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Creative Destruction and Policies in a Vintage Model of Endogenous Growth

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Abstract

This paper extends a model of endogenous growth through the introduction of a productivity-augmenting component of knowledge that makes new technologies more productive than older vintages. The paper characterizes equilibrium transitional and long-run properties for the economy. The phenomenon of creative destruction, or obsolescence, of technologies underlies the growth process. In this set up, the growth-effects of various policies are analyzed. These policies include vintage-specific subsidies to firms that produce final output, a general lump-sum tax on final output firms, and openness to trade with a less developed country. The results show the existence of growth effects that are absent in previous literature.

1 Introduction

Technological change and innovation involve the emergence of new products and technologies, along with the gradual displacement of old ones. Economic history provides many examples of this phenomenon that have been documented in, for example, Rosenberg (1982) and Mokyr (1990). That the process of replacement of new goods and sectors for older ones is a substantive ingredient of modern economic growth is not a new idea, having been first expressed in Schumpeter (1942)'s celebrated notion of *creative destruction*. In this view, economic progress is the cause as well as the consequence of the rise and decline of sectors and firms.

If innovation and obsolescence interact, then factors and policies that influence the intensity and pattern of the obsolescence process will have implications for economic growth. For example, policies in the form of taxes and subsidies that are selective in terms of the vintages targeted might alter the relative position of old and new sectors. In fact, policies are often selective. A case in place is when declining profits and employment in mature sectors prompt a protective government response. Similarly, but with an opposite slant, infant-industry policies seek to prop up new emergent economic activities.[1]

The aim of this paper is to investigate the implications of a variety of policy interventions for growth and welfare in the presence of technological obsolescence. To that end, two objectives will be pursued. The first is to gain an understanding of the features of an economy undergoing the process of gradual replacement of new for old firms and sectors. This allows to take up the second objective which is to analyze, in this context, the consequences of selective taxation and subsidies, lump-sun taxes, and openness to trade for the process of growth.

This investigation is based on the analysis of a model that extends previous work on endogenous growth theory. The model borrows its basic structure from Grossman and Helpman (1991) so that privately conducted R&D is the engine of growth. The paper

[1] Selective policies have been documented in the fast-developing experiences of Korea, Singapore and Taiwan [see Young (1992) and Westphal (1990)]. In these cases, the government has been involved in channeling credit to selected industries. Sector-specific policies are, in fact, a central theme in development policy [see, for example Kruger (1990), and Pack and Westphal (1986)]. Nonetheless, casual observation indicates that in developed countries targeted industrial policies are not rare either. There is the important literature on the political economy of trade protection that seeks to understand precisely why governments deploy policies in favor of particular economic interests, which includes Hillman (1982), Trefler (1993), Grossman and Helpman (1994), and Maggi and Rodriguez-Clare (1998).

develops a version which incorporates obsolescence of technologies. More precisely, in order to accomodate the existence of new sectors along with declining sectors or firms, this paper introduces a productivity augmenting component of knowledge that makes new technologies more productive than older vintages. Whereas in Grossman and Helpman (1991) or, for that matter, Romer (1990), there is symmetry across firms, the present framework breaks the symmetry in equilibrium outcomes. This new feature arises from a broader interpretation of the nature and effects of technological change. As in previous work, new designs for consumption varieties are the purposeful outcome of commercial research. But it is also assumed that commercial research generates as a by product knowledge that is useful in the production of output. Specifically, each new design embodies the level of technology existing at the time it is created. In this way, the distribution of productivity and output levels over the range of existing varieties will not in general be symmetric. This view produces obsolescence of existing technologies over time.

The paper characterizes equilibrium long-run properties for the economy. The economy approaches asymptotically a balanced growth path. The model exhibits transitional dynamics which differs from previous work in Grosmman and Helpman (1991). Starting from a given initial level of productive knowledge the rate of innovation decreases over time and approaches asymptotically a constant value. The behavior over the transition has a simple explanation. When the measure of differentiated goods produced is small relative to the level of technology the profitability of innovation is high. As the economy evolves, the measure of existing varieties rises faster than knowledge which reduces the relative market advantage of innovators. The long-run growth rate of innovation in this economy is compared with the one corresponding to the symmetric model in Grossman and Helpman (1991). Under the present assumption that knowledge makes new goods more productive than older varieties, the economy's aggregate growth rate is higher. Two forces with opposite sign underlie this result. The introduction of new goods embodying the leading technology erodes the profits of existing firms over time. Innovators expect more productive competitors to arrive in the future. On the other hand, however, new goods enjoy a productivity advantage over existing ones. In this model this later effect determines the net positive effect on the incentives for innovation. One can also characterize the dynamic and static profiles for employment, output and prices over existing vintages. Replacement of old technologies is gradual and employment and profits are higher in new vintages.

The equilibrium is suboptimal for exactly the same reasons as the economy without vintage effects. The market induces too little research. In this set up, I analyze the growth-effects of various policies that have a differential impact on technologies at different stages over their life-cycle. The major find is that policies may reduce or decrease the innovation rate and growth by affecting the intensity of the process of creative destruction or obsolescence.

I analyze an ad-valorem subsidy to final-output firms. When the subsidy favors production of older vintages the long-run growth rate is reduced. On the other hand, a subsidy to the more recent vintages is favorable to growth. The interpretation is that policies that amplify the destructive effect of growth by widening the advantage of new over old technologies are beneficial to growth. I also explore the role of lump-sum taxation on the producers of final goods. In the model without obsolescence, growth is unambiguously reduced because profits, and thus the value of innovating firms, is reduced. In the present model, on the contrary, profits for any firm are declining over time. A lump-sum tax will render old firms unprofitable. As the range of competing vintages in operation is narrowed, the profits of innovation may increase. Thus lump-sum taxation can be beneficial for growth. A similar mechanism underlies the beneficial effects on growth from trade with a less developed country.

This model is related to literature on endogenous growth. In that private R&D is the engine of growth and there is horizontal product differentiation, the paper relates to Romer (1990) as well as to Grossman and Helpman (1991). In that the model introduces a notion of vertical differentiation it resembles Stokey (1988), Young (1991), and Aghion and Howitt (1992). Other papers have managed to combine both vertical and horizontal differentiation, most remarkably Young (1993). The modeling strategy in Caballero and Jaffe (1993) and Lai (1998) is very close to the one in this paper. The emphasis and the questions addressed set the present paper apart from those works. The model has scale effects and is subject to the objections by Jones (1995). The choice of model has been dictated uniquely by simplicity though. The effects on the incentives for R&D activities studied in this paper should nevertheless carry over under Jones (1995)'s assumptions, as well as to an appropriate version of Howitt (1999) model of growth without scale effects. The economic mechanisms emphasized in this paper resemble the beneficial effect of recessions in which old sectors are destroyed, thereby fueling the expansion of new innovating industries. This idea has been pursued in literature that includes Caballero and Hammour (1994).

Many papers to date have discussed the effects of policies on growth. The main contribution of this paper is to demonstrate in a simple model that the mere existence of vintage effects may be significant to understand some of these effects.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 characterizes the equilibrium of the economy and its welfare properties. Section 4 introduces policies and analyzes their growth effects. Section 5 concludes the paper.

2 The Model

The two inputs in the model are a primary factor denoted by h, and technology or knowledge A. The technology input has two components. The first is disembodied (scientific or engineering) knowledge useful in the production of further knowledge. The second components consists of knowledge that is embodied in the production process of particular consumption goods. There is a measure of existing differentiated consumption-goods varieties. The measure of varieties at a given point t in time is n_t. Each variety is indexed by $i \in [0, n_t]$. Varieties i such that $i > n_t$ are not available [i.e. not discovered] at time t. The introduction of the embodied component of knowledge is the main distinctive feature of the model which I proceed to set up more formally.

The productive side of the economy is composed of two sectors. A research and development (R&D) sector and a consumption-goods sector. The R&D sector uses the primary factor h and the existing level of disembodied knowledge A_t as inputs to produce designs for new consumption varieties. The technology of the R&D firms exhibits increasing returns to scale in both inputs and is specified as

$$\dot{n}_t = \delta h_{rt} A_t \text{ with } \dot{n}_t \geq 0. \tag{1}$$

Here δ is a productivity parameter in R&D and h_{rt} denotes the amount of the primary input h employed in research at time t. Additions to the stock of disembodied knowledge result as a by-product or externality from the R&D activity that creates new goods. I choose units so that,

$$\dot{A}_t = \dot{n}_t. \tag{2}$$

The second is the consumption-goods sector which is composed of the firms that produce differentiated varieties. Each of these firms uses the primary factor as an input to physically produce a particular design brought about by the R&D sector. So, at any point t in time there is an spectrum of firms producing differentiated goods. The technology of a firm producing a variety $i \in [0, n_t]$ is given by a function which is linear in the amount of primary factor employed there $h(i)_t$,

$$\phi(A_{t(i)})h(i)_t. \tag{3}$$

Here $\phi(.)$ is a non-decreasing function of the productivity knowledge embodied in the production of this good. I assume it corresponds to the stock of knowledge existing at the time the variety i was invented $A_{t(i)}$. The motivation for this assumption is the same as for the external effect on R&D. This way, knowledge has a productivity-augmenting component which affects consumption-goods' technologies unequally.[2] This point makes a crucial difference with respect to most of the previous R&D-growth literature.[3]

This specification presumes the existence of differences across between technologies used by firms of different vintages. It also implies that once a variety appears it is restricted to be produced with a time-invariant technology [i.e. $\phi(A_{t(i)})$ does not change over time] so that no firm is allowed to update its production function. Clearly, this is a very stylized representation of a firm's dynamics. What really matters for the current purpose though is the existence of a technological gap between new and old firms. This feature would arise endogenously under reasonable assumptions in a model where firms are allowed to update technology after their creation as in Parente (1994).

The consumption side is standard. The economy is populated by a fixed measure (let it be 1) of identical infinitely-lived agents. Each of them is endowed with h units of the non-perishable primary factor which are supplied inelastically. So the endowment of this input in the economy is constant over time. Preferences are defined over paths of consumption of different varieties to the market. The preferences are represented by an utility function mapping $(\{c(i)_t\}_{i\in[0,\infty]})_{t\in[0,\infty]}$ into reals. However for each t, only existing

[2] Young's (1991) view of technological processes is very similar. There technological innovation is defined by the introduction of new and *"more advanced"* horizontally differentiated goods that are producible with lower factor requirements. The model in Caballero and Jaffe (1993) and Lai (1998) differentiates goods by their quality in a similar way.

[3] The case $\phi(.)$ a constant is Romer (1990) or Grossman&Helpman (1991).

varieties can be consumed so that, without loss of generality, the choice set can be written $(\{c(i)_t\}_{i\in[0,n_t]})_{t\in[0,\infty]}$. We assume preferences are time-separable and that at each point in time t all the differentiated varieties are imperfect substitutes with constant elasticity of substitution between any pair of goods. In particular, we adopt the CES specification due to Dixit-Stiglitz (1977) which has largely proved quite tractable and, because of imperfect substitutability, accommodates increasing diversity in consumption,

$$\left[\int_0^{n_t} c(i)_t^{\alpha} di\right]^{1/\alpha} \quad \text{with } 0<\alpha<1. \tag{4}$$

The elasticity of substitution is $1/(1-\alpha)$ which characterizes the preference for variety. It is worth noting that concerning preferences all varieties are treated symmetrically. The infinite stream of current utilities is discounted and the intertemporal elasticity of substitution between the current utilities of any two periods is assumed to be constant. Preferences are represented by

$$\int_0^{\infty} e^{-\rho t}\frac{1}{1-\sigma}\left[\int_0^{n_t} c(i)_t^{\alpha} di\right]^{(1/\alpha)(1-\sigma)} dt. \tag{5}$$

The market structure is described as follows. The market for designs arising from the R&D sector is competitive. Since the technology for R&D firms exhibits increasing returns to scale, the equilibrium must rely on some form of external effect in R&D. The existing stock of disembodied knowledge enters the R&D technology as an externality or spillover effect. Concerning final-goods markets, each innovator must purchase a design from the research sector before commencing production. This constitutes a fixed cost which prevents any rule of marginal-cost pricing from being supported in equilibrium. Then, some form of imperfect competition must be invoked. Final-goods firms behave as monopolististic competitors. Also, embodied knowledge creates increasing returns in the production of varieties. This is assumed to be an externality as well. The market for the primary input is competitive so that both final-good and R&D firms are wage takers in this market. The structure of asset markets, which includes shares on firms, is assumed to be complete. Consumers can freely lend and borrow. Any producer of a particular variety finances its initial cost by issuing equity. Consumers can save in both financial assets yielding a return r_t and shares on the firms in the economy.

3 Equilibrium

The equilibrium is defined as paths for prices, allocations of the primary input and consumption vectors such that: consumers maximize utility; research and final-goods firms maximize profits; all markets clear.

Consumer's behavior is solved in two stages. First, at any point in time t, the level of consumption expenditure must be allocated optimally among the existing set of varieties so as to maximize current utility. Let E_t denote consumption expenditure at time t, $\int_0^{n_t} p(i)_t c(i)_t di$, where $p(i)_t$ is the price of variety i at time t.[4] Then, given prices $\{p(i)_t\}_{i\in[0,n_t]}$, the static problem consists of choosing $\{c(i)_t\}_{i\in[0,n_t]}$ that, by Eq. (4), maximizes $\int_{[0,n_t]} c(i)_t^\alpha di$ provided that $E_t = \int_{[0,n_t]} p(i)_t c(i)_t$ holds. Given E_t, the solution to this problem yields a demand function for each variety:

$$c(i)_t = E_t \frac{p(i)_t^{-\theta-1}}{\int_0^{n_t} p(j)_t^{-\theta} dj} \text{ where } \theta \equiv \frac{\alpha}{1-\alpha} > 0. \tag{6}$$

Hence the indirect current-utility reads

$$E_t \left[\int_0^{n_t} p(i)_t^{-\theta} di\right]^{\frac{1}{\theta}}. \tag{7}$$

The second step in the consumer's problem is the choice of paths of consumption and assets so as to maximize the value of the discounted stream of current indirect utilities in Eq. (5). Consumer's holdings of assets consist of both financial assets yielding a return r_t, and shares on firms. A no-arbitrage condition that requires that the consumer be indifferent between financial assets and shares must be satisfied at every point in time. Denote the value at t of a firm producing a variety born at τ, $i(\tau)$, by $V[i(\tau)]_t$. We assume that the value of a firm coincides with its fundamental value, that is to say, $V[i(\tau)]_t \equiv \int_t^\infty e^{-\int_t^s r_u du} \pi(i(\tau))_s ds$, where $\pi(i(\tau))_t$ is the profit of firm $i(\tau)$ at time t. Then the no-arbitrage condition reads,

$$r_t V[i(t)]_t = \pi(i(t))_t + \dot{V}[i(t)]_t, \tag{8}$$

where the dot notation denotes the derivative with respect to the time subscript only [not t in $i(t)$]. The second right-hand side term is the gain/loss of a (given) firm $i(t)$.

[4] I have implicitly assumed that $p(i)_t = \infty$ for $i > n_t$.

The sum of the instantaneous profit and the capital gain/loss must equal the opportunity cost on equity claims of the capital invested in firm $i(t)$, $r_t V[i(t)]_t$. Provided that this condition holds and given the distribution of prices $\{\{p(i)_t\}_{i\in[0,n_t]}\}_{t\in[0,\infty]}$, the consumer selects $\{E_t\}_{t\in[0,\infty]}$ and sequences of assets that maximize $\int_{[0,\infty]} e^{-\rho t}\,(P_t E_t)^{1-\sigma}/(1-\sigma)dt$ with $P_t \equiv [\int_{[0,n_t]} p(j)_t^{-\theta}dj]^{\frac{1}{\theta}}$. The intertemporal budget constraint comes from consolidating the period-by-period constraints: $\int_0^\infty e^{-\int_0^t r_\tau d\tau}[w_t h - E_t]dt + W_0 \geq 0$, where W_0 is initial wealth and w_t is the competitive wage paid to the primary factor. The solution to the consumer's dynamic problem must satisfy the following differential equation:

$$\frac{\dot{E}_t}{E_t} = \frac{1-\sigma}{\sigma}\frac{\dot{P}_t}{P_t} + \frac{(r_t-\rho)}{\sigma}. \tag{9}$$

Research firms sell new designs in a competitive market. As said before the technology exhibits global increasing returns to scale. However, the aggregate nature of the external effect makes a a particular firm face a constant returns production function in the primary input. Denote the price of designs created at t by p_{At}. In any equilibrium profits of engaging in research must be non-positive, otherwise the demand of h by research firms would be infinite. Then, the equilibrium condition is:

$$w_t \geq p_{At}\delta A_t \text{ with equality whenever } \dot{A}_t > 0. \tag{10}$$

Because of the linearity in h_{rt} the size of R&D firms remains undetermined. The allocation of the primary input between R&D and consumption is pinned down by the final-goods firms' behavior.

Each firm producing a differentiated variety maximizes current profits at each period. Each of them faces a demand function given by consumer's optimal behavior in Eq. (6), so that profits at t of a firm producing i are defined as,

$$\pi(i)_t = E_t\frac{p(i)_t^{-\theta-1}}{\int_{A_0}^{A_t} p(j)_t^{-\theta}dj}\left[p(i)_t - \frac{w_t}{\phi(A_{t(i)})}\right]. \tag{11}$$

Given that the effect of the price choice of a particular producer on the integral is negligible, profit maximization yields,

$$p(i)_t = \frac{1+\theta}{\theta}\frac{w_t}{\phi(A_{t(i)})}. \tag{12}$$

That is, firms set prices as a mark-up over unit cost. The mark-up is constant across firms but the unit cost depends on the productivity level given by $\phi(A_{t(i)})$. The more productive a firm, the lower the price of the good. This is the source of assymetries in the economy's equilibrium. Using Eq. (12), profits and quantities in Eq. (6) and (11) can be written in a more convenient way as

$$\pi(i)_t = E_t \frac{[\phi(A_{t(i)})]^\theta}{\int [\phi(A_{t(j)})]^\theta dj} \frac{1}{1+\theta}, \tag{13}$$

$$c(i)_t = E_t \frac{[\phi(A_{t(i)})]^{1+\theta}}{\int [\phi(A_{t(j)})]^\theta dj} \frac{1}{w_t} \frac{\theta}{1+\theta}. \tag{14}$$

The market clearing condition for the primary input reads

$$\int_0^{n_t} h(i)_t di + h_{rt} = h, \tag{15}$$

with firm i's demand of factor been given by,

$$h(i)_t = \frac{c(i)_t}{\phi(A_{t(i)})} \tag{16}$$

An equilibrium condition for consumption-good firms remains to be spelled out. Each potential producer is a buyer in the market for designs which is competitive. Then the price of any new design will be bid up until it equals the value of the profits a monopolist can extract or, in other words, the value of the firm. Formally, in equilibrium, for a design appeared at t, the following must hold:

$$V[i(t)]_t = p_{At}, \tag{17}$$

so that no firm earns positive profits in a long-term sense.

Throughout the paper I will assume that $\sigma = 1$. On the other hand, one can set the time path for a nominal variable and measure prices against the chosen numeraire. I choose $p_{At} = 1$. With these simplifications, conditions (8), (9), and (10) with (17) and (13) specialize to

$$\frac{\dot{E}_t}{E_t} = r_t - \rho \tag{18}$$

$$w_t \geq \delta A_t \text{ with equality whenever } \dot{A}_t > 0, \tag{19}$$

$$r_t = \pi(i(t))_t - \theta \frac{\phi'(A_t)}{\phi(A_t)} \dot{A}_t. \tag{20}$$

Equation (20) deserves comment.[5] This condition says that the current profit of a just born firm is higher than the interest rate by a term which depends upon the effect of innovation on the new entrants' productivity. Firms earn different profits and a consumer/investor must account for the future returns on his shares. If new firms are more productive, future profits for existing firms will be decreasing over time. Hence, the required current return on a new firm must be higher than the interest rate in order to offset future losses which, in turn, depend on the rate of technological change. Hence this term captures the idea of creative destruction. In fact, condition (20) contains all the equilibrium consequences of introducing vintage-specific technological change in production. The equilibrium for the economy is completely described by equations (1), (2), (15),(16), (14), (19), (13), (18) and (20).

From the law-of-motion of knowledge (1) with the market clearing condition (15), using the factor demand functions (16) with (14), and the R&D equilibrium free-entry condition (19), an equilibrium with positive innovation must satisfy the differential equation,

$$\frac{\dot{A}_t}{A_t} = \delta h - \frac{\theta}{1+\theta} \frac{E_t}{A_t}. \tag{21}$$

From the factor-market side, a higher ratio of consumption expenditure to knowledge amounts to higher firms' demand for the primary input. The induced factor allocations then tends to lower the innovation rate. It turns out that this equilibrium equation is identical to the one arising from the model with symmetric varieties in Grossman and Helpman

[5] The total differential of the fundamental value of an innovator can be calculated as

$$\begin{aligned} \frac{dV[i(t)]_t}{dt} &= \dot{V}[i(t)]_t + \frac{dV[i(t)]_t}{di(t)} \frac{di(t)}{dt} \\ &= \dot{V}[i(t)]_t + \left[\int_t^\infty e^{-\int_t^s r_u du} \frac{d\pi[i(t)]}{di(t)} dt \right] \dot{A}_t \\ &= \dot{V}[i(t)]_t + V[i(t)]_t \theta \frac{\phi'(A_t)}{\phi(A_t)} \dot{A}_t \\ &= -\pi[i(t)]_t + \left[r_t + \theta \frac{\phi'(A_t)}{\phi(A_t)} \dot{A}_t \right] V[i(t)]_t \end{aligned}$$

where the third equality uses Eq. (13), and the fourth equality Eq. (8). The normalization of $p_{At} = 1$ and Eq. (17) lead to Eq. (20).

(1991) [i.e. with $\phi(.)$ a constant]. Although the distribution of employment over vintages is different, aggregating over production units yields the same allocation between research and goods production for given E_t/A_t.

The description of the equilibrium is completed by determining the equilibrium interest rate in the intertemporal condition (18) by using the no-arbitrage condition (20) and the expression for the firm's profits (13). To proceed, the form of $\phi(.)$ must be specified. I assume a linear specification and choose units so that

$$\phi(A) = A. \tag{22}$$

For notational convenience define

$$\Gamma(A_t, A_0) \equiv \frac{A_t^{1+\theta}}{A_t^{1+\theta} - A_0^{1+\theta}}. \tag{23}$$

where A_0 denotes the level of knowledge that is embodied in the oldest technology in operation. The profits for the producer of a just-born variety can be rewritten upon appropriate change of variables under the integral in Eq. (13) as

$$\pi(i(t))_t = \frac{E_t}{A_t}\Gamma(A_t, A_0), \tag{24}$$

With this result to solve, the equilibrium interest rate in (20) which, substituted in (18), delivers the second equilibrium equation

$$\frac{\dot{E}_t}{E_t} = \frac{E_t}{A_t}\Gamma(A_t, A_0) - \theta\frac{\dot{A}_t}{A_t} - \rho. \tag{25}$$

This equilibrium condition differs from the one in the model where obsolescence is absent for two reasons. The first right-hand side term, which is the profit rate of a new firm, reads $(1/(1+\theta))E_t/A_t\Gamma(A_t, A_0)$ with the Γ term as above but letting $\theta = 0$. The second right-hand side term simply is absent if $\phi(.)$ is constant. Equations (21) and (23) define the equilibrium by a system of two differential equations in A and E. Notice however that the term Γ introduces a lagged value for A so that the equilibrium can be viewed as a differential-difference equation system where the state variable appears with a non-constant lag. Observe that in an equilibrium with positive innovation Γ falls over time towards 1.

3.1 Long-run growth

This section is concerned with the long-run behavior of the economy. In this model, because firms, irrespective of their age, set the selling price as a mark-up over the unit cost, profits are always non-negative. Thus no firm is ever shut down. This implies that the oldest firm in operation is the one created at the beginning of times and, consequently, A_0 is constant. In an equilibrium with positive innovation the term Γ declines monotonically towards its lower bound and for arbitrarily large values of A it will be nearly constant and equal to one. So although Γ is never constant, $\Gamma(A_t, A_0) = 1$ can be taken as an approximation to long-run behavior. I assume that a positive constant-innovation rate equilibrium path does exist. Existence issues and transitional dynamics will be studied later.

Assume that the economy attains a balanced growth path displaying a constant rate of innovation $\dot{A}_t/A_t$. From the factor-market clearing equilibrium condition Eq. (21), we see that E must grow at the same rate as A. We denote the common growth constant growth rate γ. In the long-run the profit rate of an innovator is constant too and so is the interest rate. In these circumstances, the equilibrium equations(21) and (25) can be written

$$\gamma = \delta h - \frac{\theta}{1+\theta}\frac{E}{A} \tag{26}$$

and

$$\gamma = \frac{E}{A} - \theta\gamma - \rho. \tag{27}$$

Whereas (26) is identical to the model with ϕ constant, Eq. (27) is not for the reasons pointed in the discussion of Eq.(25). On one hand, the profit rate to the current innovator is higher due to its productive advantage over existing vintages. However, the pace of future innovations will reduce future profits. On net, the first *creative* effect dominates the second *destructive* effect, and so the addition of productivity vintage effects enhances innovation.[6] The intuition is simple. Returns on new firms are larger because innovators enjoy a productivity advantage over existing firms. Then consumers are more willing to invest in new firms. This *creative* effect is captured by the first right hand side term in Eq.(27) above which represents the profit rate for new innovators. It turns out that if ϕ is constant, this term is smaller by a factor $1/(1+\theta)$. There is however a countervailing force.

[6] For the symmetric case, which is exactly that in Grossman&Helpman (1991,Ch3), the balanced-equilibrium innovation rate reads, $[1/(1+\theta)][\delta H - \theta\rho]$

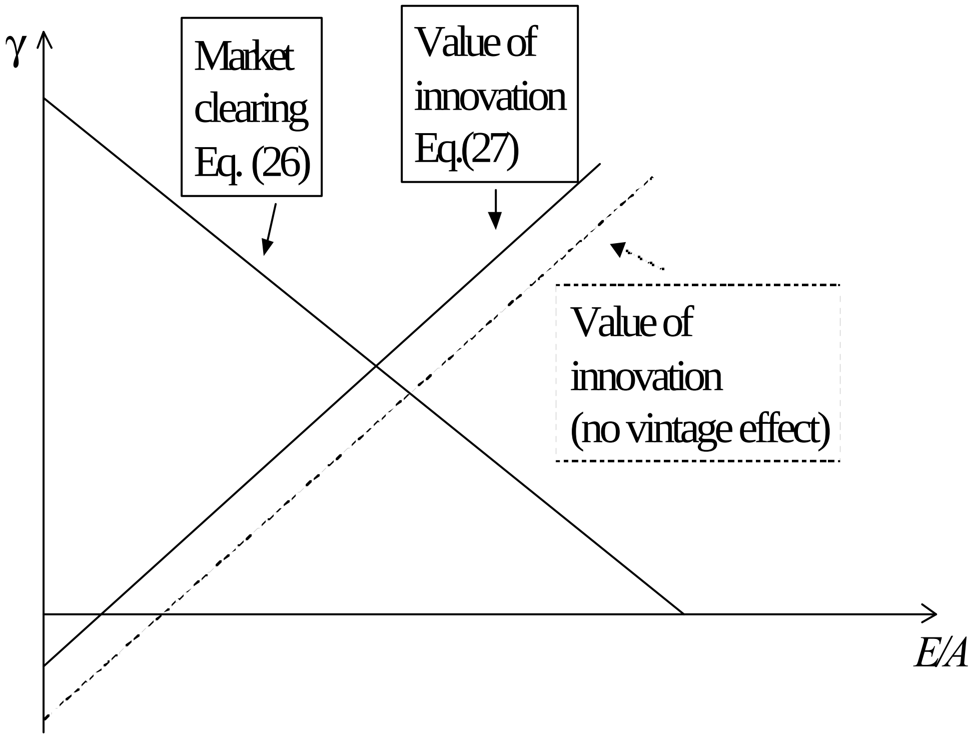

Figure 1: The balanced-growth equilibrium. The dashed line represents the symmetric case.

The ongoing introduction of new improved goods damages the prospects of any existing firm. Superior technologies are expected to come along in the future. One must account for future market losses due to the expected future entry of new superior competitors. This *destructive* component is captured in the second right hand side term. The first effect dominates so that destruction is creative indeed. This can be seen in figure 1.

Combine (24) and (25) and the balanced rate of innovation can be calculated explicitly as

$$\gamma_A = \frac{1}{1+\theta}\left[\delta H - \frac{\theta}{1+\theta}\rho\right]. \tag{28}$$

While the effect on the equilibrium innovation rate of introducing productivity improvements is unambiguous, the extent of the difference depends crucially on the parameters characterizing the preference for variety. The difference in innovation rates tends to zero as θ becomes smaller. When substitutability is low new firms, though technologically superior, enjoy a negligible market advantage. The *destruction effect* is stronger (relative to the case with higher θ) and, in the limit, the *creative effect* is completely offset.

3.2 Product Composition

The way the technology spills over productive activity gives rise to heterogeneity across firms born at different dates as well as to an evolving pattern for the variables describing the performance of every particular firm. This section characterizes the pattern for prices, quantities, profits and resource allocations both of every firm over time (the dynamic pattern) and across different firms at a given point in time (the static pattern) in the long-run. Unlike the symmetric version of the model, a number of quite appealing equilibrium properties at the industry level show up. Old goods tend to disappear, being substituted by new-born goods.

Consider first the age-distribution for prices at a given date t. A firm born at τ sets prices as a (constant) mark-up over average cost w_t/A_τ. As younger (higher τ) goods are produced at lower cost they are traded at lower prices. The extent to which cost differ across firms depends directly on the innovation path. In a balanced equilibrium with constant rate of innovation γ we can check that the price of goods decreases in the birth-date at a rate of γ. Similarly, the price of any good increases at the rate γ over time. These patterns are reflective of the relative backwardness experiencing a given good relative to the technological frontier.

Consider now the pattern for the output of final-goods. From Eq. (14), (19) and (24) one can write

$$c[i(\tau)]_t = \frac{E}{A}\frac{\theta}{\delta}\left(\frac{A_\tau}{A_t}\right)^{1+\theta}\Gamma(A_t, A_0).$$

The response of $c(i(\tau))_t$ to changes in τ only depends on how the price varies, all other things held constant. Since $p(i(\tau))_t$ decreases with τ, then $c(i(\tau))_t$ increases with τ at a proportional rate of $(1+\theta)\gamma$. As expected it depends positively on the elasticity of substitution. The time path for quantities of a particular good hinges on a number of factors. The value of aggregate consumption expenditure grows over time which exerts a positive effect. In opposite direction, as time evolves both the the number of varieties and the own price increase. We may then conclude that the quantity of a particular good over time and the quantities at a given point in time of goods of different ages decrease with time and age respectively at the common rate $-(1+\theta)\gamma$.

Concerning profits, the average cost of any firm is proportional to its price. Then profits accruing to a firm are proportional to the share of consumption expenditure on this good.

From (13), (19), and (23) the profit rate of a firm can be written as

$$\pi[i(\tau)]_t = \frac{E}{A}\left(\frac{A_\tau}{A_t}\right)^\theta \Gamma(A_t, A_0).$$

So profits fall with t and rise with τ at the rate $-\theta\gamma_A$.

The economy is endowed with a fixed amount of primary factor. In a balanced equilibrium, the splitting of resources between research and final-goods is constant. It is interesting to look now at the allocation of primary input among firms. It results from the tension between two forces. As already seen, younger firms produce a larger quantity of good. On the other hand they are more productive. We have that the first effect dominates so that more advanced firms use a larger amount of primary factor. The quantity of employed resources by individual firms in Eq.(16) then decreases with the age of the firms at a rate $-\theta\gamma_A$.

Over time however for a particular firm the demand of primary factor evolves according to the demand for its product. Then employment in a firm decreases over time at the same rate as consumption does $-(1+\theta)\gamma_A$.

Going from individual firms to age-groups of firms, the distribution of input across vintages τ is given by $\gamma c(\tau)$. The lower the age of a set of firms the higher the amount of input employed there for two reasons: first each firm individually uses a larger amount of factor and, second, the number of firms in that set is larger than in any other older set. It is worth noting that the amount of primary factor employed by a new-born firm decreases over time. The whole set of forefront firms at time t employs a constant amount $\gamma_A c(t)$ of primary factor. Since the number of new firms becomes larger, the amount employed by any of them decreases at a rate $-\gamma$.

3.3 Dynamics and existence

This subsection describes the dynamic adjustment of the economy that starts from an initial level of knowledge A_0. The analysis permits to identify existence conditions for the type of long-run outcomes we have described above. Having defined $z \equiv E/A$, it is useful to rewrite the equations for equilibrium dynamics (21) and (25) as follows

$$\gamma = \delta h - \frac{\theta}{1+\theta} z$$

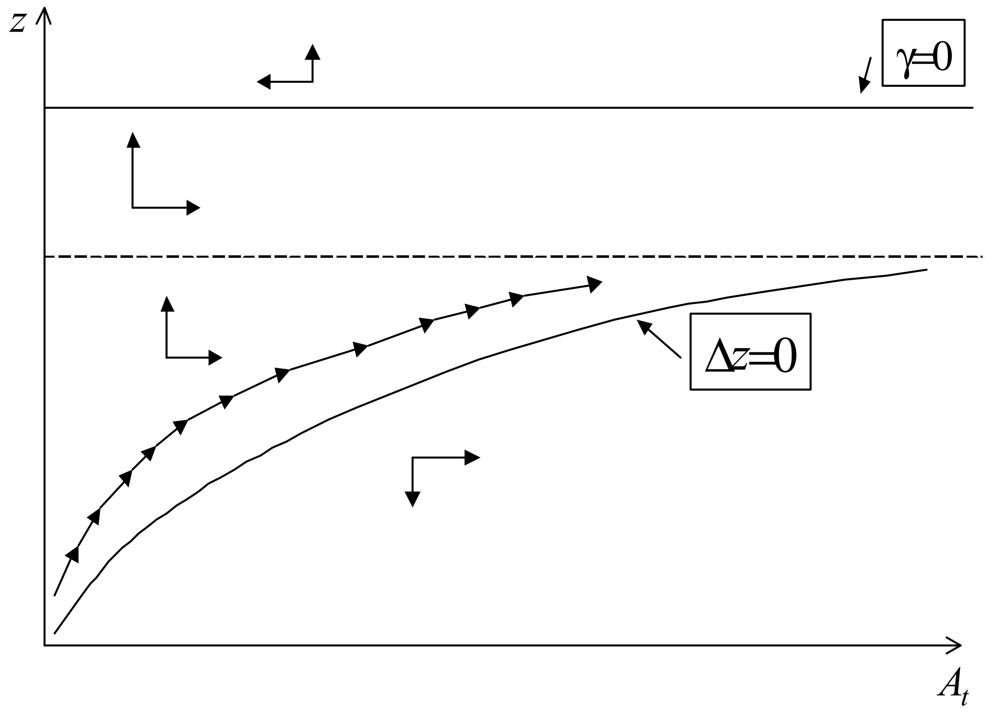

Figure 2: The dynamics od $z = E/A$.

$$\frac{\dot{z}}{z} = [\Gamma(A_t, A_0) + \theta]\, z - (1+\theta)\delta h - \rho$$

with z and γ dated at time t. Notice that the behavior on z is described by a single differential equation with non-constant coefficient. Starting from an arbitrary initial value of knowledge embodied in the oldest variety produced, the economy features a transition towards the long-run outcomes due uniquely to the fact that the term Γ changes with the value of A.[7] A graphical representation of these conditions in the A_t-z space will suffice to show the economy's behavior.

Represent the $\dot{z} = 0$ schedule as a concave, increasing and bounded from above function of A_t, with the upper bound being $[(1+\theta)\delta h + \rho]/(1+\theta)$. The arrows indicate that from a point above it, z tends to increase and the contrary case occurs from a point below. On the other hand, only points above the horizontal line at z with value $[(1+\theta)/\theta]\delta h$ are

[7] The case of Γ being constant corresponds to the assumption that the measure of varieties is the level of technological knowledge. This is the case in Grossman and Helpman (1991). Clearly, the equilibrium must feature instantaneous jump toward the BGP outcomes. Thus for there to be a transition it is necessary to distinguish the measure of goods or firms from the level of knowledge. This is done because I think that there are interesting perturbations and policies that affect the number of goods produced in the economy but not the level of knowledge.

consistent with a path with positive innovation [i.e. increasing A over time]. It is simple to see that, in the case this horizontal line falls below the upper bound for the $\dot{z} = 0$ schedule, then for any choice of initial z we have that either A falls, or E attains negative values, or innovation becomes zero in finite time. Thus a necessary condition for positive innovation to be sustainable is

$$\frac{1+\theta}{\theta}\delta h > \delta h + \frac{1}{1+\theta}\rho.$$

Under this condition the graphical analysis shows that we can find an initial value for z such that the economy follows a path of perpetual growth. This path must be above $\dot{z} = 0$ approaching it as A grows. Thus consumption expenditure grows faster than knowledge and the ratio between the two approaches the long-run value, which, consistently with out interpretation, implies that the rate of profit on new goods decreases as the measure of varieties produced increases. Accordingly the innovation rate declines monotonically to its long-run value.

3.4 Welfare

The economy contains potential effects for the edge between the market outcome and the optimal social allocations. It is not hard to show however that the market equilibrium differs from the social optimum for the same reasons as in the economy without vintage effects. In particular, the static allocation of input across vintages is optimal. On the other hand, the planner's allocation of inputs to research exceeds the equilibrium one, thus making research insufficient in equilibrium. But the wedge is determined by the standard intertemporal spillover effect. This can be seen by solving the planner's optimal control problem to find that the optimal growth rate exceeds the equilibrium growth rate by factor $1 + \theta$, just as in the symmetric version analyzed by Grossman and Helpman (1991). The details of the argument are in appendix A. Market prices and interest rate reflect the creative destruction behind obsolescence in a socially efficient way. Thus the same type of R&D subsidy would be a first best policy.[8] The next section will analyze other policies that cannot typically be studied in models where goods are all symmetric.

[8] Lai (1998) discusses at length welfare effects in a similar context. The present result suggests that the presence of new welfare effects is related to aspects of his model other than obsolescence.

4 Policies

The virtue of the model developed thus far is that clarity is preserved while incorporating obsolescence and heterogeneity of firms or sectors. This is interesting because this allows the analysis of the growth effects of policies that may have a differential impact on different vintages. Selective subsidies are observed in practice. Subsidies are an instrument that governments use to alleviate the loss of profits and employment in certain firms and sectors. In other cases, this type of interventions are deployed to prop up new emergent sectors. In the model, these policies bring about growth effects by interfering the process of creative destruction. Other policies such as lump-sum taxes and trade policies will also have consequences for the allocation of resources to growth generating activities through a similar channel.

4.1 Production Subsidies

Assume a a firm i is subsidized at the rate $\eta(i)_t$, which may change over time. Given that the demand of this good $x(i)_t$ is exactly as before profits are

$$\pi(i)_t = x(i)_t \left[p(i)_t(1+\eta(i)_t) - \frac{w_t}{\phi(A_{t(i)})} \right]$$

Profit maximization implies mark-up pricing on a variable cost $w_t/\phi((A_{t(i)})(1+\eta(i)_t))$ which now includes the fact that a positive ad-valorem subsidy allows the firm to sell at a lower price. Assuming, as before, that $\phi(A) = A$ is given as in Eq. (22), the analogous to Eq. (13) and (14) are, respectively,

$$c(i)_t = \frac{E_t}{A_t}\frac{\theta}{1+\theta}\frac{1}{\delta}\frac{(1+\eta(i)_t)^{1+\theta}A_{t(i)}^{1+\theta}}{\int^{n_t}(1+\eta(j)_t)^{\theta}A_{t(j)}^{\theta}dj}$$

and

$$\pi(i)_t = E_t\frac{1}{1+\theta}\frac{(1+\eta(i)_t)^{1+\theta}A_{t(i)}^{\theta}}{\int^{n_t}(1+\eta(j)_t)^{\theta}A_{t(j)}^{\theta}dj}$$

It is apparent that the market position and thus the value of a firm will depend on the overall distribution of subsidy rates over vintages. Certainly, most often subsidies are not set for all the goods produced in the economy, but they are designed to protect specific sectors or groups of producers. The details of the analysis are in appendix B.

4.1.1 Protection of declining sectors

For example, governments have used subsidies to help declining sectors. The target consists of sectors that have experienced a long-process of job destruction and reduction in profit rates. In terms of our model, the beneficiaries of such a policy would be the oldest vintages. Provided that this policy will certainly keep old firms from declining too fast, we want to asses the implications for the overall performance of the economy. To this end, assume that $\eta(i)_t = \eta$ for i such that $t(i) < t - T$, and $\eta(i)_t = 0$ otherwise. That is, all firms aged T and over benefit from the constant subsidy. It is convenient to define

$$\Gamma^{s1}(A_t) \equiv \frac{A_t^{1+\theta}}{(1+\eta)^{\theta}\left[A_{t-T}^{1+\theta} - A_0^{1+\theta}\right] + \left[A_t^{1+\theta} - A_{t-T}^{1+\theta}\right]} \tag{29}$$

which, as $\Gamma(.)$ in Eq.(23), reflects the competitiveness of the leading technology relative to the whole set of existing firms. In this case, the presence of η shows that older firms are now relatively more competitive thus reducing the edge of the state-of-the-art firms. I will focus on long-run balanced outcomes with constant innovation rate γ so the role of A_0 can safely be ignored. The expressions for quantities and profits in Eq.(13) and (14) can now be written as

$$c(i)_t = \frac{E_t}{A_t}\theta\frac{1}{\delta}\left(\frac{A_{t(i)}}{A_t}\right)^{1+\theta}(1+\eta(i))^{1+\theta}\Gamma^{s1}(A_t) \tag{30}$$

$$\pi(i)_t = \frac{E_t}{A_t}\left(\frac{A_{t(i)}}{A_t}\right)^{\theta}(1+\eta(i))^{1+\theta}\Gamma^{s1}(A_t) \tag{31}$$

This equations can be used to develop the two main equilibrium equations. As for the derivation of Eq. (21), the block of conditions (1), (15), (16) and (19) now leads up to a condition that reflects the market clearing allocation of the primary input to research

$$\gamma = \delta h - \frac{\theta}{1+\theta}\frac{E}{A}\frac{1+e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}{1+e^{-\gamma(1+\theta)T}(\eta^{1+\theta}-1)} \tag{32}$$

The second term on the right-hand side describes the allocation of inputs to the production of consumption goods rather than research. The expression shows that the presence of the subsidy increases the demand of inputs for final-good production and is detrimental to research. This negative effect is smaller the older the recipient firms since those firms take up relatively fewer resources. Graphically, the negatively-sloped line in figure 1 shifts downwards.

The second condition is related to the equilibrium returns to innovation activities. Again, just as for the derivation of Eq. (25), the set of conditions (18) and (8) permit to derive the following

$$\gamma = \frac{E}{A}\frac{1+e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}{1+e^{-\gamma(1+\theta)T}((1+\eta)^{\theta}-1)} - \theta\gamma - \rho \tag{33}$$

The term accompanying E/A picks up the role of the subsidy for the returns to innovation. The net sign of this effect is ambiguous. On one hand, the subsidy increases the present value of the firm and thus the return to creating a new product. However, the subsidy also increases the competitiveness of the rest of firms which tends to erode the flow of profits. The direct benefits occur in the future and are discounted, thus when discount is high or the period to qualify for the subsidy, T, is long then the subsidy scheme is more likely to be detrimental for the return to investment. In terms of figure 1, the positively sloped curve may shift either way.

In general equilibrium, nonetheless, the net effect of subsidies is unambiguously negative on growth. Combining the two above conditions Eq.(32) and (33) shows that the long-run rate of innovation is smaller than when $\eta = 0$, and that it declines with the value of η and with the value of T. Analytically, the equilibrium γ is determined by

$$\frac{1+e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}{1+e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)} = \frac{(1+\theta)\gamma+\rho}{\delta h-\gamma}\frac{\theta}{1+\theta}.$$

The forces at work are as follows. On one hand, the value of firms becomes higher because the subsidy increases profits from age T on. On the other hand, since subsidized firms increase their demand for inputs, there is less productive factor left for research. It turns out that the effect on the value of the firms is small relative to the impact on the demand for input by firms. The reason is that the former is discounted since it occurs far in the future. What is important to stress is that it is not the presence of subsidies per se that reduces the allocation of resources to growth-generating activities, but its distribution across firms of different vintages. If all goods were subsidized [i.e. $T = 0$] then the subsidy rate η would not have any bearing on equilibrium innovation.

4.1.2 Infant industry protection

The opposite case that subsidies are targeted at the newest firms is considered next. I find that such a scheme creates a wider advantage of innovators over the average existing firm. This produces a larger incentive for undertaking research. To make this case, assume that at t there is a positive subsidy rate only for firms born after $t - T$. Assume that $\eta(i)_t = \eta$ for i such that $t(i) > t - T$, and $\eta(i)_t = 0$ otherwise.

For this case, it is useful to define

$$\Gamma^{s2}(A_t) \equiv \frac{A_t^{1+\theta}}{A_{t-T}^{1+\theta} - A_0^{1+\theta} + (1+\eta)^\theta \left[A_t^{1+\theta} - A_{t-T}^{1+\theta}\right]} \tag{34}$$

Again, this is reflective of the relative market position. The qualitative effect of the subsidy is as in Γ^{s1} as defined in Eq.(28) above, but here this subsidy improves the relative competitiveness of younger rather than older vintages. The equilibrium expressions for quantities and profits in Eq. (30) and (31) also apply provided that Γ^{s1} is replaced by Γ^{s2}. Proceeding as in the previous cases, the market-clearing related condition leads to the expression

$$\gamma = \delta h - \frac{\theta}{1+\theta} \frac{E}{A} \frac{(1+\eta)^{1+\theta}(1 - e^{-\gamma(1+\theta)T}) + e^{-\gamma(1+\theta)t}}{(1+\eta)^\theta(1 - e^{-\gamma(1+\theta)T}) + e^{-\gamma(1+\theta)t}} \tag{35}$$

which describes the incentives governing the allocation of inputs to research and final-goods production. As before, for given E/A, the subsidy leads to less resources to research. Graphically, the positive curve in figure 1 shifts downwards. On the other hand, the study of the return to innovations produces the other equilibrium equation

$$\gamma = \frac{E}{A} \frac{(1+\eta)^{1+\theta}(1 - e^{-\rho T} e^{-\gamma(1+\theta)T}) + e^{-\rho T} e^{-\gamma(1+\theta)T}}{(1+\eta)^\theta(1 - e^{-\gamma(1+\theta)T}) + e^{-\gamma(1+\theta)T}} - \theta\gamma - \rho \tag{36}$$

The effect of η here is unambiguously positive. The innovator enjoys the subsidy edge right from the start of its activity, and the drop in profit due to the future withdrawal of the subsidy is discounted. Graphically, the negatively-sloped curve shifts upwards in figure 1.

In general equilibrum, the direct impact on the profitability of research dominates so that the subsidy increases the rate of innovation. This can be seen explicitly by combining Eq.(35) and (36) to obtain

$$\frac{(1+\eta)^{1+\theta}-e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}{(1+\eta)^{1+\theta}-e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}=\frac{(1+\theta)\gamma+\rho}{\delta h-\gamma}\frac{\theta}{1+\theta}.$$

Again, a subsidy that applies to everyone ($T\rightarrow\infty$) has no growth effect, showing the importance of the selective character of this policy. Whereas the growth effect is welfare improving, this subsidy also produces a static distortion. To judge whether this policy is beneficial the dynamic benefits must be balance against the losses created by the distortion.

4.2 Lump-sum Taxes

Business taxation usually has an important lump-sum component. Here I assume there is a fixed tax f collected of firms producing consumption goods. Since a firm's gross profit declines over time, this fixed tax per period will imply that current net profits for some firms will become negative. At this point, any such a firm will shut down and stop operating. The firms that will drop out of the produced set will be those aged T an above for some T that has to be determined. More formally, Eq.(12) still characterizes the pricing rule of a firm i. Provided that the range of varieties is not finite, following an analysis similar to that in section 4.1 delivers the following equations for gross profit rates and quantities

$$c(i)_t \quad = \quad \frac{E_t}{A_t}\frac{\theta}{\delta}\left(\frac{A_{t(i)}}{A_t}\right)^{1+\theta}\Gamma^f(A_t,A_{t-T}) \tag{37}$$

$$\pi(i)_t=\frac{E_t}{A_t}\left(\frac{A_{t(i)}}{A_t}\right)^{\theta}\Gamma^f(A_t,A_{t-T}) \tag{38}$$

with

$$\Gamma^f(A_t,A_{t-T})\equiv\frac{A_t^{1+\theta}}{A_t^{1+\theta}-A_{t-T}^{1+\theta}}=\frac{1}{1-e^{-\gamma(1+\theta)T}}, \tag{39}$$

where the second equality uses the constant-growth assumption.

Now one can calculate the age T at which a firm will be discontinued. To that end, set

$$\pi(i(t))_{t+T} = \frac{E_{t+T}}{A_{t+T}} \left(\frac{A_t}{A_{t+T}} \right)^{\theta} \Gamma^f (A_{t+T}, A_t) - f = 0,$$

which, on a BGP with $A_{t+T} = \exp(\gamma T) A_t$ and constant E/A delivers the relation between f and T given γ and E/A

$$\frac{E}{A} e^{-\gamma\theta T} \frac{1}{1 - e^{-\gamma(1+\theta)T}} = f. \tag{40}$$

One can show that the equation related to market clearing can be found by rearranging (1), (15), (16) and (19) to yield the same equation (26) as in the model without policies, which is reproduced again as

$$\gamma = \delta h - \frac{\theta}{1+\theta} \frac{E}{A}. \tag{41}$$

The condition related to the return from innovations will be affected though. Using Eq.(8) and the normalization $V[i(t)]_t = 1$ one can show that

$$r = \frac{E_t}{A_t} \Gamma(A_t, A_{t-T}) - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-rT} \right).$$

Comparing this with Eq.(20) with (22) reveals a number of effects of the tax f on the equilibrium return to innovations. The term $\Gamma^f(.)$ indicates that a new firm will be more profitable since the range of competitors is narrowed as consequence of the tax. The last term accounts for the negative effects of the tax on the profits firms can produce. With Eq.(18) and (38), the above expression leads to the other equilibrium equation

$$\gamma = \frac{E_t}{A_t} \frac{1}{1 - e^{-\gamma(1+\theta)T}} - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-(\gamma+\rho)T} \right) - \rho \tag{42}$$

Given the tax f, the balanced-growth equilibrium consists of values for T, γ and E/A that satisfy Eq. (40), (41) and (42). As shown in appendix C, with a positive lump-sum tax, the innovation rate will be higher than without the tax if and only if

$$\theta \frac{\gamma}{\gamma + \rho} (e^{\gamma T} - e^{-\rho T}) < 1.$$

For T small enough, γ is higher than without f. Using the shut-down condition Eq.(40), for every such a T, and the corresponding γ and E/A, a value of the tax f can be found that generates this equilibrium. It can then be concluded that for large enough f growth increases with a lump-sum tax.

This result stands in stark contrast with the one implied by the model without obsolescence and symmetric goods of Grossman and Helpman (1991). In that case, a lump-sum tax is unambiguously detrimental for growth through its negative effect on profit rates.

4.3 Trade and Openness

In this section, it is considered the case that the economy engages in trade with a foreign country that has a lower level of technological development. Formally, denote by $\tilde{A}_t$ the level of knowledge reached by this foreign partner at time t. The assumption is that $A_t/\tilde{A}_t > 1$. Since the focus is on balanced-growth situations, I will assume that this gap is constant over time so both countries are growing at the same rate. Therefore, it takes a constant period T for the foreign country to catch up with the domestic country's technology, so that $\tilde{A}_{t+T} = A_t$ all t. This implies that $A_t/\tilde{A}_t = \exp(\gamma T)$. I will further assume that the wage in the foreign economy, $\tilde{w}$, is lower than that in the domestic economy w and that the gap is such that all goods that are technologically feasible in the foreign country [i.e. $i \square \tilde{A}$] will be produced there. In other words, over time old varieties formerly produced domestically will eventually be produced in the foreign country.[9] This scenario thus captures the features of the international product cycle studied by Vernon (1966) and Stokey (1991). Firms in both countries operate under conditions of monopolistic competition, so the pricing rule for goods will now be

$$p(i)_t = \frac{1+\theta}{\theta}\frac{1}{A_{t(i)}} \times \begin{cases} w_t & i > \tilde{A}_t \\ \tilde{w}_t & i \square \tilde{A}_t \end{cases}$$

Define the term that represents the relative competitiveness of an innovator in the open domestic country as follows.

$$\Gamma^o(A_t, \tilde{A}_t) \equiv \frac{A_t^{1+\theta}}{\left(\frac{w_t}{\tilde{w}_t}\right)^\theta (\tilde{A}_t^{1+\theta} - A_0^{1+\theta}) + (A_t^\theta - \tilde{A}_t^{1+\theta})} = \frac{e^{\gamma(1+\theta)T}}{\left(\frac{w_t}{\tilde{w}_t}\right)^\theta + e^{\gamma(1+\theta)T} - 1} < 1,$$

[9] Mateos-Planas (1998) shows that this set of assumptions is consistent with the implications of a two-country general equilibrium model with imitation, rather than R&D, in the foreign country.

where the equality follows from the assumptions made about the technological gap, and the inequality is from the assumption $\tilde{w} < w$. Observe that for the closed economy analyzed in section 3, the balanced-growth value of Γ in Eq. (23) is unity. This indicates that a domestic innovator in the open economy faces fiercer competition from low-wage producers located in the foreign country. On its own, that would tend to have a detrimental effect on the incentives to conduct research. The net effect depends on a larger number of interactions. To work out these effects, start again with the relation related to clearing in the domestic market for the primary input. This leads to

$$\begin{aligned} \gamma &= \delta h - \frac{\theta}{1+\theta}\frac{E}{A}\Gamma^o(A_t, \tilde{A}_t)\left[1 - \left(\frac{\tilde{A}_t}{A_t}\right)^{1+\theta}\right] \\ &= \delta h - \frac{\theta}{1+\theta}\frac{E}{A}\Gamma^o(A_t, \tilde{A}_t)(1 - e^{-\gamma(1+\theta)T}) \end{aligned}$$

Comparison with Eq.(26) corresponding to the closed economy indicates that in the open economy foreign competition in the production of final goods frees resources for research. In terms of figure 1, the curve with negative slope shifts upwards with trade.

The second equilibrium condition that relates to the market value of innovation for the open economy reads

$$\gamma = \frac{E}{A}\Gamma^o(A_t, \tilde{A}_t)(1 - e^{-(r+\gamma\theta)T}) - \theta\gamma - \rho.$$

This differs from Eq.(27) that corresponds to the closed economy. As mentioned earlier, trade with a low-wage country reduces current profits as well as the lifetime span of an innovating firm in the high-wage domestic economy. Graphically, the curve with positive slope in figure 1 will shift downwards with openness. Combining the two last equations, the net effect of trade can be determined. It is convenient to define

$$\varphi(T, \gamma) \equiv \frac{1 - e^{-\gamma(1+\theta)T}}{1 - e^{-(\rho+(1+\theta)\gamma)T}}.$$

Then the equilibrium growth rate must satisfy

$$\gamma = \frac{1}{1+\theta\varphi(T,\gamma)}\left[\delta h - \frac{\theta}{1+\theta}\rho\varphi(T,\gamma)\right],$$

which is to be compared with the closed-economy expression in Eq.(28). Since $\varphi(T,\gamma) < 1$, growth for the open economy is higher. This outcome is driven by the diversion of resources from final-goods production into research that occurs when foreign competition intensifies the process of economic obsolescence of existing domestic technologies.

5 Conclusions

This paper studies a model of growth where technological change has an embodied component that makes new vintages of firms more productive. The phenomenon of obsolescence is a feature of the growth process whereby new sectors or firms replace existing ones. The transitional and long-run properties of the economy are investigated. The analysis identifies policies that have effects on growth through their influence on the course of the *creative destruction* associated with the ongoing process of emergence and obsolescence of firms and sectors.

This paper shows that subsidies targeted at young innovative sectors or firms will have a positive growth effect. On the contrary, subsidies aimed at older vintages will be detrimental for growth. These growth effects arise entirely from the selective nature of the policies. A uniform non-selective subsidy would have no growth effect. Similarly, a lump-sum tax on firms in the economy that forces the old less-profitable firms to shut down may have a positive effect on growth. It is interesting that, in the absence of obsolescence, a lump-sum tax of the type analyzed here would reduce the growth rate unambiguously. The degree of openness to international trade of a more developed economy with a less developed partner will also be growth-enhancing for similar reasons. Foreign competition moves the production with old technologies to the foreign country and releases resources for innovation at home.

The main contribution of this paper is to demonstrate that, once obsolescence is accounted for, new and potentially important growth-effects of policies show up. The study of these effects necessarily demands a model that, like the one proposed in this paper, accommodates heterogeneous firms and sectors.

This paper stops short of producing a detailed welfare analysis of the policies considered. In particular, selective subsidies to innovative sectors have a dynamic welfare benefit but produce a static loss. The net welfare effects of this and other policies deserve to be investigated in future work.

The model used in this paper is very stylized and is thus bound to have limitations. The analytical simplicity of the model comes at the cost of sacrificing the possibility of relating the model to observable data on sectors and industries. But a more quantitatively oriented approach would be required to assess the practical significance of the effects of the policies considered in this paper. Such an approach would also be necessary to evaluate the net welfare effects of these policies.

References

[1] Aghion, P. and P. Howitt. A Model of Growth Through Creative Destruction. *Econometrica, 60*, 1992.

[2] Caballero R. J. and M. Hammour. The Cleansing Effect of Recessions. *American Economic Review, 84, 5, December*, 1994.

[3] Caballero R. J. and A. B. Jaffe. How High Are the Giants' Shoulders: an Empirical Assesment of Knowledge Spillovers and Creative Destruction in a Model of Economic Growth. *NBER Macroeconomics Annual*, 1993.

[4] Dixit, A. and J. Stglitz. Monopolistic Competition and Optimal Product Diversity. *American Economic Review, 67*, 1977.

[5] Grossman G. and E. Helpman. Protection for Sale. *American Economic Review, 84*, 1994.

[6] Grossman G. M. and E. Helpman. *Innovation and Growth in the Global Economy. M.I.T. Press*, 1991.

[7] Hillman, A. L. . Declining Industries and Political SupportProtectionist Motives. *American Economic Review, 72*, 1982.

[8] Jones, C. I. R&D Models of Endogenous Growth. *Journal of Political Economy, 103*, 1995.

[9] Howitt, P. Steady Endogenous Growth with Population and R&D Inputs Growing. *Journal of Political Economy, 107*, 1999.

[10] Krueger, A. O. Government Failures in Development. *Journal of Economic Perspectives, 4, 3*, 1990.

[11] Lai, E. Schumpeterian Growth with Gradual Product Obsolescence. *Journal of Economic Growth, 3*, 1998.

[12] Maggi, G. and E. Rodriguez-Clare. Import Penetration and the Politics of Trade Protection. *NBER Working Paper 6711*, 1998.

[13] Mateos-Planas, X. Trade, the Product Cycle, and the International Production Pattern. *Mimeo*, 1998.

[14] Mokyr, J. *The Lever of Riches: Technological Creativity and Economic Progress*, Oxford University Press, 1990.

[15] Pack, H. and L. E. Westphal. Industrial Strategy and Technological Change: Theory versus Reality. *Journal of Development Economics, 22*, 1986.

[16] Parente, S. Technology Adoption, Learning-by-Doing and Economic Growth. *Journal of Economic Theory, 63*, 1994.

[17] Romer, P. Endogenous Technological Change. *Journal of Political Economy, 98*, 1990.

[18] Rosenberg, N. Inside the Black Box: Technology and Economics. *Cambridge University Press*, 1982.

[19] Schumpeter, J. A. *Capitalism, Socialism and Democracy*, New York: Harper, 1942.

[20] N. Stokey. Learning by Doing and the Introduction of New Goods. *Journal of Political Economy, 96*, 1988.

[21] N.Stokey. The Volume and Composition of Trade Between Rich and Poor Countries. *Rev Ec Studies*, 1991.

[22] Trefler, D. "Trade Liberalization and te Theory of Endogenous Protection. *Journal of Political Economy, 101*, 1994.

[23] Vernon. International Investment and International Trade in the Product Cycle. *Quarterly Journal of Economics, 80*, 1966.

[24] Westphal L. E.. Industrial Policy in an Export-Propelled Economy: Lessons from South Korea's Experience. *Journal of Economic Perspectives, 4, 3*, 1990.

[25] Young, A. Invention and Bounded Learning by Doing. *Journal of Political Economy, 101*, 1993.

[26] Young, A. A Tale of Two Cities: Factor Accumulation and Technical Change in Hong-Kong and Singapore. *NBER Macroeoconomics Annual*, 1992.

[27] Young, A. Learning by Doing and the Dynamic Effects of International Trade. *Quarterly Journal of Economics, 106*, 1991.

A Welfare Analysis

The planner's static problem consists of maximizing

$$\int_0^n c(j)^\alpha dj$$

subject to the constraint

$$h_c = \int_0^n \frac{c(j)}{A_{t(j)}} dj,$$

where h_c denotes the amount of primary input available in the consumption good sector. The solution to this problem delivers the optimal allocation of this input across existing sectors at a time t:

$$c(i)_t = h_c \frac{A_{t(i)}^{1+\theta}}{\frac{1}{1+\theta}(A_t^{1+\theta} - A_0^{1+\theta})}.$$

In the market equilibrium the same condition holds provided that there $h_c = (\theta/(1+\theta))(h + (1/(1+\theta))(\rho/\delta))$. This allows the computation of the indirect utility as

$$\int_0^{n_t} c(i)_t di = h_c^{\theta/(1+\theta)} A_t \left(\frac{1}{1+\theta}\right)^{1/(1+\theta)} = h_c^\alpha A_t (1-\alpha)^{\alpha/(1-\alpha)}$$

where the 2nd equality uses the definition of θ in Eq. (6). For the sake of comparison, in the symmetric case the exponent on A is smaller $(1-\alpha)/\alpha$.

The planner's dynamic problem is the choice of the paths for A and h_c that maximize the representative households utility. The Hamiltonian associated with this problem is

$$H = e^{\rho t} \log((1-h_r)A^{1/\alpha}) + \mu(\delta h_r A).$$

The solution on a BGP leads to the optimal

$$h_r^* = h - \frac{\alpha}{\delta}\rho = h - \frac{\theta}{1+\theta}\frac{\rho}{\delta}.$$

The resulting optimal growth rate is

$$\gamma^* = \delta(h - \alpha\frac{\rho}{\delta}).$$

Comparing with the market growth rate γ in Eq. (31) shows that $\gamma^*/\gamma = 1/(1-\alpha)$. The same holds for the symmetric economy analyzed by Grossman and Helpman (1991).

B Subsidies

Consider production is subsidized at rates $\eta(i)_t$. As with the basic model, the equilibrium has two main relations. The first comes from market clearing that determines the amount of labor to R&D. This is still given by equations (1), (15), (16), and (19). The other relation follows from the equilibrium valuation of innovations. Again, this consists of equations (18) and (8) [or (20)]. These relations are incomplete though. To close the model, one needs to determine the effect of the subsidy on the amount of the differentiated consumption goods and the profit rate.

Equation (6) still holds for the demand of each intermediate. If i is subsidized at the rate $\eta(i)$ then profits are

$$\pi(i)_t = x(i)_t \left[p(i)_t(1+\eta(i)_t) - \frac{w_t}{\phi(A_{t(i)})}\right]$$

The FOC for profits maximization becomes,

$$x(i)_t[-(1+\theta)(1+\eta(i)_t)+\frac{w_t}{p(i)_t\phi(A_{t(i)})}(1+\theta)+(1+\eta(i)_t)]=0$$

implies mark-up pricing,

$$p(i)_t=\frac{1+\theta}{\theta}\frac{1}{1+\eta(i)_t}\frac{w_t}{\phi(A_{t(i)})}$$

Assuming, as before, that $\phi(A)=A$, the quantity produced and sold by the firm is,

$$x(i)_t=E_t\frac{\theta}{1+\theta}\frac{1}{w_t}\frac{(1+\eta(i)_t)^{1+\theta}A_{t(i)}^{1+\theta}}{\int^{n_t}(1+\eta(j)_t)^{\theta}A_{t(j)}^{\theta}dj}$$

With this equations, the profit rate becomes

$$\pi(i)_t=E_t\frac{1}{1+\theta}\frac{(1+\eta(i)_t)^{1+\theta}A_{t(i)}^{\theta}}{\int^{n_t}(1+\eta(j)_t)^{\theta}A_{t(j)}^{\theta}dj}$$

Now the expressions for quantities and profits can be used in the derivation of the two main equilibrium equations. This will be done for the different types of policies separately.

Protection of declining sectors

Assume that $\eta(i)_t=\eta$ for j such that $t(i)<t-T$, and $\eta(i)_t=0$ otherwise. That is, all firms over age T benefit from the constant subsidy. With $\Gamma(A_t,A_0)$ as defined in the main text Eq.(23)

$$\Gamma(A_t)\equiv\frac{A_t^{1+\theta}}{(1+\eta)^{\theta}\left[A_{t-T}^{1+\theta}-A_0^{1+\theta}\right]+\left[A_t^{1+\theta}-A_{t-T}^{1+\theta}\right]}$$

We will focus on long-run balanced outcomes with constant innovation rate γ. In this context

$$\Gamma(A_t\rightarrow\Gamma_{\infty}=\frac{1}{(1+\eta)^{\theta}\left(e^{-\gamma(1+\theta)T}-e^{-\gamma(1+\theta)t}\right)+1-e^{-\gamma(1+\theta)T}}$$

From the abov expression for $c(i)$,

$$\begin{aligned}c(i)_t=E_t\frac{\theta}{1+\theta}\frac{1}{w_t}&\frac{(1+\eta(i))^{1+\theta}A_{t(i)}^{1+\theta}}{\frac{1}{1+\theta}\left\{(1+\eta)^{\theta}\left[A_{t-T}^{1+\theta}-A_0^{1+\theta}\right]+\left[A_t^{1+\theta}-A_{t-T}^{1+\theta}\right]\right\}}\\
&=E_t\theta\frac{1}{w_t}\frac{(1+\eta_i)^{1+\theta}A_{t(i)}^{1+\theta}}{(1+\eta)^{\theta}\left[A_{t-T}^{1+\theta}-A_0^{1+\theta}\right]+\left[A_t^{1+\theta}-A_{t-T}^{1+\theta}\right]}\\
&=E_t\theta\frac{1}{w_t}\left(\frac{A_{t(i)}}{A_t}\right)^{1+\theta}(1+\eta)^{1+\theta}\Gamma(A_t)\end{aligned}$$

We can recover the expression for profits as,

$$\begin{aligned}\pi(i)_t&=\frac{1}{\theta}\frac{w_t}{A_{t(i)}}x(i)_t\\
&=E_t\frac{(1+\eta(i))^{1+\theta}A_{t(i)}^{\theta}}{(1+\eta)^{\theta}\left[A_{t-T}^{1+\theta}-A_0^{1+\theta}\right]+\left[A_t^{1+\theta}-A_{t-T}^{1+\theta}\right]}\\
&=\frac{E_t}{A_t}\left(\frac{A_{t(i)}}{A_t}\right)^{\theta}(1+\eta)^{1+\theta}\Gamma(A_t)\end{aligned}$$

Market clearing

The block of equilibrium equations (1), (15), (16), and (19) can now be developed. The factor-market clearing condition (15)

$$h_{rt} + \int^{n_t} h(i)_t di = H$$

with the law-of-motion of knowledge (1) with (2) ,$\frac{\dot{A}_t}{A_t}\frac{1}{\delta} = h_{rt}$, R&D equilibrium condition (19) $w_t = \delta A_t$ and factor demand functions (16) $h(i)_t = \frac{x(i)_t}{A_{t(i)}}$, yield a condition for any equilibrium with positive innovation. First,

$$\begin{aligned}
\int^{n_t} h(i)_t di &= E_t \theta \frac{1}{w_t} \Gamma(A_t) \left(\frac{1}{A_t}\right)^{1+\theta} \\
&\quad \times \left\{ \int^{t-T} A_\tau^\theta (1+\eta)^{1+\theta} \dot{A}_\tau d\tau + + \int_{t-T}^{t} A_\tau^\theta \dot{A}_\tau d\tau \right\} \\
&= E_t \theta \frac{1}{w_t} \Gamma(A_t) \left(\frac{1}{A_t}\right)^{1+\theta} \frac{1}{1+\theta} \\
&\quad \times \left\{ (1+\eta)^{1+\theta} \left[A_{t-T}^{1+\theta} - A_0^{1+\theta} \right] + \left[A_t^{1+\theta} - A_{t-T}^{1+\theta} \right] \right\}
\end{aligned}$$

Using that $w = \delta A$ in Eq.(19), and considering long-run outcomes market clearing can be posed,

$$\begin{aligned}
h &= \gamma \frac{1}{\delta} + \frac{\theta}{1+\theta} \frac{1}{\delta} \frac{E_t}{A_t} \Gamma(A_t, A_0) \\
&\quad \left[(1+\eta)^{1+\theta} \left(e^{-\gamma(1+\theta)T} - e^{-\gamma(1+\theta)t} \right) + 1 - e^{-\gamma(1+\theta)T} \right]
\end{aligned}$$

For graphical representation we will use the BGP relation,

$$\frac{E}{A} = \frac{\delta h - \gamma}{\Gamma_\infty \left[(1+\eta)^{1+\theta} e^{-\gamma(1+\theta)T} + 1 - e^{-\gamma(1+\theta)T} \right]} \frac{1+\theta}{\theta}$$

Market valuation

The other equilibrium equation analogous to Eq. (23) comes from the analysis of firms' valuation and the no-arbitrage condition. Letting $V(\tau)_t$ denote the present (fundamental) value of a firm in cohort τ at time t. I find that

$$\frac{dV(t)_t}{dt} = \dot{V}(t)_t + \gamma V(t)_t + e^{-\int_t^{t+T} r_u du} \pi(t)_{t+T}^- - e^{-\int_t^{t+T} r_u du} \pi(t)_{t+T}^+,$$

with

$$\pi(t)_{t+T}^+ = (1+\eta)^{1+\theta} \pi(t)_{t+T}^-,$$

and

$$\pi(t)_{t+T}^- = \frac{E_{t+T}}{A_{t+T}} \left(\frac{A_t}{A_{t+T}}\right)^\theta \Gamma(A_{t+T}).$$

We use the normalization:

$$V(t)_t = 1.$$

In a BGP with constant r and γ

$$\begin{aligned}\frac{dV(t)_t}{dt} &= \dot{V}(t)_t + \theta\gamma + e^{-rT}(\pi(t)^{-}_{t+T} - \pi(t)^{+}_{t+T}) = \\ &= \dot{V}(t)_t + \theta\gamma + e^{-rT}\frac{E_{t+T}}{A_{t+T}}\left(\frac{A_t}{A_{t+T}}\right)^{\theta}\Gamma(A_{t+T})(1-(1+\eta)^{1+\theta}) = 0\end{aligned}$$

With the no-arbitrage condition

$$r = \pi(t)_t + \dot{V}(t)_t$$

and the fact that

$$\pi(t)_t = \frac{E_t}{A_t}\Gamma(A_t)$$

we get

$$r = \frac{E_t}{A_t}\Gamma(A_t) - \theta\gamma + e^{-rT}\frac{E_{t+T}}{A_{t+T}}\left(\frac{A_t}{A_{t+T}}\right)^{\theta}\Gamma(A_{t+T})((1+\eta)^{1+\theta}-1)$$

Finally, intertemporal optimality gives the behavior of consumption expenditures

$$\frac{\dot{E}}{E} = r - \rho,$$

so that in a BGP we have,

$$\gamma = \frac{E_t}{A_t}\Gamma(A_t) - \theta\gamma + e^{-rT}\frac{E_{t+T}}{A_{t+T}}\left(\frac{A_t}{A_{t+T}}\right)^{\theta}\Gamma(A_{t+T})((1+\eta)^{1+\theta}-1) - \rho.$$

Or, rearranging,

$$(1+\theta)\gamma = \frac{E}{A}\Gamma_\infty\left[1 + e^{-(r+\gamma\theta)T}((1+\eta)^{1+\theta}-1)\right] - \rho,$$

In the long-run consumption must grow at the same rate as knowledge and then $r = \gamma + \rho$. Therefore, after rearranging, we get

$$\frac{E}{A} = \frac{(1+\theta)\gamma + \rho}{\Gamma_\infty[1 + e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)]}$$

Balanced growth equilibrium

Combining the two above conditions

$$\frac{1 + e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}{1 + e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)} = \frac{(1+\theta)\gamma + \rho}{\delta h - \gamma}\frac{\theta}{1+\theta}.$$

Infant industry protection

To make this case, we assume that there a positive subsidy rate only for firms born after $t-T$. For this case, it is useful to redefine

$$\Gamma(A_t) \equiv \frac{A_t^{1+\theta}}{A_{t-T}^{1+\theta} - A_0^{1+\theta} + (1+\eta)^{\theta}\left[A_t^{1+\theta} - A_{t-T}^{1+\theta}\right]}$$

which along long-run balanced growth paths reads

$$\Gamma_\infty = \frac{1}{e^{-\gamma(1+\theta)T} + (1+\eta)^\theta(1-e^{-\gamma(1+\theta)T})}$$

Market clearing

With the definiton of $\Gamma(A_t, A_0)$ in the text, market-clearing now requires,

$$\begin{aligned} h &= \gamma\frac{1}{\delta} + \frac{\theta}{1+\theta}\frac{1}{\delta}\frac{E_t}{A_t}\Gamma(A_t) \\ &\quad \left[e^{-\gamma(1+\theta)T} + (1+\eta)^{1+\theta}(1-e^{-\gamma(1+\theta)T})\right] \end{aligned}$$

which more conveniently can be written as,

$$\frac{E}{A} = \frac{\delta h - \gamma}{\Gamma_\infty\left[(1+\eta)^{1+\theta} - e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)\right]}\frac{1+\theta}{\theta}$$

Market valuation

On the valuation side, normalizing value of innovations to one, we find that,

$$\frac{dV}{dt} = V_t' + \theta\gamma + e^{-rT}\frac{E}{A}\left(\frac{A_t}{A_{t+T}}\right)^\theta \Gamma_\infty((1+\eta)^{1+\theta}-1) = 0.$$

Provided that $\pi(t)_t = (E/A)\Gamma_\infty(1+\eta)^{1+\theta}$, the no-arbitrage condition $r = \pi(t)_t + V_t'$ implies that

$$r = (E/A)\Gamma_\infty\left[(1+\eta)^{1+\theta} - e^{-rT}e^{-\gamma\theta T}((1+\eta)^{1+\theta}-1)\right] - \theta\gamma.$$

Using that in BGP $r = \gamma + \rho$ we arrive at

$$\frac{E}{A} = \frac{(1+\theta)\gamma + \rho}{\Gamma_\infty\left[(1+\eta)^{1+\theta} - e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)\right]}$$

Balanced growth equilibrium

From the two long-run equilibrium equations we can study the determination of γ to determine that larger η increases the rate of innovation. More explicitly,

$$\frac{(1+\eta)^{1+\theta} - e^{-\rho T}e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)}{(1+\eta)^{1+\theta} - e^{-\gamma(1+\theta)T}((1+\eta)^{1+\theta}-1)} = \frac{(1+\theta)\gamma+\rho}{\delta h - \gamma}\frac{\theta}{1+\theta}.$$

C Lump-sum taxation

Let T denote the age of the oldest frim in operation. Following analysis similar to that in section 3 delivers the following equations for gross profit rates and quantities

$$x(i)_t = \frac{E_t}{A_t}\frac{\theta}{\delta}\left(\frac{A_{t(i)}}{A_t}\right)^{1+\theta}\Gamma(A_t, A_{t-T})$$

$$\pi(i)_t = \frac{E_t}{A_t}\left(\frac{A_{t(i)}}{A_t}\right)^{\theta}\Gamma(A_t, A_{t-T})$$

with

$$\Gamma(A_t, A_{t-T}) \equiv \frac{A_t^{1+\theta}}{A_t^{1+\theta} - A_{t-T}^{1+\theta}} = \frac{1}{1 - e^{-\gamma(1+\theta)T}}$$

where the second equality uses the constant-growth assumption.

Now one can calculate the age T at which a firm will be discontinued. To that end, set

$$\pi(i(t))_{t+T} = \frac{E_{t+T}}{A_{t+T}} \left(\frac{A_t}{A_{t+T}}\right)^{\theta} \Gamma(A_{t+T}, A_t) - f = 0,$$

which, on a BGP with $A_{t+T} = \exp(\gamma T) A_t$ and constant E/A delivers the relation between f and T given γ and E/A

$$\frac{E}{A} e^{-\gamma\theta T} \frac{1}{1 - e^{-\gamma(1+\theta)T}} = f.$$

One can show that the equation related to market clearing can be found by rearranging (1), (15), (16) and (19) to yield the same equation as in the model without policies,

$$\gamma = \delta h - \frac{\theta}{1+\theta} \frac{E}{A}.$$

The condition related to the return from innovations will be affected. Differentiate the fundamental value of the firm to obtain

$$\begin{aligned}
\frac{dV[i(t)]_t}{dt} &= \dot{V}[i(t)]_t + \theta\gamma \int_t^{t+T} e^{-r(s-t)} [\pi[i(t)]_s + f] ds \\
&= \dot{V}[i(t)]_t + \theta\gamma V[i(t)]_t + \theta\gamma f (-1/r) \left[e^{-r(s-t)}\right]_t^{t+T} \\
&= \dot{V}[i(t)]_t + \theta\gamma V[i(t)]_t + \theta\gamma f \frac{1}{r} \left(1 - e^{-rT}\right)
\end{aligned}$$

Using Eq. (8) and the normalization $V[i(t)]_t = 1$ constant it follows

$$r = \frac{E_t}{A_t} \Gamma(A_t, A_{t-T}) - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-rT}\right).$$

With Eq. (18) then

$$\begin{aligned}
\gamma &= \frac{E_t}{A_t} \Gamma(A_t, A_{t-T}) - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-(\gamma+\rho)T}\right) - \rho \\
&= \frac{E_t}{A_t} \frac{1}{1 - e^{-\gamma(1+\theta)T}} - \theta\gamma - \theta\gamma \frac{1}{r} f \left(1 - e^{-(\gamma+\rho)T}\right) - \rho
\end{aligned}$$

Combine the market-clearing and the valuation conditions to substiture E/A out to obtain

$$\frac{(1+\theta)\gamma + \rho}{(\delta h - \gamma)\frac{1+\theta}{\theta}} = \frac{1}{1 - e^{-\gamma(1+\theta)T}} - \frac{\theta\gamma \frac{1}{r} f (1 - e^{-(\gamma+\rho)T})}{(\delta h - \gamma)\frac{1+\theta}{\theta}}$$

On the other hand, the shut-down condition with the market-clearing condition can be written as

$$\frac{f}{(\delta h - \gamma)\frac{1+\theta}{\theta}} = \frac{e^{-\gamma\theta T}}{1 - e^{-\gamma(1+\theta)T}}.$$

Combining the two last expressions gives an expression in γ and T

$$\frac{(1+\theta)\gamma + \rho}{(\delta h - \gamma)\frac{1+\theta}{\theta}} = \frac{1}{1 - e^{-\gamma(1+\theta)T}} \left[1 - \theta \frac{\gamma}{\gamma + \rho} e^{-\gamma\theta T} (1 - e^{-(\gamma+\rho)T})\right]$$

In the case without the tax f, the equilibrium γ is characterized by this equation when the left-hand side is equal to 1. Therefore, under the lump-sum tax, the innovation rate will be higher if and only if

$$\theta \frac{\gamma}{\gamma+\rho} e^{-\gamma\theta T}(1-e^{-(\gamma+\rho)T}) < e^{-\gamma(1+\theta)T}$$

or, after rearranging,

$$\theta \frac{\gamma}{\gamma+\rho}(e^{\gamma T}-e^{-\rho T}) < 1.$$

The left side is an incresing function of T, and for T small enough the inequality holds. It follows that for T smaller than some value, γ is higher than without f. Using the shut-down condition, for every such a T, and corresponding γ and E/A, an f can be found that generates this equilibrium. So the result is that there exist large enough values for f such that growth increases with a lump-sum tax.