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The Demographic Transition in Europe: a Neoclassical Dynastic Approach

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Abstract

This paper investigates the factors that shaped the demographic transition in a number of European countries (Sweden, England, and France) since the mid 18th century. The analytical framework is a version of the neoclassical growth model with dynastic preferences calibrated to match the Swedish experience. This setup is studied quantitatively to asses the contribution of various factors to the explanation of the observed demographic patterns, both over time and across countries. The factors considered are mortality changes, technological progress, and the evolution of the cost of children. The analysis suggests that the contribution of observed mortality rates and technology is only partial. A substantial part of the demographic-transition facts must be attributed to unobservable variation in the cost of children, both over time and across countries.

Keywords: altruism, growth, demographic transition, mortality, fertility.

JEL classification: J1, O0

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1 Introduction

The mid 18th century witnessed the onset of unprecedented demographic changes in West European populations. Differences in timing and speed across countries notwithstanding, these transformations in reproductive behavior involved the eventual shift from the historical high to the current low levels of mortality and fertility. The declines in mortality generally antedated the fertility declines.[1] As result of this lag between mortality and fertility reductions, the natural rate of growth of European populations accelerated markedly between 1750 and 1850, with peak growth rates recorded over the second half of the 19th century. Subsequently, lasting declining trends in reproduction rates gathered pace, thereby leading to the modest natural rates of increase observed in recent times.

Behind this general picture lies a diversity of national experiences. Chesnais (1992,p223), Anderson (1996,p217-224), and Woods (1996,p305-307) concur in identifying differentiated models according to the timing and amplitude of the changes undergone up to the late 19th century. The Northern model is characterized by mortality declines driving the early surge of natural rates of increase in the presence of stable or mildly declining fertility. It is regarded as the pattern that best conforms to the description offered by the traditional model of the demographic transition. Sweden, in particular, is the classical textbook example of this theory. England (and Wales) represent a model where the early mortality declines were accompanied by appreciable increases in fertility. Therefore, the burst of population growth there exhibited a larger amplitude and longer duration, relative to the Swedish case. Whereas the Swedish and English experiences share the presence of booming populations after 1750, the French case is deemed as atypical in that there the transition occurred smoothly, with only a moderate and brief burst of population growth. The peculiarity that sets the French experience apart is that falling death rates were accompanied by compensating falls in fertility, thus failing to produce the manifest acceleration of natural rates observed elsewhere within the century after 1750.

The aim of this paper is to investigate the role of different factors in shaping the demographic transformations underway since the mid/late 18th century in European countries. The focus is on three such factors: mortality, technological progress, and the costs of bearing and rearing children. The specific goal is to assess the importance of each of these

[1] As argued in detail by Chesnais (1992,p142) and Livi-Bacci (1997,p115).

factors in explaining both the features of the demographic transition over time and the observed differences across the Swedish, English and French national experiences.

The analysis is conducted within the framework of a competitive neoclassical economy where- as in Barro and Becker (1989) - dynastic altruism is identified as the motivating force for capital accumulation and fertility choices. The approach is quantitative and the conclusions are judged by their ability to quantitatively match the observed paths for selected economic and demographic variables. A parametric setting is considered such that the model's equilibrium is consistent with empirically motivated figures for a typical Western economy in its pre-transition steady-state. Similarly, another version of the economy is calibrated to observations that correspond to the typical developed economy in the post-transition steady-state. Then the paths for mortality rates, technological change and the child-cost parameter are calibrated so that the transition between these two steady-states features the best match to Swedish productivity growth, while accurately replicating the observed paths for the natural rate of increase of population and mortality rates

Two types of experiments are conducted on this setup. The first experiment considers alternative paths for mortality, technological change and child-costs to evaluate the importance of the changes in each for the explanation of the Swedish transition. The second experiment is similar but uses direct observations on vital statistics for England and France to study how differences in the behavior of mortality, technology and child-costs provide an interpretation of the diversity of patterns in the natural growth rate of population.

A brief summary of the main results is as follows. Throughout the period 1740-1985, accompanying the rise in the growth rate of technology and the decline in mortality rates, the cost of children must have featured a pronounced U-shape that reaches its trough between 1880-1914. Concerning national experiences, differences in the speed of mortality declines can explain some of the differences in the observed transitions. A significant part of the differences, though, must be attributed to the diversity in the evolution of the child-cost factor.

The main contribution of this paper is to provide a quantitatively-oriented interpretation of the historical demographic transition in Europe within a version of Barro and Becker (1989) model. The quantitative approach to demographic analysis is also a characteristic of Moe (1998), Hansen and Prescott (1998), Fernandez-Villaverde (1999), and Eckstein et al (1999). The focus on historical data is a feature of the last three papers. Unlike the present paper, Fernandez-Villaverde (1999) does not analyze quantitatively the transition and focuses on a more restricted set of facts. Eckstein et al. (1999) use detailed Swedish data on mortality and fertility to estimate and analyze a non-altruistic life-cycle partial-equilibrium model with exogenous wages and no capital accumulation. In the present paper, I use instead low-frequency observations for three countries to calibrate and simulate the transitional dynamics of a dynastic general-equilibrium model of capital accumulation. This literature will be further discussed at the end of the paper.

This paper also relates to literature on the demographic transition that departs from the neoclassical technology and/or dynastic preferences assumed here. This literature includes Becker et al. (1990), Ehrlich and Lui (1991), Dahan and Tsiddon (1998), Galor and Weil (1996), and Galor and Weil (2000). These works contain important insights into the relation between economic and demographic changes, but their implications have not been evaluated with a quantitative approach.

The rest of the paper is organized as follows. Section 2 documents the facts to be explained. Section 3 presents the model and characterizes the household's optimal choices. Section 4 characterizes the equilibrium and outlines some steady-state effects of parameter changes. Section 5 calibrates the model's benchmark economy. Section 6 reports the results of the numerical experiments and discusses the findings. Section 7 ends the paper with conclusions and directions for future research.

2 The Facts

The natural rate of increase (NRI) is the rate of increase in population per 1000 population over one year due to the natural processes of births and deaths only. The NRI is thus calculated as the difference between the crude birth rate (CBR) and crude death rate (CDR). The CBR is the number of births in a year per thousand population. The CDR is calculated in a similar way. These measures are crude indeed, but are the most widely available indicators across regions and periods. The sources of the data used in this section are described in Appendix A.

The earliest date for which complete yearly figures of crude rates are available for England, Sweden and France is 1740. For England there is ample evidence extending further back to 1541, and I will use this evidence to help characterize the conditions prevailing in the pre-transitional period. Figure 1 displays the HP-filtered (with parameter 100) long annual series of CBR's and CDR's for England. Before 1740, there does not seem to be any lasting trend, while fluctuations in crude rates appear to be large. 35-year average annual rates of natural growth per 1000 population are 7.136, 8.809, 5.953, 1.275, 2.755 and 3.428 over the periods 1541-1575, 1576-1610, 1611-1645, 1646-1680, 1681-1715 and 1716-1750 respectively. The average is some 4.9 per thousand population. This figure seems to be highly influenced by the sharp drop in mortality during the century after 1575. It is difficult to say whether this is a permanent feature of the pre-transitional regime or part of a cycle that that can be traced further back in time. If, to remove this influence, attention is focused on the late part of that period, the average NRI over 1646-1750 is 2.5 per 1000, more in line with the figure of 3 of Lucas (1998) [reported in Hansen and Prescott (1998)]. A more modest rate seems also to be consistent with the impressionistic accounts of an almost perfect balance between mortality and fertility characteristic of the times preceding the modern world in, for example, Coale and Watkins (1986).

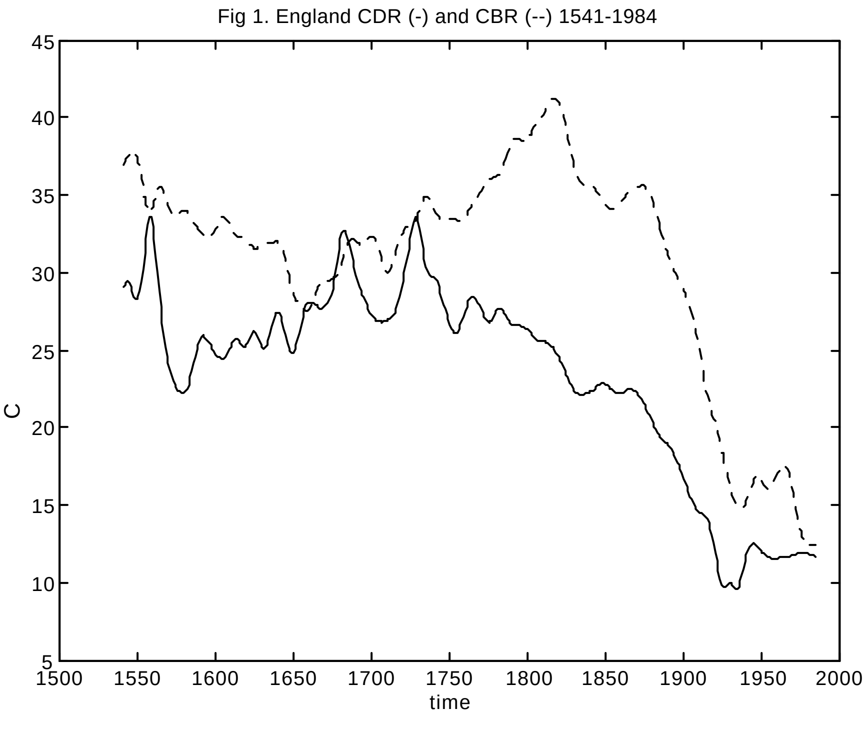

Figure 1: CDR and CBR for England, 1541-1984. HP-filtered annual series with parameter 100.

Figure 1 also shows that the mid 18th century constitutes a turning point in the secular behavior of demographic series that will lead the way towards lasting reductions in fertility and mortality rates. That this is also a momentous period for Sweden and France cannot be read from analogous data, but the consensus among demographers is that the breakthrough must be dated around 1750. Hence this paper focuses on the developments that set off at that time.

Figures 2 through 4 display the HP-filtered annual CBR's and CDR's over 1740-1985. In the three countries, there is a secular trend towards lower mortality. Concerning fertility, the differences across the three countries are more apparent. In England, the fertility rate rises and stays above pre-transitional values for over a century before the onset of the declining trend. In Sweden, instead, the fertility rate remains fairly stable during the period that precedes the decided downturn underway since 1875. Whereas in England and Sweden the paths for the CBR and CDR are dissociated over much of the period, the pattern in France is of a near balance between births and deaths. The declines in death rates are matched by falls in fertility right from the outset.

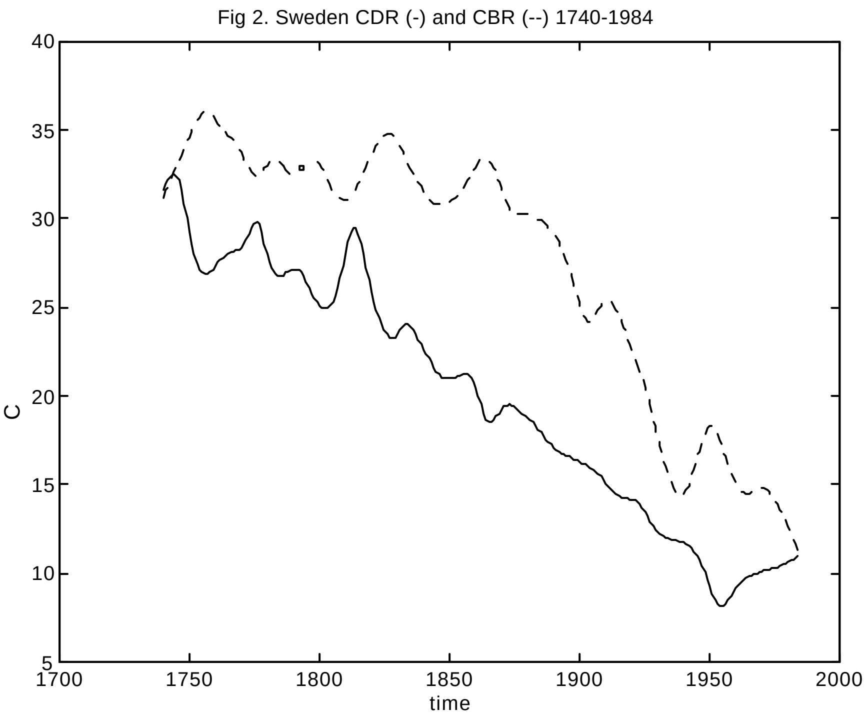

Figure 2: CDR and CBR for Sweden, 1740-1984. HP-filtered annual series with parameter 100.

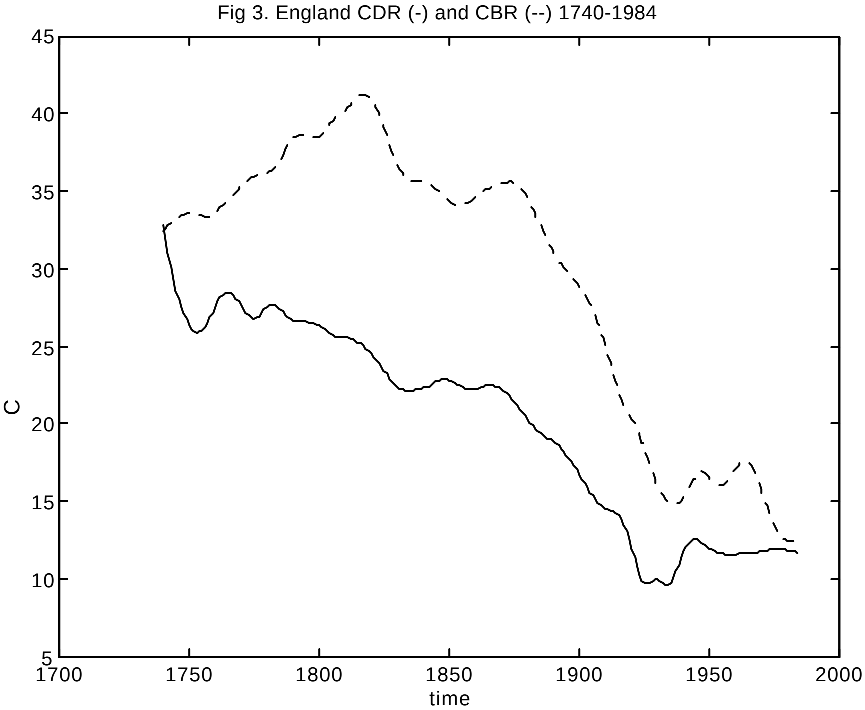

Figure 3: CDR and CBR for England, 1740-1984. HP-filtered annual series with parameter 100.

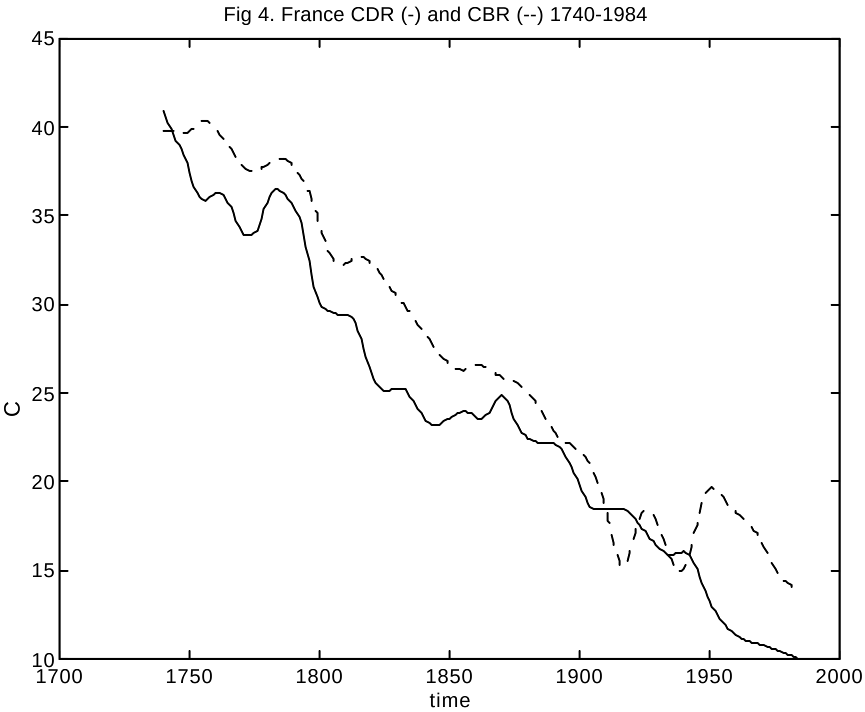

Figure 4: CDR and CBR for France, 1740-1984. HP-filtered annual series with parameter 100.

The separate experiences in crude rates are reflected in the implied paths for the NRI. These are represented in figure 5 below. Sweden displays moderately rising NRI's until the mid 1800's, then an acceleration that subsides by 1914 and gives way to the characteristic declining trend since. Compared with the Swedish case, England's experience is notorious by its early acceleration in the NRI, which can be traced back to the mid 1700's. The English NRI's move into step with the Swedish figures by the mid 1800's. France undergoes a comparatively stable transition in terms of the NRI, with only a brief and modest burst in the 1820's.

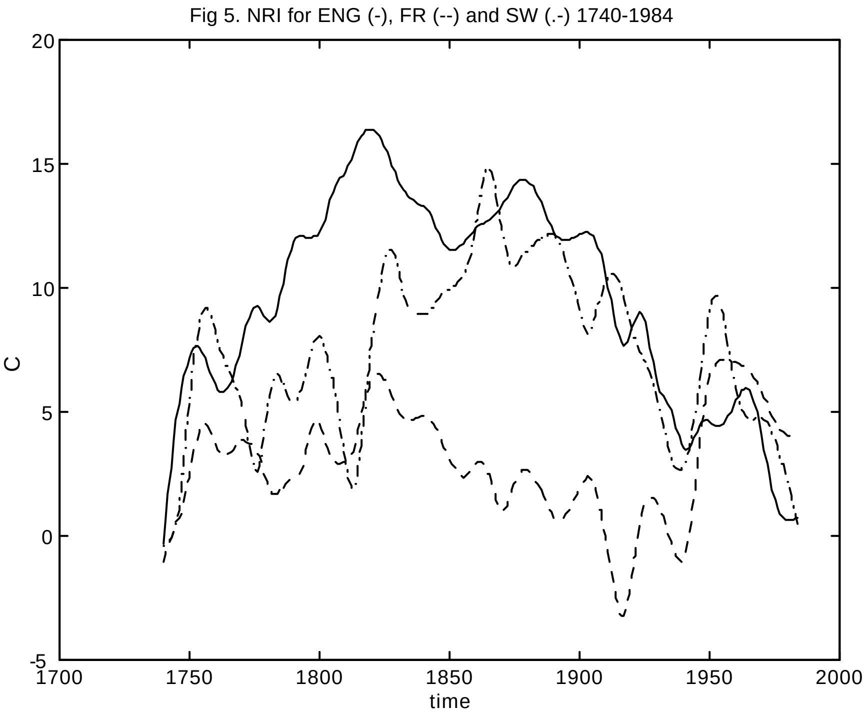

Figure 5: NRI for Sweden, England and France, 1740-1984. HP-filtered annual series with parameter 100.

For the analysis of this paper, it will prove convenient to focus on 35-year average figures. Following this convention, the data displayed in figure 5 is summarized in Table 1.

Table 1. Natural Rate of Increase (NRI).
35-year annual averages per 1000, 1740-1984

period	England	France	Sweden
1740-1774	6.09	2.71	5.16
1775-1809	11.35	2.93	5.84
1810-1844	14.73	4.99	7.9
1845-1879	12.68	2.49	11.61
1880-1914	12.02	1.14	10.56
1915-1949	6.04	0.39	5.94
1950-1984	3.55	5.79	5.12

One of the factors of interest in explaining NRI's will be the changes in mortality. The

CDR provides a rough approximation to the history of vital statistics, but it is a measure plagued with composition effects. For the purposes of the analysis, it will be convenient to gather direct information on survival probabilities of agents within different age groups. In particular, life-tables provide the data that permits to construct the probability of survival between ages 1 and 40 for the corresponding cohort. Consistently with the notation to be used later in the model, I let π denote this probability. The probability that a member of the cohort dies after age 15, conditional on dying between ages 1 and 40, γ, can also be calculated. The series that can be constructed from available sources are summarized in table 2 below. Figures in *italics* for England indicate entries that were constructed with indirect information as documented in Appendix A.

Table 2. Survival probability between age 1 and 40 (π) and probability that non-survivors die past age 15 (γ)

period	π	γ
	Sweden	
1778-1782	0.607	0.509
1783-1812	0.614	0.568
1813-1847	0.678	0.608
1848-1882	0.698	0.546
1883-1917	0.774	0.624
1918-1952	0.881	0.700
1953-1965	0.967	0.756
	England	
1740-1779	*0.641*	*0.627*
1780-1809	*0.678*	*0.650*
1861-1881	0.673	0.604
1891-1911	0.772	0.577
1921-1947	0.884	0.687
1950-1963	0.965	0.776
	France	
1851-1878	0.628	0.599
1879-1913	0.727	0.651
1920-1947	0.846	0.744
1949-1965	0.953	0.795

Against the gradual trend in Sweden, the improvements in England were rapid over the second half of the 1700's, but were then interrupted over much of the 19th century. In France, survival rates seem to catch up at least since the mid 1800's. Inspection of CDR's in figure 6 shows a similar picture. In Sweden the decline appears to be continuous whereas England seems to go through stages, becoming relatively flat between 1840-1880 and steeper afterwards. In France, the lasting declines in mortality seem to have started earlier and from higher levels. Although France tends to catch up with its neighbors by 1840, the mortality rate remains consistently higher throughout.

It is also instructive to look at the series for infant survival rates (probability of surviving to age 1) which extend back to periods for which complete life-tables are not available. As figure 7 shows, secular improvements in Sweden start by the early 1800's. In England there is an earlier improvement in the second half of the 18th century, which flattens out during much of the 19th century, only to resume the rising trend after 1880 or so. France starts out with the lowest infant survival rates in the mid 1800's, although the gap has been narrowing down since.

The patterns of mortality change that have been reported here are consistent with the accounts in the specialized literature such as Vallin (1991), Perrenoud (1991), Chesnais (1992, p55), and Anderson (1996,p217-24).

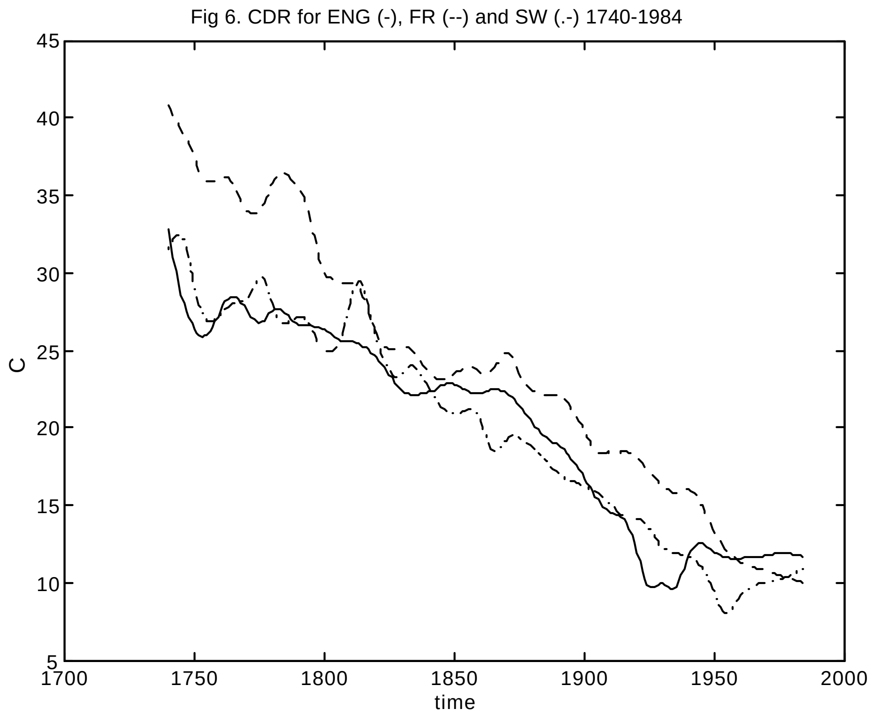

Figure 6:CDR for Sweden, England and France, 1740-1984. HP-filtered annual series with parameter 100.

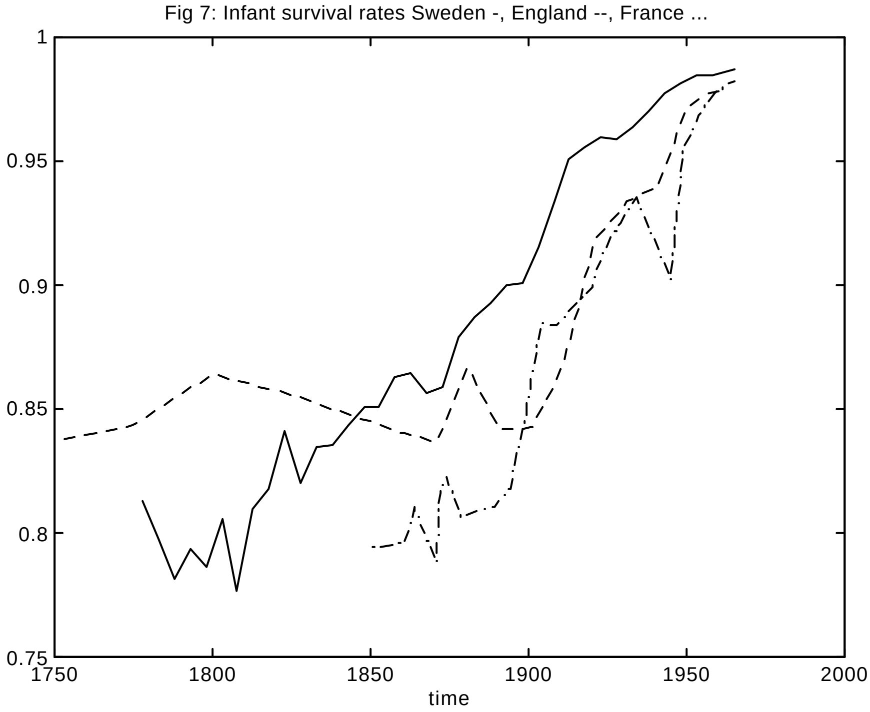

Figure 7: Infant survival rates for Sweden, England and France.

Finally, the period considered is one of unprecedented improvements in living standards. Table 3 displays growth rates of productivity. Casual evidence suggests that productivity surged first in England, the fruits of the Industrial Revolution spreading to the rest of countries with some lag. It is visible, however, that France and Sweden exhibit faster growth of productivity at least since the mid 1800's.

Table 3. Productivity Growth

	England		Sweden		France	
period	annual%	35-year factor	annual%	35-year factor	annual%	35-year factor
1700-1780	0.27	1.1				
1780-1820	0.43	1.16				
1820-1870	1.156	1.495				
1870-1913	1.225	1.532	1.744	1.831	1.584	1.733
1913-1950	1.583	1.733	2.767	2.599	1.927	1.950
1950-1987	2.866	2.688	3.348	3.166	4.202	4.223
1870-1987	1.855	1.903	2.573	2.433	2.514	2.385

3 The model

The analytical framework is a version of the neoclassical model of growth with endogenous fertility in Barro and Becker (1989). The two modifications are the introduction of mortality-risk and a simplified treatment of the cost of children. This section presents the assumptions of the model and analyzes the decision problem of the households.

3.1 Demographic structure

The agents in the economy can live for two periods: young age and adulthood. The economy is populated by a continuum of households, each consisting of possibly one adult and his heirs. I denote generations by the period when their members are born. At time $t-1$ a number N_{t-1} of children are born and become young. This value is net of early infant mortality. A member of generation $t-1$ survives to period t with probability π_{t-1}. Thus only $\pi_{t-1}N_{t-1}$ agents belonging to generation $t-1$ will reach adulthood. In turn, generation $t-1$ bears N_t effective children. The adult population growth factor between two consecutive generations t and $t+1$, g_t, is thus

$$g_t \equiv \frac{\pi_t N_t}{\pi_{t-1}N_{t-1}} \tag{1}$$

For the sake of defining consistent measures, let a model's period correspond to 35 years. Then, the NRI per thousand population between t and $t+1$ can be computed as

$$NRI_t = \left[\frac{N_t\pi_t + N_{t+1}}{N_{t-1}\pi_{t-1} + N_t} - 1\right]\frac{1000}{35} = \left[\frac{g_t\pi_t\left(1+\frac{g_{t+1}}{\pi_{t+1}}\right)}{g_t+\pi_t} - 1\right]\frac{1000}{35}. \tag{2}$$

The behavior of this variables is the primary focus of this paper. Note first that, given the paths for g_t and survival rates π_t, the definition of the NRI does not require to measure infant mortality rates.[2] Second, without further assumptions, the ambiguous effect of g_t and π_t on NRI_t is reflective of natural compositional effects.

A member of generation $t-1$ plans to have given birth to n_t children by the end of his

[2] The values of CBR and CDR do depend on the infant mortality rate, but it nets out when calculating the NRI as CBR-CDR.

first life-time period $t-1$. This planned fertility is net of infant mortality. If he survives to adulthood, with probability π_{t-1}, then the plan will be effectively implemented. However, conditional on that he dies before completing adulthood, which occurs with probability $1-\pi_{t-1}$, the fertility plan can only be carried out if premature death does not occur too soon, which is the case with probability γ_{t-1}. That is, γ_{t-1} is the probability of carrying out the planned fertility conditional on not reaching adulthood. It follows that the motion for the flow of births is

$$N_t = N_{t-1} n_t [\pi_{t-1} + \gamma_{t-1}(1-\pi_{t-1})] \tag{3}$$

As a consequence, the adult population growth factor between t and $t+1$ in Eq.(1) can be written as

$$g_t = \frac{\pi_t}{\pi_{t-1}} n_t [\pi_{t-1} + \gamma(1-\pi_{t-1})] \tag{4}$$

When mortality and planned fertility have constant values the growth rate of total population, $NRI \times (35/1000)$, coincides that of adult population, $g-1$.

3.2 Technology

A single sector produces final output. The total amount of capital, K_t, and labor units, L_t, are the inputs employed in period t to produce total output through a neoclassical production function, $F(K_t, A_t L_t)$. It will be assumed that $F(.,.)$ is Cobb-Douglas with $0 < \theta < 1$ being the output share of capital, and A_t representing labor-augmenting technology,

$$F(K_t, A_t L_t) \quad = \quad K_t^{\theta}(A_t L_t)^{1-\theta} \tag{5}$$

Technology is assumed to grow at a rate $x_t - 1$ between period t and $t+1$,

$$x_t = \frac{A_{t+1}}{A_t} \tag{6}$$

Output produced at t can be used for consumption C_t, for accumulation of next-period capital K_{t+1}, or for producing children. Capital depreciates at the rate $\delta < 1$. Each birth is assumed to imply a goods-cost. This cost amounts to $A_t \eta_t$ for every born child. Since the number of births at t is N_t, total child-rearing expenditures amount to $A_t\eta_t N_t$.

Finally, it remains to specify the technology for rearing children. Many authors argue that child-rearing is intensive in time. In this case, the term η_t should reflect the opportunity cost of the parental time, as in Barro and Becker (1989). Here it is assumed that children entail a cost in terms of goods only. This cost may depend on the level of the economy-wide capital per worker though. With some abuse of notation, then one can can write

$$\eta_t = \eta_t(\hat{k}_t) \tag{7}$$

with $\hat{k}_t$ being capital per effective worker, $\hat{k}_t = K_t/(A_t L_t)$. Note this formulation admits that the child-cost be proportional to the technology-adjusted wage as in Barro and Becker (1989). By assuming it is a goods-cost rather than a time-cost, however, I rule out the impact of changes of fertility on hours of labor supplied. This simplifies the explicit computation of the model's transitional dynamics. The time index in $\eta_t(.)$ accounts for the possibility that the parameters of this function change over time.

3.3 Households

Individual agents make choices on fertility and capital transfers. This section describes the household's environment and characterizes the optimal choices.

The budget constraint

An agent of generation $t-1$ receives at $t-1$ a claim on the amount of wealth k_t from his parents. In the first period of his life, this wealth is allocated between two assets named conditional annuities, s_t, and simple annuities, a_t. Simple annuities will pay at t a rate of return R_t^a only if he survives. A conditional annuity will pay a rate of return R_t^s except if he dies and, additionally, does not have any children, in which case it will pay nothing. For example, if $\gamma_{t-1} = 1$ conditional annuities yield a return with certainty and are equivalent to simple annuities. If the agent dies before period t but leaves descendants, the return on conditional annuities is devoted to cover the rearing cost of his heirs, $n_t A_t \eta_t$, and to provide them with *accidental* bequests, adding up to $n_t k_{t+1}^A$. If he survives to period t, then he receives the returns from his total wealth and labor income and spends the revenues on own consumption, child-rearing and wealth accumulation in the form of voluntary bequests,

$n_t k^V_{t+1}$.

More formally, the portfolio choice at time $t-1$ by the agent of cohort $t-1$ must satisfy the constraint $s_t + a_t = k_t$. In the event that she dies and has children, which happens with probability $(1-\pi_{t-1})\gamma_{t-1}$, accidental bequests to generation t are determined by $R^s_t s_t = n_t k^A_{t+1} + n_t A_t \eta_t$, where $A_t\eta_t$ is the expenditure on each child and k^A_{t+1} represents the wealth received by each child that survives up to period $t+1$. Here and after all prices are expressed in terms of the final good. If she lives up to the second period t, then she faces the constraint $c_t = w_t + R^s_t s_t + R^a_t a_t - n_t k^V_{t+1} - n_t A_t \eta_t$. Here c_t is her consumption, w_t is labor income and k^V_{t+1} is the bequest received by each of her children. The budget set for an agent of cohort $t-1$ is given by the three last expressions. They can be combined to get a more compact expression,

$$c_t = w_t + R^a_t k_t - n_t \frac{R^a_t}{R^s_t} A_t \eta_t - n_t k^V_{t+1} - n_t k^A_{t+1} \left(\frac{R^a_t}{R^s_t} - 1 \right). \tag{8}$$

Preferences and the decision problem

The preferences of an individual of cohort $t-1$ reflect a concern for the welfare of all future descendants. Generations are linked through parental altruism which is materialized in the form of intergenerational transfers of wealth. Let V_{t-1} denote the utility attained by a typical member of generation $t-1$ from the perspective of time $t-1$. The following separable specification is assumed,

$$\begin{aligned} V_{t-1} &\equiv E\left[u(c_t) + \beta n_t^{1-\epsilon} V_t \mid t-1\right] \\ &= \pi_{t-1} u(c_t) + \beta E\left[n_t^{1-\epsilon} V_t \mid t-1\right] \end{aligned} \tag{9}$$

where E is the expectation operator. This formulation assumes that a parent's utility depends on her own consumption, on the number of children, and the utility of each child. As of period $t-1$, there is uncertainty about outcomes, so utility is evaluated in expected terms. Expenditures on children do not enter utility but only adult consumption does through $u(.)$.[3] The discount term reflects that individuals care about the welfare of every

[3] For the sake of tractability, models with altruistic preferences often assume that the number of children enters separately with the utility of the children [see, for instance Razin and Ben-Zion (1975) and Palivos (1995)]. On the other hand, the assumption that parents' instantaneous utility does not depend on the number of children is relaxed in Barro and Sala-i- Martin (1995,Ch7). Actually, in their formulation, dropping this additional component of utility from children precludes any dynamics for fertility, at least

child. It is assumed that $\epsilon > 0$, and $\epsilon > 1$ if utility is negative.[4] The instantaneous utility function $u(.)$ is assumed to take on the following CRRA specification,

$$u(c) = \frac{1}{1-\sigma} c^{1-\sigma}, \quad \text{with } \sigma > 0. \tag{10}$$

Households take as given labor income, asset returns and mortality risk. It is assumed that the per-child cost is also exogenously given to the household.[5] Now members of generation $t-1$ make their choices on the first period $t-1$ under uncertainty. For the preferences in (9) and (10), given a value k_t of wealth, the values c_t, k^A_{t+1}, k^V_{t+1} and n_t are determined as the solution to the Bellman equation,

$$\begin{aligned} V_{t-1}(k_t) &= \max \left\{ \pi_{t-1} u(c_t) + \beta E[n_t^{1-\epsilon} V_t(k_{t+1})] \right\} \\ &= \max \left\{ \pi_{t-1} u(c_t) + \beta n_t^{1-\epsilon} [\pi_{t-1} V_t(k^V_{t+1}) + (1-\pi_{t-1})\gamma_{t-1} V_t(k^A_{t+1})] \right\} \end{aligned} \tag{11}$$

subject to the budget constraint (8). Although the problem is stationary I keep the subscripts in the value function for clarity.

One can show (see appendix B) that, with competitive insurance market, the equilibrium asset returns R^a and R^s are actuarially fair. There is then full insurance against lifetime uncertainty so that $k_{t+1} = k^A_{t+1} = k^V_{t+1}$ and the FOC's for the choice of k_{t+1} can be written as,

$$n_t u'(c_t) = \beta n_t^{1-\epsilon} R_{t+1} u'(c_{t+1}) \tag{12}$$

where R_{t+1} is the return on one-period securities.[6]

when $\eta_a = 0$.

[4] Barro and Becker (1989) assume $\epsilon < 1$. The current formulation allows us to accommodate relative risk aversion greater than one, a point also made by Alvarez (1999,ft2).

[5] In Barro and Sala-i-Martin (1995,Ch7) the alternative assumption is made. I think the assumption made here accords better with existing theories.

[6] An extended-family arrangement, as in Ehrlich and Lui (1991), could also have been assumed as an alternative to markets for annuities. Each extended family is composed of a large number of identical households with independent prospects of survival. This mechanism would ensure bequests to every child whether he is orphan or not by assuming that an implicit contract concerning intra-family transfers is enforced. In a different model, Ehrlich and Lui (1991) show that, with a trigger-strategy penalty on the defectors, such type of contracts are self-enforcing. The parallel analysis has not been carried out here.

As one referee rightly pointed out, either assumption rules out a potentially important role for capital market imperfections, and the assumptions made are justified on grounds of analytical convenience. In the absence of such insurance institutional arrangements, life-time uncertainty would command variation in the distribution of wealth. See Loury (1981) and Fuster (1999) on this matter.

The budget constraint (8) can be posed as,

$$c_t = w_t + R_t \frac{k_t}{\pi_{t-1}} - \left(\frac{A_t \eta_t}{\pi_t} + \frac{k_{t+1}}{\pi_t} \right) \frac{\pi_t}{\pi_{t-1}} n_t (\pi_{t-1} + (1 - \pi_{t-1}) \gamma_{t-1}). \tag{13}$$

The present full-insurance setup implies that, for a given real return on equity, mortality risk does not influence optimal intertemporal choices. The net result is that the condition for intergenerational optimality (12) is not influenced by mortality risk.[7]

Fertility choice

Since there is full insurance, one can write the value function in (11) as,

$$V_{t-1}(k_t) = \max \left\{ \pi_{t-1} u(c_t) + \beta n_t^{1-\epsilon} (\pi_{t-1} + (1 - \pi_{t-1}) \gamma_{t-1}) V_t(k_{t+1}) \right\},$$

the maximization being subject to the constraint in (13). For the choice on n_t to be well defined, the condition $(1-\sigma)/(\sigma - \epsilon) > 0$ has to be satisfied. Therefore, $\epsilon > \sigma$ if and only if $\sigma > 1$. The optimal choice of n_t balances the cost of an additional descendant in terms of parent's utility and the gain in future generations' utility. Using the specification of $u(.)$ in (10), appendix B shows that the solution is characterized by

$$c_{t+1} = \frac{1-\sigma}{\sigma - \epsilon} \left[R_{t+1} \frac{A_t \eta_t}{\pi_t} - w_{t+1} \right]. \tag{14}$$

The conditions (12-(14) characterize the household's behavior provided that the implied utility value is bounded.

4 Equilibrium

For analytical convenience, using Eq.(4) the equilibrium is defined in terms of the growth rate of the labor force, g_t, and per-unit-of-effective-worker variables.

Definition: Given $f(.)$, δ, β, σ, ϵ, $\{\eta_t(.), x_t, \pi_t, \gamma_t\}_{t=0}^{\infty}$ and $\hat{k}_0$, a *perfect foresight (PF) competitive equilibrium* consists of sequences of quantities for capital per effective worker

[7] In Ehrlich and Lui (1991) the return from investing in children's human capital is positively related to survival probabilities and this determines the positive effect of mortality rates reductions on economic outcomes and the tendency of fertility to decline over time. On the contrary, in the current setup the relevant return on investment is independent of survival rates. Preston (1980, p.324-326)'s calculations imply that, to a first approach, this assumption may not be grossly misleading.

$\hat{k}_t$, consumption per effective worker $\hat{c}_t$, adult population growth g_t and prices w_t, r_t, R_t for $t = 0, 1, 2, ...$ such that: taking r_t and w_t as given, firms maximize profits; taking R_t and w_t as given, households maximize their dynastic utility; returns on capital and debt are equalized (no-arbitrage); all markets clear.

Market clearing implies that total wealth equals the aggregate capital stock. Since all households are assumed to be identical, $K_t = N_{t-1}k_t$. Each adult provides one unit of labor so that $L_t = N_{t-1}\pi_{t-1}$. Then $\hat{k}_t \equiv K_t/(A_tL_t) = k_t/(A_t\pi_{t-1})$. Output per effective worker is then $f(\hat{k}_t) \equiv F(\hat{k}_t, 1)$. The no-arbitrage condition implies that $R_t = 1 - \delta + r_t$. Maximization by competitive firms leads to $r_t = f'(\hat{k}_t)$ and $w_t = A_t[f(\hat{k}_t) - \hat{k}_t f'(\hat{k}_t)]$. Thus we can represent the equilibrium wage rate and the rental rate of capital as $Aw(\hat{k})$ and $r(\hat{k})$, respectively.

Then clearing in output market, $F(K_t, A_tL_t) + (1-\delta)K_t = C_t + K_{t+1} + N_tA_t\eta_t$ or, equivalently, the household's budget constraint (13), can be written as

$$f(\hat{k}_t) + (1-\delta)\hat{k}_t = \hat{c}_t + g_t\left(\hat{k}_{t+1}x_t + \frac{\eta_t(\hat{k}_t)}{\pi_t}\right), \quad t = 0, 1, 2, ..., \tag{15}$$

Optimal fertility in (14) dictates

$$\hat{c}_t = \frac{1-\sigma}{\sigma-\epsilon}\left[(1-\delta+r(\hat{k}_t))\frac{\eta_{t-1}(\hat{k}_{t-1})}{x_{t-1}\pi_{t-1}} - w(\hat{k}_t)\right], \quad t = 1, 2, ... \tag{16}$$

and the first-order condition for intertemporal allocation of consumption (12) can be posed as

$$g_t = \frac{\pi_t}{\pi_{t-1}}(\pi_{t-1} + \gamma_{t-1}(1-\pi_{t-1}))[\beta(1-\delta+r(\hat{k}_{t+1})]^{1/\epsilon}\left(\frac{\hat{c}_{t+1}}{\hat{c}_t}x_t\right)^{-\sigma/\epsilon}, \quad t = 0, 1, 2, ... \tag{17}$$

where use has been made of (4) to substitute n_t away. The term $\eta_t(\hat{k})$ will now be specified as

$$\eta_t(\hat{k}) = \eta_{at} + \eta_{bt}w(\hat{k}), \quad \text{with} \quad \eta_{at}, \eta_{bt} \geq 0. \tag{18}$$

Thus an equilibrium is described by paths for $\hat{k}_t$, $\hat{c}_t$ and g_t that satisfy the four equations (15)-(18) above for given initial capital intensity, $\hat{k}_0$ and a $\hat{c}_0$ consistent with the appropriate transversality condition. Under the functional forms assumed, $f(\hat{k}) = \hat{k}^{\theta}$, $r(\hat{k}) = \theta\hat{k}^{\theta-1}$ and $w(\hat{k}) = (1-\theta)\hat{k}^{\theta}$. The description of the equilibrium is completed by a condition ensuring that the household's utility is not unbounded for the paths implied by the above equations. The discussion of this condition will be conducted in terms of steady-state outcomes.[8]

Define a steady-state as an equilibrium where all per unit-of-effective-worker variables, prices and fertility and mortality rates remain constant over time. A steady-state is characterized by the following set of equations,

$$\Lambda(\hat{k}) \equiv f(\hat{k}) + (1-\delta)\hat{k} - \hat{c}(\hat{k}) - (\hat{k}x + \frac{\eta(\hat{k})}{\pi})g(\hat{k}) = 0 \tag{19}$$

with

$$\hat{c}(\hat{k}) = \frac{1-\sigma}{\sigma-\epsilon}\left[(1-\delta+r(\hat{k}))\frac{\eta(\hat{k})}{x\pi} - w(\hat{k})\right] \tag{20}$$

$$g(\hat{k}) = (\pi + \gamma(1-\pi))[\beta(1-\delta+r(\hat{k}))]^{1/\epsilon}x^{-\sigma/\epsilon} \tag{21}$$

and

$$\eta(\hat{k}) = \eta_a + \eta_b w(\hat{k}). \tag{22}$$

Bounded utility requires the condition shown in appendix C that

$$g(\hat{k})x < 1-\delta+r(\hat{k}). \tag{23}$$

As in Barro and Becker (1989), there may be multiplicity of steady-states. Although the number of steady-states cannot be bounded analytically, none of the numerical setups considered in this research has been found to have more than two steady-states. The

[8] The equilibrium has a similar structure to the one studied in Barro and Becker (1989). The presence of the life-uncertainty terms, π and γ, makes one difference. The other difference concerns the specification of the determination for the child-cost term $\eta(.)$ as a goods cost, which simplifies the economy's dynamic structure.

equilibrium system is of second-order as long as η_{bt} is positive.[9] Local stability analysis has been performed by studying the two roots of the linearized system. Although analytical results have not been obtained, the linearized system has been investigated numerically. For all the economies considered in this research, the condition $\Lambda'(\hat{k}) > 0$ [see definition in Eq.(19)] characterizes saddle-path stable steady-states.[10]

The steady-state effects of a change in the rate of technical progress and the child-cost are already discussed in Barro and Becker (1989). A higher rate of technical progress, x, implies, by Eq.(17), a steeper profile for consumption which, for given r, calls for lower discount or lower fertility. But x may also lead to lower capital accumulation and a higher interest which exerts the opposite effect on equilibrium fertility. The effects of changes in the child-cost parameters on population growth are straightforward. The treatment of mortality here is more elaborated than in Barro and Becker (1989), which opens new theoretical possibilities. On one hand, higher survival probabilities have a direct impact on the population growth rate [see Eq.(4)]. In this model, stable steady-states are such that the marginal value of the increased quantity of children declines relative to the return from quality. Therefore parents tend to increase the quality of the children which expresses itself in larger transfers of capital per capita and, possibly, lower fertility. On the other hand, however, increasing the survival rate implies a reduction of the perceived per-parent cost of raising and endowing a given amount of children. This price-effect induces a substitution of quantity for quality of children which tends to increase fertility rates. The net result depends on the relative importance of the two effects. When mortality drops happen to be concentrated at advanced adult ages (γ large), the impact on population growth is small and the cost reduction or substitution effect dominates.[11] On the contrary, when young people is the group experiencing major drops (γ small), then it is more likely that increasing survival rates drive the economy to higher levels of output per head along with, possibly, lower planned fertility rates. The latter is clearly a necessary condition for adult population growth to fall after a reduction of mortality risk. In general, specific statements about the implications of changes in parameters can only be made in a quantitative version of the

[9] In Barro and Becker (1989), where children involve a time cost, the system is of third order.

[10] These results are consistent with those conjectured in Barro and Becker (1989). Discussion of existence and stability are available from the author.

[11] The implicit assumption in Barro and Becker (1989)'s discussion is that $\gamma = 1$

model.[12]

5 Calibration

This section determines the parameters of the benchmark economy that will be used for the analysis. The working hypothesis is that the economies under study were all at a steady-state before the changes operated at about 1740, and that they converge towards a new steady-state that is consistent with post-transition figures. Furthermore, it is assumed that the three economies [Sweden, England, and France] are alike at both the pre-transitional and the post-transitional steady-states. This will permit to focus the analysis on the factors shaping the transition between the two steady-states. The parameters to be determined thus are two sets of steady-state parameters, and sequences for the parameters that are allowed to change over time. These time-varying parameters are the child-cost parameters, the survival probabilities, and the rate of technological change. Whereas the steady-state parameters are chosen to match the average behavior of a typical western economy, the sequences of parameters are calibrated to transition observations for Sweden. Sweden is chosen as the benchmark for the evidence reviewed in section 2 indicates this is the "normal" case that lies between the two more extreme experiences of England and France.

5.1 Steady-state calibration

At a steady-state, the parameters of the model are π, γ, θ, δ, x, σ, β, ϵ, η_a, and η_b. The procedure to pin down these numbers for the final, or post-transition, steady-state is the following.

1. Assume that the model's period corresponds to 35-years. Set directly $\pi = 1$ (so γ becomes irrelevant), $\theta = 0.4$, $\delta = 1.0$, $\sigma = 3$ and $x = 2.0$.

2. Set targets $g = 1$ and, for the investment-output ratio,

$$I/Y = [gx + (1 - \delta)]\hat{k}^{1-\theta} = 0.15$$

[12] Detailed analysis of the consequences of changes in π and γ can be found in an early working paper version available from the author.

to derive $\hat{k}$ in the steady-state. Use (21) and $R = r+1-\delta = \theta\hat{k}^{\theta-1}+1-\delta = (1+0.049)^{35}$ to calculate

$$\beta = \frac{x^{\sigma}}{R} = 1.501$$

3. Set target 0.588 for the share of cost of one child on adult consumption. Using (19) and (20),

$$0.588 = \frac{\eta(\hat{k})(g/\pi)}{\frac{1-\sigma}{\sigma-\epsilon}\left[(1-\delta+r(\hat{k}))\frac{\eta(\hat{k})}{\pi x} - w(\hat{k})\right]},$$

or

$$\epsilon = \sigma - 0.588(1-\sigma)\frac{(1-\delta+r(\hat{k}))\frac{\eta(\hat{k})}{\pi x} - w(\hat{k})}{\eta(\hat{k})}.$$

Calibrate $\eta(.)$ and ε according to these conditions and that the steady-state produces the target $g = 1$. For the specification in Eq.(18), $\eta(\hat{k}) = \eta_0 + \eta_1(1-\theta)\hat{k}^{\theta}$, there exists one degree of freedom for the choices of η_a or η_b. I set $\eta_a = 0$.

The choices in step 1 are justified as follows. I will consider a final steady-state where survival rates are virtually hundred percent. The capital share is assumed to be 40% as in Cooley and Prescott (1997). Given the period length of 35 years, a depreciation rate of hundred percent seems reasonable. The coefficient of risk-aversion of 3 is in the middle range of values used in the literature. The assumption of 2% long-run growth per year leads to $x = (1+0.02)^{35} = 2$. In step 2, I target a zero-growth population. The investment-output ratio is close to the average for developed countries over the last two-decades. Note the implied gross return of 5% is not off the mark. Step 3 uses the equivalence Oxford scale, which weights the first adult as 1, the second by 0.7, and each children as 0.5 [see Van Praag and Warnaar (1997)]. Since the model has single households, the share of the cost of a child in adult consumption can be calculated as $(0.5 \times 2)/(1+0.7) = 0.588$. To resolve the indeterminacy, I will arbitrarily assume $\eta_a = 0$ for the benchmark case. The arbitrary choices in this calibration are thus $\eta_a = 0$ and $\sigma = 3$. They should be the focus of robustness analysis.

I will analyze the transition of economies towards the long-run post-transition steady-state just described, starting from the pre-transition steady-state. The assumption is that only the parameters π, γ, $\eta(.)$ and x have changed since the mid 18th century. To calibrate their values at the initial steady-state, one needs observations about the pre-transitional situation in England, France and Sweden. Motivated by the discussion on pre-industrial figures in section 2, a target for the population growth increase of 3.0 per 1000 per year seems reasonable. For the model's 35-year periods, the target is thus set to $g = 1.11$. On the other hand, $x = 1.0$ as output has remained largely stagnant before the Industrial Revolution. The benchmark observations for Sweden in the mid 1700's in table 2 point to a choice $\pi = 0.6$ and $\gamma = 0.50$. Note here π is taken to represent survival probability between age 1 and 40, and γ is interpreted as the conditional probability of dying after age 15 provided that death occurs between ages 1 and 40. Finally, the benchmark economy will be characterized by a choice of η_b that is consistent with the target for g. This calibration involves an annual interest rate of 2.4%, smaller than the 5% of the final steady-state. Other studies, such as Hansen and Prescott (1998), entertain similar implications for the interest rate. Table 4 summarizes the choices of steady-state parameters.

Table 4. Calibration steady-states

Constant	σ	θ	δ	β	ε
	3.0	0.4	1.0	1.1488	3.78
Changing	x	π	γ	η_a	η_b
Initial $g = 1.11$:	1.0	0.6	0.50	0	0.335
Final $g = 1.0$:	2.0	1.0	1.0	0	0.5

The changes between the two steady-states reflect the interplay of some key comparative-static effects of the benchmark economy. The effect of the increase in survival probabilities between the initial and final stead-states would, on its own, lead to higher population growth and lower capital intensity. Thus, in spite of the theoretical case made at the end of section 4, the model rules out the possibility that the reduction in mortality can explain the broad characteristics of the demographic transition. This result appears to be robust to changes in parameter settings. Other factors must have contributed to the net transformations operated. The observed rise in technical progress alone does indeed lead to lower population growth along with lower capital intensity. But the net effect of the observed joint changes in mortality and technical change leads to a final population growth rate that exceeds the benchmark zero-growth case. Thus, a net increase in the child-cost parameter is needed. This tends to partly balance the negative effect of faster technical change and lower mortality on capital intensity.

5.2 Calibrating the transition

This section describes the procedure leading to the choice of the paths $\{\pi_t, \gamma_t, \eta_{bt}, x_t\}$ over the transition period. One has to establish a correspondence between the model's periods and historical dates. I will choose to map the model's periods to historical 35-year intervals for which the figures of NRI were averaged and calculated in table 1. This is shown in the following table 5.

Table 5. Correspondence model's periods and historical dating

Model	Observations
-1	1670-1704
0	1705-1739
1	1740-1774
2	1775-1809
3	1810-1844
4	1845-1879
5	1880-1914
6	1915-1949
7	1950-1984
8	1985-2020

Periods -1 and 0 correspond to the pre-transition stage. The benchmark paths π_t and γ_t are set according to the information gathered in table 2 for Sweden. Those observation are dated in the model using the correspondence in table 5 to match the dates of the historical observations in table 2. Table 6 shows the calibrated paths.

Table 6. Calibrated Vital Statistics

period	π_t	γ_t
-1	0.600	0.500
0	0.600	0.500
1 1778-1782	0.607	0.509
2 1783-1812	0.614	0.568
3 1813-1847	0.678	0.608
4 1848-1882	0.698	0.546
5 1883-1917	0.774	0.624
6 1918-1952	0.881	0.700
7 1953-1965	0.967	0.756
8	1.000	1.000

Now, whereas the paths for π_t and γ_t have been determined from direct observations, the two remaining paths for x_t and η_{bt} are calibrated so that the implied transition is consistent with targets for the NRI and productivity growth. The target for the path of the NRI is constructed from the Swedish figures in table 1. I assume the NRI becomes zero after two periods following 1984. Concerning average productivity growth, I will use the 35-year average figures in Table 3. However, figures on early periods are missing for Sweden. I will approximate these missing figures using some indirect information. The figures for GDP per capita growth in Maddison (1991) indicate that during 1820-1870 Sweden's productivity growth rate must have been lower than England's. Also, since technological progress from England spread with some delay, one could assume a lagged start of productivity increases in Sweden. Therefore I assume low productivity growth for the periods 1780-1820 and earlier in Sweden. Chesnais (1992,p455) dates the first stages of industrialization and growth for Scandinavian countries at the turn of the 19th century. Early writers date the beginning of economic growth in England around 1760 [Chesnais (1992,p447, ft3)]. Scaling down the English figures in table 3, the calibration target for productivity adopted for the Swedish benchmark obtains. Table 7 below shows the chosen targets for productivity growth and the NRI. The entries in italics indicate the figures that have been constructed from indirect evidence.

Table 7. Calibration targets for productivity growth and the NRI

period	NRI	Productivity growth
0	3.14	1.0
1	5.16	*1.011*
2	5.84	*1.011*
3	7.90	*1.25*
4	11.61	*1.40*
5	10.56	1.83
6	5.94	2.60
7	5.12	3.17
8	3.00	
9	1.00	
10	0.00	

Given the paths for π_t and γ_t, and the target for NRI_t, Eq.(2) permits to calculate the path for g_t that is consistent with that target. From a practical point of view, the equivalent exercise of targeting g_t rather than NRI_t proves to be more convenient because of the compositional effects involved in the latter variable. A second practical observation is that productivity growth between t and $t+1$ is calculated as $x_t(\hat{k}_{t+1}/\hat{k}_t)^\theta$. Third, calibrating η_{bt} and x_t requires to be able to compute the transition of the model when the system is forced by changes in π_t, γ_t, η_{bt} and x_t over time. To perform these calculations, I assume perfect foresight so that the changes set off after period 0 are known and anticipated by agents as of period 1. This seems a natural assumption given the low frequency of the model's implications. The system is saddle-path stable and the equilibrium path is found using an iterative Gauss-Seidel type of algorithm. Details are provided in appendix D.

Now one can find the paths for the child cost and technology growth that are consistent with the calibration targets in table 7 given the vital statistcs in table 6, which correspond to the Swedish demographic transition. The paths for η_b and x have been calibrated in order to produce an accurate match of the NRI's. There is a tension with matching the figures of productivity growth though. Indeed, only paths of productivity that underestimate the targeted figures are consistent with a good fit for the NRI. Conversely, paths that are consistent with the high observed productivity growth rates will lead to NRI's way below the observations. The compromise adopted here is to pick out a setting that, being consistent with the target NRI's, gets as close as possible to the pattern of the target path for productivity growth. One choice that satisfies this constraint is reproduced in table 8 below. For the sake of completeness, I also add paths of parameters that have already been calibrated.

Table 8. Benchmark Paths for Parameters.

period	π	γ	x	η_b
-1	0.600	0.500	1.000	0.335
0	0.600	0.500	1.000	0.335
1	0.607	0.510	1.011	0.335
2	0.614	0.568	1.100	0.170
3	0.678	0.608	1.350	0.220
4	0.698	0.546	1.450	0.200
5	0.774	0.624	2.200	0.130
6	0.881	0.700	2.300	0.180
7	0.967	0.756	2.200	0.280
8	1.000	1.000	1.750	0.320
9	1.000	1.000	1.850	0.440
10	1.000	1.000	1.900	0.500
11	1.000	1.000	2.001	0.500

The parameters in table 4 and the paths in table 8 characterize the benchmark choice.

To provide a visual impression, these benchmark paths are depicted in figure 8 below. Figures 8a and 8b simply display the historical Swedish figures for survival rates. Figure 8c shows that strong growth of technology starts in period 3 (1810-1844) and increases to reach a peak that exceeds the long-run rate in period 6 (1915-1949). In periods 7 and 8 (1950-2000), the growth of technology slows down to hit a low of 1.6% a year. Subsequently, technological change moves up towards its long-run value of 2%. As displayed in figure 8d, the evolution of the child-cost parameter has a clear U-shaped profile. Until period 5 (1880-1914) there is a marked reduction of the child-cost. After that, sharp increases set the child-cost component well above its historical level by period 8 (1985-2020) towards its long-run value.

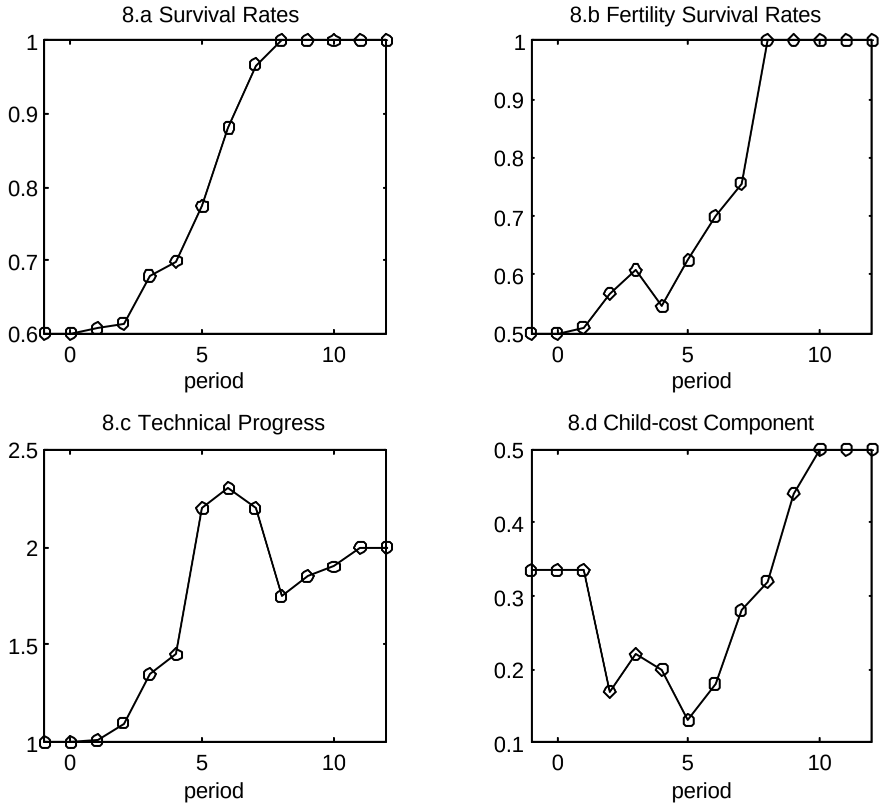

Figure 8. Exogenous benchmark parameters. (8a) π, (8b) γ, (8c) x, and (8d) η_b.

Figure 9 below represents the paths for the key endogenous variables of the model that are implied by the calibration considered, along with their targeted counterparts. In figure 9a capital per efficient unit of labor falls sharply up to period 6 (1915-1949). This decline is particularly dramatic in period 2 (1775-1809). Then it rises moderately towards its long-run value. The simulated paths for g and the NRI in figures 9b and 9c closely match their targets. The main feature of the path for the NRI is a marked hump shape with a peak in period 4 (1845-1879). The growth rate of productivity exhibits a similar pattern, the peak occurring in period 7 (1950-84). The match for productivity growth is far from perfect, thus reflecting the tension in the calibration mentioned above.

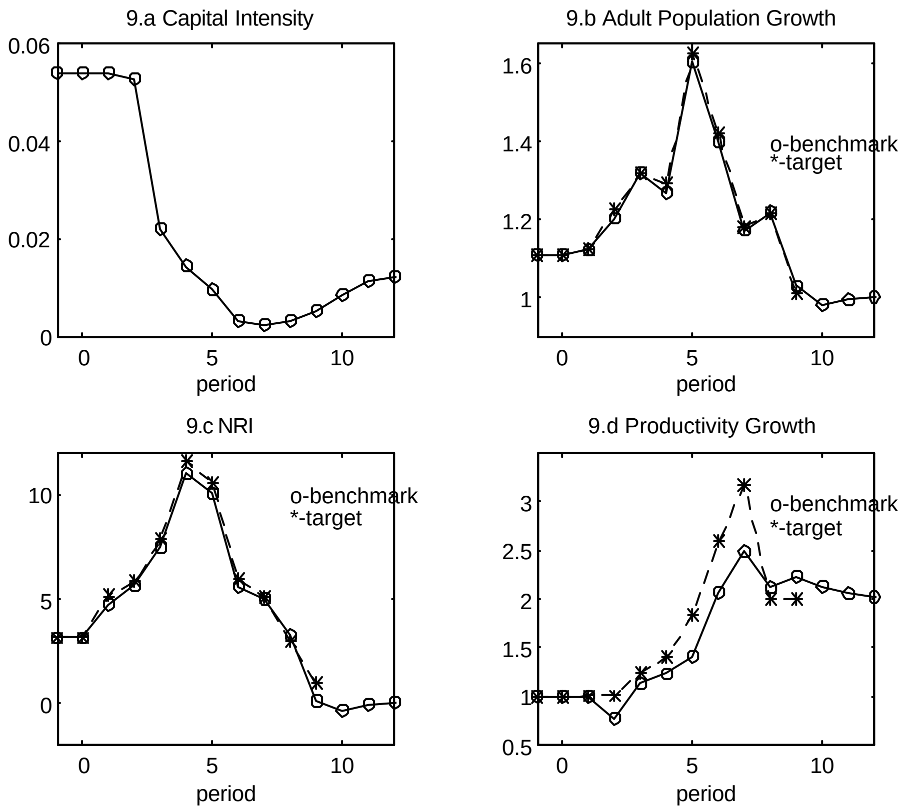

Figure 9. Endogenous benchmark paths. (9a) $\hat{k}$, (9b) g, (9c) NRI, and (9c) productivity growth. The values indicated with * correspond to the benchmark calibration targets.

6 Numerical Experiments

This section explores the importance of the three exogenous factors considered in this model— mortality, technological change, and child-costs— for the demographic transitions that European countries have undergone. In the first part, I will focus on the Swedish case as characterized by the benchmark economy calibrated in the previous section. The exercise considers deviations from the benchmark parametric paths in order to evaluate how the trajectories for the NRI and productivity growth are modified. The second exercise is similar but concerned with comparisons across national experiences. I will modify the benchmark economy using data on vital statistics for England and France and information of their productivity growth performances. This exercise will serve to assess how much of the differences in NRI's relative to Sweden must be attributed to these two factors and how much to differences in the evolution of the child-cost component.

6.1 Comparative dynamics

In this section, I will replace, one at a time, the benchmark trajectories for x, η_b, π and γ by smoothed paths that are still consistent with the benchmark initial and final steady-states. I consider smooth paths that obtain by applying constant 35-year rates of increase of 0.09, 0.05, 0.07 and 0.09 to x, η_1, π and γ respectively between period 0 and period 8. The comparison of the outcomes with the ones in the benchmark case will provide an indication of how important the particular features of the calibrated paths are for the evolution of the NRI and productivity growth.

Figure 10a shows that in the benchmark setup technical change has been particularly fast since 1880. Figure 10b suggests that the accelerating path for technological productivity has helped keep population rises under check. With a more uniform path for the growth of technical change, fertility would have remained well above the benchmark values since the late 19th century. Naturally, figure 10c shows that the fast pace of technological progress goes to explain part the observed high productivity-growth performance, but not all. Accelerated productivity growth since the late 19th century is still a pervasive feature of the transition even under the smooth path for technical change. In any case, the basic pattern characteristic of the benchmark trajectories for the NRI and productivity growth seems to be robust to variations in the evolution of technical change.

In figure 11a, the benchmark path for the child-cost parameter differs dramatically from its smoothed version. Figure 11b reveals the paramount role of the pattern for the child-cost parameter. The non-monotonic U-shaped profile assumed in the benchmark case is strictly necessary to produce the hump-shaped profile for the NRI. Assuming that the cost of children has risen gradually at an uniform pace implies a trajectory for the NRI that is counterfactual even in its qualitative features. As shown in figure 11c, the sharp early declines in the child-cost component contributed to moderate the rate of productivity growth during the period before 1880. Conversely, the large increases in the child-cost since the late 1800's positively contributed to the acceleration of productivity growth observed since.

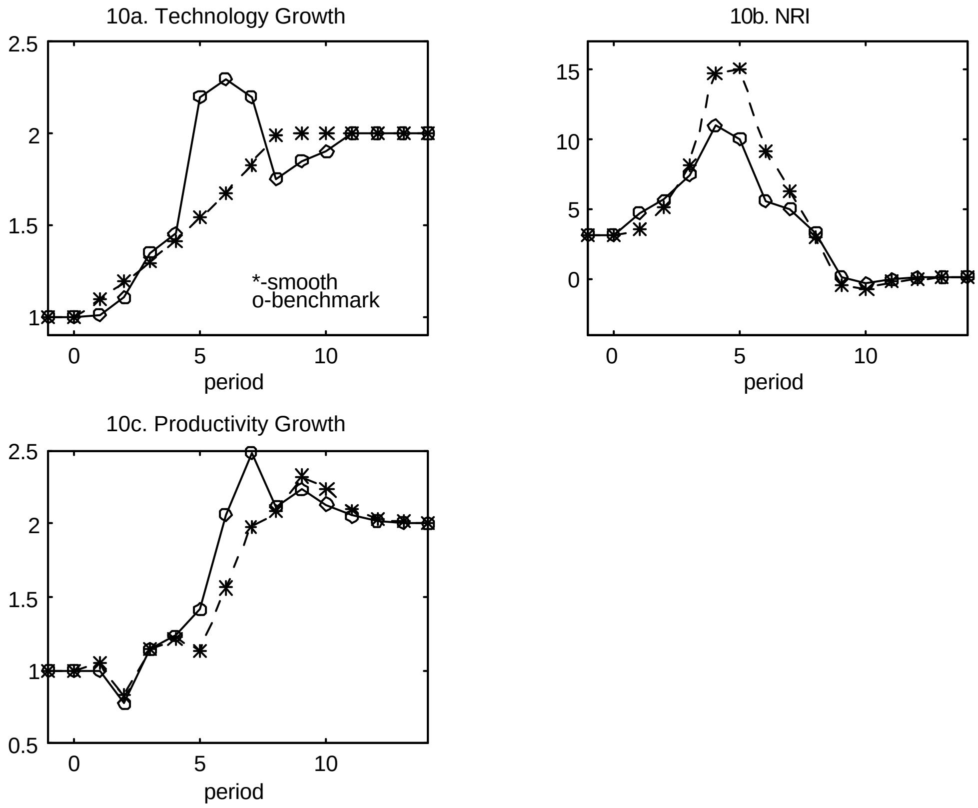

Figure 10. Comparison of benchmark paths (o) with the paths produced by an alternative path for the rate of technical change x (*). (10a) x, (10b) NRI, and (10c) productivity growth.

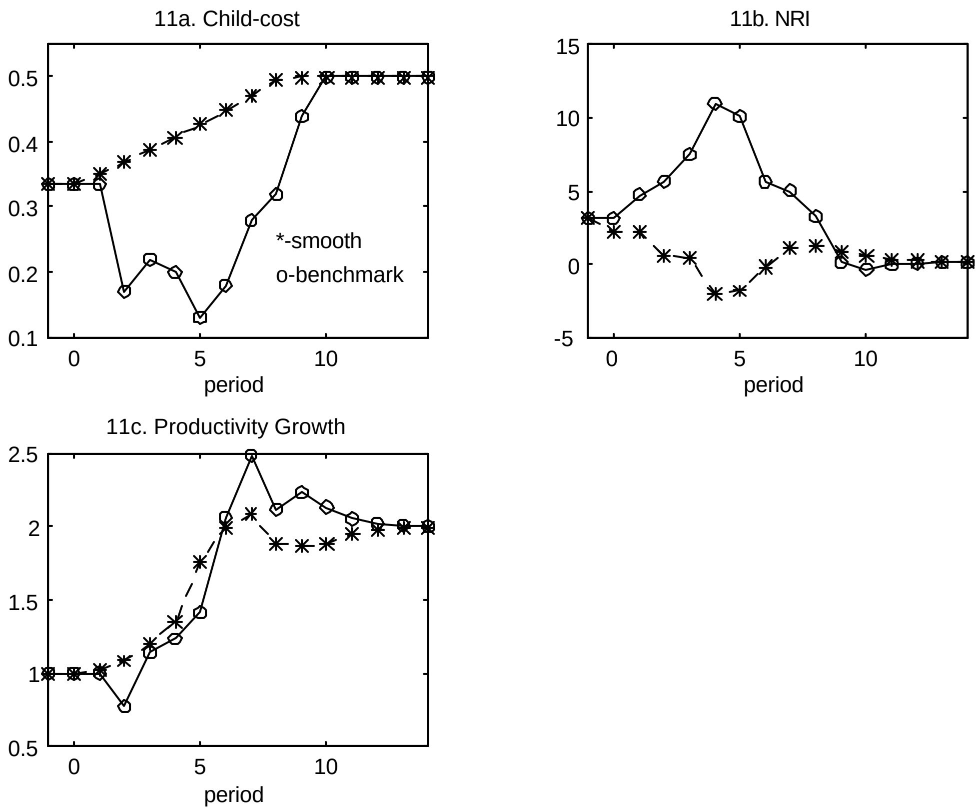

Figure 11. Comparison of benchmark paths (o) with the paths produced by an alternative path

for the child-cost parameter η_b (*). (11a) η_b, (11b) NRI, and (11c) productivity growth.

Concerning survival probabilities, figure 12a shows that the benchmark calibration implies a relatively slow improvement over the first few periods of the transition, followed by relatively fast increases afterwards. The survival rate remains, however, relatively low throughout. This also applies to the comparison of the benchmark trajectory for γ and its smoothed counterpart shown in figure 13a. As displayed in figures 12b and 13b, the specific pattern of mortality does have a bearing on the behavior of population growth. The smooth paths for survival probabilities lead to relatively higher NRI throughout the central periods of the transition. Quantitatively, the effect of γ appears to be smaller than that of π. The effect on population growth of changes in path for mortality goes beyond the direct impact of mortality itself. This is shown in figure 12d that compares the path of planned fertility n in the benchmark calibration to the one obtained under the smooth path for survival rates. Lower mortality rates increase net planned fertility. Finally, figures 12c and 13c reveal that the economic consequences of the pattern of mortality changes over the transition are minor.

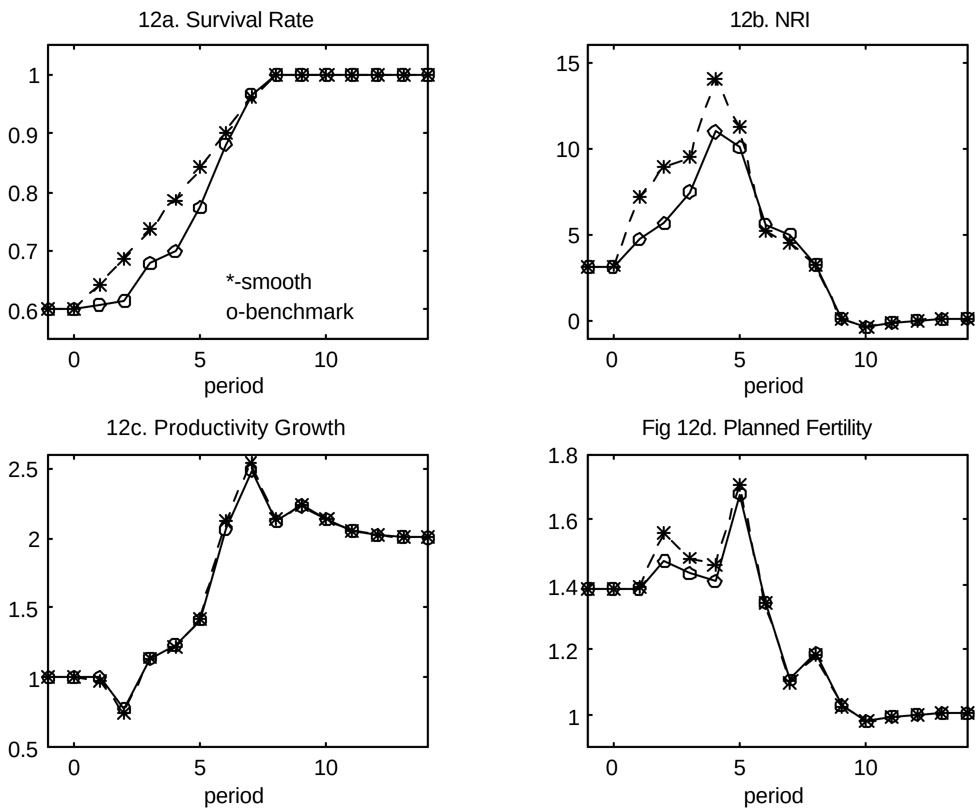

Figure 12. Comparison of benchmark paths (o) with the paths produced by an alternative path for the survival rate π (*). (12a) π, (12b) NRI, (12c) productivity growth, and (12d) n.

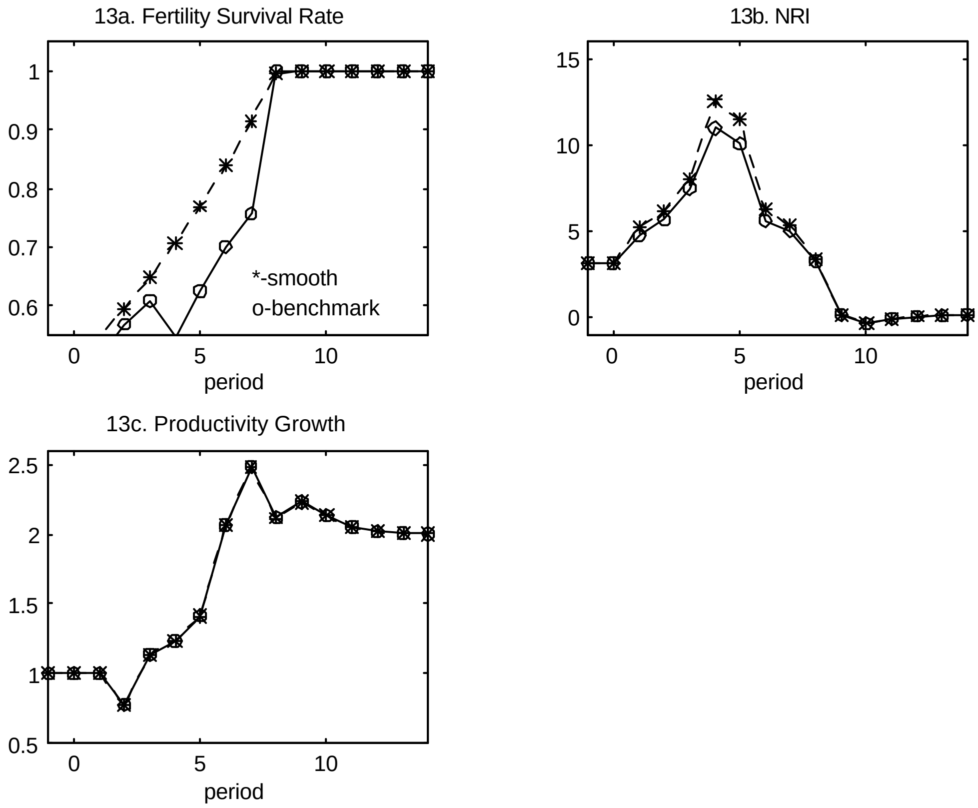

Figure 13. Comparison of benchmark paths (o) with the paths produced by an alternative path for the conditional survival rate γ (*). (13a) γ, (13b) NRI, and (13c) productivity growth.

The main insights from preceding analysis can be summarized as follows. The existence of a burst of population growth cannot be explained in the absence of sharp declines in the child-cost component over the first periods of the transition. In effect, departures from the benchmark U-shaped child-cost path lead to counterfactual implications for the qualitative profile of the NRI. This is because the contribution to population growth of the rising rate of technical exchange is negative. Given that the effect on the NRI of observed mortality declines does not prove powerful enough, the size of reductions in the child-cost must have been large at least until about 1880. Subsequently, rising costs have decisively contributed to the falls in population growth and, to some extent, to the acceleration of productivity growth through capital accumulation. In any case, mortality declines tend to boost the NRI and, hence, their contribution to the observed size of early population rises must have been positive. More rapid drops in mortality than observed would have delivered higher NRI's over the transition.

6.2 Comparing national experiences

Using data on vital statistics for England and France, I modify the benchmark economy to identify the contribution of differences in mortality paths to the different behavior of the NRI in these countries relative to Sweden. After informally accounting for technology disparities, the remaining differences will have to be attributed to dissimilar trajectories for the child-costs. Figure 14 below displays the summary data on NRI's for Sweden, England and France contained in table 1. According to the convention adopted (see table 5), the first observation corresponds to period 1 in the model.

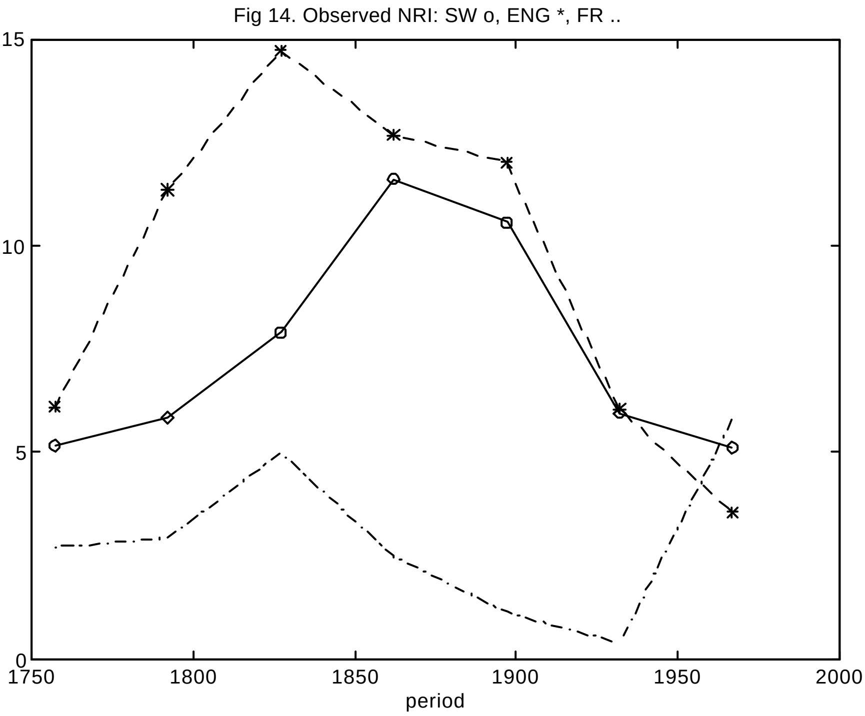

Figure 14. Historical NRI for Sweden, England, and France. The figures represented are 35-year averages contained in Table 1.

6.2.1 The English Case

Here the focus is on interpreting the faster and larger burst in population growth in England relative to Sweden. The following figure 15 compares the survival probabilities of the benchmark economy (Sweden) and the English economy. The figures from England are taken to be those shown in table 2 above, with interpolated values for the missing observations in period 3. These interpolated values are 0.675 and 0.630 for π and γ respectively. It is apparent that, relative to Sweden, England features faster early rises for about two periods, an interruption for two further periods, and a decided and lasting upward trend afterwads.

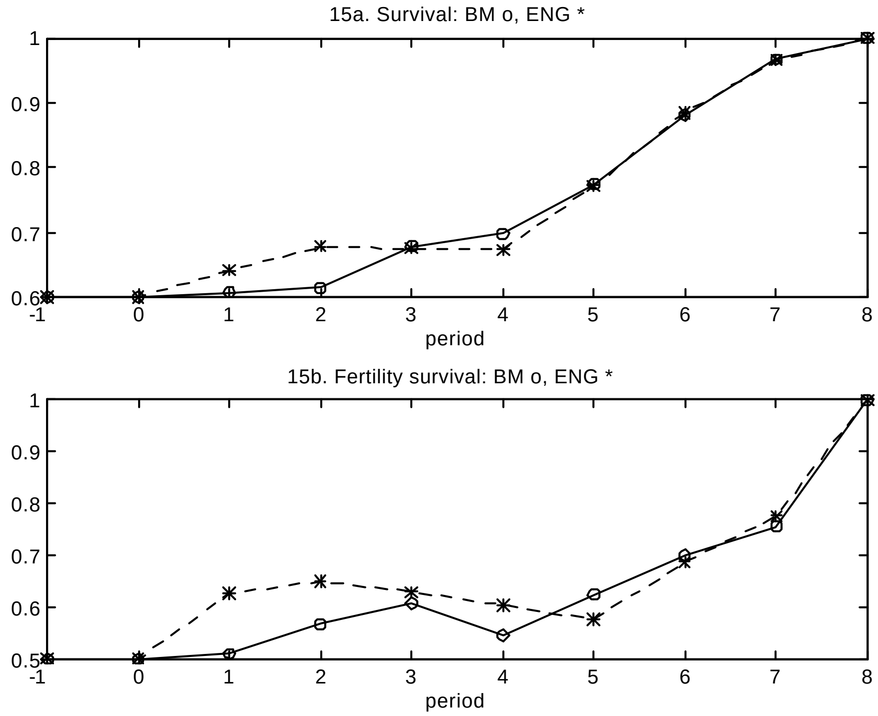

Figure 15. Survival rates in England (*) and the Swedish benchmark (o).

(15a) π, and (15b) γ.

Using these English vital statistics in the otherwise benchmark model produces the outcomes shown in figure 16. The NRI associated with England stays higher for the first two periods of the transition. Subsequently, the Swedish and English trajectories evolve very closely. This result is not very surprising given that differences in mortality rates are most visible during periods 1 and 2. The direction of the effect responds to the model's implications of positive effects of higher survival on the NRI. The naked-eye comparison of the figures for the English NRI in figure 14 and those reproduced in figure 16 show a rough resemblance for the period 1740-1809. After that, however, the benchmark economy modified with English mortality indicators fails to predict the persistence of NRI's above those observed in Sweden in figure 14. Thus the complete explanation of the English demographic transition after 1809 requires to resort to unobservable differences in the child-cost component and/or rates of technical change.

I begin with the former. The failure to predict the English NRI's in period 3 and 4 suggests that η_b should be reduced in either or both of these periods. Indeed, reducing

η_{b3} and η_{b4} from 0.22 and 0.20 to 0.17 and 0.13, respectively, produces a close fit to the English experience. This choice of η_{bt} is shown against the benchmark path in figure 17. The corresponding path for the NRI is depicted along the benchmark counterpart in figure 18 below. Note, incidentally, that increasing the child-cost for the English economy tends to moderate the excessive jump of the NRI in period 1 observed in figure 16, thus improving the quantitative matching to the observations in figure 14.

This story is also consistent with the conclusion reached by authors that the initial rises in reproduction rates in Sweden where mainly driven by mortality reductions rather than increases in fertility behavior. In England, instead, the population explosion was fueled by large rises in fertility. This point is made in figure 19 below that depicts the benchmark economy's planned fertility n against the one produced for the English parameters in figures 15 and 17.

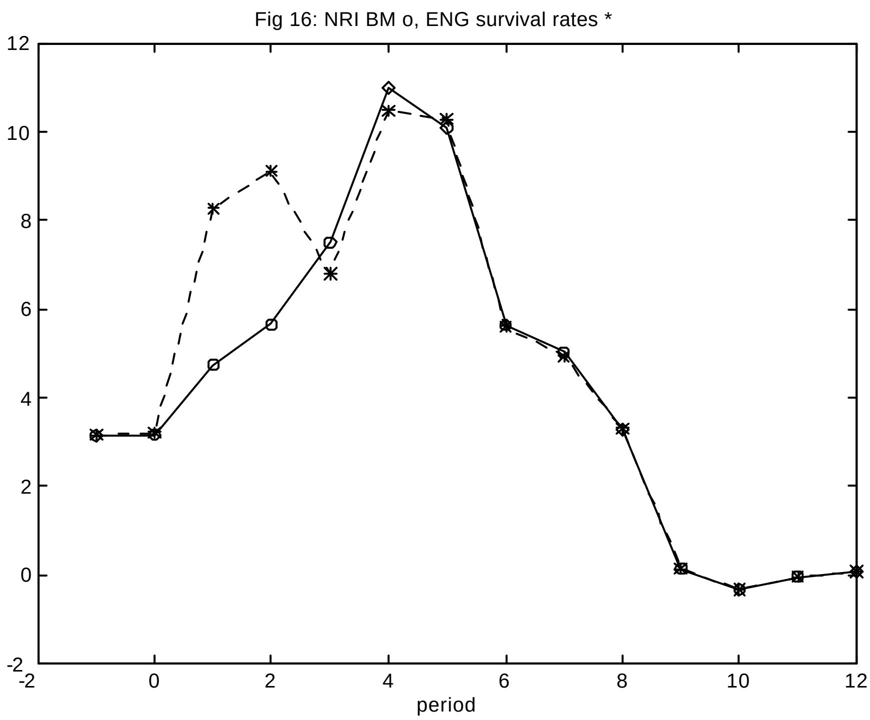

Figure 16. The NRI implied by English survival rates (*) against the Swedish benchmark (o).

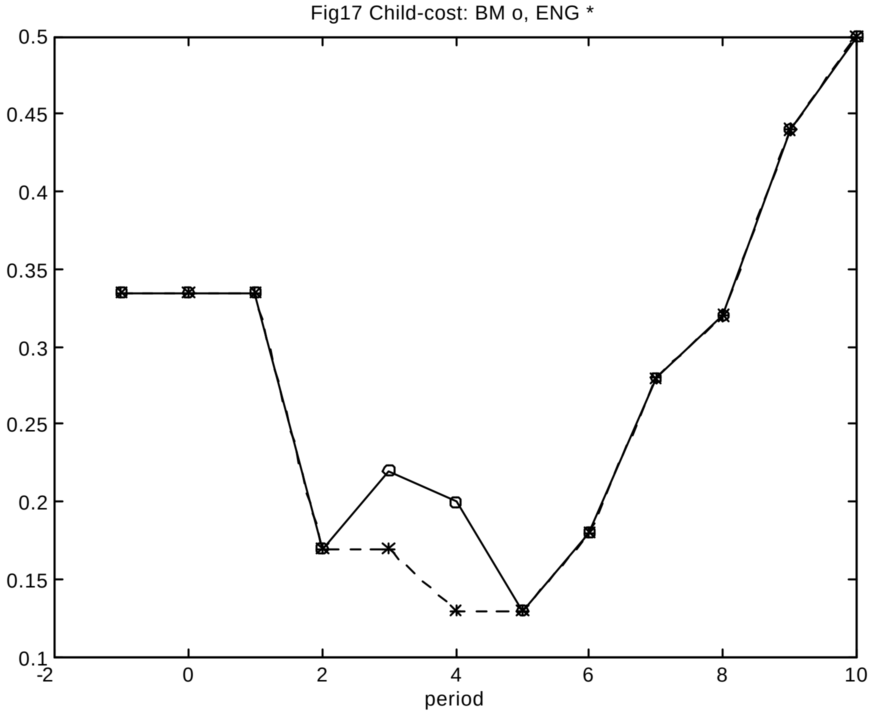

Figure 17. The child-cost η_b for England (*) against the Swedish benchmark (o).

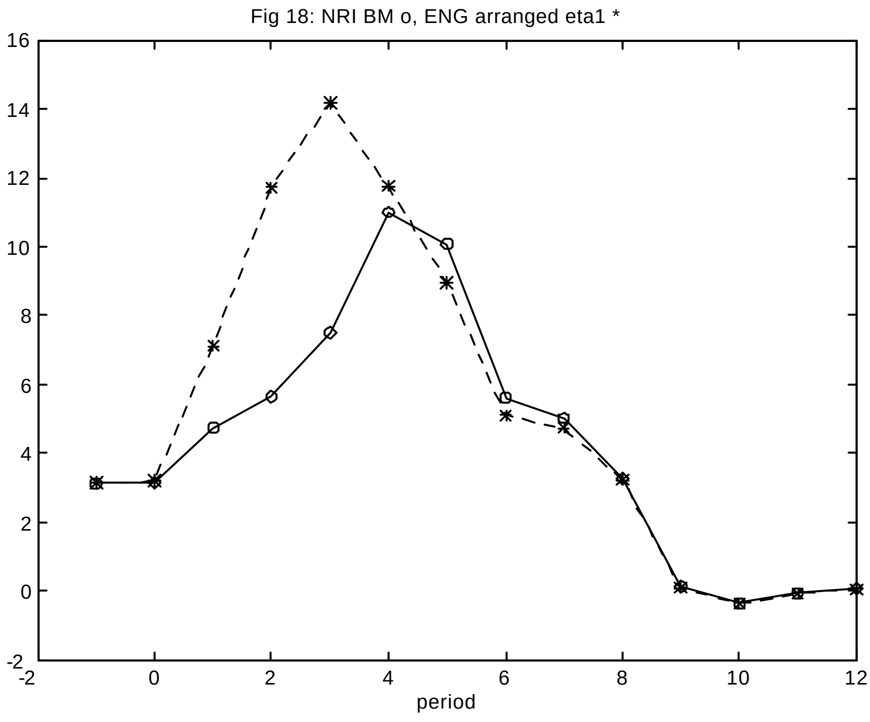

Figure 18. The NRI implied by English survival rates and child-costs (*) against the Swedish benchmark (o).

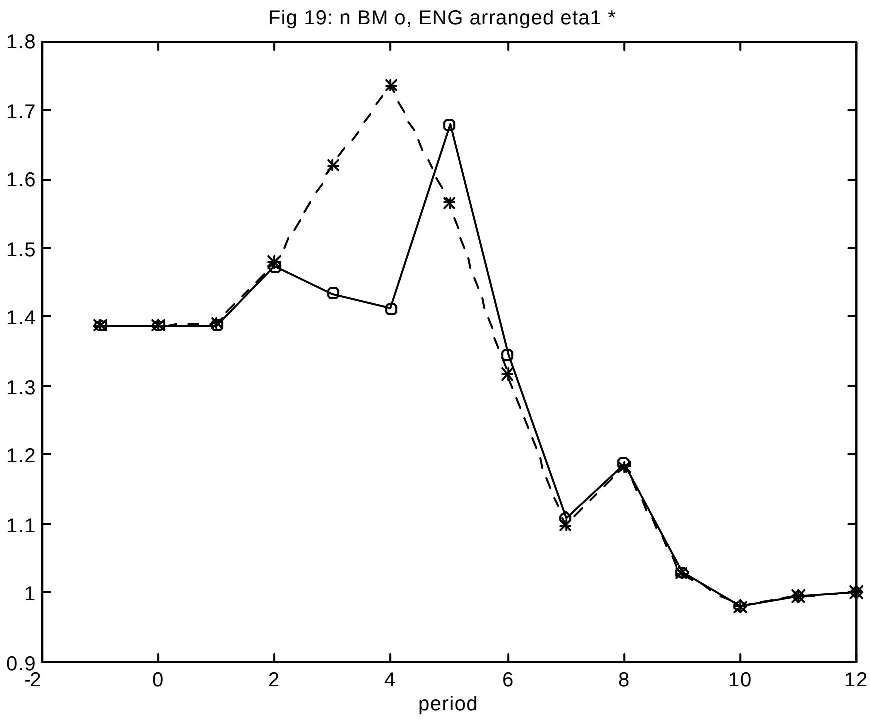

Figure 19. Planned fertility n implied by English survival rates and child-costs (*) against the Swedish benchmark (o).

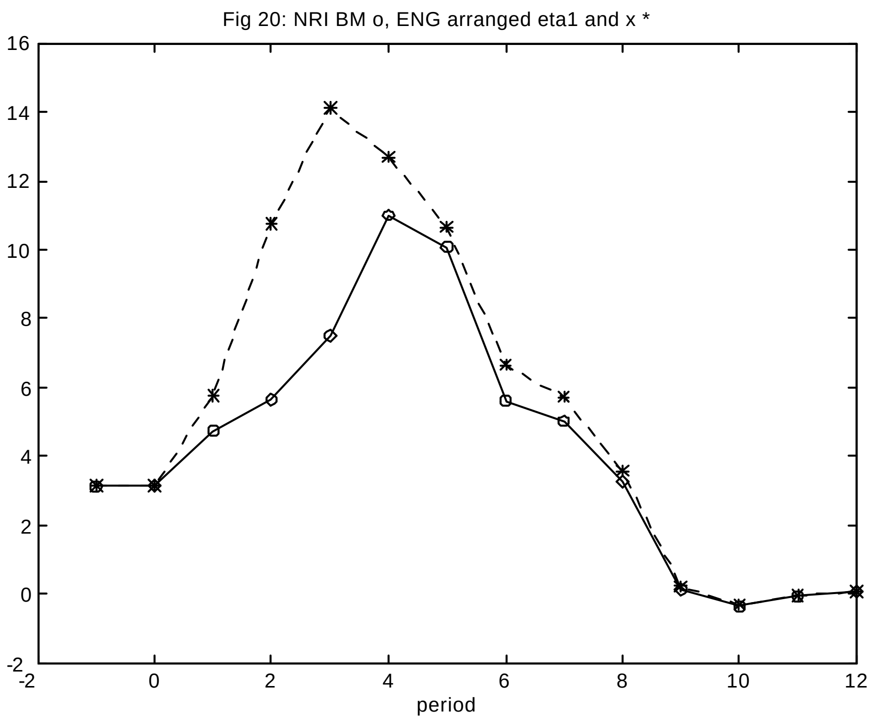

Figure 20. The NRI implied by English survival rates, child-cost, and productivity growth (*) against the Swedish benchmark (o).

Finally, a quantitatively better match to the English evidence can be achieved after some well justified adjustments in the path of technical change. Productivity growth in

England after period 4 (1870) became slower than in Sweden, whereas it had been faster in the preceding period. This suggests to fine-tune the rate of technological change as follows: raise x_1 and x_2 from 1.011 and 1.1 to 1.1 and 1.16 respectively; lower x_5 and x_6 from 2.2 and 2.3 respectively to 2.0. Doing so leads to a NRI for England that is above the Swedish one throughout, thus producing a close match of the English series in figure 14. This is displayed in figure 20.

To sum up, earlier and faster drops of mortality in England explain part of its initial more rapid NRI's relative to Sweden. But this alone cannot account for the persistence and size of the differences. A complete explanation requires to add that the technology for children became about 0.17/0.22 or 0.13/0.20 (23% or 35%) cheaper in England than in Sweden during the period 1810-1879 (periods 3 and 4). In addition, a pattern of technology growth that is consistent with differences in productivity growth between the two countries completes the comparative interpretation of the demographic transition in England.

6.2.2 The French Case

It is said that France did not undergo the standard pattern of demographic transition observed elsewhere. As documented is section 2 and figure 14, the NRI remained within a relatively narrow band throughout.

As in the previous section, I will use information on French vital statistics to have a sense of its importance to explain the particular behavior of the NRI in France. As it transpires from table 2, the data on survival probabilities for France is incomplete. For periods 1 through 3 I will have to make an educated guess. There are clear indications—like the data represented in figures 6 and 7 for CDR's and infant survival rates—that, in the early periods, France was lagging behind its neighbors. Let us then suppose that France had a path for survival probabilities π that remained below the benchmark ones between periods 1 and 3. Tentatively I set $\pi_1 = 0.6$, $\pi_2 = 0.605$ and $\pi_3 = 0.621$ instead of the benchmark 0.607, 0.0614, and 0.678, respectively. For lack of better information, I set γ as in the benchmark economy. For periods 4 through 7, table 2 provides information directly. The rest of parameters are as in the benchmark economy. Figure 21 below depicts the choice of π and γ for France and, for comparison purposes, the benchmark Swedish case.

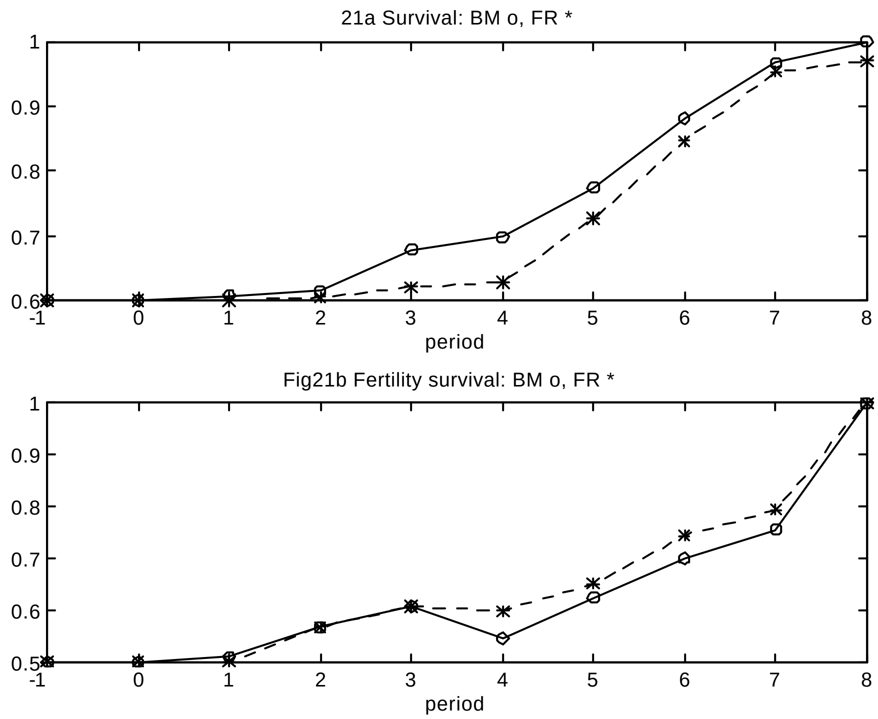

Figure 21. Survival rates in France (*) and the Swedish benchmark (o).

(21a) π, and (21b) γ.

The implications of the pattern of mortality declines assumed for France is illustrated in figure 22. Between periods 1 and 4, the NRI is lower in France. Low survival rates in France over this period certainly contributed to moderate population growth. However, although somehow delayed, the pattern under French mortality conditions still shows a counterfactual surge of population growth in periods 4 and 5. In the light of section 2, the assumption that technology growth in France is similar to that in Sweden appears a reasonable one. Therefore, I will concentrate now on finding the differences in the child-cost relative to the Swedish benchmark that are required to explain the French experience.

Figure 23 depicts the required path for the child-costs that matches the French experience under the local mortality conditions. Compared with the benchmark path also displayed, France must have experienced less abrupt initial reductions and, thereby, a smoother profile. Figure 24 represents the implied path for the NRI which accurately matches the corresponding French observations represented in figure 14 above.

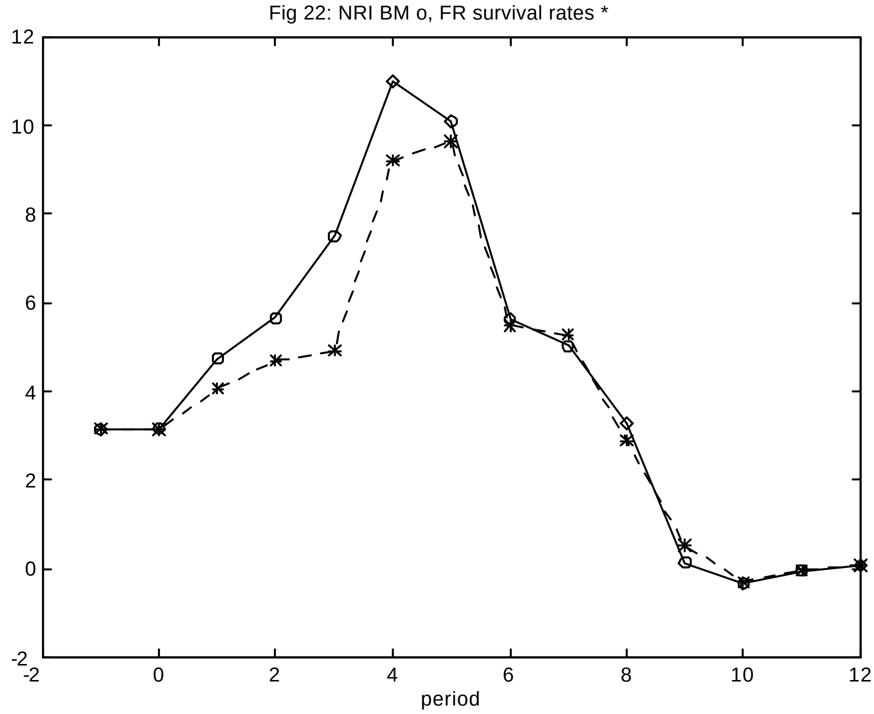

Figure 22. The NRI implied by French survival rates (*) against the Swedish benchmark (o).

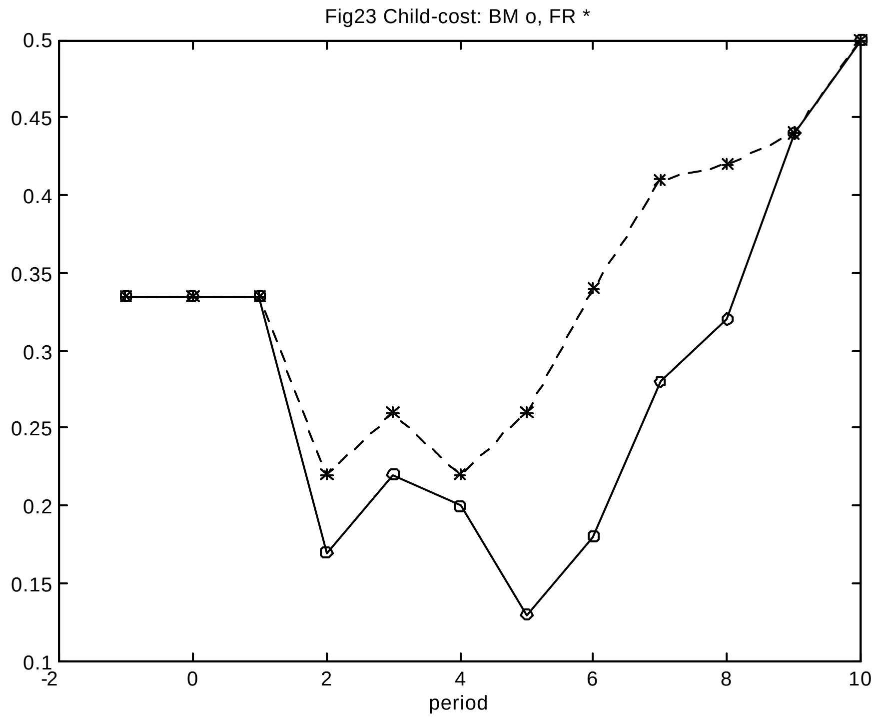

Figure 23. The child-cost η_b for France (*) against the Swedish benchmark (o).

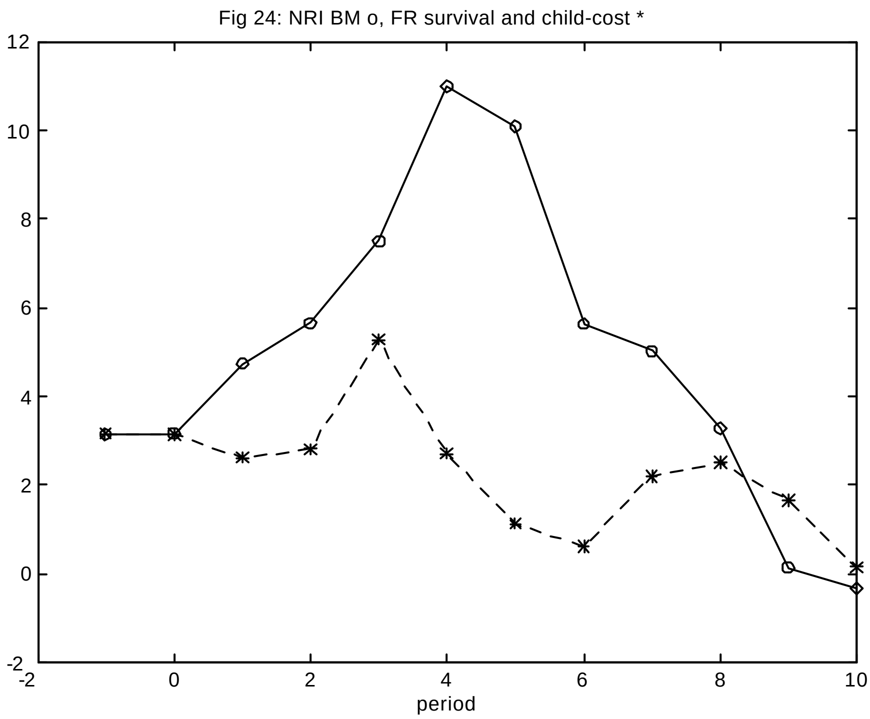

Figure 24. The NRI implied by French survival rates and child-costs (*) against the Swedish benchmark (o).

To sum up, higher mortality contributed to the comparatively lower NRI in France over periods 1 and 2 (1740-1809). However under the benchmark conditions for the child-cost component, the model still produces NRI's that are much too high and shifting relative to the French experience. The explanation of the smoother evolution of the NRI in France requires to bring in a path for the child-cost that shows less abrupt changes than the benchmark one and remains above it throughout.

6.3 Discussion

The exercises in section 6.1 and 6.2 can be regarded as the basis for the interpretation of the demographic-economic transformations under study. This tentative interpretation revolves around three elements: mortality, technological change, and child-costs. Whereas mortality and, to some extent, technological progress can be related to observed variables, the model's child-cost component cannot. Therefore, the contribution of the child-cost to the interpretation of the facts under study is a measure of the importance of unknown factors.

The findings reported here suggest that the importance of those unknown factors may be significant. The pattern of rapid drops at the beginning and subsequent rises appears to have a primary role in shaping the typical hump-shaped trajectory for the NRI. Besides, the late increases appear to account for part of the high rates of productivity growth observed in the more recent times. Concerning the comparative analysis, differences in those factors must have accounted for some of the disparities in NRI between Sweden and England over the period 1810-1879, and for differences between Sweden and France over a much longer period. There is certainly a role for mortality and technology. The point is that the explanation provided would miss some prominent features of the data without the presence of the special patterns for the child-cost parameter found in the analysis, both over time and across countries.

But since the model's child-cost component does not have a clear empirical counterpart, it becomes difficult to judge whether the paths for this parameter implied by the analysis make empirical sense. This is a question that must be confronted if one wants to evaluate the quantitative relevance of the neoclassical model with dynastic preferences for the study of the demographic transition. This calls for investigating versions of the model where the cost of children can be related to meaningful observable factors. A formulation that reduces the importance of unobservable factors will be a step towards the success of the neoclassical model with dynastic preferences. If such an endeavor proves elusive (after all, the current framework might not a be a useful abstraction for the questions at hand) then perhaps an alternative framework of analysis should be considered. The existing literature has done some, while partial, progress in those directions. In the rest of this section, I want to highlight the relation of this literature with the present paper.

An important set of contributions includes Becker et al. (1990), Ehrlich and Lui (1991), Galor and Weil (1996), Dahan and Tsiddon (1998) and Galor and Weil (2000). These are works where human capital and distribution considerations are placed at the centre. They derive the child-cost from primitive assumptions. For example, in Becker, Murphy and Tamura (1990) and Ehrlich and Lui (1991) it is the cost of time that rises as human capital accumulates. In Galor and Weil (1996) gender-gap model, technological assumption are laid out such that women's wages rise to a larger proportion than households income. In Dahan and Tsiddon (1998) the fraction of low-fertility skilled people increases over the transition. However, the ability of these models to quantitatively match the observations has not been explored yet. On the other hand, these papers show that departures from the neoclassical growth model and/or dynastic preferences may have a profound effect for the predictions of the theory [for example, Ehrlich and Lui (1991)]. Hence they may well have implications for the cost of children that differ for the ones found here both quantitatively and, perhaps, qualitatively.

On the other hand, the works that build on the neoclassical model happen to share a quantitative approach. Moe (1998) introduces human capital and non-market activities in a model similar to the present one. She applies that model to data for a developing country. Hansen and Prescott (1998) study the English historical demographic transition and the shift from the pre-industrial regime to the modern mode of production. In their model, fertility and population follows a pre-specified functional form though.

Fernandez-Villaverde (1999) is closest to the present analysis. That paper studies the English demographic transition. In a 4-period lifetimes version of the present framework, the author introduces human capital and skill-biased technological change. Changes in this form of technical change then induce changes in relative prices that can be associated with changes in the cost of children. As in the present paper, the demographic transition requires changes in those relative prices. Unlike the present paper, his quantitative analysis refers to the steady-state, whereas only the qualitative features of the transitions since the late 19th century are studied. In contrast, in the present paper the focus in on the quantitative properties of the transition within and across countries since the mid 18th century. Fernandez-Villaverde (1999) claims some success in explaining the English demographic transition appealing to growth in skill-biased productivity. Two remarks are in order. First, the emphasis of that paper is on the late stages of the transition that feature secular declining fertility and population growth. Then, just as in the present analysis, technical change discourages fertility. In addition, the skill-biased nature of this change amplifies the effect and helps get the net numbers right. Thus perhaps the surge of the child-cost that I find since the late 19th century could just be interpreted as the effect of the skill-bias. However, this explanation does not carry over to explain the experience of the preceding period starting in the mid 18th century. I have documented the large rises in CBR's in Britain during most of that period. Skill-biased technical progress alone is inconsistent with those observations and, relative to the present paper, would require even larger drops in other unknown components of the child-cost.

The second remark concerns the design of the experiments. In Fernandez-Villaverde

(1999) technical change is considered to occur in isolation, with constant mortality rates throughout. The present paper shows that the rapid increases in survival probabilities since the late 19th century create a pressure towards higher fertility. In my model, the rapid increases in labor-augmenting technical change are not enough to exactly match the observed declines over the period. It remains to be seen if, in the presence of realistic changes in mortality, Fernandez-Villaverde (1999) would require to assume an unrealistically faster rate of skill-biased technical change or, equivalently, a greater role for increases in the unobservable child-cost component.

Eckstein et al. (1999) focus on explaining the fertility transition in Sweden based on the long (1751-1990) time-series of mortality and wages in a life-cycle model. They found that in the estimated model, wages and mortality alone accurately explain the evolution of total fertility. Aside from the fact that theirs is not an equilibrium model nor does have capital accumulation choices, the main difference with the present analysis is the absence of altruism in preferences. My result that technology change (that drives wages) and mortality are not enough suggests that the specification of preferences may be a critical aspect. Also the success of Eckstein et al. (1999) could be partly attributed to the replacement-effect of infant mortality changes on total fertility. The present paper focuses on the NRI rather than total fertility and, therefore, the influence of infant mortality changes is modest or nil (see footnote 2).

I want to finish this discussion with a few remarks of caution regarding the present paper. One concern is the accuracy in the measurement of observations and selection of targets that have been used in the calibration analysis. For some parameters, there are simply no observations available and their values have had to be approximated using the best information available to me. Even for the figures documented in the literature, considerable dispute remains among historical demographers.

In another respect, the exact matching of the targeted paths for productivity growth has proved elusive. Thus the results cannot be evaluated in terms of the ability to precisely match this observation. The compromise adopted here has been to deal with productivity changes more informally. In particular it has been assumed to be the same in Sweden and France; for England, it has been shifted somehow arbitrarily in view of the observed productivity growth evidence. But this evidence itself has to be approached with reservations.

7 Conclusion and final remarks

In this paper the neoclassical model of growth with dynastic preferences is analyzed quantitatively to identify the role of different factors in shaping the demographic transition since the mid 18th century in a set of representative European countries. The factors considered are the evolution of mortality, technical change, and the costs of rearing children.

A benchmark is selected such that, given the historical path of mortality rates in Sweden, the model matches the low frequency time series for the natural rate of increase and productivity growth in that country. It emerges that the cost of children must have first decreased between 1740 and 1914 to, subsequently, rise towards current values. To explain the relatively larger burst of population growth in England until about 1879, the amplitude of the fall in the child-cost component must have been larger there between 1810-1879. The relatively mild shifts of the NRI observed in France suggest that the child-cost must have remained relatively high and stable there throughout the period. The model also identifies a role for the observed differences in mortality changes in explaining part of the faster initial population boom in England during 1740-1809 and the more moderate pattern in France. Differences in mortality rates appear to have tiny effects on productivity growth though.

This paper demonstrates that an understanding of the determinants of the cost of children may be an important ingredient in the explanation of the fertility transition and the different patterns observed across countries. The explanatory power of the model relies largely on the existence of child-costs differences —both across countries and over time— in precisely the direction and order of magnitude found in the analysis. By assumption, though, this paper does not provide an explanation of the factors that drive the behavior of the child-cost parameter and, therefore, cannot assess whether the quantitative implications find empirical support. This paper, however, may be a useful guide in the search for theories that endogeneize the cost of children. It will be down to those theories to relate their fundamental parameters to observations and test their explanatory power.

This paper builds on a stylized model and is thus bound to have limitations. In order to focus on the effects of mortality and allow for the computation of the transition, a particularly simple form for the child-rearing technology has been assumed. Another assumption is the existence of markets providing full life-insurance. While simplifying the treatment of uncertainty, this view minimizes the economic impact of mortality rates by ruling out the direct effect of survival on the expected returns to investment. The demographic structure of this paper is very simple. A natural extension of this work is to consider a richer more realistic setup. Finally, the analysis has considered the framework of the neoclassical exogenous-growth model with dynastic preferences. The literature shows that alternative specifications of technology and/or preferences may provide useful insights. Future research should pursue these leads with the systematic quantitative approach adopted in this paper.

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A Data Appendix

- Figure 1: CBR and CDR for England. Period 1541-1849 from Wrigley and Schofield (1981,TabA3.3). Period 1850-1984 from official registrations as reported in Chesnais (1992).
- Figure 2: CBR and CDR for Sweden 1740-1984. Official registrations in Chesnais (1992,TabA1.1-A1.7).
- Figure 3: CBR and CDR for England 1740-1984. Period 1740-1849, Wrigley and Schofield (1981,TabA3.3). Period 1850-1984, official registrations in Chesnais (1992,TabA1.3-A1.7).
- Figure 4: CBR and CDR for France 1740-1984. Official registrations in Chesnais (1992,TabA1.1-A1.7).
- Figure 5: NRI calculated from sources as in figures 2,3 and 4.
- Table 1: 35-year annual averages 1740-1984 for NRI. Sources as for figures 5.
- Table 2: from variable l_x (number of surviving to exact age x out of 100,000 born) for $x = 1, 15, 40$ in Keyfit and Flieger (1968) for Sweden, England and France, survival probabilities can then be calculated $\pi(0-1)$, $\pi(0-15)$ and $\pi(0-40)$. From there,

  $$\pi(1-40) = \frac{\pi(0-40)}{\pi(0-1)}.$$

  Similarly,

  $$\gamma \equiv \frac{1-\pi(15-40)}{1-\pi(1-40)} = \left(1 - \frac{\pi(0-40)}{\pi(0-15)}\right) \frac{1}{1-\pi(1-40)}.$$

  The figures are averaged over the periods represented and across sexes.

  For England, this source only provides data from 1861 on. To reconstruct early data, the periods 1740-1779 and 1780-1809 are based on Wrigley et al (1997, Table6.1,p215 and Tab6.19,p290)'s figures for ${}_nq_x$, which is the probability of dying for an individual of exact age x, before reaching age $x+n$. These provide 1q0, 4q1, 5q5, 5q10, 5q25, 5q30, and 5q35, for 1740-9, 1750-9, 1760-9, 1770-9, 1780-9, 1790-9, 1800-9 (child rates go a little farther). But figures for mortality rates between ages 15 and 25 (${}_{10}q_{15}$) are missing. Thus ${}_5q_{15}$ and ${}_5q_{20}$ will be calculated as a proportion of ${}_1q_0$, the proportion being the average factor of proportion observed in periods 1861, 1871, and 1881 in Keyfit and Flieger (1968) life-tables: ${}_5q_{15}/{}_1q_0 = 0.191497919$ and ${}_5q_{20}/{}_1q_0 = 0.24447499$.
- Figure 6: CDR Sweden, England and France 1740-1984. Source as for figures 2, 3 and 4.
- Figure 7: Infant survival England 1580-1837 Wrigley et al (1997), and 1861-1965, Keyfit and Flieger (1968). Infant survival Sweden 1861-1965, Keyfit and Flieger (1868). Infant survival France, Keyfit and Flieger (1868)
- Table 3: from productivity figures from Maddison (1991, Tab C11-C12, p274-76). Also Tables 3.1 and 3.3. For England, figures adjusted for 3 first periods as in Hansen and Prescott (1998).

B Optimal choices

Optimal investment, annuities and uncertain life-times

It is expositionally convenient to introduce at this stage some equilibrium properties of prices in assets markets. This permits to characterize further the solution to the household's problem as well as to simplify the definition of equilibrium below.

There is individual life-time uncertainty but there is no aggregate risk. At the end of $t-1$, a proportion π_{t-1} of members of generation $t-1$ survive, the proportion $\gamma(1-\pi_{t-1})$ dies and leaves children whereas the fraction $(1-\gamma)(1-\pi_{t-1})$ dies and leaves no child at all. Hence there is room for operative competitive annuities markets. Assume that there are insurance firms offering these assets. Free entry drives profits of operating firms to zero. It turns out that these firms offer actuarially fair contracts. At t, the revenues of the firms issuing simple annuities are $R_t N_{t-1} a_t$ and the outlays are $\pi_{t-1} N_{t-1} R_t^a a_t$, where R_t is the rate of return on equity. Book balancing implies then that $R_t = \pi_{t-1} R_t^a$. Likewise, zero profits for firms trading in conditional annuities implies that $[\pi_{t-1} + (1-\pi_{t-1})\gamma] R_t^s = R_t$. Consequently,

$$\frac{R_t^a}{R_t^s} = \frac{\pi_{t-1} + (1-\pi_{t-1})\gamma}{\pi_{t-1}}.$$

We now show that the existence of such market provides an insurance mechanism which greatly simplifies the treatment of uncertainty. An interior solution to the household's investment problem implies two FOC's:

$$\begin{aligned} k_{t+1}^V : \quad & n_t u'(c_t) = \beta n_t^{1-\epsilon} R_{t+1}^a \pi_t u'(c_{t+1}^V) \\ k_{t+1}^A : \quad & n_t \left(\frac{R_t^a}{R_t^s} - 1\right) \pi_{t-1} u'(c_t) = \beta n_t^{1-\epsilon} (1-\pi_{t-1})\gamma R_{t+1}^a \pi_t u'(c_{t+1}^A), \end{aligned}$$

where use has been made of the Envelope Theorem. With fair returns on annuities $c_{t+1}^V = c_{t+1}^A = c_{t+1}$ and $k_{t+1}^V = k_{t+1}^A = k_{t+1}$, where the second statement follows from a standard monotonicity argument. This leads to Eq.(12) in the text.

Fertility

Optimal choice on n_t implies the condition,

$$(A_t \eta_t + k_{t+1}) u'(c_t) = \beta(1-\epsilon) n_t^{-1} n_t^{1-\epsilon} V_t(k_{t+1}).$$

The planned number of descendants is chosen so as to balance the cost of an additional descendant in terms of parent's utility (LHS) and the marginal utility (RHS).

Use the intergenerational optimality condition above to substitute $u'(c_t)$ in order to express the LHS of the FOC for fertility in terms of generation-t utility,

$$(\eta_t + k_{t+1}) R_{t+1} u'(c_{t+1}) = (1-\epsilon) V_t(k_{t+1}).$$

Now replace the term $k_{t+1} R_{t+1}$ through the budget constraint and expand the RHS,

$$\begin{gathered} [R_{t+1}\eta_t \pi_t (c_{t+1} - w_{t+1})] u'(c_{t+1}) + \\ [n_{t+1}(\pi_t + (1-\pi_t)\gamma)(\eta_{t+1} + k_{t+2})] u'(c_{t+1}) = \\ (1-\epsilon)\left[\pi_t u(c_{t+1}) + \beta n_{t+1}^{1-\epsilon}(\pi_t + (1-\pi_t)\gamma) V_{t+1}\right]. \end{gathered}$$

Use again the FOC for fertility one period ahead to write,

$$\begin{gathered} [R_{t+1}\eta_t + \pi_t (c_{t+1} - w_{t+1})] u'(c_{t+1}) + \\ (\pi_t + (1-\pi_t)\gamma)\beta(1-\epsilon)(n_{t+1})^{1-\epsilon} V_{t+1} = \\ (1-\epsilon)\pi_t u(c_{t+1}) + \\ (1-\epsilon)\beta n_{t+1}^{1-\epsilon}(\pi_t + (1-\pi_t)\gamma) V_{t+1}, \end{gathered}$$

the last terms in both sides cancel so that,

$$[R_{t+1} A_t \eta_t + \pi_t (c_{t+1} - w_{t+1})] u'(c_{t+1}) = (1-\epsilon)\pi_t u(c_{t+1}).$$

The LHS of this equation represents the marginal cost of n_t in terms of utility of adults at $t+1$. The first term is the direct outlay on the additional child conveniently capitalized at the interest rate. The second component is the reduction in net consumption $c_{t+1} - w_{t+1}$ due to the lower share of each descendant on the bequests adjusted by the probability that the additional child survives. The RHS is the marginal dynastic utility of n_t which consists of the gain in adulthood utility at $t+1$. The terms in both sides capturing effects relative to cohorts older than the $t+1$th cancel out. Using the specification of $u(.)$, it can be rearranged to yield Eq.(10) in the text.

C Bounded utility value

Lemma B1. *At the steady-state it must necessarily hold that* $g(1+x) < 1 - \delta + r$.

Proof: In a steady-state, dynastic utility can be written in extended form as,

$$V = \pi c^{1-\sigma} \sum_{t=0}^{\infty} \left[\beta n^{(1-\epsilon)} (\pi + \gamma(1-\pi))(1+x)^{1-\sigma} \right]^t .$$

This sum converges if $\beta n^{1-\epsilon}(\pi + \gamma(1-\pi))(1+x)^{1-\sigma} < 1$. This can be rewritten as,

$$\beta g \left(\frac{\pi + \gamma(1-\pi)}{g} \right)^{\epsilon} (1+x)^{1-\sigma} = \beta g \left((\beta(1-\delta+r))^{-1/\epsilon} (1+x)^{(-\sigma)/\epsilon} \right)^{1/\epsilon} (1+x)^{1-\sigma} < 1,$$

where I use of eq (4) and eq (21) in the text. Rearrangement concludes. *Q.E.D.*

D Computation of the Transition

This appendix briefly outlines the procedure to compute the transitions when the pre-transitional steady-state is disturbed by changes at time T. In the paper's timing, $T = 1$. I will assume the economy is initially on a steady-state with constant π_i, x_i, η_{bi} and γ_i, and $\hat{k}_i$ and $\hat{c}_i$. I consider that at time T the path for these parameters changes and these changes are known as of T. Note that the new values of x_T, γ_T and π_T are known at time T before they are realized at time $T+1$. There is perfect foresight about the future course of events. The only aspect that has to be worked out is how the economy is set on a new saddle-path after the initial changes. It is instructive to summarize the equilibrium given by Eq.(15)-(17) as a 2nd order difference equation $\Lambda(...|...) = 0$. For example, at time t, and under perfect foresight, $\hat{k}_{t+1}$ is determined by

$$\Lambda(\hat{k}_{t-1}, \hat{k}_t, \hat{k}_{t+1} | \pi_{t-1}, \gamma_{t-1}, x_{t-1}, \eta_t, \eta_{t-1}, \pi_t, x_t) = 0. \tag{D1}$$

and consumption

$$\hat{c}_{t+1} = \hat{c}(\hat{k}_t, \hat{k}^e_{t+1} | \pi_t, x_t, \eta_t) \tag{D2}$$

With this, the procedure is as follows:

1. At time $t = T-1$, $\hat{k}_{T-1} = \hat{k}_i$, $\hat{c}_{T-1} = \hat{c}_i$. Then, by (D1) and (D2), $\hat{k}_T = \hat{k}_i$ and $\hat{c}_T = \hat{c}_i$.
2. At time $t = T$, again by by (D1) and (D2), $\hat{k}_{T+1} \neq \hat{k}_i$ and determined by

   $$\Lambda(\hat{k}_{T-1}, \hat{k}_T, \hat{k}_{T+1} | \pi_{T-1}, \gamma_{T-1}, x_{T-1}, \eta_T, \eta_{T-1}, \pi_T, x_T) = 0.$$

   Correspondingly

   $$\hat{c}_{T+1} = \hat{c}(\hat{k}_T, \hat{k}_{T+1} | \pi_T, x_T, \eta_T) \neq \hat{c}_i.$$

3. At time $t = T+1$, since the initial $\hat{k}_{T+1} \neq \hat{k}_i$, the value $\hat{k}_{T+2}$ must be the one that places the economy on its new stable saddle path. The next step shows how this is done.

   Consumption $\hat{c}_{t+2}$ is then determined by (D2).
4. Choose a $\hat{k}_{T+2}$. At time $t = T+2$, find $\hat{k}_{T+3}$ through (D1) by solving

   $$\Lambda(\hat{k}_{T+1}, \hat{k}_{T+2}, \hat{k}_{T+3} | \pi_{T+1}, \gamma_{T+1}, x_{T+1}, \eta_{T+2}, \eta_{T+1}, \pi_{T+2}, x_{T+2}) = 0.$$

   Iterate from $t = T+3$ on using

   $$\Lambda(\hat{k}_{t-1}, \hat{k}_t, \hat{k}_{t+1} | \pi_{t-1}, \gamma_{t-1}, x_{t-1}, \eta_t, \eta_{t-1}, \pi_t, x_t) = 0.$$

   If the path $\hat{k}_t$ converges to the steady-state, then this is the equilibrium. Otherwise, a new value for $\hat{k}_{T+2}$ must be selected and the iterations repeated.

The procedure to implement step 4 consists of specifying a large time horizon so that the economy must have reached the steady-state by then. The chosen path is the one that gets $\hat{k}$ close to the steady-state value within an accuracy criterion. To speed up computations, 20 periods were selected first. The paths reported in the paper are not sensitive to increases in the number of periods.

The solution for $\hat{k}_{t+1}$ in each period through the non-linear mapping $\Lambda(...|...)$ has used a Newton-Rapson algorithm. The choice of the initial values that lead to the solution proved to be an issue.