2$, then

$$\frac{(1+2\varphi\Lambda)}{(1+\varphi\Lambda^2)} > 1. \tag{37}$$

Moreover we have that

$$\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)} \geq 1; \tag{38}$$

if

$$\varphi \geq \widetilde{\varphi} = \frac{1 - \beta\Lambda}{2\beta\Lambda^3}. \tag{39}$$

Expression (39) constitutes a sufficient condition for the existence of an optimal announcement for the government, as we clearly have $\widetilde{\varphi} > \widehat{\varphi}$. It is possible to see that the highest possible value of $\widetilde{\varphi}$ is for $\Lambda \to 1$ and for $\beta \to .5$. In this case we find that $\widetilde{\varphi} \to .5$. So we can conclude that $0 < \widehat{\varphi} < .5$. QDE.

So proposition 4 shows that under delegation optimal monetary policy can be more credible than under the conduction of monetary policy directly by the government. If the weight on reappointment costs is sufficiently high, but not necessarily infinite, there always exists an announcement available for the government such that the central banker is able to sustain zero inflation as a reputational equilibrium no matter whether the government deviates or not from the announcement made. As the central banker behaviour cannot be influenced and reappointments are costly, the government never deviates and will stick to the announcement made. Thus, McCallum's criticism of the delegation approach does not hold provided that the costs of changing monetary institutions are sufficiently high. Reappointment costs play a crucial role in the delegation process but in contrast to the contracting approach to delegation, based on the static one-shot game framework, also the central banker's reputation for being committed to low inflation has a fundamental role for the credibility of optimal delegation.

4.3 Comparison with Jensen's analysis

Now, in order to compare our analysis with that of Jensen we examine the case where the government focuses only on the incentive scheme that would be optimal in the static one-shot game and announces $f_0^a = \lambda\alpha\overline{y}$. Thus in our framework the requirement for the patience of the central banker that must be satisfied for the credibility of optimal delegation is only (25), which now becomes

$$\beta \geq \widehat{\beta}^{b**}\left(\lambda\alpha\overline{y};\varphi\right) \equiv \frac{\left(1+\varphi\Lambda^2\right)^2}{\left(1+\varphi\Lambda\right)^2\Lambda\left(1+2\varphi\Lambda^2\right)}. \tag{40}$$

Even if condition (40) is referred to the central banker instead of the government, it is similar to the condition analysed by Jensen (which is given in

our framework by (14)). Its fulfilment implies that, if the government delegates monetary policy to a central banker announcing the incentive scheme $f_0^a = \lambda\alpha\overline{y}$, optimal monetary policy will be credible. The main difference is that in our framework the fulfilment of the government's announcement is related to the central banker's reputation for low inflation. On the contrary in Jensen's analysis the fulfilment of the government's announcement is related to the government's reputation for low inflation.

Let's compare the condition for the credibility of optimal monetary policy when the government conducts by itself monetary policy with the condition that secures the credibility of optimal monetary policy under delegation.

From condition (40) it is possible to derive the following corollary:

Corollary 5 *:*

$$\begin{aligned} \lim_{\varphi\to+\infty} \widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) &= 0; \\ \lim_{\varphi\to 0} \widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) &= \widehat{\beta}. \end{aligned}$$

Moreover it is possible to prove the following proposition which is in our framework the equivalent of proposition 1:

Proposition 6 . *For $\beta < \widehat{\beta}$ and all $\varphi > 0$, (i) $\partial\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi)/\partial\varphi < 0$ and (ii) $\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) < \widehat{\beta}$.*

PROOF. From condition (40) follows that

$$\frac{\partial\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi)}{\partial\varphi} = -\frac{2\left(1+\varphi\Lambda^2\right)\left[\varphi\Lambda^2\left(3-\Lambda+\varphi\Lambda^2\right)+1\right]}{\left(1+\varphi\Lambda\right)^3\left(1+2\varphi\Lambda^2\right)^2} < 0 \qquad (41)$$

for all $\varphi > 0$ and $1 < \Lambda < 2$ (implied by $\beta < \widehat{\beta}$). Using corollary 1 immediately follows also that $\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) < \widehat{\beta}$. QDE.

So, in contrast with what is stated in proposition 1, according to proposition 5 the condition for the credibility of optimal monetary policy under delegation becomes weaker as φ increases.

The intuition for this different result is the following. In our framework it is possible to see that the punishment (for the central banker) following

a deviation at t, $\left(L_{t+1}^{b,NCD} - L_{t+1}^{b,PR}\right)$, becomes weaker the higher is φ. As in the case of Jensen (but referred to the government's loss) the reason is because $L_{t+1}^{b,PR}$ is independent of φ and $L_{t+1}^{b,NCD}$ is a decreasing function of φ. But the penalty for the central banker from inflating is twice higher then the cost for the government from deviating from the announcement made at t. We can see that under the discretionary solution we have $2f_{t+1}^{NCD}\pi_{t+1}^{NCD} = 2\varphi\left(f_{t+1}^{NCD} - f_{t+1}^{a,NCD}\right)^2$. Thus the punishment will be higher in our case for a given value of φ.

Now consider the gain from deviation in period t. Here again $L_t^{b,PR}$ is independent of φ while $L_t^{b,DD}$ increases with φ. Consequently, the temptation to deviate, expressed by $\left(L_t^{b,PR} - L_t^{b,DD}\right)$, decreases also with φ. However, the increase of $L_t^{b,DD}$ is higher than in the case of Jensen (again referred to the government's loss) as $2f_{t+1}^{DD}\pi_{t+1}^{DD} = 2\varphi\left(f_{t+1}^{DD} - f_{t+1}^{a,DD}\right)^2$. So in our case for a given value of φ the incentive to deviate will be lower than in the case considered by Jensen.

These differences relative to Jensen's analysis ensure that if reappointment costs become more important the decrease in the temptation to deviate will be higher than the decrease in the punishment from deviating.

5 Conclusions

The analysis developed has shown that, contrary to Jensen's analysis, institutional arrangements based on incentive structures which delegate monetary policy to an independent and far-sighted central banker and are costly to change, under certain circumstances which are not extreme, may be more credible than the conduct of monetary policy without delegation. In particular, our results suggest that if the weight assigned to reappointment costs in the loss function of the government is relatively high, but not necessarily infinite, McCallum's criticism of the delegation approach does not hold. This result is due to the presence of reappointment costs but there is an important distinction with respect to the standard theory of monetary delegation, based on the static one-shot game. It crucially depends on the influence on the behaviour of the government of the central banker's reputation for being committed to low inflation.

The recent literature on the credibility of optimal monetary delegation

when delegation can be changed, as exemplified by the works of Jensen (1997), Herrendorf (1998), and al-Nowaihi and Levine (1996), has shown that the delegation solution for time inconsistency can be conducive to reputation building for the government and hence is not an alternative to the reputational solution, as is usually claimed in the standard theory, but is at best supplementary. However this new body of literature, by assuming that the central banker behaves always in a discretionary fashion, has focused exclusively on the reputational enforcement of the government for being committed to the announced institutional arrangements or to low inflation. This assumption is based on the view, formalised in the standard theory, that incentive schemes or policy targets are introduced in order to constrain the behaviour of the central banker according to the objectives of the government.

The view of the delegation process formalised in our analysis is quite different, as one of the main effects of the introduction of an incentive structure that is costly to change is to enhance the central banker's reputation. Moreover, via his commitment to the announced inflation target the central banker indirectly constrains the behaviour of the government. Hence we agree with McCallum (1997, p.109) when he insightfully argues about the possible positive implications of contract or incentive arrangements for central bankers: ".. the main effect of such arrangements [as those of New Zealand's] is not principally to constrain the central bank to act in accordance with the government's objectives, but rather to constrain the government by increasing the difficulty of its bringing pressure to inflate upon the central bank.... Arrangements such as those of New Zealand's, therefore, give the central banks an increased opportunity to behave in a rule-like, committed fashion".

As clarified by our analysis, the role played by incentive schemes in strengthening the central banker's reputation is crucial for the importance of reappointment costs. In particular the constraint that the central banker's reputation imposes on the government's temptation to deviate from the announced incentive scheme may significantly reduce the amount of reappointment cost required for disciplining the government's behaviour.

6 Appendix: proof that $\underline{\phi}$ and $\underline{\theta}$ are positive

We first recall that the assumptions about the parameters are the following:

$$\begin{aligned} .5 &< \beta < 1; \\ \beta &< \widehat{\beta} = \frac{1}{\Lambda}; \\ \Lambda &= 1 + \lambda\alpha^2; \end{aligned} \tag{42}$$

which imply that

$$\begin{aligned} 1 &< \Lambda < 2; \\ \beta\Lambda &< 1. \end{aligned} \tag{43}$$

Given these assumptions, $\underline{\theta}$ and $\underline{\phi}$ are always positive if we can prove that

$$1 + \varphi\Lambda^2 - \sqrt{\beta\Lambda\left(1 + 2\varphi\Lambda^2\right)} > 0. \tag{44}$$

This inequality can be rewritten as

$$1 + \varphi\Lambda^2 - \sqrt{\beta\Lambda\left[\left(1 + \varphi\Lambda^2\right)^2 - \left(\varphi\Lambda^2\right)^2\right]} > 0. \tag{45}$$

As from the assumed parameter values we have that$\sqrt{\beta\Lambda} < 1$, it is clear that the term outside the square root is always greater than that inside. Hence the inequality is always satisfied.

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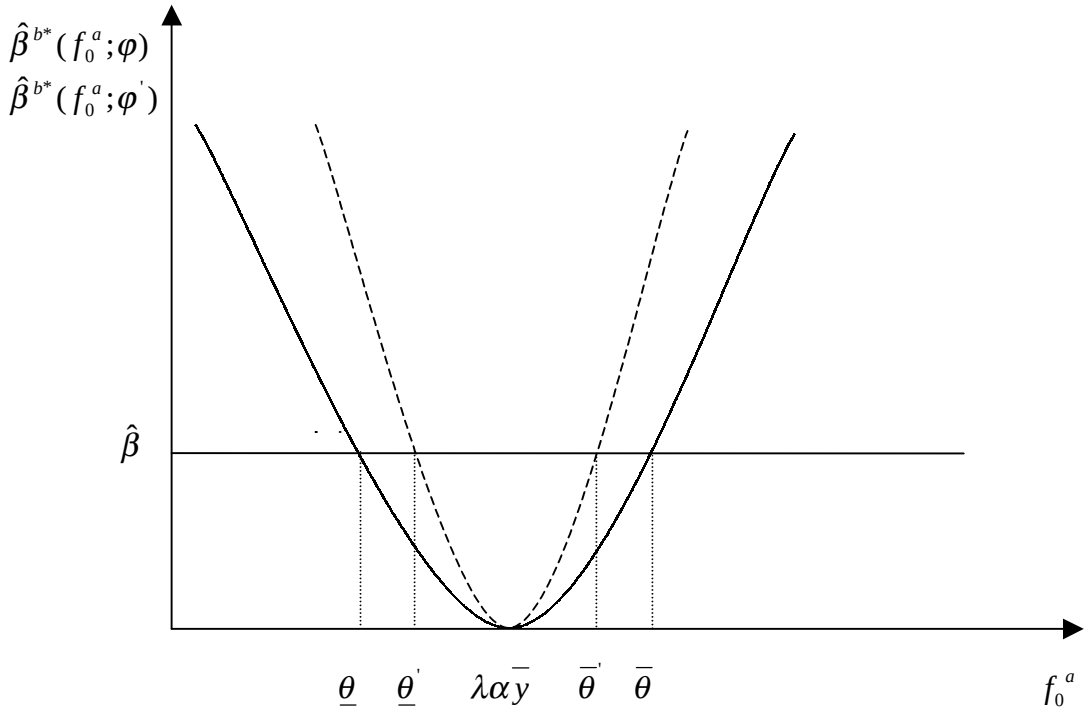

Figure 1. An increase of the weight on reappointment costs.

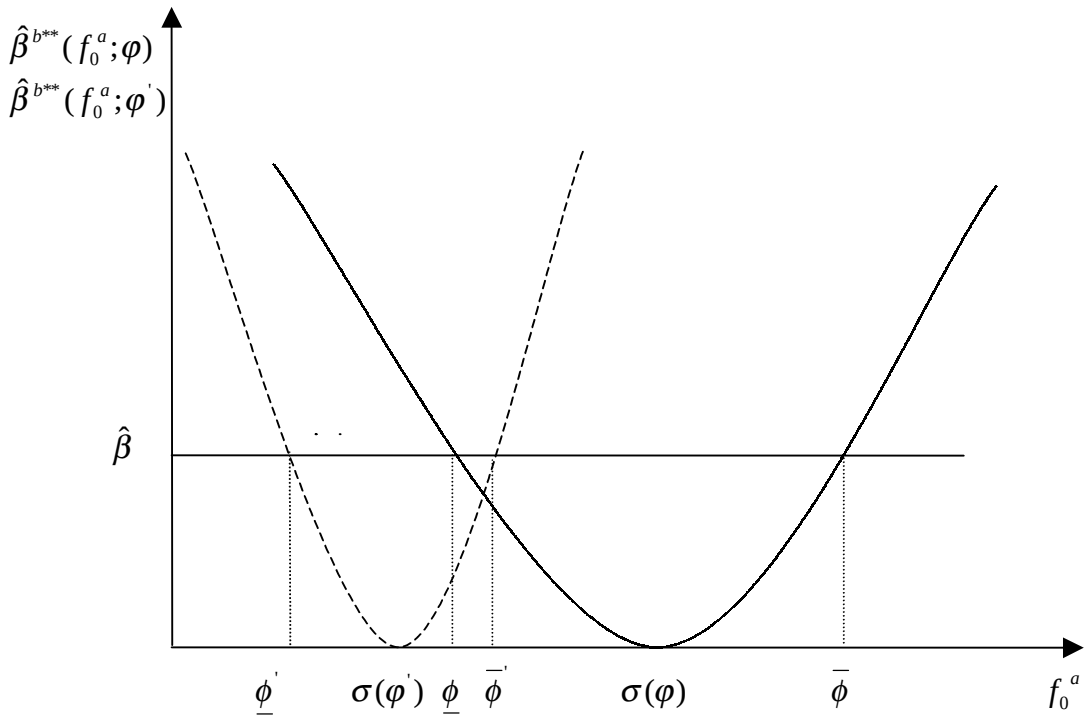

Figure 2. An increase of the weight on reappointment costs.

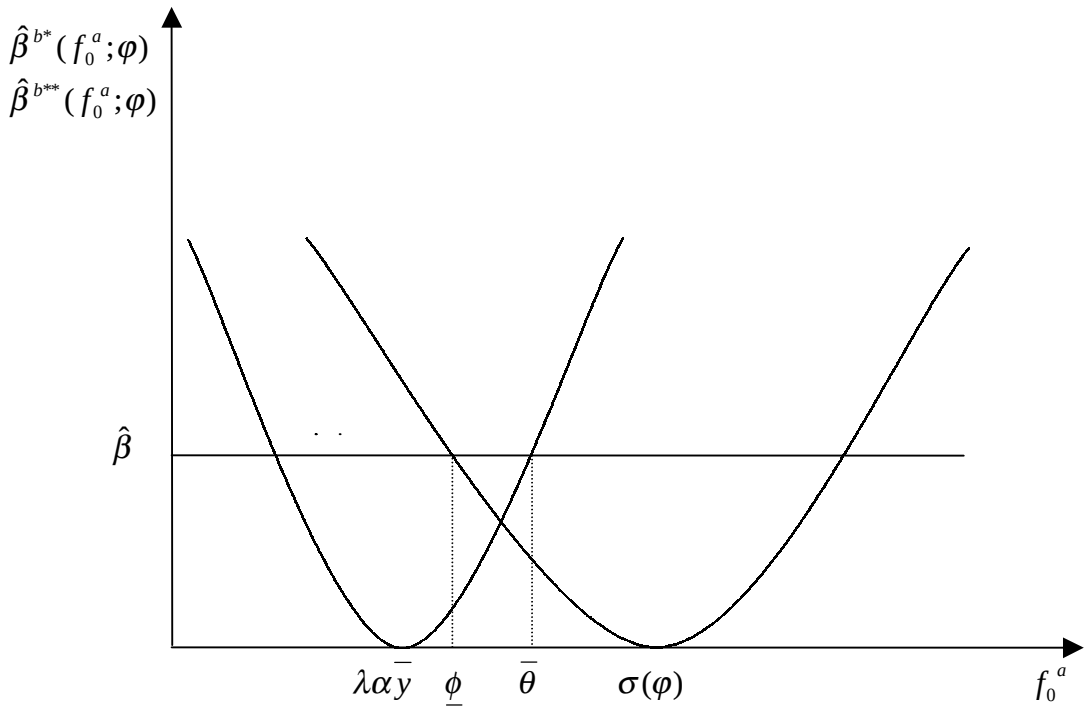

Figure 3. The set of optimal announcements available for the government.