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Designing instrument rules for monetary stability: the optimality of interest-rate smoothing

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Abstract

A key issue in monetary policy is that on the importance of following systematic behaviours. The paper revisits the classic debate on rules versus discretion focusing on the design of instrument rules in a manner that push discretionary policy choices in the direction of the commitment equilibrium. It is shown that an instrument rule with an optimal degree of monetary inertia may render negligible the inflationary bias associated with discretion without necessarily implying a trade-off between flexibility and commitment. The rationale for this surprising finding is found in the disciplining effect played by interest-rate smoothing on the incentive to create surprise inflation by reducing suddenly the interest rate within the time horizon of existing nominal contracts. If the degree of gradualism is high it may enhance the credibility of optimal monetary policy as it contrasts the incentive to fool private sector.

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1 Introduction

Real world central bankers do not seem to believe in the desirability of tying their hands by fixing policy choices according to a given formula for setting the instrument. Blinder (1998), speaking at the same time as a practitioner and as an academic, expresses clearly this view when he writes: "the real world cure to the alleged "inflation bias" problem did not come from adopting rigid precommitment ("rules") or other institutional changes, as Kydland-Prescott and Barro-Gordon suggested. It came from determined but discretionary application of tight money." However, despite of this reluctance to commit to follow fixed rules, there exists strong empirical evidence supporting the existence of a rule-like behaviour by central banks in the setting of interest rates.

Barro and Gordon viewed the time inconsistency explanation of the inflationary bias as an argument for rules over discretion, along Friedman lines.[1] With the policy maker committed to a fixed monetary policy rule it is not possible to create surprise inflation and the problem of dynamic inconsistency disappears.

But there is an important distinction from Friedman's case for a constant growth rate of money, who believed that the presence of long and variable lags between adjustments to monetary policy instruments and their real effects on the economy implies potentially destabilising impacts of an active monetary policy. Actually, the Barro and Gordon argument for a monetary policy rule can be also maintained in a framework where there is room for an activist feedback monetary policy rule. If we allow for the presence of information asymmetry between policy maker and private sector regarding the realisations of supply shocks, then the equilibrium policy rule in the commitment regime involves a potential role for an active monetary policy, offsetting shocks and therefore helping to stabilise inflation and output.

However, as observed by Alesina (1988), Persson and Tabellini (1990) and Lohmann (1992), in this Barro-Gordon framework extended to incorporate supply shocks the ex-ante optimal monetary policy rule is contingent on the state of the world. Unfortunately, in practice policy makers cannot easily commit to a state-contingent monetary policy rule. If, instead of a state contingent rule, a simple rule is pursued (e.g. a Friedman type rule), then

[1] Barro and Gordon (1983) wrote in the abstract of their celebrated paper: "The value of these commitments - which amount to long-term contracts between the government and the private sector - underlies the argument for rules over discretion."

it will dominate a discretionary rule only if output shocks are small and rare. In unstable periods there is more scope for stabilisation policies and a discretionary behaviour is preferable.

The idea that there is a trade-off between the benefits of avoiding the inflationary bias of discretionary policy and the potential costs of being bound to follow a monetary rule that is no longer appropriate has led some economists to find some compromises. Flood and Isard (1989) have proposed, for instance, the formulation of simple rules with explicit escape clauses. They argue that society might improve on the outcomes achieved under a discretionary regime, by motivating the central bank to follow a hybrid policy: in normal times the central bank follows a simple rule, while it responds to unusual circumstances at its discretion.

Some other economists increasingly have viewed rules not as constraints imposed on central banks externally, but as time-consistent means of operating internally - for example, as explicit starting points for consideration of current policy option. So in that sense there need not necessarily be a trade-off between flexibility and commitment. Recent efforts in this direction are those of Feldstein and Stock (1994), Hall and Mankiw (1994), and McCallum (1994). They have proposed alternative monetary rules to be considered by central banks whose policies are not strictly limited by exchange rate commitments. In particular McCallum formulates a more sophisticated monetary feedback rule, which takes account of changes in the velocity of circulation. He proposes a rule that sets the growth rate of money base at 3 per cent annum, with adjustments for the change in base velocity over the past four years and also for the deviation of nominal GNP from a defined steady noninflationary path. This rule, which treats nominal GNP as the target variable and the monetary base as the instrument, is shown to produce good inflation and output performance in several small econometric models. In the context of fixed rules, the McCallum type can provide a useful compromise with flexibility.

An alternative view on the argument is provided by Taylor (1993). According to him the rules versus discretion dilemma is rather a semantic issue, in the sense that in practice a policy rule can be defined more generally as a systematic behaviour. Hence there is no need to follow mechanically an algebraic formula. Moreover, as Taylor argues: "with this broader definition of policy rules, comparing the performance of different rules becomes more challenging".

In his celebrated article he formulates the so called Taylor rule: a representative interest rate rule that captures relatively well the Fed's behaviour during the 1987-1992 period. In order to make operational this rule, which obviously is not practical to follow in a rigid way, he considers two possibilities. One is to include the specific formula in the list of key elements that form the basis for monetary policy decisions. A second is that of making use of general characteristics of the given rule without referring explicitly to the algebraic formula for the central bank's decision-making process.

McCallum (1993), in answering to Taylor's proposal of broadening the definition of policy rules beyond a specific formula, shows that the presence or absence of systematic behaviour is not sufficient for separating between discretion and a behaviour based on an rule. What is also required is that "the policy authority [....] must also design the systematic response pattern to take account of the private sector's expectational behaviour". By making use of the analytical distinction between commitment and discretion on which is based the time-inconsistency literature he shows that also under a discretionary regime the policy maker may follow a systematic behaviour. In particular he expresses the policy maker problem in terms of choosing the parameter values of a given formula. Nevertheless the presence of this systematic behaviour under a discretionary regime does not prevent the economy from the arising of an inflationary bias.

In the present paper we will focus on the criticism of McCallum to Taylor's definitional issue and explore the importance of adopting systematic behaviours in the setting of monetary instruments. Contrary to McCallum's finding, we show that optimally designed instrument rules may render negligible the inflationary bias associated with time consistent monetary policy without prejudice to stabilisation of shocks. Furthermore, our framework sheds also some light on the puzzling issue of the inertial behaviour followed by central bankers in the setting of interest rates, i.e. the practice of interest-rate smoothing.

The order of the exposition is the following. Section 2 provides an analytical distinction between instrument rules and target rules. Section 3 discusses the empirical evidence on interest rate rules. Section 4 illustrates the model. Section 5 considers the simple case of no interest-rate smoothing and replicates McCallum's finding. The case of sluggish adjustments of the interest rate is examined in section 6. Here are exposed the most innovative results of the analysis. Section 7 concludes.

2 Instrument rules versus target rules

In our analysis we will focus our attention on instrument rules and in particular on interest rate rules. Thus a first step should be to define formally an instrument rule. To the aim of defining rigorously what are instrument rules it might be useful first to compare them with an alternative kind of rules: target rules.

Svensson (1996) proposes the following distinction: "Setting the instrument to make the inflation forecast equal to the inflation target is an example of a *target rule* which, if applied by the monetary authority, would result in an endogenous optimal reaction function expressing the instrument as a function of the available relevant information. This is different from an *instrument rule* that directly specifies the reaction function for the instrument in terms of the current information."

This dichotomy is not generally accepted. McCallum (1997), for example, argues that the distinction between instrument rules and target rules is merely theoretical. Judging from a practical perspective, the importance of a target rule that is not expressed in terms of a feasible instrument variable is debatable. Consequently, he proposes the following definition: "a monetary policy rule is a formula that specifies instrument settings, with the choice of a target variable and path constituting only one ingredient".

In the time-inconsistency literature there is a standard formalisation of instrument rules and target rules. Instrument rules are identified with a fixed formula that specifies the decision rule of the policy maker and are interpreted in terms of the analytical distinction between commitment to a rule and discretion. The definition of target rules is found instead within the normative side of the literature, rather than in the positive one.

Rogoff's (1985) is considered the pioneer of strategic delegation in monetary policy. The proposal of changing monetary institutions for dealing with the issue of time inconsistency and the alleged inflationary bias is the core idea behind the approach of monetary delegation. Recently the delegation approach has been extended by the work of Walsh (1995), Persson and Tabellini (1993) and Svensson (1997) among others, with the introduction of incentive schemes or policy targets in order to completely remove the inflationary bias. As pointed out by Rogoff the optimality of delegation of monetary policy to a "weight-conservative" central banker suggests also an alternative interpretation of the delegation process. In particular he shows that this kind of solution to the inflationary bias problem can be interpreted also as an inflation targeting scheme based on punishments and rewards. Similar considerations have been made on Walsh contracting solution too. For example, Persson and Tabellini (1997) have stressed the close relationship between inflation targeting schemes and contracts based on penalties conditional on realised inflation.[2] Also Svensson's "target-conservative" central banker can be indirectly related to an inflation targeting scheme as it is possible to show that an optimal inflation target is equivalent to an optimal linear inflation contract.

Other types of target rules have been considered by Rogoff: monetary targeting, interest-rate targeting, etc.. But in all the cases considered the target rule has been modelled by including in the central bank's objective function some weight on achieving the specified target.

In the following sections we will consider a general definition of an instrument rule for the interest rate which yields some interesting insights on the role that can be played by instrument rules in monetary policy.

3 The empirical evidence on interest rate rules

Despite of the great importance of the interest rate as a policy instrument only recently the empirical and theoretical literature has focused on interest rate rules.[3] The path breaking work of Taylor (1993) started the debate.[4] There has been a large body of literature describing the macroeconomic implications of interest-rate smoothing, which assumes the presence of an interest-rate smoothing or alternatively an interest-rate targeting objective in the loss function of the policy maker.[5] However, in this line of research interest-rate smoothing is not related to an explicit rule for setting interest rates.

[2]See also Walsh (1997) for a broader discussion on inflation targeting regimes.

[3]In the words of Blinder (1998): "Returning to Poole's dichotomy [on the choice of monetary instrument]...in the end, real-world events, not theory, decided the issue. Ferocious instabilities in estimated LM curves in the United States, United Kingdom, and many other countries, beginning in the 1970s and continuing to the present day, led economists and policymakers alike to conclude that money-supply targeting is simply not a viable option....So interest rates won by default".

[4]See Taylor (1999) for a recent state-of-the-art appraisal of the fundamental issues facing central banks with a particular focus on monetary policy rules.

[5]Reviews on this literature are provided by Cukierman (1992), Goodhart (1996), Walsh (1998), Clarida, Galì and Gertler (1999), Sack and Wieland (1999).

The empirical evidence on the interest rate rule followed by central banks in the last two decades can be summarised by the following form

$$r_t = \gamma r_{t-1} + (1-\gamma)\,\bar{r}_t; \tag{1}$$

with

$$\bar{r}_t = \chi_0 r^* + \chi_1 (\pi_t - \pi^*) + \chi_2 (y_t - y^*); \tag{2}$$

where $0 < \gamma < 1$, r_t is the nominal interest rate and the variable $\bar{r}_t$ is an operative target given by expression (2). In this specification the operative target is a function of both inflation π_t and output y_t expressed as deviations from trend levels; r^* is trend nominal interest rate and χ_0, χ_1, χ_2 are positive parameters. This policy rule implies that the interest rate reacts to inflation and output gap but there is only partial adjustment to these variables due to the presence of an interest-rate smoothing component. The common feature of all estimates of the degree of inertia in the central bank's response, γ, is that they are large and highly significant, normally on the order of .8 or .9.[6]

Using the words of Clarida, Galì and Gertler (1999): "The existing theory, by and large, does not readily account for why the central bank should adjust rates in such a sluggish fashion. Indeed, understanding why central banks choose a smooth path of interest rates than theory would predict is an important unresolved issue".

One important exception is constituted by Woodford (1998). He provides a rationale for a central banker with an interest-rate smoothing objective in terms of an optimal monetary delegation problem. Among other relevant contributions, his model provides also a New Keynesian perspective of the issue of time inconsistency.[7] However his analysis is based on two crucial assumptions. He postulates that in the social loss there is an interest-rate targeting motive and that there exists a central banker that prefers for interest rate to deviate farther from its target. In this latter case, as observed by Woodford, the interest rate target can hardly be interpreted as a target. These assumptions are needed to show that it can be advantageous for society to delegate monetary policy to a central banker who includes in his loss function an interest-rate smoothing objective.

[6] See the review in Clarida, Galì and Gertler (1999). Sack (1998) estimates for γ a value of 0.63, with a standard error of 0.08. Higher values are found by Orphanides (1998) and Clarida, Galì and Gertler (1998a,b).

[7] See also Clarida, Galì and Gertler (1999) for a similar attempt.

Unfortunately these assumptions seem quite *ad hoc*. As we will see later on, our analysis provides also an alternative view on the inertia observed in the response by central banks to changes in macroeconomic variables, without using the restrictive assumptions considered by Woodford.

4 The model

The analysis is based on a stochastic rational expectations IS-LM model. Aggregate supply is given by a standard expectations augmented Phillips curve

$$y_t = y_n + \alpha\left(\pi_t - \pi_t^e\right) + v_t; \tag{3}$$

where y_t is the level of output, π_t the inflation rate and v_t is a random disturbance normally distributed with mean zero and variance σ_v^2. Private sector's inflation expectations, π_t^e, are formed rationally. The parameter α is positive.

Aggregate demand is given by

$$y_t = y_n - \beta\left(r_t - \pi_t^e - \rho\right) + u_t; \tag{4}$$

where r_t is the nominal interest rate, ρ is the long-run equilibrium real interest rate and u_t is a stochastic disturbance normally distributed with mean zero and variance σ_u^2 . The stochastic disturbances u_t and v_t are assumed to be independent of each other. The parameter β is positive.[8]

By equating (3) and (4), after some manipulations we can obtain an expression for inflation as a function of the nominal interest rate, inflation expectations and exogenous variables

$$\pi_t = \frac{\beta}{\alpha}\rho + \frac{\alpha+\beta}{\alpha}\pi_t^e - \frac{\beta}{\alpha}r_t - \frac{1}{\alpha}v_t + \frac{1}{\alpha}u_t. \tag{5}$$

This equation can be expressed also in terms of interest rate expectations in the following way. After taking expectations of (5) we get

$$\pi_t^e = r_t^e - \rho. \tag{6}$$

Substitution of (6) in (5) implies

[8] The specification of the aggregate demand with current period inflation expectations has been used also by Clark, Goodhart and Huang (1999).

$$\pi_t = -\rho + \frac{\alpha+\beta}{\alpha} r_t^e - \frac{\beta}{\alpha} r_t - \frac{1}{\alpha} v_t + \frac{1}{\alpha} u_t; \tag{7}$$

consequently the aggregate demand equation can be rewritten as

$$y_t = y_n - \beta \left(r_t - r_t^e \right) + u_t. \tag{8}$$

Turning to preferences, as usual it is assumed that the government's preferences coincide with those of society. The preferences are represented by the government's loss function

$$V = E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{9}$$

where δ, with $0 < \delta < 1$, is the discount factor and the government's period loss function L_t is given by

$$L_t = \pi_t^2 + \lambda \left(y_t - \overline{y} \right)^2; \tag{10}$$

where the parameter λ is a relative weight. In order to introduce the issue of time inconsistency it is assumed that the government wishes to increase output above the natural level, with $\overline{y} > y_n$.

Monetary policy is delegated to a central banker whose preferences are given by:

$$V^b = E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{11}$$

where it is assumed that the central banker's period loss function is identical to expression (10).

Last but not least, an important novel element of our model concerns the way monetary policy is implemented. The interest rate is the instrument used by the monetary authority for achieving its goals. In the following analysis we will consider efficient instrument rules for setting the nominal interest rate. The general specification of these instrument rules will be

$$r_t = \gamma \widehat{r}_{t-1} + (1-\gamma) \overline{r}_t; \tag{12}$$

$$\bar{r}_t = \mathsf{a} + \mathsf{b} \bar{r}_t^e + \mathsf{c} \widehat{r}_{t-1} + \mathsf{m} v_t + \mathsf{n} u_t; \tag{13}$$

$$\widehat{r}_t = \mathsf{a} + \mathsf{b}\overline{r}_t^e + \mathsf{c}\widehat{r}_{t-1}; \tag{14}$$

where the interest rate level chosen by the central banker is expressed as a convex combination of two components, with $0 < \gamma < 1$. One is a targeting component in the sense that the variable $\overline{r}_t$ is an operative target which expresses the level of the interest rate in terms of a specified formula. The other is a partial adjustment component, with the interest rate r_t dependent upon the past values of $\widehat{r}_t$.

This general specification for an interest rate rule is consistent with the empirical evidence on interest-rate smoothing, as expressed by (1). But in contrast with the specifications usually considered in empirical and theoretical literature it is based only on a component of the lagged interest rate. Actually, as we can see from the definition of $\widehat{r}_t$, the inertial term is referred only to the systematic component of the operative target $\overline{r}_t$ while stochastic shocks are excluded. Thus according to (12) current policy decisions depend also on past decisions but only on those referred to the systematic component of the operative target.

The rule adopted in the analysis reflects the recurrent use of operating targets for the interest rates controlled by central banks. However, a typical specification for the operating target would be to express $\overline{r}_t$ in terms of the current level of inflation and a proxy for real activity, as in equation (2). Here we consider an alternative specification for $\overline{r}_t$, which takes account of all the relevant state variables present in the model. The term $\overline{r}_t^e$ in expressions (13) and (14) represents private sector's expectations on the operative target. If we add in the framework also some persistence in inflation and output, then the specification of $\overline{r}_t$ would resemble more closely the specifications of $\overline{r}_t$ examined in empirical analysis.[9]

If the degree of monetary inertia is zero, that is $\gamma = 0$, and $\widehat{r}_{t-1}$ is eliminated from (13) we get an expression which is analogous to that examined by McCallum (1993) for discussing the importance of the presence of a systematic behaviour in order to distinguish between rule-like and discretionary behaviour. He supposes that the policy maker's optimisation problem consists in choosing the parameters a, b,m and n.[10]

[9] See Clark, Goodhart and Huang (1999) for a similar consideration in a framework where inflation persistence is explicitly introduced in the analysis.

[10] McCallum (1993) does not consider explicitly shocks in his policy rule but the line of reasoning is the same: the systematic procedure followed in the implementation of policy

This operating procedure is an institutional feature of monetary policy and is common knowledge among all the players of the policy game examined. Moreover, this procedure is operative under both discretion and commitment. Of course the optimal parameter values may change according to the given monetary regime under which the central banker operates. However, it bears repeating that the policy rule operates despite the fact that the policy regime is discretionary. As suggested by McCallum, the presence of this policy rules simply reflects the idea that the policy maker adopts a systematic behaviour in setting the monetary instrument.

The question he poses is whether the presence of this systematic behaviour under a discretionary regime may imply outcomes that are distinct from the case where this systematic behaviour is absent and, in particular, if these outcomes are the same as those pertaining to a regime with commitment. We investigate the same question but with the introduction of an additional element concerning the specification of the policy rule examined by McCallum. In particular we consider the possibility that the policy maker adopts also a systematic inertial behaviour in choosing the level of the interest rate.

As we have seen the presence of monetary inertia in central banks behaviour is empirically well documented. We assume that the inertial component in the policy rule is constituted only by the systematic component of the operative target $\overline{r}_t$. The rationale for this restriction is rather intuitive: smoothing interest rate in proportion to r_{t-1} or $\overline{r}_{t-1}$ would affect the stabilisation of shocks in a suboptimal way, while the time-inconsistency problem we are seeking to tackle is related only to the systematic component of the policy rule.

So we suppose that monetary authorities act gradually with a certain degree of monetary inertia for some reasons. Candidate reasons that have been adduced in the literature are several: forward-looking behaviour by private agents, measurement errors associated with macroeconomic variables, uncertainty about structural parameters, concern for the financial stability of the banking system, adverse reactions of financial markets to frequent modifications in the direction of short-term interest rates.[11]

In contrast to the standard approach for modelling an interest-rate smoothing motive, we do not include in the central bank's loss function an objective

consists in choosing optimally the value of the parameters which relates macroeconomic variables to the level of the control variable.

[11] For a recent review see for example Sack and Wieland (1999).

of minimising the deviations of current level of interest rate from previous period levels, as done for instance by Woodford (1998). We postulate instead that this interest-rate smoothing motive is explicitly incorporated in the policy rule adopted by the central bank.

How plausible is this alternative formalisation? Sack (1998) for instance says that: "many empirical studies of monetary policy incorporate an explicit interest-rate smoothing incentive in the objective function of the Fed. However, introducing this argument has little justification beyond matching the data. Furthermore, the above statistics provide evidence of gradualism only if the Fed would otherwise choose a random-walk policy in absence of an interest-rate smoothing objective. Therefore, while establishing that the funds rate is not a random walk, these statistics do not necessarily provide evidence of gradualism in monetary policy."[12] Moreover Sack and Wieland (1999) offer several arguments, discussing the empirical evidence supporting them, that explain why central banks may have an incentive to smooth interest rates without assuming an interest-rate smoothing objective in their loss function.

We interpret interest-rate smoothing as a feature of the implementation of monetary policy in practice and not as an object in the policy maker's loss function. While gradualism is a stylised fact, on the contrary there is no evidence of central bank's having as goal or as intermediate target the minimisation of the deviations of current level of interest rate from previous period levels. Moreover the motive for smoothing interest rate should be modelled explicitly in the loss function and not by simply inserting the by-product of the desired main goal.[13]

Hence, in the present framework we assume that central bank's behaviour is characterised by some degree of monetary inertia, defining γ between zero and one, and that this stylised fact is incorporated in the policy rule followed. A crucial assumption is that the systematic behaviour followed by the policy

[12] The issue whether interest-rate smoothing is the result of the presence of an additional objective in the policy maker's loss function or alternatively is a by-product of the particular characterisation of the structure of the economy (which implies some partial adjustment mechanism) is an open debate. In order to have a clearer understanding on this issue further empirical evidence is needed. An interesting empirical contribution is that of Favero and Rovelli (1999) who propose an approach which allows to identify the parameters that pertain to the policy maker's preferences independently from those relative to the structure of the economy.

[13] For instance see Cukierman (1992) for an attempt in the direction of including a financial stability motive directly in the central bank's loss function.

maker and embodied in the policy rule (12) is generally understood by the public.

Moreover we assume also that the policy maker can choose ex-ante the specific value of γ (within the defined range) in order to stabilise inflation and output optimally. This means that the presence of inertia is introduced exogenously and justified by the above arguments, but the degree of gradualism in the adjustment of the interest rate is chosen endogenously by the central banker. In particular we assume that the parameter γ represents and additional institutional feature of the systematic operating procedure chosen ex-ante for setting the interest rate.

It is possible to observe that the described process for setting the monetary instrument resembles the case when the monetary authority does not control perfectly the instrument. For instance Cukierman and Meltzer (1986), Letterie (1997) and Swank (1994) assume that the volatility of multiplicative control errors is an institutional feature of the implementation of monetary policy that can be chosen ex-ante optimally. However conservatism in the use of the monetary instrument (as implied by Brainard's uncertainty) does not necessarily mean gradualism (as implied by a partial adjustment process).

In the following sections we will suppose that the implementation of monetary policy follows three stages. In the first stage, the government delegates monetary policy to a central banker and instructs him to follow an assigned instrument rule, given by (12). In other words the choice made by government consists to direct the central banker to follow a systematic behaviour in the setting of interest rates. In the second stage, the central banker makes the ex-ante choice of γ, that is he chooses the degree of monetary inertia introduced in the economy. In the third stage, the parameter values in $\overline{r}_t$ are derived from the optimisation problem of the central banker. Of course the stages become two in the case of no interest rate-smoothing.

5 No interest-rate smoothing

5.1 Commitment

In order to replicate McCallum's results, we start first with the case when the monetary authority does not change smoothly the interest rate, i.e. $\gamma = 0$ in the instrument rule (12). In this case we have that the interest rate is given by

$$r_t = \bar{r}_t. \tag{15}$$

In our framework, due to the presence of an instrument rule of the type of (12), the actual choice variable of the monetary authority is $\bar{r}_t$. Hence the relevant expectations are those that the private sector forms on $\bar{r}_t$. From (15) we have

$$r_t^e = \bar{r}_t^e. \tag{16}$$

Following Svensson (1997) and Clark, Goodhart and Huang (1999), in the commitment solution the central banker internalises in his minimisation problem the impact of his decisions on the interest rate on private sector's expectations. So let's formalise the optimization problem of the central banker when he is able to commit in advance to a decision rule for the choice variable $\bar{r}_t$. We have:

$$\min_{\bar{r}_t, \bar{r}_t^e} E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{17}$$

$$\text{s.t. Eqs. } (7), (8), (10), (15), (16) \text{ and } \bar{r}_t^e = E_{t-1}\left[\bar{r}_t\right];$$

which is equivalent to solve the static problem of minimising the expected period loss function

$$\min_{\bar{r}_t, \bar{r}_t^e} E_{t-1} L_t; \tag{18}$$

$$\text{s.t. Eqs. } (7), (8), (10), (15), (16) \text{ and } \bar{r}_t^e = E_{t-1}\left[\bar{r}_t\right].$$

Differentiating L_t with respect to $\bar{r}_t$ and $\bar{r}_t^e$ we get the following first order conditions

$$\begin{aligned} -2\frac{\beta}{\alpha}\pi_t - 2\lambda\beta\left(y_t - \overline{y}\right) + \quad _{t-1} &= 0; \\ E_{t-1}\left[2\frac{\alpha+\beta}{\alpha}\pi_t + 2\lambda\beta\left(y_t - \overline{y}\right) - \quad _{t-1}\right] &= 0; \end{aligned} \tag{19}$$

where $\quad _{t-1}$ is the Lagrange multiplier of $\bar{r}_t^e = E_{t-1}\left[\bar{r}_t\right]$. Eliminating the multiplier yields

$$-2\frac{\beta}{\alpha}\pi_t - 2\lambda\beta\left(y_t - \overline{y}\right) + 2\frac{\alpha+\beta}{\alpha}E_{t-1}\pi_t + 2\lambda\beta\left(E_{t-1}y_t - \overline{y}\right) = 0. \tag{20}$$

Now, we can postulate the following reaction function for the central banker:

$$\bar{r}_t = \mathsf{a} + \mathsf{b}\bar{r}_t^e + \mathsf{m}v_t + \mathsf{n}u_t, \tag{21}$$

where there is no lagged variable.

Substitution of (21) in (20) implies that for optimality we must have following values for the coefficients of the decision rule (21)

$$\begin{aligned}
\mathsf{a} &= \frac{\rho\alpha^2}{\left(1+\lambda\alpha^2\right)\beta^2}; \\
\mathsf{b} &= \frac{\left(1+\lambda\alpha^2\right)\beta^2 - \alpha^2}{\left(1+\lambda\alpha^2\right)\beta^2}; \\
\mathsf{m} &= -\frac{1}{\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{n} &= \frac{1}{\beta}.
\end{aligned} \tag{22}$$

The private sector's expectations are found by taking expectations at $t-1$ over the reaction function of the central banker, which gives

$$\bar{r}_t^{e,c} = \rho. \tag{23}$$

Substituting the private sector's expectation in the central banker's reaction function gives the equilibrium value of $\bar{r}_t$ under a regime with commitment

$$\bar{r}_t^c = \rho - \frac{1}{\left(1+\lambda\alpha^2\right)\beta}v_t + \frac{1}{\beta}u_t. \tag{24}$$

Inserting (24) in (15) gives the equilibrium value of the interest rate r_t^c. While substitution of the equilibrium values for $\bar{r}_t^c$ and $\bar{r}_t^{e,c}$ back in equation (7) and (8) gives the equilibrium values of inflation and output:

$$\pi_t^c = -\frac{\alpha\lambda}{1+\lambda\alpha^2}v_t; \tag{25}$$

$$y_t^c = y_n + \frac{1}{1+\lambda\alpha^2} v_t. \tag{26}$$

>From (15) and using (24), it is possible to see that average nominal interest rate is equal to the long-run equilibrium real interest rate, ρ. While from (25) we can see that under a regime with commitment average inflation is equal to zero.

5.2 Discretion

Now consider the optimization problem of the central banker when he chooses $\bar{r}_t$ in a discretionary manner. In this case the coefficients in $\bar{r}_t$ are chosen period by period, rather than once and for all, and private sector's expectations are no longer a control variable. Here the policy maker solves the following problem

$$E_{t-1} \min_{\bar{r}_t} L_t; \tag{27}$$

$$\text{s.t. Eqs. } (7), (8), (10), (15), (16);$$

where, as observed by Persson and Tabellini (1998), it is possible to conclude that the expectations operator becomes redundant.

Differentiating L_t with respect to $\bar{r}_t$ we get the following first order condition

$$-2\frac{\beta}{\alpha}\pi_t - 2\lambda\beta\left(y_t - \overline{y}\right) = 0. \tag{28}$$

Again, we assume that the central banker's reaction function is represented by (21). This yields the following optimal values for the coefficients of the decision rule considered

$$\begin{aligned}
\mathsf{a} &= -\frac{\alpha\left[\rho + \alpha\lambda\left(\overline{y} - y_n\right)\right]}{\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{b} &= \frac{\left(1+\lambda\alpha^2\right)\beta + \alpha}{\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{m} &= -\frac{1}{\left(1+\lambda\alpha^2\right)\beta};
\end{aligned}$$

$$\mathsf{n} \;=\; \frac{1}{\beta}. \tag{29}$$

Comparing the optimality conditions (29) with those correspondent to the case of commitment, we can observe that now a is negative instead of positive, while in the numerator of b the term $(\beta - \alpha)$ is disappeared. The private sector's expectations are found by taking expectations at $t-1$ over the reaction function of the central banker, which gives

$$\bar{r}_t^{e,d} = \rho + \alpha\lambda\left(\overline{y} - y_n\right). \tag{30}$$

Repeating all the substitutions made in the case of commitment we can get the following equilibrium values for the case of discretion

$$\bar{r}_t^d = \rho + \alpha\lambda\left(\overline{y} - y_n\right) - \frac{1}{\left(1 + \lambda\alpha^2\right)\beta}v_t + \frac{1}{\beta}u_t; \tag{31}$$

Inserting (31) in (15) gives the equilibrium value of the interest rate r_t^d. While substitution of the equilibrium values for $\bar{r}_t^d$ and $\bar{r}_t^{e,d}$ back in equation (7) and (8) gives the equilibrium values of inflation and output:

$$\pi_t^d = \alpha\lambda\left(\overline{y} - y_n\right) - \frac{\alpha\lambda}{1 + \lambda\alpha^2}v_t; \tag{32}$$

$$y_t^d = y_n + \frac{1}{1 + \lambda\alpha^2}v_t. \tag{33}$$

>From (15) and using (31), it is possible to see that average nominal interest rate is higher than in the case of commitment, as it is equal to $\left[\rho + \alpha\lambda\left(\overline{y} - y_n\right)\right]$. While from (32) we can derive the famous result that under a regime with discretion average inflation is characterised by an inflationary bias, equal to $\left[\alpha\lambda\left(\overline{y} - y_n\right)\right]$. Thus, as observed by McCallum, the presence or absence of systematic behaviour is not enough to distinguish between rule-like behaviour and discretion, as the presence of a systematic behaviour under a discretionary regime does not remove the inflationary bias.

6 Interest-rate smoothing

6.1 Commitment

Let's consider the case of interest-rate smoothing. Consider again first a regime where a commitment strategy is available to the central banker. In this case the instrument rule we will consider for the interest rate is (12).

Expression (12) implies that private sector's expectations on the interest rate are given by

$$i_t^e = \gamma \widehat{r}_{t-1} + (1-\gamma)\,\bar{r}_t^e. \tag{34}$$

The optimization problem of the central banker when he is able to commit in advance to a decision rule for the choice variable $\bar{r}_t$ can be expressed as

$$\min_{\bar{r}_t, \bar{r}_t^e} E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{35}$$

$$\text{s.t. Eqs. } (7), (8), (10), (12), (34) \text{ and } \bar{r}_t^e = E_{t-1}[\bar{r}_t].$$

This is now a dynamic programming problem with two control variables, $\bar{r}_t$ and $\bar{r}_t^e$, and one state variable, the lagged variable $\widehat{r}_{t-1}$. As shown by Lockwood and Philippopoulus (1994) and Lockwood, Miller and Zhang (1994) and (1998), the solution can be obtained by solving the following equation with the value function $V(\widehat{r}_{t-1})$:

$$V(\widehat{r}_{t-1}) = \min_{\bar{r}_t, \bar{r}_t^e} E_{t-1}\left[L_t^b + \delta V(\widehat{r}_t)\right]; \tag{36}$$

$$\text{s.t. Eqs. } (7), (8), (10), (12), (34) \text{ and } \bar{r}_t^e = E_{t-1}[\bar{r}_t].$$

As we have a linear-quadratic problem, $V(\widehat{r}_t)$ must also be quadratic. Without loss of generality, we can set

$$V(\widehat{r}_t) = \theta_0 + 2\theta_1 \widehat{r}_t + \theta_2 \widehat{r}_t^2. \tag{37}$$

Now using the fact that $\widehat{r}_t = \bar{r}_t - (\mathsf{m} v_t + \mathsf{n} u_t,)$, we have

$$V_{\bar{r}}(\widehat{r}_t) = 2(\theta_1 + \theta_2 \widehat{r}_t). \tag{38}$$

Differentiating $V(\widehat{r}_{t-1})$ with respect to $\bar{r}_t$ and $\bar{r}_t^e$ we get the following first order conditions

$$
\begin{aligned}
-2\frac{\beta}{\alpha}\left(1-\gamma\right)\pi_t - 2\lambda\beta\left(1-\gamma\right)\left(y_t-\overline{y}\right) + 2\delta\theta_1 + 2\delta\theta_2\widehat{r}_t + \quad {}_{t-1} &= 0; \\
E_{t-1}\left[2\left(1-\gamma\right)\frac{\alpha+\beta}{\alpha}\pi_t + 2\lambda\beta\left(1-\gamma\right)\left(y_t-\overline{y}\right) - \quad {}_{t-1}\right] &= 0;
\end{aligned} \tag{39}
$$

where $\quad_{t-1}$ is the Lagrange multiplier of $\bar{r}_t^e = E_{t-1}\left[\bar{r}_t\right]$. Eliminating the multiplier yields

$$
\begin{aligned}
0 &= -2\frac{\beta}{\alpha}\left(1-\gamma\right)\pi_t - 2\lambda\beta\left(1-\gamma\right)\left(y_t-\overline{y}\right) + 2\delta\theta_1 + 2\delta\theta_2\widehat{r}_t \\
&\quad +2\left(1-\gamma\right)\frac{\alpha+\beta}{\alpha}E_{t-1}\pi_t + 2\lambda\beta\left(1-\gamma\right)\left(E_{t-1}y_t-\overline{y}\right).
\end{aligned} \tag{40}
$$

Here we have a dynamic framework. Thus in order to find the optimal reaction function for the central banker we postulate a decision rule like expression (13).

Substitution of (13) in (40) implies that for optimality we must have following values for the coefficients of the decision rule (13)

$$
\begin{aligned}
\mathsf{a} &= \frac{\alpha^2\left[\rho\left(1-\gamma\right)-\delta\theta_1\right]}{\left(1+\lambda\alpha^2\right)\beta^2\left(1-\gamma\right)^2+\delta\theta_2\alpha^2}; \\
\mathsf{b} &= \frac{\left(1-\gamma\right)^2\left(\beta^2-\alpha^2+\lambda\beta^2\alpha^2\right)}{\left(1+\lambda\alpha^2\right)\beta^2\left(1-\gamma\right)^2+\delta\theta_2\alpha^2}; \\
\mathsf{c} &= -\frac{\alpha^2\gamma\left(1-\gamma\right)}{\left(1+\lambda\alpha^2\right)\beta^2\left(1-\gamma\right)^2+\delta\theta_2\alpha^2}; \\
\mathsf{m} &= -\frac{1}{\left(1-\gamma\right)\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{n} &= \frac{1}{\left(1-\gamma\right)\beta}.
\end{aligned} \tag{41}
$$

The private sector's expectations are found by taking expectations at $t-1$ over the reaction function of the central banker:

$$
\bar{r}_t^{e,c} = \frac{\rho\left(1-\gamma\right)-\delta\theta_1}{\left(1-\gamma\right)^2+\delta\theta_2} - \frac{\gamma\left(1-\gamma\right)}{\left(1-\gamma\right)^2+\delta\theta_2}\widehat{r}_{t-1}. \tag{42}
$$

Substituting the private sector's expectations in the central banker's reaction function gives the equilibrium value of $\bar{r}_t$ under a regime with commitment:

$$\bar{r}_t^c = \frac{\rho(1-\gamma)-\delta\theta_1}{(1-\gamma)^2+\delta\theta_2} - \frac{\gamma(1-\gamma)}{(1-\gamma)^2+\delta\theta_2}\widehat{r}_{t-1} - \frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}\upsilon_t + \frac{1}{(1-\gamma)\beta}u_t. \tag{43}$$

Inserting (43) in (12) gives the equilibrium value of the interest rate r_t^c. While substitution of the equilibrium values for $\bar{r}_t^c$ and $\bar{r}_t^{e,c}$ back in equation (7) and (8) gives the equilibrium values of inflation and output:

$$\pi_t^c = -\frac{\delta[\rho\theta_2+(1-\gamma)\theta_1]}{(1-\gamma)^2+\delta\theta_2} + \frac{\gamma\delta\theta_2}{(1-\gamma)^2+\delta\theta_2}\widehat{r}_{t-1} - \frac{\lambda\alpha}{1+\lambda\alpha^2}\upsilon_t; \tag{44}$$

$$y_t^c = y_n + \frac{1}{1+\lambda\alpha^2}\upsilon_t. \tag{45}$$

>From (44) emerges an important difference with respect to the static case, without interest-rate smoothing. We can observe from the equilibrium value of inflation that interest-rate smoothing allows the policy maker to smooth inflation over a number of periods.

Following Svensson (1997), in order to find θ_1 and θ_2 we can apply the envelope theorem on (36), which combined with (38) implies

$$V_{\bar{r}}(\widehat{r}_{t-1}) = 2(\theta_1+\theta_2\widehat{r}_{t-1}) = E_{t-1}[2\gamma(\pi_t)]; \tag{46}$$

or

$$2(\theta_1+\theta_2\widehat{r}_{t-1}) = -2\delta\left\{\frac{\gamma[\theta_1(1-\gamma)+\rho\theta_2]}{(1-\gamma)^2+\delta\theta_2} - \frac{\gamma^2\theta_2}{(1-\gamma)^2+\delta\theta_2}\widehat{r}_{t-1}\right\}. \tag{47}$$

Identification of θ_1 and θ_2 gives

$$\theta_1 = -\frac{\delta\theta_2\gamma\rho}{(1-\gamma)\,\delta\gamma + (1-\gamma)^2 + \delta\theta_2}; \tag{48}$$

$$\theta_2 = 0; \tag{49}$$

$$\theta_2 = \frac{\delta\gamma^2 - (1-\gamma)^2}{\delta}. \tag{50}$$

If $\theta_2 = 0$, then also $\theta_1 = 0$ and the value function (37) becomes equal to a constant. Hence in the case of $\theta_2 = 0$ the optimization problem is not any more a dynamic programming problem but is reduced to a static one-period issue. In this case the monetary authority cannot use the interest rate to smooth inflation over a number of periods. The same consideration can be made for the second value of θ_2 when γ has the following values

$$\gamma_1 = \frac{1-\sqrt{\delta}}{1-\delta}; \tag{51}$$

and

$$\gamma_2 = \frac{1+\sqrt{\delta}}{1-\delta}; \tag{52}$$

where the value γ_2 given by (52) can be excluded by definition as, for $0 < \delta < 1$, it is greater than one. Thus it cannot be a possible value of γ, being it defined as $0 < \gamma < 1$. Let's see how we can select the relevant solution.

For convenience we focus the analysis on the parameters of the decision rule given by $\bar{r}_t$. Without loss of generality, we can write the expression of the equilibrium value of $\bar{r}_t$ as

$$\bar{r}_t = \phi_0 + \phi_1\widehat{r}_{t-1} + \phi_2 v_t + \phi_3 u_t. \tag{53}$$

Using (43), it is possible to see that the values of θ_2 given by (49) and (50) imply respectively the following values for the coefficient of $\widehat{r}_{t-1}$

$$\phi_1 = -\frac{\gamma}{1-\gamma}; \tag{54}$$

and

$$\phi_1 = -\frac{1-\gamma}{\delta\gamma}. \tag{55}$$

In order to eliminate one of these solutions we can consider the argument used by Lockwood and Philippopoulos (1994), Svensson (1997), Clark, Goodhart and Huang (1999). They recommend to choose the smaller solution as the relevant one. This is based on the fact that the smaller solution has the nice property to be stable, in the sense that when a small disequilibrium deviation is introduced the system returns to the equilibrium value of ϕ_1 under a revision rule which is consistent with the recursive nature of the optimization problem. Now, it is possible to see that when $\gamma > \gamma_1$ the solution (55) is lower in absolute value than that expressed by (54) and the opposite occurs when $\gamma < \gamma_1$. Moreover, if $\gamma > [1/(1+\delta)] > \gamma_1$, the solution (55) will be less than one in absolute value; the same occurs for the solution (54) when $\gamma < .5$. In the appendix A it is shown what is in our framework the revision rule that can be used for analysing the stability requirement.

In the following sections we will consider the solution (55) as the relevant one, assuming for the moment that the central banker chooses ex-ante a degree of monetary inertia included between the range $[1/(1+\delta)] < \gamma < 1$.

Substitution of (48) and (50) in expressions (43), (44) and (45) yields

$$\bar{r}_t^c = \frac{[(1-\gamma)+\delta\gamma]}{\gamma\delta}\rho - \frac{(1-\gamma)}{\gamma\delta}\widehat{r}_{t-1} - \frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}v_t + \frac{1}{(1-\gamma)\beta}u_t; \tag{56}$$

$$\pi_t^c = -\frac{\left[\delta\gamma^2-(1-\gamma)^2\right]}{\gamma\delta}\rho + \frac{\left[\delta\gamma^2-(1-\gamma)^2\right]}{\gamma\delta}\widehat{r}_{t-1} - \frac{\lambda\alpha}{1+\lambda\alpha^2}v_t; \tag{57}$$

and

$$y_t^c = y_n + \frac{1}{1+\lambda\alpha^2}v_t. \tag{58}$$

Here it is possible to see that average inflation and interest rate under a commitment regime with interest-rate smoothing are the same as under a commitment regime without monetary inertia, that is

$$\begin{aligned} E\left[\pi_t^c\right] &= 0, \\ E\left[r_t^c\right] &= \rho. \end{aligned} \tag{59}$$

The derivation of these results is found in appendix B. Moreover, from the comparison with the analogous expressions for the case of commitment without interest-rate smoothing, it is possible to observe that stabilisation of shocks is still optimal. So smoothing interest rate, with a degree of inertia γ chosen ex-ante in the interval $[1/(1+\delta)] < \gamma < 1$, is still optimal under a commitment regime.

6.2 Discretion

The optimization problem of the central banker when a commitment technology is not available can be formalised as

$$V\left(\widehat{r}_{t-1}\right) = E_{t-1} \min_{\bar{r}_t} \left[L_t + \delta V\left(\widehat{r}_t\right)\right]; \tag{60}$$

$$\text{s.t. Eqs. } (7), (8), (10), (12), (34) \; .$$

Differentiating $V\left(\widehat{r}_{t-1}\right)$ with respect to $\bar{r}_t$ we get the following first order condition

$$-2\frac{\beta}{\alpha}\left(1-\gamma\right)\pi_t - 2\lambda\beta\left(1-\gamma\right)\left(y_t - \overline{y}\right) + 2\delta\theta_1 + 2\delta\theta_2\widehat{r}_t = 0. \tag{61}$$

Substitution of the postulated reaction function (13) in (61) implies that for optimality we must have following values for the coefficients of the decision rule (13)

$$\begin{aligned}
\mathsf{a} &= -\frac{\beta\alpha\left(1-\gamma\right)\left[\alpha\lambda\left(\overline{y}-y_n\right)+\rho\right]+\delta\theta_1\alpha^2}{\beta^2\left(1-\gamma\right)^2\left(1+\lambda\alpha^2\right)+\delta\theta_2\alpha^2}; \\
\mathsf{b} &= \frac{\beta\left(1-\gamma\right)^2\left(\lambda\beta\alpha^2+\beta+\alpha\right)}{\beta^2\left(1-\gamma\right)^2\left(1+\lambda\alpha^2\right)+\delta\theta_2\alpha^2}; \\
\mathsf{c} &= \frac{\beta\gamma\alpha\left(1-\gamma\right)}{\beta^2\left(1-\gamma\right)^2\left(1+\lambda\alpha^2\right)+\delta\theta_2\alpha^2}; \\
\mathsf{m} &= -\frac{1}{\left(1-\gamma\right)\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{n} &= \frac{1}{\left(1-\gamma\right)\beta}.
\end{aligned} \tag{62}$$

The private sector's expectations are found by taking expectations at $t-1$ over the reaction function of the central banker:

$$\bar{r}_t^{e,d} = -\frac{\beta(1-\gamma)[\alpha\lambda(\overline{y}-y_n)+\rho]+\alpha\delta\theta_1}{\alpha\delta\theta_2-\beta(1-\gamma)^2}+\frac{\beta\gamma(1-\gamma)}{\alpha\delta\theta_2-\beta(1-\gamma)^2}\widehat{r}_{t-1}. \tag{63}$$

Following the same substitutions made previously for the case of commitment we can get the equilibrium values of $\bar{r}_t$, inflation and output:

$$\begin{aligned}\bar{r}_t^{d} &= -\frac{\beta(1-\gamma)[\alpha\lambda(\overline{y}-y_n)+\rho]+\alpha\delta\theta_1}{\alpha\delta\theta_2-\beta(1-\gamma)^2}+\frac{\beta\gamma(1-\gamma)}{\alpha\delta\theta_2-\beta(1-\gamma)^2}\widehat{r}_{t-1}\\ &\quad -\frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}v_t+\frac{1}{(1-\gamma)\beta}u_t;\end{aligned} \tag{64}$$

$$\begin{aligned}\pi_t^{d} &= -\frac{\alpha\delta[\rho\theta_2+(1-\gamma)\theta_1]+\lambda\alpha\beta(1-\gamma)^2(\overline{y}-y_n)}{\alpha\delta\theta_2-\beta(1-\gamma)^2}+\frac{\gamma\alpha\delta\theta_2}{\alpha\delta\theta_2-\beta(1-\gamma)^2}\widehat{r}_{t-1}\\ &\quad -\frac{\alpha\lambda}{1+\lambda\alpha^2}v_t;\end{aligned} \tag{65}$$

$$y_t^d = y_n+\frac{1}{1+\lambda\alpha^2}v_t. \tag{66}$$

>From (65), we can conclude that also here interest-rate smoothing allows the policy maker to smooth inflation over a number of periods.

In order to find θ_1 and θ_2 we can apply the envelope theorem on (60) which combined with (38) implies

$$\begin{aligned}V_{\bar{r}}(\widehat{r}_{t-1}) &= 2(\theta_1+\theta_2\widehat{r}_{t-1}) =\\ &\quad 2\gamma\frac{\left[\delta\theta_2\alpha^2+\beta^2(1-\gamma)^2\right]}{\alpha\left[\delta\theta_2\alpha-\beta(1-\gamma)^2\right]}\pi_t+2\lambda\gamma\frac{\beta^2(1-\gamma)^2}{\delta\theta_2\alpha-\beta(1-\gamma)^2}(y_t-\overline{y});\end{aligned} \tag{67}$$

identification of θ_1 and θ_2 gives

$$\theta_1 = -\frac{\gamma\delta\theta_2\left\{\beta(1-\gamma)^2\left[\beta\rho+\alpha\lambda(\beta+\alpha)(\overline{y}-y_n)\right]+\rho\delta\theta_2\alpha^2\right\}}{\alpha\delta\theta_2(1-\gamma)(\delta\alpha\gamma+2\gamma\beta-2\beta)+\beta^2(1-\gamma)^3(1-\gamma+\delta\gamma)+\alpha^2\delta^2\theta_2^2}; \tag{68}$$

$$\theta_2 = 0; \tag{69}$$

$$\theta_2 = \frac{\delta\gamma^2\alpha+2\beta(1-\gamma)^2+\gamma\sqrt{\delta\left[\delta\gamma^2\alpha^2+4\beta(1-\gamma)^2(\beta+\alpha)\right]}}{2\delta\alpha}; \tag{70}$$

$$\theta_2 = \frac{\delta\gamma^2\alpha+2\beta(1-\gamma)^2-\gamma\sqrt{\delta\left[\delta\gamma^2\alpha^2+4\beta(1-\gamma)^2(\beta+\alpha)\right]}}{2\delta\alpha}. \tag{71}$$

Here again the case of $\theta_2 = 0$ implies $\theta_1 = 0$ and thus the value function (37) becomes a constant. Using the general expression (53) of the equilibrium value of $\bar{r}_t$ and the coefficient values in (64), the values of θ_2 given by (69), (70) and (71) imply respectively the following values for the coefficient of the lagged $\widehat{r}_t$

$$\phi_1 = -\frac{\gamma}{1-\gamma}; \tag{72}$$

and

$$\phi_1 = -\frac{\sqrt{\delta}\alpha\gamma-\sqrt{\delta\gamma^2\alpha^2+4\beta(1-\gamma)^2(\alpha+\beta)}}{2\sqrt{\delta}(\alpha+\beta)(1-\gamma)}; \tag{73}$$

and

$$\phi_1 = -\frac{\sqrt{\delta}\alpha\gamma+\sqrt{\delta\gamma^2\alpha^2+4\beta(1-\gamma)^2(\alpha+\beta)}}{2\sqrt{\delta}(\alpha+\beta)(1-\gamma)}. \tag{74}$$

It is easy to see that the value of ϕ_1 given by (73) is in absolute value always smaller than that implied by (74). Hence the choice is between the values given by (72) and (73).

As before, the first of these values implies that in the instrument rule (12) the lagged $\widehat{r}_t$ disappears. Now it is straightforward to show using l'Hopital's Rule that

$$\lim_{\gamma\to1}\left(-\frac{\sqrt{\delta}\alpha\gamma-\sqrt{\delta\gamma^2\alpha^2+4\beta(1-\gamma)^2(\alpha+\beta)}}{2\sqrt{\delta}(\alpha+\beta)(1-\gamma)}\right)=0. \tag{75}$$

Hence it is always possible to find a value of γ that makes the solution (73) smaller than that expressed by (72), as this latter solution becomes greater than one in absolute value for $\gamma > .5$.

Substituting (68) and (70) in equations (64), (65) and (66) allows to find the equilibrium values of inflation, output and interest rate. In general average inflation will not be zero. However, an interesting property of interest-rate smoothing emerges if we take the limit of average inflation for $\gamma \to 1$. It is possible to see that we have in this case

$$\lim_{\gamma \to 1} \left[E\pi_t^d \right] = 0. \tag{76}$$

To understand why we have this effect on steady state inflation as $\gamma \to 1$ it is useful to consider also the limit of average interest rate. It is possible to show that we have

$$\lim_{\gamma \to 1} E\left[r_t^d \right] = \rho. \tag{77}$$

As $\gamma \to 1$ average interest rate tends to the level of average interest rate prevailing under a commitment regime without monetary inertia. The intuition for this striking effect of monetary inertia can be found from the first order condition, by ignoring for simplicity stochastic shocks. From expression (61) it is possible to see that as $\gamma \to 1$ the systematic component of the optimal policy rule is actually found by minimising the value function (37), as the trade-off between the marginal benefit of higher surprise inflation and the marginal cost of higher inflation becomes less important as the degree of inertia increases. In the steady state, if γ is close to 1, the optimal level of the interest rate will be equal to the lowest possible level, which in the present rational expectations IS-LM model is given by the long-run equilibrium real interest rate. This implies that the presence of a very high degree of monetary inertia under discretion is equivalent to committing to achieving a low average nominal interest rate in order to obtaining low average inflation.

These results are derived in greater detail in the appendix B. Moreover from the comparison of (65) and (66) with the analogous equilibrium reaction function for the case of discretion without interest-rate smoothing it is possible to observe that stabilisation of shocks is still optimal.

Hence, if the degree of monetary inertia is sufficiently high, the inflationary bias associated to time consistent monetary policy becomes negligible without implying the arising of a trade-off between credibility and flexibility.

The rationale for this striking result is found in the contrasting effect played by interest-rate smoothing on the incentive to create surprise inflation by reducing suddenly interest rates within the time horizon of existing nominal contracts.

7 Conclusion

The present analysis has re-examined the debate on rules versus discretion and, in particular, the commitment versus flexibility dilemma that is associated with the adoption of policy rules. In the literature instrument rules are opposed to discretion and commonly identified with a fixed algebraic formula to which are ex-ante mechanically tied the choices of the policy maker. However there is no need to interpret instrument rules more restrictively than target rules, which are usually formalised as a constraint limiting discretionary choices.

In our framework the policy maker may follow an explicit instrument rule also under a discretionary regime, where discretion is understood as a situation where the policy maker's optimisation is done for each period after observing shocks. We interpret instrument rules as a systematic behaviour followed by the policy maker in the setting of the instrument. In particular the policy maker optimisation problem consists in choosing some parameters of a specified instrument rule that relates macroeconomic variables to the level of the instrument.

The definition of instrument rules used is in logical agreement with Taylor's alternative view on the rules versus discretion debate. He observes that in practice a policy rule can be defined more broadly as a systematic behaviour and there is no need to follow mechanically an algebraic formula. This intuition of Taylor is questioned by McCallum's by using the Barro-Gordon framework. He shows that an inflationary bias will still emerge under a discretionary regime even if monetary authorities follow a systematic behaviour in the setting of the instrument. Hence the presence or absence of a systematic pattern in the choice of the instrument is not sufficient for separating discretionary behaviour from rule-like behaviour.

Contrary to McCallum, we show that it may be advantageous for society to delegate monetary policy to a central banker following a systematic behaviour in the setting of the monetary instrument. We express the systematic pattern in the implementation of policy in terms of an optimally designed instrument rule for setting the interest rate. The crucial feature of the instrument rule examined is the presence of a certain degree of monetary inertia. It is postulated that monetary authorities smooth interest rates by means of a partial adjustment mechanism where past decisions constitute an important determinant of the current level of the interest rate. If the process of implementation of monetary policy is transparent and generally understood by the public, the adoption of the specified instrument rule may render negligible the inflationary bias associated with a discretionary regime, without necessarily introducing a trade-off between commitment and flexibility. It is shown that this favourable circumstance is associated with the presence of a sufficiently high degree of monetary inertia introduced institutionally ex-ante by the policy maker.

Our analysis is based on the idea, reflected in the empirical evidence on actual policy making, that central banker's current decisions on the interest rate are a function of both past decisions and current information. A crucial assumption for obtaining our result is that the policy maker's inertial behaviour is systematic and common knowledge among all players. Hence the present analysis can be interpreted as an argument for transparency of the process of policy decisions.

Monitoring the process of implementation of policy can be relatively easier than monitoring policy choices and central bank's external communication may play an important role in reducing the uncertainty on the way monetary policy is conducted. Al-Nowaihi and Levine (1996) have shown that, unless we take the simple non-stochastic model considered in much of the early literature, the public may face a severe signal extraction problem when monitoring the policy maker's action. On the contrary, if gradualism is an institutional feature of the implementation of policy, then what is actually needed for making credible monetary policy is making explicit the systematic process underlying policy decisions. In that perspective following systematic behaviours simplifies private sector's process of central bank watching.

It bears underlying that in our framework private sector's conditional expectations of inflation may diverge from the socially optimal rate even when monetary policy is said credible, where credibility is understood as low and stable unconditional expectation of inflation. Moreover, Barro-Gordon's result of the superiority of rules over discretion holds also in our framework, but it is reinforced as the superiority of rules is demonstrated also within a regime where a commitment technology is not available.

The finding that with interest-rate smoothing optimal monetary policy can be more credible than in the case without monetary inertia is new in the literature. As observed, for example, by Walsh (1998): "Central banks have often been criticized, however, for smoothing interest rates. During the late 1960s and 1970s, the Fed's attempts to prevent interest rates from rising in the face of increasing inflation served to exacerbate subsequent inflation. Thus, an understanding of the consequences of interest-rate smoothing is important".

In the present analysis we provide a theoretical support for the optimality of interest-rate smoothing. The intuition for the surprising result that gradualism enhances credibility can be found in the view, expressed by the time-inconsistency literature, that the main problem of monetary policy is the excessive activism of central bankers seeking to exploit employment and output gains deriving from inflation surprises. In this perspective gradualism can be optimal as it contrasts the incentive to fool private sector.

Our analysis provides an explanation of why real world central banks may deliberately choose to move their short-term interest rates in sequence of small steps in the same direction and that reversals in its direction are relatively infrequent. In particular our formalisation is capable of rationalising the very slow adjustment of interest rates in practice, which cannot be attributed to systematic policy responses to persistency in the evolution of output or inflation. As we observed, if monetary authorities inertial behaviour is systematic, acting is such a sluggish fashion may reinforce the credibility of monetary policy. The crucial determinant for this positive outcome is the transparency of the systematic process of implementation of policy, in the sense that the public must understand the specific systematic behaviour followed by the policy maker.

The analysis could be extended in a number of ways. Allowing for errors in the control of the instrument, for forward-looking behaviour by the private sector and for some persistence in inflation or unemployment would add greater realism. It would be of interest to explore whether in these cases it would be possible to derive an optimal policy rule with the same features of the popular rule examined by Taylor. However, the mentioned extensions are likely to add the complexity of the analysis without affecting the result that regimes based on instrument rules aimed at monetary stability may represent an alternative to regimes based on target rules.

Our finding has interesting policy implications. The well established international empirical evidence on instrument rules may suggest that from the point of view of implementation following instrument rules, that is systematic behaviours, is likely to be more feasible than the adoption of the other solutions proposed in the literature for eliminating the inflationary bias. In the real world only New Zealand's monetary regime is closest to the kind of delegation formalised by the contracting approach. While Svensson's "target-conservative" central banker has been criticised as been unrealistic, as in practice the countries that have adopted an inflation targeting regime do not seem to set their inflation targets below the socially optimal level of inflation rate. Moreover, Rogoff's "weight-conservative" central banker, apart from not being supported by the empirical evidence on central banks independence, has also been questioned for its real feasibility. In practice the government might find it difficult to appoint a central banker with exactly the right degree of conservatism.

Appendix A

In this appendix we provide the revision rule used in chapter 4 for analysing the stability of the multiple solutions described in the text. In particular here we follow the approach of Clark, Goodhart and Huang (1999).[14]

First we consider the general form for the equilibrium decision rule of r_t

$$r_t = \phi_0 + \phi_1 r_{t-1} + \phi_2 v_t + \phi_3 u_t. \tag{A.1}$$

>From (A.1) we have

$$r_t^e = \phi_0 + \phi_1 r_{t-1}. \tag{A.2}$$

Now substituting both equations in $E_{t-1}V\left(r_{t-1}\right)$ and using the value function

$$V_{t-1}\left(r_{t-1}\right) = \theta_{0,t-1} + 2\theta_{1,t-1} r_{t-1} + \theta_{2,t-1} r_{t-1}^2, \tag{A.3}$$

we can derive the general relationship between $\theta_{0,t-1}, \theta_{1,t-1}, \theta_{2,t-1}$ and $\phi_{0,t}, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}$. Identification of $\theta_{2,t-1}$ leads to

$$\theta_{2,t-1} = \frac{\left[\left(1-\gamma\right)\phi_{1,t} + \gamma\right]^2}{1 - \delta\phi_{1,t}^2}. \tag{A.4}$$

This expression is general and holds for both the values of $\phi_{1,t}$ obtained under commitment and discretion. They are given by

$$\phi_{1,t}^c = -\frac{\gamma\left(1-\gamma\right)}{\left(1-\gamma\right)^2 + \delta_b\theta_{2,t}}; \tag{A.5}$$

and

$$\phi_{1,t}^d = -\frac{\beta\gamma\left(1-\gamma\right)}{\beta\left(1-\gamma\right)^2 - \delta_b\theta_{2,t}\alpha}. \tag{A.6}$$

Now, by inserting (A.4) in the expressions (A.5) and (A.6) taken at $t-1$, ϕ_1 can be revised by iteration backward as t goes to $-\infty$.

[14] They thank in a note Lars Svensson for suggesting them this approach.

Appendix B

This appendix provides the derivation of the results obtained in chapter 4. Here we derive steady state or unconditional expectation of inflation and nominal interest rate when monetary authorities smooth interest-rates under both discretion and commitment.

Inserting the expressions

$$\bar{r}_t = \phi_0 + \phi_1 \widehat{r}_{t-1} + \phi_2 v_t + \phi_3 u_t, \tag{B.1}$$

$$\bar{r}_t^e = \phi_0 + \phi_1 \widehat{r}_{t-1}, \tag{B.2}$$

in the expression of inflation given by (4.7), we can write the equation of inflation in terms of the coefficients ϕ of the decision rule in the following way

$$\begin{aligned} \pi_t \quad = \quad & [(1-\gamma)\phi_0 - \rho] + [\gamma + (1-\gamma)\phi_1]\widehat{r}_{t-1} - \frac{1+(1-\gamma)\beta\phi_2}{\alpha} v_t \\ & - \frac{(1-\gamma)\beta\phi_3 - 1}{\alpha} u_t. \end{aligned} \tag{B.3}$$

Now recalling that

$$\widehat{r}_t = \phi_0 + \phi_1 \widehat{r}_{t-1}, \tag{B.4}$$

we can express $\widehat{r}_t$ as

$$\widehat{r}_t = \phi_1^t \widehat{r}_0 + \phi_0 \sum_{i=0}^{t-1} \phi_1^i. \tag{B.5}$$

Substituting (B.5) in (B.3) allows to compute steady state inflation as

$$E[\pi_t] = \frac{\phi_0 - \rho(1-\phi_1)}{1-\phi_1}. \tag{B.6}$$

Now consider first the case of commitment. Here we have the following values

$$\phi_0 = \frac{[(1-\gamma)+\delta\gamma]}{\gamma\delta}\rho, \tag{B.7}$$

$$\phi_1 = -\frac{(1-\gamma)}{\gamma\delta}, \tag{B.8}$$

which after substitution in (B.6) imply that

$$E\left[\pi_t^c\right] = 0. \tag{B.9}$$

In order to understand this result it is useful to compute average nominal interest rate. We can see that under commitment with interest-rate smoothing average interest rate will be equal to average interest rate under commitment without monetary inertia, that is

$$E\left[r_t^c\right] = \gamma\left(\frac{\phi_0}{1-\phi_1}\right) + (1-\gamma)\left[\phi_0 + \phi_1\left(\frac{\phi_0}{1-\phi_1}\right)\right] = \rho. \tag{B.10}$$

Now examine the case of discretion. Here the expression of average inflation is more complicate as in this case we have

$$\phi_0 = -\frac{\beta(1-\gamma)\left[\alpha\lambda\left(\overline{y}-y_n\right)+\rho\right]+\delta\theta_1\alpha}{\delta\theta_2\alpha-\beta(1-\gamma)^2}; \tag{B.11}$$

$$\phi_1 = \frac{\beta\gamma(1-\gamma)}{\delta\theta_2\alpha-\beta(1-\gamma)^2}; \tag{B.12}$$

where θ_1 and θ_2 are given respectively by expressions (4.68) and (4.70). However an important result can be found. Taking the limit of (B.6) for $\gamma \to 1$ yields

$$\lim_{\gamma\to 1} E\left[\pi_t^d\right] = 0. \tag{B.13}$$

In order to find the limit of this complicated expression we have used Maple©. However this result can be derived also using the following simpler limits

$$\lim_{\gamma\to 1}\theta_1 = -\rho; \tag{B.14}$$

$$\lim_{\gamma\to 1}\theta_2 = 1; \tag{B.15}$$

$$\lim_{\gamma\to 1}\phi_1 = 0; \tag{B.16}$$

$$\lim_{\gamma \to 1} \phi_0 = \rho. \tag{B.17}$$

Also here it is possible to see that as γ tends to one, the steady state level of interest rate under discretion with interest-rate smoothing tends to the steady state value of the interest rate under commitment without interest-rate smoothing, that is

$$\lim_{\gamma \to 1} E\left[r_t^d\right] = \lim_{\gamma \to 1} \left\{ \gamma \left(\frac{\phi_0}{1-\phi_1} \right) + (1-\gamma) \left[\phi_0 + \phi_1 \left(\frac{\phi_0}{1-\phi_1} \right) \right] \right\} = \rho. \tag{B.18}$$

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