Department of Economics
University of Southampton
Southampton SO17 1BJ
UK

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Time Consistent Monetary Policy Reconsidered: May We Have a Deflationary Bias Too?

Zeno Rotondi
University of Southampton and Banca di Roma*

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Abstract

The celebrated inflationary bias of time consistent monetary policy is re-examined. To this end we consider an extended version of the simple Barro and Gordon framework featuring important aspects of actual policy making such as imperfect instrument control, overlapping wage contracts, policy lags and interest rate control. The model developed provides a counterexample to the standard theory as it yields the result that a deflationary bias may be possible as well. The rationale for this surprising result is found in the distortion caused by instrument uncertainty in the trade-off between the costs and benefits associated with surprisingly lower interest rates faced at the margin by the policy maker. If the size of uncertainty is relatively large the distortion created may imply an optimal choice for the instrument which trades off the marginal benefit of lower deflation against the marginal cost of higher than optimal output. The implications of imprecise instrument control for welfare are discussed too.

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1 Introduction

The notion of time inconsistency, introduced in the economic literature by Kydland and Prescott (1977) and examined in detail by Barro and Gordon (1983) on its implications for monetary policy, is one of the explanations given by economists for the apparent inflationary bias of economic policy.[1] However, despite its popularity the issue of time inconsistency seems to be quite controversial. There is general agreement on the presence of an inflationary bias as in most countries inflation has risen above any conceivable optimal rate during their economic history.[2] On the contrary the importance of the issue of time inconsistency in explaining it does not seem to share the same level of agreement. The reasons for disagreement are various.

On one hand Taylor (1982, 1983) has suggested that, as societies have found solutions to the time inconsistency problems arising in other areas (e.g. patent law), then it is likely that the credibility problem of monetary policy might not be particularly severe or even present. It seems unlikely that a time consistent solution would prevail in situations where it is widely recognised the superiority of the optimal rule. In other words the inflationary bias may not be quantitatively relevant and therefore it does not need to be tackled and eliminated.

On the other hand McCallum (1995, 1997) has suggested that even if the inflationary bias might be quantitatively relevant (i.e. if societies have not found a solution to it) central bankers can be trusted not to be tempted to create inflation surprise as they know that this will lead to a worse equilibrium. According to this kind of criticism the Barro and Gordon model is not a plausible positive model of inflation because the absence of precommitment technologies does not prevent a central banker from behaving in a committed fashion and private individuals to rationally expect such optimal behaviour.

[1] See for example Fischer (1994) and Cukierman (1992) for a review of the alternative explanations given for the apparent inflationary bias of economic policy.

[2] A zero inflation rate target is usually considered optimal based on the many costs of inflation (crf. Driffill, Mizon and Ulph 1990). However Fischer (1994), for example, discusses the cases in favor of a socially optimal inflation rate target between 1-3 per cent. Recently Akerlof, Dickens and Perry (1996) and Groshen and Schweitzer (1997) also support the idea that the optimal inflation rate should be positive as this would help the policymaker to adjust real shocks more easily in presence of a downward rigity of nominal wages. Conversely Feldstein (1996) has suggested that the optimal inflation rate might even be negative, as the tax distortions created by inflation may reduce permanently the level of output.

Another kind of criticism concerns the specification of the Barro and Gordon model. In particular it has been shown that the finding of excessively high average inflation under discretionary monetary policy might be related to the simplicity of the model used. Here we have three major contributions.

First the inflationary bias associated with time consistent monetary policy has been questioned on the ground of lack of realism of the model used. Goodhart and Huang (1998) show that if, in to the Barro and Gordon framework, lags are introduced in the transmission of the effects of monetary policy, the inflationary bias disappears completely. The inflationary bias reappears only when overlapping nominal wage contracts are incorporated in the analysis and contracts with a length greater than the length of the policy lag are pervasive. But in this latter case the scale of the inflationary bias is considerably reduced and the explanatory power of the time-inconsistency answer to the apparent inflationary proclivities of industrialised countries is weakened.

Second, the Barro an Gordon model has been questioned also on the ground of lack of microfoundations. Nicolini (1998) has shown that in a general equilibrium monetary model the divergence between average inflation in equilibrium and the socially optimal level of inflation may not necessarily be of positive sign. In his framework the policy maker may find optimal to deviate by choosing inflation rates lower than expected and hence a disinflationary bias may arise. Clearly this striking results eliminates completely the issue of the importance of the time-inconsistency answer to the apparent inflationary bias of economic policy.

A third point has been made by Nobay and Peel (1998). In particular, they argue against the use of quadratic or linear preferences for examining optimal policy, as is done by Barro and Gordon. By exploiting a procedure used in Bayesian analysis, the Linex form, they analyse the implications of asymmetric preferences. It is shown that in this case the standard inflationary bias result under discretion does not hold unambiguously and there might be a deflationary bias as well. Moreover, a deflationary bias prevails unambiguously under precommitment and in this case the deviation of inflation from the socially optimal level is larger than under the case of discretion.

These criticisms have been partially left unanswered. Canzoneri (1985) has shown that Taylor's criticism does not hold when the policymaker has private information and his action cannot be monitored perfectly. The criticism concerning the size of the inflationary bias has been answered for example by Walsh (1998, pp.369-375). In examining the issue of the importance of the inflationary bias when there are policy lags and overlapping nominal wage contracts he observes that the presence of a reduced inflationary bias does not mean that the issue of time inconsistency is unimportant. The simple model used in the time inconsistency literature may not explain all observed inflation but nevertheless it raises the important issue of the incentives to deviate from optimal rules faced by policy makers.

In the present analysis we explore more deeply the third kind of criticism described above and related to specification issues. In particular we focus on the issue of the lack of realism of the Barro and Gordon framework. Thus we will examine how robust is the time-inconsistency explanation of the apparent inflationary bias of economic policy to modifications in the model's specification. Following Goodhart and Huang we extend the simple model of Barro and Gordon by incorporating policy lags and overlapping wage contracts. However, in contrast to them we focus on optimal interest rate rules, rather than optimal money growth rules, and introduce also the issue of instrument uncertainty. Therefore our framework is closer to actual policy making where central banks adjust their short-term interest rate in response to deviations of inflation and output from given targets and the control of monetary aggregates has been progressively abandoned.[3] Moreover a further degree of realism is added with the introduction of instrument uncertainty. Real world policy makers are subject to considerable uncertainty about the effects of policy on target variables and on instruments themselves. Therefore there is also non negligible uncertainty about policy multipliers.

One of the main results of our analysis is that under certain circumstances it is possible that a disinflationary bias may emerge as well as an inflationary one. Hence our model provides a confirmation for the findings of Nicolini and Nobay and Peel. This surprising result implies that the time-inconsistency explanation for the apparent positive inflationary bias becomes a qualified one. In the subsequent sections we will show in detail under which circumstances Barro and Gordon's celebrated result still holds in our framework.

The organisation of the exposition is the following. In section 2 we describe the model. In section 3 the equilibrium values are determined. Section 4 derives and discusses the main results of the analysis and compares them

[3] In the words of Blinder (1998, pp.26-29): "Returning to Poole's dichotomy [on the choice of monetary instrument]...in the end, real-world events, not theory, decided the issue. Ferocious instabilities in estimated LM curves in the United States, United Kingdom, and many other countries, beginning in the 1970s and continuing to the present day, led economists and policymakers alike to conclude that money-supply targeting is simply not a viable option....So interest rates won by default".

with those obtained in the previous literature. Finally, in section 5 some concluding observations are made.

2 The model

Following Fischer (1977) we assume two-period overlapping nominal wage contracts, which imply an aggregate supply function of the form

$$y_t = y_n + \frac{\alpha}{2}(\pi_t - E_{t-1}\pi_t) + \frac{\alpha}{2}(\pi_t - E_{t-2}\pi_t); \tag{1}$$

where y_n is the natural level of output, π_t is the realised rate of inflation and $E_{t-1}\pi_t$ and $E_{t-2}\pi_t$ are wage setters' inflation expectations.[4] Expectations are formed rationally using all available information at the end of period $t-1$ and $t-2$ respectively. The aggregate demand is given by a standard IS function

$$y_t = y_n - \beta(r_{t-1} - E_{t-1}\pi_t - \rho); \tag{2}$$

where ρ is the long-run real interest rate, r_{t-1} is the nominal interest rate. Here it is assumed that the interest rate, which is the instrument used for conducting monetary policy, affects output with a one-period lag.[5] Moreover we assume for simplicity that the monetary authority sets the instrument in terms of deviations of the nominal interest rate from the constant long-run real interest rate, $i_{t-1} = r_{t-1} - \rho$. This assumption ensures that when instrument uncertainty is introduced planned and actual level of the nominal

[4] As discussed in the introduction Goodhart and Huang (1998) and Walsh (1998) examine also the case of overlapping nominal wage contracts (along the lines of Fischer 1977) with a one-period lag in the effect of monetary policy. But in contrast to the present analysis they consider money supply as the instrument and the effects of interest rate changes on aggregate demand are ignored. Moreover the monetary authority is assumed to control perfectly the money supply and the issue of instrument uncertainty does not arise.

[5] This specification of the IS curve has been adopted for example by Ball (1997) and Svensson (1997) in a recent analysis of optimal interest rate rules when the are lags in the effect of monetary policy. Unlike the present analysis they assume also that monetary policy affects inflation with a two-period lag . This assumption complicates the analysis as it implies that in order to have a time-inconsistency problem we should introduce three-period labor contracts. However this complication would not change the basic insights of the analysis.

interest rate are with high probability non negative . [6]

As we want to examine the case of instrument uncertainty we assume that the actual level of the instrument is determined in the following way:

$$i_t = \varphi_t i_t^P + \psi_t; \tag{3}$$

which says that the monetary authority does not control the instrument perfectly and the planned level will differ from the actual level due to the presence of multiplicative control errors, φ_t, and additive control errors ψ_t. The assumption that the monetary authority does not control the instrument perfectly may reflect the fact that the interest rate that can be controlled more accurately is typically a short-term interest rate whereas in the aggregate IS-curve the relevant interest rate is of long-term, which may not be determined only by short-term interest rate movements.

At the same time the presence of control errors in the choice of the instrument may also reflect the possibility that there is uncertainty about the effects of the policy variable. This view is related to the stochastic optimization literature with uncertainty about policy effects started by Brainard (1967). In particular Brainard has shown that uncertainty about the parameters in the relationship between the policy variable and the target variable leads to a conservative use of the policy instrument.

In order to introduce a role for stabilisation policy, we assume that before the planned level of the interest rate is chosen the monetary authority receives a signal ε_t about the shock ψ_t.[7] Hence we have

[6] In the present framework the nominal interest rate can be negative for a small number of periods depending on the realisations of the stochastic control shocks. The issue of a non-negative nominal interest rate constraint is discussed in Lebow (1993), Cecchetti (1997) and Rudebush and Svensson (1998). In practice a negative nominal interest rate is not feasible but theoretical analysis usually does not exclude it, mainly in order to avoid the complexity of introducing a non-negativity constraint. An example is Clark, Huang and Goodhart (1999) where the level of the interest rate in the optimal decision rule for the monetary authority is not constrained to be non-negative.

Following Rudebush and Svensson (1998), we examine the potential power of central banks in conducting expansionary monetary policy and assume that there are always other instruments available (e.g. unsterilised interventions or increasing liquidity by means of open market purchases of Treasury securities at all maturities) when the nominal interest rate is near to zero.

[7] In the present framework in order to keep the analysis as simple as possible we ignore aggregate demand and supply shocks. As the model considered is static and assuming that monetary authorities at period t have no advance information about the shocks in period

$$\psi_t = \varepsilon_t + \nu_t. \tag{4}$$

We assume for analytical convenience that the shocks follow a multivariate normal distribution

$$\begin{pmatrix} \varphi_t \\ \varepsilon_t \\ \nu_t \end{pmatrix} \sim N_3 \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varphi^2 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & \sigma_\nu^2 \end{pmatrix} \right);$$

where multiplicative control shocks have mean 1 and variance σ_φ^2 while additive shocks have mean zero and variances σ_ε^2 and σ_ν^2.[8]

The timing of the actions is the following: the first half of the workers fix their nominal wages for two periods using information gathered at the end of period $t-2$; at the beginning of period $t-1$ the monetary authority receives a signal about the additive control error and sets the planned level of the interest rate; subsequently the multiplicative and additive control errors are realised; finally the second half of the workers fix their nominal wages for two periods using information up to the end of period $t-1$. Thus in the present model the monetary authority can potentially fool only the proportion of workers that fix their nominal wage at the end of period $t-2$, before monetary policy is chosen. The workers that fix their nominal wages at the end of period $t-1$ can observe perfectly the actual level of the interest rate.

Now equating output in (1) and (2) we have

$$\pi_t = -\frac{\beta}{\alpha} i_{t-1} + \frac{1}{2} E_{t-1}\pi_t + \frac{\beta}{\alpha} E_{t-1}\pi_t + \frac{1}{2} E_{t-2}\pi_t. \tag{5}$$

From equation (5) we can derive the relationship between inflation and the policy instrument. As there are no current shocks, current inflation is affected one period in advance by the lagged interest rate, inflation expectations of half of the wage setters based on available information at $t-2$ and inflation expectations formed at the end of period $t-1$. This implies that at the end

$t+1$, then the role of aggregate supply and demand shocks would be similar to that of the unforecastable component of additive control shocks. The monetary authorities are not able to stabilise these shocks and the optimal instrument feed-back rule will not depend on them.

[8]See Schellekens (1998) for the same assumption on the stochastic structure of the shocks in the context of Brainard uncertainty and time inconsistency. Differently from here he considers also the possibility that control shocks are not independent from each other with the covariance being different from zero.

of period $t-1$ wage setters and investors can predict exactly the level of the period t inflation rate. The implication for monetary policy is that now the policy maker can no longer take expectations formed at the end of period $t-1$ as given. Actually the policy maker's choice made at $t-1$ can affect expectations immediately and therefore his ability to create surprise inflation is weakened. Thus, after taking expectations at $t-1$, (5) becomes

$$\pi_t = -\frac{2\beta}{\alpha - 2\beta} i_{t-1} + \frac{\alpha}{\alpha - 2\beta} E_{t-2}\pi_t; \tag{6}$$

with $\beta \neq \alpha/2$.

Considering (3) and (4) and taking expectations at $t-2$ of (6) we obtain

$$E_{t-2}\pi_t = E_{t-2} i^P_{t-1}. \tag{7}$$

Substituting (7) back into (6) we can express inflation in terms of the control variable i^P_t and the wage setters' expectations of the level of the control variable itself. Shifting one period forward we have

$$\pi_{t+1} = -\frac{2\beta}{\alpha - 2\beta} \left(\varphi_t i^P_t + \varepsilon_t + \nu_t\right) + \frac{\alpha}{\alpha - 2\beta} i^{Pe}_t; \tag{8}$$

where $i^{Pe}_t = E_{t-1} i^P_t$, in order to simplify the notation.

Substituting (6) and (7) in (2) and shifting one period forward, output can be expressed as

$$y_{t+1} = y_n + \frac{\alpha\beta}{\alpha - 2\beta} \left[i^{Pe}_t - \left(\varphi_t i^P_t + \varepsilon_t + \nu_t\right)\right]. \tag{9}$$

By examining (8) and (9) it is possible to see that the effects of changes in the interest rate on output and inflation are ambiguous and depend on the assumed parameter values in the model. In particular, an increase in the actual level of the interest rate will reduce (increase) both output and inflation if $\beta < \alpha/2$ (if $\beta > \alpha/2$). On the contrary an increase in the level of the interest rate expected by the private sector will increase (reduce) both output and inflation if $\beta < \alpha/2$ (if $\beta > \alpha/2$).

Nevertheless it is possible to show that the system represented by equations (1) and (2) will be stable and converge to the long-run equilibrium only if $\beta < \alpha/2$. To confirm this take expectations through equations (1) and (2) at $t-1$ with the policy action i^P_{t-1} and $E_{t-2}\pi_t$ both fixed. In this way we have that expected output $E_{t-1}y_t$ is an increasing function of expected inflation

$E_{t-1}\pi_t$ in both the supply and demand equations. Since the equilibrium is a RE equilibrium, it is clear that there is a unique equilibrium and that it occurs at the intersection of the two curves, no matter whether β exceeds or is less than $\alpha/2$. Now one can see that, if the expected demand curve (with slope equal to β) is flatter than the expected supply curve (with slope equal to $\alpha/2$), then higher than equilibrium expected inflation induces expected excess supply. Conversely if, the expected demand curve is steeper than the expected supply curve, then higher than equilibrium expected inflation induces expected excess demand. Both cases imply adjustments over time of the inflation rate towards its market-clearing value.

However the dynamic process of inflation will converge to its long-run equilibrium value and at the same time lead to a dampening of the output gap only if $\beta < \alpha/2$. Assuming that the economy is composed of many small firms, with each producer setting its own price, in presence of market excess demand each firm would wish to charge a higher than market price; on the contrary with excess market supply. Price setting by individual traders only leads to a stable RE equilibrium if higher expected price reduces expected excess demand.

The convergence requirement implies that the parameter that determines the effect of the expected real interest rate on goods demanded, β, must be sufficiently small compared to the elasticity of goods supplied with respect to inflation surprises, α.[9] In the following analysis we assume that the convergence requirement holds. Hence we have also that the impacts of monetary policy on the target variables are the same of the standard IS-LM analysis: an increase in the actual level of interest rate reduces both inflation and output.

Finally the model is closed by the preferences of the monetary authority. The policy maker chooses i_t^P to minimise the following period loss function

$$L_{t+1} = \pi_{t+1}^2 + \lambda \left(y_{t+1} - \overline{y}\right)^2; \tag{10}$$

which is the standard objective function considered in the literature on the issue of time inconsistency in monetary policy. As usually, society is assumed

[9] The empirical literature on the expectations augmented Phillips curve has evidenced some problems in isolating a significantly positive effect of price surprises on goods supplied. See for example the recent empirical analysis provided by Barro and Broadbent (1997). Moreover, to our knowledge, Fischer's model of overlapping nominal wage contracts has not been empirically tested. So the empirical literature does not provide clear insights into the relative importance of the two parameters examined.

to have the same preferences as the policy maker. Thus (10) represents also society's period loss function. The policy maker weights deviations of inflation and output from specified target values with $\lambda > 0$. Here a standard assumption is that the policy maker has an output target greater than the natural level of output, i.e. $\overline{y} > y_n$.

3 Time consistent equilibrium

The discretionary solution of the model is derived by minimising the expected value of the loss function (10) with respect to i_t^P, conditional on the information set of the policy maker when i_t^P is chosen,

$$\min_{i_t^P} E\left[L_{t+1} \mid \varepsilon\right]; \tag{11}$$

subject to (8) and (9) and taking as given the private sector's expectations i_t^{Pe} . We have the following first order condition:

$$\begin{aligned}
&\left(-\frac{2\beta}{\alpha-2\beta}\left(\varphi_t i_t^P+\varepsilon_t+\nu_t\right)+\frac{\alpha}{\alpha-2\beta}i_t^{Pe}\right)\left(-\frac{4\beta\varphi}{\alpha-2\beta}\right)\\
&+\lambda\left(y_n+\frac{\alpha\beta}{\alpha-2\beta}\left(i_t^{Pe}-\varphi_t i_t^P-\varepsilon_t-\nu_t\right)-\overline{y}\right)\left(-\frac{2\beta\alpha\varphi}{\alpha-2\beta}\right)=0;
\end{aligned} \tag{12}$$

which becomes, after taking expectations conditional on the signal ε_t and simplifying:

$$\begin{aligned}
&2\left(-\frac{2\beta}{\alpha-2\beta}\left(\left(1+\sigma_\varphi^2\right)i_t^P+\varepsilon_t\right)+\frac{\alpha}{\alpha-2\beta}i_t^{Pe}\right)\\
&+\lambda\alpha\left(y_n+\frac{\alpha\beta}{\alpha-2\beta}\left(i_t^{Pe}-\left(1+\sigma_\varphi^2\right)i_t^P-\varepsilon_t\right)-\overline{y}\right)=0;
\end{aligned} \tag{13}$$

where we have used the fact that $E\left[\varphi^2\right]=1+\sigma_\varphi^2$. Rearranging the first order condition we get the monetary authority reaction function

$$i_t^P = \frac{(2\beta-\alpha)\lambda\alpha\left(\overline{y}-y_n\right)}{\beta\left(4+\lambda\alpha^2\right)\left(1+\sigma_\varphi^2\right)}+\frac{\alpha\left(2+\lambda\alpha\beta\right)}{\beta\left(4+\lambda\alpha^2\right)\left(1+\sigma_\varphi^2\right)}i_t^{Pe}-\frac{1}{1+\sigma_\varphi^2}\varepsilon_t. \tag{14}$$

The private sector's expectations are found by taking expectations at time $t-1$ over the policy maker's reaction function which yields

$$i_t^{Pe} = \frac{(2\beta - \alpha)\,\lambda\alpha\,(\overline{y} - y_n)}{2\,(2\beta - \alpha) + \beta\sigma_\varphi^2\,(4 + \lambda\alpha^2)}. \tag{15}$$

Thus the optimal instrument feed-back rule will be

$$i_t^{P} = \frac{(2\beta - \alpha)\,\lambda\alpha\,(\overline{y} - y_n)}{2\,(2\beta - \alpha) + \beta\sigma_\varphi^2\,(4 + \lambda\alpha^2)} - \frac{1}{1+\sigma_\varphi^2}\varepsilon_t; \tag{16}$$

which implies that after the control error shocks are realised the actual level of the nominal interest rate will be

$$i_t = \frac{\varphi_t\,(2\beta - \alpha)\,\lambda\alpha\,(\overline{y} - y_n)}{2\,(2\beta - \alpha) + \beta\sigma_\varphi^2\,(4 + \lambda\alpha^2)} - \frac{\left[\varphi_t - \left(1+\sigma_\varphi^2\right)\right]}{1+\sigma_\varphi^2}\varepsilon_t + \nu_t. \tag{17}$$

Now we are able to find the equilibrium values for inflation and output. Substituting (15) and (16) back into the expressions for inflation and output we get

$$\pi_{t+1} = \frac{(2\beta\varphi_t - \alpha)\,\lambda\alpha\,(\overline{y} - y_n)}{2\,(2\beta - \alpha) + \beta\sigma_\varphi^2\,(4 + \lambda\alpha^2)} - \frac{2\beta\left[\varphi_t - \left(1+\sigma_\varphi^2\right)\right]}{(2\beta - \alpha)\left(1+\sigma_\varphi^2\right)}\varepsilon_t + \frac{2\beta}{2\beta - \alpha}\nu_t; \tag{18}$$

and

$$y_{t+1} = \frac{\alpha\beta\,(\varphi_t - 1)\,\lambda\alpha\,(\overline{y} - y_n)}{2\,(2\beta - \alpha) + \beta\sigma_\varphi^2\,(4 + \lambda\alpha^2)} - \frac{\alpha\beta\left[\varphi_t - \left(1+\sigma_\varphi^2\right)\right]}{(2\beta - \alpha)\left(1+\sigma_\varphi^2\right)}\varepsilon_t + \frac{\alpha\beta}{2\beta - \alpha}\nu_t. \tag{19}$$

4 Implications of multiplicative instrument uncertainty

Here we investigate the implications of the introduction of multiplicative uncertainty comparing the results obtained in the previous section with the case when there is only additive uncertainty. When the policy maker faces imperfect information about the shocks hitting the economy it should respond on the basis of its best forecast of these shocks. However Brainard (1967) showed that this is no longer true when there is multiplicative uncertainty in the parameters of the relationship between the level of the policy instrument and the goal variable. In this case uncertainty implies that the policy choices affect the shape of the distribution of the goal variable and that it is optimal to adjust less than completely to the disturbances.

As in Cukierman and Meltzer (1986) and Swank (1994) and Letterie (1997), in the next sections it is assumed that σ_{φ}^2 is an institutional feature of the implementation of monetary policy. The volatility of multiplicative control errors is chosen by the policy maker ex-ante and cannot be modified after the private sector's expectations about the instrument are set. This assumption is based on the idea that, unlike policy choices, institutional operating procedures can be changed only with a time lag which is greater than the horizon of existing nominal contracts.

4.1 Inflation

Following the same algorithm used for deriving the discretionary solution it is easy to find that without multiplicative uncertainty the optimal instrument feed-back rule and the equilibrium inflation rate are given respectively by

$$i_t^{AD} = \frac{\lambda\alpha\left(\overline{y} - y_n\right)}{2} - \varepsilon_t; \tag{20}$$

$$\pi_{t+1}^{AD} = \frac{\lambda\alpha\left(\overline{y} - y_n\right)}{2} + \frac{2\beta}{2\beta - \alpha}\nu_t; \tag{21}$$

where i_t^{AD}, π_{t+1}^{AD} are the level of the monetary instrument and the inflation rate when there is only additive uncertainty.

By taking unconditional expectations we can get the inflationary bias in the two cases examined.[10] We have

[10] The bias in average inflation under a time consistent monetary policy is defined relative to a hypothetical regime where the policy maker is able to credibly precommit in advance to a rule for setting the monetary instrument. The precommitment solution can be found assuming that the policy maker sets the interest rate according to the following rule: $i_t^P = \overline{\phi} + \phi\varepsilon_t$. The policymaker minimises his expected loss with respect to both the systematic and the state contingent components of the rule, respectively $\overline{\phi}$ and ϕ. In contrast to a discretionary regime, in a regime with precommitment the policy maker internalises in its optimisation problem the effects of its decision rule on expectations by

$$E\left[\pi^{AD}\right]=\frac{\lambda\alpha\left(\overline{y}-y_n\right)}{2}; \tag{22}$$

and

$$E\left[\pi\right]=\frac{\left(2\beta-\alpha\right)\lambda\alpha\left(\overline{y}-y_n\right)}{2\left(2\beta-\alpha\right)+\beta\sigma_{\varphi}^2\left(4+\lambda\alpha^2\right)}. \tag{23}$$

The variances of inflation are given by

$$Var\left[\pi^{AD}\right]=\left(\frac{2\beta}{2\beta-\alpha}\right)^2\sigma_{\nu}^2; \tag{24}$$

and

$$\begin{aligned} Var\left[\pi\right] &= \frac{\left[2\beta\lambda\alpha\left(\overline{y}-y_n\right)\right]^2\sigma_{\varphi}^2}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_{\varphi}^2\left(4+\lambda\alpha^2\right)\right]^2}+\frac{\left(2\beta\right)^2\sigma_{\varphi}^2}{\left(1+\sigma_{\varphi}^2\right)\left(2\beta-\alpha\right)^2}\sigma_{\varepsilon}^2 \\ &\quad +\left(\frac{2\beta}{2\beta-\alpha}\right)^2\sigma_{\nu}^2; \end{aligned} \tag{25}$$

where we used that $Var\left[\pi\right]=E\left[\pi^2\right]-\left(E\left[\pi\right]\right)^2$ and given the properties of the trivariate normal distribution considered here we can compute the following joint moments $E\left[\varphi^2\varepsilon\right]=E\left[\varepsilon\varphi\nu\right]=0$; $E\left[\varphi\varepsilon^2\right]=\sigma_{\varepsilon}^2$ and $E\left[\varphi^2\varepsilon^2\right]=\left(1+\sigma_{\varphi}^2\right)\sigma_{\varepsilon}^2$.

From (22) and (23) it is possible to derive one of the main results of the present analysis:

Proposition 1 . *Multiplicative uncertainty has an ambiguous effect on average inflation: if the amount of multiplicative uncertainty is relatively large (small) it implies a deflationary (inflationary) bias.*

From (23) it is straightforward to verify that, for

$$\sigma_{\varphi}^2>2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^2\right), \tag{26}$$

setting $i_t^{Pe}=\overline{\phi}$. Deriving the first order conditions and taking unconditional expectations it is possible to see that in equilibrium average inflation will be equal to zero, independently of whether multiplicative uncertainty is present or not. However it is possible to see that, if the socially optimal level of inflation is greater than zero, in the case of commitment we may have a deflationary bias too.

average inflation under a time consistent monetary policy will be negative; otherwise it will always be positive.

The intuition for this surprising result is as follows. The result obtained is independent of the additive component of the control errors. So assume first that there is no instrument uncertainty and then introduce only multiplicative uncertainty. If there is no instrument uncertainty both the planned level of the instrument (here equal to the actual level) and average inflation are positive and are respectively given by (20), without the stochastic term, and (22). From the first order condition (13) it is possible to see that when $\sigma_\varphi^2 = 0$ and $\varepsilon_t = 0$ the policymaker will set the planned level of the instrument at the point where the marginal cost of higher inflation with a lower (surprise) interest rate compensates exactly the marginal benefit (with negative sign) of higher output with a lower (surprise) interest rate.

Now introduce multiplicative uncertainty with σ_φ^2 sufficiently large. In this case both the planned level of the instrument and average inflation have negative sign and are respectively given by (16), without the stochastic term, and (23). From the first order condition it is possible to see that the introduction of a small increase of σ_φ^2 distorts at the margin both the costs of higher inflation and benefits of higher output from creating surprisingly lower interest rates. In order to compensate these distortions a further reduction of the instrument with respect to the given expectations is required. If σ_φ^2 is sufficiently large the policy maker will end up trading-off the marginal benefits of lower deflation with the marginal costs of higher than optimal output deriving from surprisingly higher negative levels of the instrument. Thus, in the end, the reason for our new result is that multiplicative instrument uncertainty distorts the trade-off faced at the margin by the policy maker when choosing the optimal level of the instrument.

Still from (23) we can prove the following proposition:

Proposition 2 . *Multiplicative uncertainty always worsens the stabilisation role of the instrument but the effect on the credibility of monetary policy is ambiguous: if the amount of multiplicative uncertainty is relatively large (small) it improves (deteriorates) credibility.*

Comparing the instrument feed-back rules (16) and (20) is clear that multiplicative uncertainty makes it optimal for the policymaker to stabilise less the forecast additive control error and to adopt greater caution in conducting monetary policy. The effect of multiplicative uncertainty on average inflation can be analysed from the following first derivative

$$\frac{\partial E[\pi]}{\partial \sigma_\varphi^2} = -\frac{2\beta\sigma_\varphi^2 (2\beta-\alpha)(4+\lambda\alpha^2)\lambda\alpha(\overline{y}-y_n)}{\left[2(2\beta-\alpha)+\beta\sigma_\varphi^2(4+\lambda\alpha^2)\right]^2} > 0; \tag{27}$$

with

$$\sigma_\varphi^2 \neq 2(\alpha-2\beta)/\beta\left(4+\lambda\alpha^2\right). \tag{28}$$

Here we have the following cases:

i) for $0 < \sigma_\varphi^2 < 2(\alpha-2\beta)/\beta(4+\lambda\alpha^2)$ the inflationary bias under multiplicative uncertainty is increased as σ_φ^2 increases;

ii) for $\sigma_\varphi^2 > 2(\alpha-2\beta)/\beta(4+\lambda\alpha^2) > 0$ the deflationary bias under multiplicative uncertainty is reduced as σ_φ^2 increases.

This ambiguous effect of multiplicative uncertainty on the credibility of monetary policy is new in the time inconsistency literature. The standard result, as exemplified by Swank (1994), Pearce and Sobue (1997), Letterie (1997), Letterie and Lippi (1997) and Schellekens (1998), is that multiplicative uncertainty unambiguously improves the credibility of discretionary monetary policy by inducing a more cautious stance of monetary policy. Thus the standard result is that imperfect monetary control constrains the temptations of the policymaker to surprise the private sector. However in all these models the policy instrument is the supply of money and the issues of lags in the transmission of the effects of monetary policy and overlapping nominal wage contracts are excluded from the analysis. Here we show that in a more complex model, closer to actual policy making, the effect of multiplicative uncertainty on credibility is ambiguous and depends on the level of the volatility of multiplicative control errors. This ambiguity stems from the possibility in our model of having under the time consistent monetary policy both a deflationary bias and an inflationary bias.

Finally we consider the implications of the introduction of multiplicative uncertainty for the variance of inflation. We have the following proposition:

Proposition 3 . *The introduction of a marginal increase of multiplicative uncertainty unambiguously increases the volatility of inflation.*

This proposition follows from the first derivative of (25) which can be showed to be

$$\left.\frac{\partial Var[\pi]}{\partial \sigma_\varphi^2}\right|_{\sigma_\varphi^2=0} = \frac{\beta^2\left[(\lambda\alpha)^2(\overline{y}-y_n)^2+4\sigma_\varepsilon^2\right]}{(2\beta-\alpha)^2} > 0. \tag{29}$$

4.2 Output

Without multiplicative uncertainty equilibrium output is

$$y_{t+1}^{AD} = y_n - \frac{\alpha\beta}{\alpha - 2\beta}\nu_t. \tag{30}$$

Let's take unconditional expectations of (19) and (30). In both cases expected output will be equal to the natural level

$$E[y] = E\left[y^{AD}\right] = y_n; \tag{31}$$

while the variances of output are given by

$$Var\left[y^{AD}\right] = \left(\frac{\alpha\beta}{\alpha - 2\beta}\right)^2 \sigma_\nu^2; \tag{32}$$

and

$$\begin{aligned} Var[y] &= \frac{\left[\beta\alpha\lambda\alpha\left(\overline{y} - y_n\right)\right]^2 \sigma_\varphi^2}{\left[2\left(2\beta - \alpha\right) + \beta\sigma_\varphi^2\left(4 + \lambda\alpha^2\right)\right]^2} + \frac{\left(\alpha\beta\right)^2 \sigma_\varphi^2}{\left(1 + \sigma_\varphi^2\right)\left(2\beta - \alpha\right)^2}\sigma_\varepsilon^2 \\ &+ \left(\frac{\alpha\beta}{2\beta - \alpha}\right)^2 \sigma_\nu^2. \end{aligned} \tag{33}$$

Here we have the following proposition:

Proposition 4 . *The introduction of a marginal increase of multiplicative uncertainty unambiguously increases the volatility of output.*

This proposition follows from the first derivative of (33) which can be shown to be

$$\left.\frac{\partial Var[y]}{\partial\sigma_\varphi^2}\right|_{\sigma_\varphi^2=0} = \frac{\beta^2\alpha^2\left[\left(\lambda\alpha\right)^2\left(\overline{y} - y_n\right)^2 + \sigma_\varepsilon^2\right]}{\left(2\beta - \alpha\right)^2} > 0. \tag{34}$$

4.3 Social welfare

Let's analyse the implications of multiplicative uncertainty for social welfare. The unconditional expectation over society's period loss function can be expressed in the following convenient way

$$E[L] = (E[\pi])^2 + Var[\pi] + \lambda \overline{y}^2 + \lambda Var[y]. \tag{35}$$

After substituting the relative expressions for the variances and unconditional expectations under the two cases here considered we have

$$\begin{aligned} E[L] &= \frac{\left[(2\beta-\alpha)^2 + \beta^2\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]\left[\alpha\lambda\left(\overline{y}-y_n\right)\right]^2}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]^2} + \lambda\overline{y}^2 \\ &\quad + \frac{\left(4+\lambda\alpha^2\right)\beta^2\sigma_\varphi^2}{\left(1+\sigma_\varphi^2\right)\left(\alpha-2\beta\right)^2}\sigma_\varepsilon^2 + \frac{\left(4+\lambda\alpha^2\right)\beta^2}{\left(\alpha-2\beta\right)^2}\sigma_\nu^2; \end{aligned} \tag{36}$$

and

$$E\left[L^{AD}\right] = \frac{\left[\lambda\alpha\left(\overline{y}-y_n\right)\right]^2}{4} + \lambda\overline{y}^2 + \frac{\left(4+\lambda\alpha^2\right)\beta^2}{\left(\alpha-2\beta\right)^2}\sigma_\nu^2. \tag{37}$$

Now it is possible to show the following proposition:

Proposition 5 . *If the amount of multiplicative uncertainty is relatively small its introduction is welfare decreasing. On the contrary if its amount is sufficiently large multiplicative uncertainty has an ambiguous effect on the expected social loss which depends on the relative importance of the credibility problem with respect to the flexibility problem: the larger is the credibility problem with respect to the flexibility problem the more likely multiplicative uncertainty improves social welfare.*

The overall effect of multiplicative uncertainty on social loss can be examined from the following first derivative

$$\begin{aligned} \frac{\partial E[L]}{\partial \sigma_\varphi^2} &= -\frac{\left[2\left(\beta-\alpha\right)\left(2\beta-\alpha\right)+\beta^2\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]\left(4+\lambda\alpha^2\right)\beta\left[\alpha\lambda\left(\overline{y}-y_n\right)\right]^2}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]^3} \\ &\quad + \frac{\left(4+\lambda\alpha^2\right)\beta^2}{\left(1+\sigma_\varphi^2\right)^2\left(2\beta-\alpha\right)^2}\sigma_\varepsilon^2; \end{aligned} \tag{38}$$

with

$$\sigma_\varphi^2 \neq 2\left(\alpha - 2\beta\right)/\beta\left(4 + \lambda\alpha^2\right). \tag{39}$$

Here we have two cases depending on the dimension of multiplicative uncertainty:

i) for $\sigma_\varphi^2 < 2\left(\alpha - 2\beta\right)/\beta\left(4 + \lambda\alpha^2\right)$, we have

$$\frac{\partial E\left[L\right]}{\partial\sigma_\varphi^2} > 0; \tag{40}$$

ii) for $\sigma_\varphi^2 > 2\left(\alpha - 2\beta\right)/\beta\left(4 + \lambda\alpha^2\right)$, we have

$$\frac{\partial E\left[L\right]}{\partial\sigma_\varphi^2} \lesseqqgtr 0 \mathit{iff} \ \frac{\overline{y}}{\sigma_\varepsilon} \gtreqqless \Delta\left(\sigma_\varphi^2\right); \tag{41}$$

with

$$\Delta\left(\sigma_\varphi^2\right) \equiv \frac{\sqrt{\beta\left[2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]^3}}{\alpha\lambda\left(1+\sigma_\varphi^2\right)\left(\alpha-2\beta\right)\sqrt{2\left(\alpha-2\beta\right)\left(\alpha-\beta\right)+\beta^2\sigma_\varphi^2\left(4+\lambda\alpha^2\right)}}. \tag{42}$$

Thus, if the credibility problem is large enough relatively to the flexibility problem, the introduction of sufficient multiplicative uncertainty is likely to reduce the social loss compared to the case when there is only additive instrument uncertainty. A similar result has been found, for example, also by Schellekens (1998), Letterie (1997) and Letterie and Lippi (1997). They found contrary to Swank (1994), where multiplicative uncertainty always improves social welfare, that the effect of multiplicative uncertainty on social welfare is ambiguous depending on the size of the credibility problem. Devereux (1987) has provided the same result also for the case when there is only additive uncertainty. Using a model with endogenous wage indexing, along the lines of Gray (1976), he has shown that uncertainty is more likely to increase welfare when the credibility problem becomes more important. The main difference of our analysis with respect to the previous literature is that now when multiplicative uncertainty becomes more advantageous at the same time the occurrence of a deflationary bias is more likely.

5 Conclusions

In the present analysis we have tried to re-examine the issue of the inflationary bias associated with discretionary monetary policy by using an extended version of the Barro and Gordon framework, closer to actual policymaking. The model developed has yielded some results that question the previous findings. In particular we have shown that time inconsistency does not necessarily imply an inflationary bias, but may yield a deflationary one instead. In this respect our framework under specified circumstances provides a counterexample to Barro and Gordon's famous result.

This surprising finding implies that the current use of the time-inconsistency paradigm as a possible explanation of episodes of persistent and excessively high inflation rates should be more cautious. Actually, our model predicts that economies which feature a relatively large incentive to increase output above its long-run level are more likely to be plagued by a deflationary bias. In this case the implementation of policy should be characterised by a relatively more imprecise control of the policy instrument. An Inflationary bias is more likely to be present in economies where the credibility problem is relatively less serious. Here the implementation of policy should be characterised by a relatively better control of the policy instrument.

Some recent works by Nicolini, Nobay and Peel have shown that Barro and Gordon's inflationary bias result is not robust to interesting modifications of the original framework. In particular they have examined the implications of the introduction of microfoundations, with a general equilibrium model, and asymmetric central bank preferences. Our analysis has confirmed the above finding.

However, in contrast to Nicolini we have used a framework closer to that used by Barro and Gordon, while unlike Nobay and Peel we do not obtain a deflationary bias under a regime with commitment and the superiority of the ex ante optimal monetary policy still holds in our model. The latter result crucially depends on the assumptions about the socially optimal level of inflation. If we assume an inflation target greater than zero in the policy maker's loss function, we may have in our framework a deflationary bias also in the case of commitment.

References

Akerlof, G.A., W.T. Dickens, and G.L. Perry (1996). "The Macroeconomics of Low Inflation", Brookings Papers of Economic Activity 1:1-59.

Ball, L. (1997). Efficient Rules for Monetary Policy, NBER Working paper N. 5952.

Barro, R. and D. Gordon (1983). A Positive Theory of Monetary Policy in a Natural Rate Model, *Journal of Political Economy*, 91:589-610.

Blinder, A.S. (1998). "Central Banking in Theory and in Practice", the MIT Press, Cambridge, Massachusetts, USA.

Brainard, W. (1967). Uncertainty and the Effectiveness of Policy, *American Economic Review*. 57: 411-425.

Broadbent, B. and R. Barro (1997). Central Bank Preferences and Macroeconomic Equilibrium, *Journal of Monetary Economics*, 39:17-43.

Canzoneri, M. (1985). Monetary policy games and the role of private information, *American Economic Review,* 75:1056-1070.

Cecchetti, S.G. (1997). Central bank policy rules: conceptual issues and practical considerations, NBER Working Papers, N.6306.

Clark, P.B., C.A.E. Goodhart and H. Huang (1999). Optimal Monetary Policy Rules in a Rational Expectations Model of the Phillips Curve, *Journal of Monetary Economics,* 43:497-520.

Cukierman, A. (1992). "Central Bank Strategy, Credibility and Independence - Theory and Evidence", the MIT Press, Cambridge, Massachusetts, USA.

Cukierman, A. and Meltzer, A.H. (1986). A Theory of Ambiguity, Credibility and Inflation Under Discretion and Asymmetric Information, *Econometrica*. 54:1099-1128.

Devereux, M. (1987). The Effect of Monetary Variability on Welfare in a Simple Macroeconomic Model, *Journal of Monetary Economics*. 19: 427-435.

Driffill, J., G.E. Mizon and A. Ulph (1990), "Costs of Inflation",. In B. Friedman and F. Hahn (eds.), The Handbook of Monetary Economics, Vol.2, pp. 1021-1066. New Yprk: North-Holland.

Feldstein, M. (1996). "The costs and benefits of going from low inflation to price stability", in C. Romer and D. Romer (eds.): "Reducing Inflation". Chicago: University of Chicago Press, PP. 123-66.

Fischer, S. (1977). Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule, *Journal of Political Economy.* 85: 191-205.

Fischer, S. (1994). Modern central banking, in F. Capie, C.A.E. Goodhart, S. Fischer and N. Schnadt (eds.), "The Future of Central Banking", Cambridge University Press, Cambridge, UK.

Goodhart, C.A.E.and H. Huang (1998). Time Inconsistency in a Model with Lags, Persistence, and Overlapping Wage Contracts, *Oxford Economic Papers*, 50:378-396.

Gray, J.A. (1976). Wage Indexation: a Macroeconomic Approach, *Journal of Monetary Economics.* 2: 221-235.

Groshen, E. and M. Schweitzer (1997), Identifying inflation's grease and sand effects in labour market, NBER Working Papers, N. 6061.

Kydland, F. and E. Prescott (1977). Rules rather than discretion: the inconsistency of optimal plans, *Journal of Political Economy*, 85:473-490.

Lebow, D.E. (1993). Monetary Policy at Near Zero Interest Rates. Working Paper No. 136, Division of Research and Statistics, Federal Reserve Board.

Letterie, W. (1997). Better Monetary Control May Decrease the Distortion of Stabilisation Policy: A Comment, *Scandinavian Journal of Economics.* 99: 463-470.

Letterie, W. and F. Lippi (1997). Excessive Activism or Passivism of Monetary Policy, Temi di Discussione del Servizio Studi, Banca d'Italia No. 299.

McCallum, B.T. (1995). Two fallacies concerning central bank independence, *American Economic Review Papers and Proceedings.* 85: 207-211.

McCallum, B.T. (1997). Crucial issues concerning central bank independence, *Journal of Monetary Economics.* 39: 99-112.

Nobay, A.R. and D.A. Peel (1998). Optimal monetary policy in a model of asymmetric central bank preferences, LSE Financial Market Group, Discussion Papers, N.306.

Pearce D. and M. Sobue (1997). Uncertainty and the Inflation Bias of Monetary Policy, *Economics Letters.* 53: 203-207.

Rudebusch, G. D. and L.E.O. Svensson (1998). Policy Rules for Inflation Targeting, NBER Working Papers, No.6512.

Schellekens, P. (1998). Caution and Conservatism in Monetary Policymaking, Discussion Paper No. 284, Financial Market Group, LSE, London.

Svensson, L.E.O (1997). Inflation Forecast Targeting: Implementing and Monitoring Inflation Targets, *European Economic Review,* 41: 1111-1146.

Swank, O. (1994). Better Monetary Control May Increase the Inflationary Bias of Policy, *Scandinavian Journal of Economics*. 96: 125-131.

Taylor, J.B. (1982). Establishing Credibility: A Rational Expectations Viewpoint, *American Economic Review*, 72: 81-85.

Taylor, J.B. (1983). Comment on "Rules, discretion and reputation in a model of monetary policy" by R.J. Barro and D.B. Gordon, *Journal of Monetary Economics*, 12: 123-125.

Walsh, C.E. (1998). "Monetary Theory and Policy", the MIT Press, Cambridge, Massachusetts, USA.