Department of Economics
University of Southampton
Southampton SO17 1BJ
UK

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INTERNATIONAL ENVIRONMENTAL AGREEMENTS, UNCERTAINTY AND LEARNING – THE CASE OF STOCK DEPENDENT UNIT DAMAGE COSTS

Alistair Ulph

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INTERNATIONAL ENVIRONMENTAL AGREEMENTS , UNCERTAINTY AND LEARNING – THE CASE OF STOCK DEPENDENT UNIT DAMAGE COSTS

ALISTAIR ULPH
(University of Southampton)

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ABSTRACT

In Ulph (2002) I analysed how the possibility of future resolution of uncertainty about damage costs affected the incentives and timing for countries to join a self-enforcing international environmental agreement (IEA). I analysed two membership rules – fixed (countries commit whether to join an IEA for all periods) and variable (countries decide each period whether to join). While total damage costs depended on the stock of pollution, average damage costs did not. One consequence was that, in the variable membership model the number of countries who joined in the current period could not be affected by future learning. In this paper I allow unit damage costs to depend on the stock of pollution. I show that while this complicates the analysis, all the results of the previous paper are essentially unaffected. In particular, with variable membership future learning has almost no affect on current membership.

Key words: International Environmental Agreements, uncertainty, learning, fixed membership, variable membership.

JEL Classification: F02, F18, Q4.

1. INTRODUCTION

Trying to reach an international agreement on problems such as climate change is beset by the difficulties of dealing with a problem which is truly global, and hence ideally requires actions by all countries, by the time scale over which current reductions in greenhouse gas emissions might affect future climate, by the still considerable uncertainty about the possible benefits and costs of actions to reduce greenhouse gas emissions, both in aggregate and in terms of their distribution across countries, and the fact that we will get better information on these costs and benefits in the future. The recent IPCC (2001) reports give an excellent summary of the major uncertainties about the potential impacts of climate change and their associated degree of confidence, and the sequence of IPCC reports is an excellent illustration of how scientific information has improved over time.

The possibility of future learning leads to the important *timing* question: should we delay reductions in emissions until we get better information on impacts, or should we accelerate emissions reductions in case we learn bad news and have little time to respond to it? There is also the important question of how does the answer to this question depend on the significant irreversibilities involved, both in the accumulation of greenhouse gases and the accumulation of capital. There is now a significant literature on these questions both theoretical[1] and with empirical applications to climate change[2]. In general the theoretical arguments are ambiguous and the empirical literature suggests small effects, leading to the claim by Karp and Zhang (2001) that the issue of uncertainty and learning is a red herring for climate change policy and policy-makers should base their current emissions strategy on the best scientific information currently available[3].

However all this literature assumes there is a single global decision-maker. But a crucial feature of global environmental problems is the absence such a world authority, and hence the need to tackle such problems by means of international environmental agreements (IEAs), which, given the sovereignty of independent nation states, have to be designed to be self-enforcing. This leads to a second, and perhaps more important, timing question: how does the possibility that uncertainty will be resolved through future learning affect the incentives for countries to join international environmental agreements, both now and in the future. In this context it is worth noting that uncertainty about the impacts of climate change and the need for further research was one of the reasons the US gave for not ratifying the Kyoto Agreement.

In an earlier paper (Ulph (2002)) I reviewed the very small literature that had addressed this question, and introduced a model which significantly extended previous analyses.

[1] See Fisher , Hanemann and Narain (2001) for an excellent survey.

[2] See, for example, Manne and Richels (1992), Peck and Teisberg (1993), Grubb, Chapuis and H-Duong (1995), Kolstad (1996), Nordhaus and Popp (1997), Ulph and Ulph (1997), Kelly and Kolstad (1999), Karp and Zhang (2000, 2001).

[3] The possibility of catastrophic risks such as the disintegration of the West Antartic or Greenland ice sheets or partial shutdown of the North Atlantic thermohaline circulation system does suggest the need for stronger current action to reduce emissions, see Fisher and Narain (2001) and Gjerde, Grepperud and Kverndokk (1999).

The model was an explicit model of IEA membership which extended a model by Rubio and Ulph (2002a) to include uncertainty and learning, and the model by Rubio and Ulph had in turn extended the model of Barrett (1994) by allowing for a stock pollutant. Unlike Na and Shin (1998), I allowed for an arbitrarily large number of countries and the solution concept employed allowed, in principle, for any number of signatories between 2 and the grand coalition of all countries. Because the model was dynamic I could address issues which could not be addressed in the static framework of Na and Shin, or indeed in much of the literature on IEAs. In a dynamic framework there were two possible models of how countries decided whether or not to join an IEA. Countries could decide at the outset whether or not to join an IEA and are committed to that decision for all future time periods and states of the world; I called this the fixed membership model. More consistent with the notion of national sovereignty, countries could decide in each period and each state of the world whether to join an IEA; I called this the variable membership model. This allowed analysis of how membership varies over time as information is refined. By comparing the fixed and variable membership models I could address the question - is it better to have countries commit at the outset to their membership decision or to allow them to decide each period?

I showed that if membership is fixed, then, if expected damage costs are high and there is a relatively high degree of uncertainty about damage costs, learning leads to lower membership and lower global welfare than no learning; but otherwise learning leads to more members and higher welfare than if there is no prospect of learning. On the other hand if membership is variable, then, for the special case I considered, first period membership is unaffected by whether or not there is learning, but second period membership is (on average) higher with learning than no learning, but global welfare is lower. As to whether it is better to have fixed or variable membership, I showed that fixed membership results in higher expected global welfare than variable membership if it leads to at least as many signatories who abate pollution in each period and each state of the world. Otherwise, variable membership yields higher expected global welfare.

However, as just indicated, the model I employed had a special feature. While total damage costs in any period depended on the total stock of pollutants, average damage costs were independent of the stock of pollution. It was this assumption that implied that in the variable membership model first period membership was unaffected by whether or not countries would get better information in the future. Since an important question is how the possibility of *future* learning might affect *current* incentives to join an IEA, this was an undesirable feature of that model. In this paper I keep all the other features of the model in the previous paper, but now allow unit damage costs to depend on the stock of pollution. That now ensures that in principle current membership depends on whether or not countries will get better scientific information in the future. However, while using this more general model complicates analysis of the model (for example by leading to the possibility of multiple stable IEAs for a given set of parameters), I show that the results of Ulph (2002) are virtually unaffected. In particular, it turns out that in the variable membership model, current membership is only very slightly affected by whether or not there is future learning. So the fact that there may be better scientific information available in the future does not markedly affect current incentives to join an IEA. Thus it appears that for second timing question the possibility of getting better scientific information in the future may also be something of a 'red herring'.

In section 2 I set out the basic model. In section 3 I analyse what stable IEAs will form in four cases: No Learning with Fixed Membership; No Learning with Variable Membership, Learning with Fixed Membership and Learning with Variable Membership. In section 4 I use these results to compare IEA membership and global welfare when there is No Learning and Learning and when there is Fixed Membership or Variable Membership. Section 5 concludes and suggests obvious lines for further extensions.

2. THE BASIC MODEL.

The model is an extension of Barrett (2002), Rubio and Ulph (2002a) and Ulph (2002). There are N identical countries indexed by $i = 1,\ldots, N$. There are two time periods, t =1, 2, which I think of as the present and the future. For the moment I ignore uncertainty. Denote by q_{it} the amount of pollution emitted by country i in period t. To keep things simple I assume that q_{it} can take one of only two values, which I normalise to be 0 or 1 and interpret as *abate* or *pollute*[4]. I denote by Q_{it} the total emissions of all countries other than i and by Q_t the total emissions of all N countries in period t.

I deal with a global stock pollution problem, and assume that total damage costs in each country in period t depend on the stock of pollution $Z_t = Z_{t-1} + Q_t$ at the end of the period. Note that, for simplicity, I am ignoring any natural decay in the stock of the pollutant (I will also ignore discounting). I normalise by assuming that $Z_0 = 0$. I assume that the unit damage cost in period t depends on the global stock of pollution at the beginning of the period as follows: $c_t = c + kZ_{t-1}$. I denote the net benefit function of country i in period t by:

where b is the (constant) benefit a country derives from a unit of emissions. I henceforth normalise b to 1, and define parameters which I interpret as damage cost parameters *relative* to unit benefits of emissions.

To introduce uncertainty, I assume that the damage cost parameters (relative to unit benefits) are not known with certainty. For simplicity, I assume that there are only two states of the world, high damage cost and low damage cost which occur with equal probability, 0.5. I denote the damage cost parameters in the high cost state by $\gamma\eta \equiv \gamma + \delta; \kappa\eta = \kappa + \lambda$, and in the low cost state by .
Expected damage cost parameters are γ and κ, while δ and λ are measures of the dispersion of damage costs. Note that I continue to assume that all countries are identical, so that damage costs are perfectly correlated across countries. To capture learning, again I assume the simplest form of learning (following (Ulph and Ulph (1996), Na and Shin (1998)) namely that if learning takes place then between period 1 and 2 all countries learn with certainty the true value of their damage cost parameters. If there is no learning then

[4] We make this simplification because even in the one-period model with certainty, allowing for continuous emission levels, for example by having quadratic damage and abatement costs, makes the analysis of IEA stability quite complicated. In Barrett' s seminal 1994 paper he resorted to numerical simulations to derive his results. Moreover he did not take account of the need to ensure that emissions must be non-negative. Taking proper account of such restrictions by allowing for corner solutions complicates the analysis further (see Rubio and Ulph (2002c)). Since my aim is to extend the basic Barrett model by having a stock pollutant and uncertainty and learning, I follow Rubio and Ulph (2002a) in taking the simplest version of the Barrett model which has discrete emission levels. I conjecture that the results are not sensitive to this simplification, but it will be important for future research to test that conjecture.

countries have to make their period 2 decisions before they know the true value of their damage cost parameters[5]. Finally I assume that all countries are risk neutral. Note that the model in Ulph (2002) is a special case of this model in which , i.e. unit damage costs do not depend on the stock of pollution.

Putting the above together, I now define for country *i* the expected present value of net benefits over the two periods when learning takes place as:

(1a)

If learning does not take place, then and (1a) becomes:

(1b)

For reasons which will become clear shortly I need to make further assumptions about parameter values:

Assumption 1:

To ensure the constraints on parameters are satisfied, and to slightly reduce the number of parameters, I choose parameters in the following sequence: (i) choose *N*; (ii) choose (iii) choose (iv) choose

So the parameter set can be thought of as or, more simply, as .

It is straightforward to derive[6]:

Result 1 (Non-cooperative and Cooperative Equilibria) *Given Assumption 1: (i) if all countries act non-cooperatively then the dominant strategy for each country is to pollute in each period and each state of the world (if learning is possible) and the expected payoff to each country is* $2-N(3\gamma + 2N\kappa) < 0$, *whether learning occurs or not; (ii) if all countries act cooperatively (jointly maximise aggregate net benefits) then each country*

[5] Since I assume that damages occur in period 1 and 2, with the same damage cost parameters, it might be asked why observation of damages in period 1 does not reveal the true state of the world, so learning always takes place. One response is that it takes time to process the information about damage costs that occurred in period 1. Another is that this is just a very simple model and should not be interpreted too literally.

[6] All proofs are in the Appendix.

abates in each period and each state of the world, and receives a net payoff of 0, whether learning occurs or not; (iii) the gain to cooperation (difference in payoff if all cooperate and if all do not cooperate) for each country is $G \equiv N(3\gamma + 2N\kappa)-2 > 0$, which is increasing in N, γ and κ.

The first part of Result 1 ensures that the non-cooperative global pollution game is a Prisoner's Dilemma. The last part of Result 1 says that the gains to cooperation will be greater the more countries there are, and the more damaging is pollution relative to the benefits from emissions of pollution.

Result 1 covers the two extremes where all countries either cooperate or do not cooperate. In the next section I analyse the intermediate case in which some countries join an International Environmental Agreement. I complete this section by setting out three general points about how I model an International Environmental Agreement.

The first general point is that an IEA is modelled as a two-stage game. In the second-stage (the Emission Game), countries decide their emission levels, with each non-signatory country (denoted by *f* – fringe or free-rider) choosing its emission level to maximise its net-benefit, taking as given emissions of all other countries, while the signatory countries (denoted by *s*) choose their emissions to jointly maximise their aggregate net benefit, taking as given the emissions of non-signatory countries[7]. Denote by the *equilibrium* payoffs to a signatory and non-signatory country respectively from the Emissions Game when there are *n* signatories, $2 \leq n \leq N$. In the first-stage (the Membership Game) each country has to decide whether to join or not to join. The key point is that there is no international agency to compel countries to join such an agreement, so any agreement has to be *self-enforcing*, i.e. it has to be a country's individual interest to join an agreement. The concept of self-enforcing IEA used here is the one introduced by Carraro and Siniscalco (1993), Barrett (1994) among others and which borrows from the literature

Definition 1 *An IEA of size n is* ***stable*** *or* ***self-enforcing*** *if it satisfies the two conditions:* ***Internal Stability****:* ***External Stability****:*

Internal stability is just the condition that no signatory country wishes to leave the IEA and become a non-signatory, while external stability is the condition that no non-signatory country wants to join the IEA and become a signatory. These conditions can also be interpreted as the conditions for a Nash equilibrium of the membership game in which each country takes as given the membership decisions of all other countries.

[7] In the case of continuous emission levels, it is important to distinguish between the case where the signatory countries take as given the emission levels chosen by the non-signatories (the Cournot assumption) and the case where signatories take as given the *reaction functions* of non-signatory countries (the Stackelberg assumption) – see Finus (2000), Rubio and Ulph (2002c). In this simpler model of two emission levels, the dominant strategy of non-signatories is to pollute no matter what signatories do, so there is no difference between Cournot and Stackelberg.

The second general point is that in the dynamic context employed here, there are two ways we can think of countries joining an IEA. The first is that at the start of period 1 each country decides whether or not to join and is committed to this decision for both periods and for both states of the world (if learning takes place); I call this the fixed membership model. In this model, in the emissions game each country has to determine its strategy for emissions over all periods and states of the world. The second approach is that in each period and each state of the world each country decides whether or not to join an IEA. I call this the variable membership model. This is perhaps more consistent with the notion that there is no external agency to enforce commitments on membership, so one is looking for a sub-game perfect sequence of membership and emission decisions.

The final general point is that I am going to analyse four cases of stable IEA depending on whether there is fixed membership (*fm*) or variable membership (*vm*) and on whether there is learning (*l*) or no learning (*nl*). In the next section I analyse the stable IEAs in the four cases, and in section 4 I compare the outcomes to assess how learning affects the incentives to join an IEA and whether it is better to have fixed or variable memberships.

3. Self-Enforcing IEAs with Uncertainty and Learning

In this section I analyse the stable IEAs in four cases in the following sequence: fixed membership, no learning (*fmnl*), variable membership, no learning (*vmnl*), fixed membership, learning (*fml*) and variable membership, learning (*vml*).

3.1 Fixed Membership, No Learning.

As noted in the previous section, I model this as a two-stage game in which in stage 1 countries decide whether or not to join the IEA, to which they are committed for 2 periods, while in stage 2 they choose their emission levels over both periods. I start with the second-stage game.

I define:

.

It is straightforward to show that . I also define:

denotes the smallest integer not less than the real number x. In Ulph (2002), for the case where $\kappa = 0$, I defined two critical membership levels . It is straightforward to show that , with equality if $\kappa = 0$. Then:

Lemma 1 (Emission Game– Fixed Membership No Learning).

(i) For any number of signatories, n, the optimal strategy of the non-signatories is to pollute in both periods, no matter what the signatory countries do.

(ii) The optimal strategy of the signatories is for each to abate in both periods if $n \geq n^{**}$; to abate in period 1 and pollute in period 2 if ; and to pollute in both periods if .

(iii) The expected payoffs to signatories and non-signatories from these strategies are:

The intuition behind these results is as follows. The strategy for the non-signatories just follows from Result 1, that if a country acts non-cooperatively, its dominant strategy is to pollute in both periods. For signatories, it will pay for each country to abate if the benefit it foregoes is less than the savings in damage costs to all members; so it requires a relatively large number of members (at least n^{**}) to justify abating in both periods; if the numbers fall below some lower bound ( ), it will not be worth abating in any period; and for an intermediate number of signatories, it will not be worth abating in the second period (when, in this model, pollution is less costly because it only lasts one period), but it will pay to abate in period 1.

I now turn to the first-stage membership game.

Result 2 (Membership Game – Fixed Membership, No Learning)

(i) *There can be no stable IEA larger than n^{**} or smaller than* □

(ii) *For $\gamma \geq 0.4$ the unique stable IEA of the fixed membership game with no learning has n^{**} members;*

(iii) *For $\gamma < 0.4$, n^{**} will be stable for large values of κ (which get larger as γ decreases), while* □ *will be stable for smaller values of κ.*

When $\kappa = 0$, then I showed in Ulph (2002) that there would be a unique stable IEA, with $n^* = n^{**}$ members if $\gamma \geq 0.4$, and □ members otherwise. This is consistent with Result 2. But when $\kappa > 0$, then it remains the case that the unique stable IEA has n^* members when $\gamma \geq 0.4$, but when $\gamma < 0.4$, then n^{**} may still be the unique stable IEA, for large values of κ. So introducing stock-dependent costs makes it more likely that the higher membership IEA will be stable. The intuition is that the threat of switching from abating in both periods to abating only in period 1 if membership falls below n^{**} is more of a deterrent when there are large values of κ.

However, part (iii) of Result 2, deliberately, leaves open the possibility that, when $\gamma < 0.4$ there may be no stable IEA or there may be two stable IEAs. Since it is not possible to get further analytically, to determine what happens I have run a number of numerical simulations[8]. The result is shown in Figure 1(a) and summarised as follows:

Result 2′ *For the fixed membership game with no learning, for any N there are three possible outcomes depending on the region in which the parameters γ, μ (equivalently γ, κ) lie as follows (see Figure 1):*
A The unique stable IEA has n^{**} members;
B The unique stable IEA has □ *members;*
C The IEAs with n^{**} and □ *members are both stable.*

The shapes of Regions A and B are consistent with Result 2. Regions A, B and C occupy (approximately[9]): 40%, 55% and 5% of (γ, μ) parameter space respectively. Note there are no parameters for which there is no stable IEA. For future purposes, for parameter values for both n^{**} and □ are stable, I will select □ as the stable IEA, so that Region C in Fig 1 (a) merges with Region B as in Figure 1(b), where Region A accounts for 40% of parameter space and Region B 60%.

For future reference I will refer to the two types of stable IEAs for the fixed membership, no learning case as FMNL(i), in which there are n^{**} signatories who abate in both periods, and FMNL(ii), in which there are □ signatories who abate only in period 1.

3.2 Variable Membership, No Learning

[8] The numerical simulations used 6 values of $N = 25, 50, \ldots, 150$, 1000 values of γ between $1/N$ and 1, and 1000 values of μ between 0 and 1.

[9] The percentage vary very slightly depending on N.

In this case I continue to assume that there is no learning, but now allow that in each time period countries are free to join or leave the IEA. Of course countries in period 1 will need to understand how their decisions affect the stock of emissions and hence the membership and emission decisions and their associated payoffs in period 2. So I work backwards.

3.2.1 Period 2 of Variable Membership, No Learning

At the start of period 2 the world inherits a stock of pollution, , from period 1. Define as the unit damage cost in period 2; by Assumption 1, . Then the period 2 expected payoff to any country *i* is :

The last term is a constant as far as period 2 decisions are concerned. So the model is isomorphic to a static model with certain cost-benefit parameter , so, from the results of Barrett (2000), Rubio and Ulph (2002a), we have[10]:

Lemma 2 (Period 2 Emissions Game) *In the period 2 emissions game with n signatories for the variable membership model with no learning, the optimal strategy for non-signatories is to pollute, and for signatories is for each to abate as long as and otherwise to pollute.*

The strategy for non-signatories follows immediately from Result 1, in particular the fact that polluting is always a dominant strategy for a country acting non-cooperatively. The strategy for signatories follows immediately by observing that if one signatory abates it loses 1 unit of benefit, but saves the collection of *n* signatories an amount in damage costs. Define , the smallest size of IEA membership for which the signatories abate pollution. Then:

Lemma 3 (Period 2 Membership Game) *In the period 2 membership game of the variable membership model with no learning, the unique stable IEA has members. The expected payoffs to signatories and non-signatories are:*

The intuition is that any IEA with membership strictly greater than cannot be internally stable, because any member knows (from Lemma 2) that if it leaves the IEA the remaining members will continue to abate, so the consequence of defection is to increase pollution by 1 unit, which gains it 1 unit of benefit, but loses it in damage costs. Since , defection pays. But when membership is , any further defection will cause the remaining signatories to stop abating and pollute. So now defection by one

[10] Since these results are standard and quite obvious we omit proofs and just provide intuition. For proofs see Rubio and Ulph (2002).

signatory would cost it in damage costs, which, by definition of , exceeds the benefit it gets from polluting. Note that, if κ = 0, as in Ulph(2002), stable IEA membership in period 2 is n^*, which is independent of .

It will be important to know the properties of . Before deriving these, there is one other issue I need to address, and that is what a country believes about how it's decisions in period 1 affect the payoff it will get in period 2. Obviously one link is through the effect of period 1 decisions on . But that is not enough to determine period 2 payoff, because as Lemma 3 notes, that payoff will also depend on whether a country is a signatory or not in period 2. Rubio and Ulph (2002a), discuss a number of ways a country in period 1 might form beliefs about whether it will be a signatory or not in period 2. As in Ulph(2002) I shall take their simplest approach – the Random Assignment Rule - of assuming that there is a random process of choosing signatories in period 2 such that each country has the same probability, namely , of being a signatory. A justification for this approach might be that, with homogeneous countries, all that the stability analysis can explain is how many countries will sign the IEA, it cannot explain which countries sign up. The implicit assumption that a country's chances of being a signatory in period 2 are independent of whether it was a signatory or not in period 1 reflects a view that countries are unable to make commitments about membership from one period to the next. As noted, Rubio and Ulph (2002a) explore other ways of modelling these beliefs, including one in which a country's chances of being a signatory in period 2 depend strongly on whether it was a signatory in period 1. I leave it for further research to explore the implications of using these other approaches in a model with uncertainty and learning.

Thus, under the Random Assignment Rule, every country will have the same expected period 2 payoff function:

I now turn to the properties of

Lemma 4 (i) (ii)
(iii) .

That the size of the stable period 2 IEA decreases as the stock of pollution rises, is due to the increase in the unit damage cost; it is a well-know feature of the static Barrett model that as damage cost rises, the number of signatories falls. The reduction in the expected payoff to signatories and non-signatories arises for 3 reasons: there is the direct effect of an increase in the stock of pollution; this also increases the unit damage cost; and, as just noted, this reduces the number of signatories and hence increases period 2 pollution. Finally, when considering the average expected value function, averaged over signatories and non-signatories, there is a fourth effect which goes in the other direction – there is an increase in the proportion of non-signatories and since they are better off than signatories this acts to increase the average expected payoff. But, as the proof makes clear, this effect is too small to offset the other three effects of an increase in the stock of pollution inherited from period 1.

3.2.2 Period 1 of Variable Membership, No Learning

The expected payoff to country *i* in period 1 is:

Then it is straightforward to show that:

Lemma 5 (Period 1 Emissions Game - Strategies) *In the period 1 emission game of the variable membership model with no learning with n signatories the optimal strategy for non-signatories is always to pollute, and for signatories to abate as long as* $\eta(n) \geq 0$*, where* .

Again Assumption 1 guarantees that the dominant strategy for a country acting non-cooperatively is to pollute. The condition for the signatories has the usual interpretation. If all signatories abate, each gives up one unit of benefit; but the savings in damage costs to each of then are the immediate savings γn, and the expected benefit in period 2 .

Now, using Assumption 1, it is straightforward to see that $\eta(0) = -1$, $\eta(N) > 0$, and, from Lemma 4, . So there exists a unique integer , which is the smallest integer such that . Then:

Lemma 6 (Period 1 Emissions Game – Payoffs) *In the period 1 emissions game of the variable membership model with no learning with n signatories, the expected payoffs to signatories and non-signatories are:*

Then it is readily shown that:

Result 3 (Membership Game) *The unique stable IEA of the variable membership model with no learning has members in period 1 and members in period 2, with expected payoffs to signatories and non-signatories:*

When κ = 0, I showed in Ulph (2002) that , so that, allowing for integer problems, , and both are decreasing functions of γ, for reasons already explained. Numerical simulations show that, broadly speaking, these properties carry over to the case of stock dependent costs. are decreasing functions of both γ and κ, and , except when γ is reasonably large (say > 0.4), when, typically .

.

3.3 Fixed Membership, Learning.

Between period 1 and period 2, countries learn whether damage costs are high or low, so countries are able to condition their emissions in period 2 on what they have learned. So now each country decides on 3 emission levels: . The expected payoff to country *i* was given in (1a).

I start with the second-stage emission game, where now countries can fine-tune their emissions in period 2 to the information they have gained. I define two new critical threshold membership levels as follows:

In Ulph (2002) I defined two critical threshold levels of membership . It is straightforward to show that , and there is equality if κ = 0. With these definitions, I now derive:

Lemma 7 (Emission Game – Fixed Membership Learning) *For the emissions game with fixed membership and learning with n signatories:*

- *(i)* *The optimal strategy for the non-signatories is to pollute in both periods and both states of the world no matter what the signatories do.*
- (ii) *For signatories, the optimal strategy is to abate in both periods and in both states of the world if ; to abate in period 1 and in period 2 if damage costs are high, but pollute in period 2 if damage costs are low if ; to abate in period 1 and pollute in period 2 in both states if ; finally to pollute in both periods and both states of the world if .*
- (iii) *Ignoring a common term, , the expected payoffs to signatories and non-signatories are:*

The intuition behind these results is the same as for Lemma 1. The strategy for non-signatories follows from Result 1: that for a country acting non-cooperatively the dominant strategy is to pollute in all periods and states of the world. The strategy for signatories follows from the fact to justify abating pollution requires that the benefits foregone be less than collective damage to all signatory countries, so it requires a large number of signatories to justify abating in all periods; as the number of signatories falls below that level, the signatories cease abating in increasing order of damage costs: first polluting only in the low damage cost state in period, then in both states in period 2, and finally in both states and both periods.

Having defined the optimal strategies and payoffs for the Emissions Game for all possible values of *n*, I now turn to the Membership Game. The following result summarises what can be said analytically:

Result 4

(i) *There cannot be a stable IEA of size greater than or less than .*

(ii) *If is stable (and (γ, μ) parameter values lie within Region A of Figure A); otherwise will be stable if:*

(iii) *If is stable (and (γ, μ) parameter values lie within Region B of Figure 1(b)); otherwise ( ) it is stable if:*

(iv) * is externally stable if ; if it is internally stable if:*

* is internally stable if and otherwise it is internally stable if:*

The rationale behind Result 4(i) and the first parts of Result 4(ii) and (iii) is that the payoffs to for the case of fixed membership and learning are the same as the payoffs to for the case of fixed membership and no learning, and so there are parameter values for which the existence of the intermediate threshold membership level makes no difference to the stability analysis of Result 2. However, Result 4 does not rule out the possibility that for some parameter values there may be either no stable IEA or multiple stable IEAs. To make progress I have again used numerical calculations for a wide range of parameter values. These confirm that for all parameter values there is always at least one stable IEA, and, for a very small set of parameter values (a subset of

Region C in Figure 1(a) for low values of θ - less than 0.5% of parameter space) both are stable. In what follows, for these parameter values, as in the case of fixed membership with no learning, I will select as the stable IEA. Then, it turns out that, for any N and θ, Regions A and B of (λ, μ) space in Figure 1(b) can be split into two-subregions: A(i) in which is stable, A(ii) in which is stable; B(i) in which is stable and B(ii) in which is stable. I formalise this as:

Result 4′ *For the case of fixed membership with learning, for each set of parameter values for N, γ, μ (κ) and θ (δ and λ) there is a unique stable IEA which will be one of: FML(i): signatories who abate in all periods and states of the world; FML(ii): signatories who abate in period 1 and in period 2 if there are high damage costs but pollute if there are low damage costs; or FML(iii): signatories who abate in period 1 and pollute in period 2 no matter which state arises . For any N and θ the regions of (γ, μ) parameter space in which each is the unique stable IEA are as follows: Region A(i): FML(i); Region A(ii): FML(ii); Region B(i): FML(iii); Region B(ii): FML (ii).*

The size and shape of the four regions depends crucially on the value of θ[11], the parameter which measures the degree of uncertainty about damage costs. Table 1(a) shows the proportions of (γ, μ) parameter space[12] which are accounted for by the four regions for values of $\theta = 0.1,\ldots,0.9$. Since this paper focuses on the impact of introducing stock dependent costs into the model of Ulph (2002) in Table 1(b) I show what proportions of γ parameter space[13] are accounted for by the four regions for low (0.1), medium (0.5) and high (0.9) values of μ and θ. The rationale behind these results is the following. From Result 4 (ii) the first sufficient condition for to be stable ( ) only arises for very low values of θ. From Result 4(iii) the first sufficient condition for to be stable ( ) will also be more likely to be satisfied for lower values of θ, but for very small values of γ the gaps between the three values will be large and so will be stable for low values of γ no matter what value θ takes.

To see what shape the four regions take, I have plotted the four regions for the low, medium and high values of θ = 0.1, 0.5 and 0.9 in Figure 2 (a) – (c) respectively. The obvious point to note is that Regions A(ii) and B(ii) are not connected regions of parameter space. This reflects the fact that the second sufficient conditions in Results 4 (ii) – (iv) depend on the value of functions which in turn depend on how close are to through the operator .

In summary, the possibility of learning allows countries to condition second period emissions strategy on whether damage costs are high or low. Non-signatories make no

[11] Parameter N has much less impact.
[12] Using 100 values for each of γ and μ.
[13] Using 1000 different values of γ.

use of this, but signatories do, and the strategy of abating in period 1 and abating in period 2 if damage costs are high, but polluting if damage costs are low, is one which signatories will adopt for an intermediate range of sizes of membership. This strategy raises the critical membership size needed for signatories to decide to abate in both states of the world in period 2 (relative to the critical membership size with no learning, i.e.). It also introduces another candidate for a stable IEA, FML (ii), which is intermediate between FML(i) (the analogue of FMNL(i)) and FML(iii) (the analogue of FMNL(ii)). FML(i) replaces FML(i) as the stable IEA in a subset of Region A (which can be the whole of Region A for > 0.2), and replaces FML (iii) as the stable IEA in a subset of Region B.

3.4 Variable Membership, Learning

The analysis is almost identical to that in Section 3.2 for variable membership with no learning, so I shall omit the proofs.

3.4.1. Period 2 of Variable Membership, Learning

By the start of period 2, countries will know whether damage costs are high or low, and will know the stock of pollution, , inherited from period 1. Define as the unit damage cost in state *j* = *l,h* in period 2. Then the payoff function to country *i* in period 2 in state *j=l,h* is:

.

The last term is a constant as far as period 2 state *j* decisions are concerned, so the model is again iso-morphic to the simple static model with certainty. So, by analogy with Lemma 2:

Lemma 8 (Period 2 State *j* Emissions Game) *In the emissions game in state j in period 2 for the variable membership game with learning and n signatories, the optimal strategy for non-signatories is to pollute and for signatories is for each to abate as long as and otherwise to pollute.*

The rationale is exactly the same as for Lemma 2. I now define as the smallest size of IEA membership for which signatories abate pollution in state *j* in period 2. Note that if damage costs are high, there will be *fewer* signatories to an IEA in period 2 than if damage costs are low. This simply reflects the property of the static Barrett model that IEA membership is inversely related to the gains from cooperation. Then:

Lemma 9 *The unique stable IEA in period 2, state j=l,h of the model with variable membership and learning has members. Payoffs to signatories and non-signatories are:*

The rationale is the same as for Lemma 3. In Ulph (2002) I showed that when = 0 then the stable IEA in state *j* in period 2 had membership: , so membership was independent of . It is clear that when > 0, membership in state *j* in period 2 will be lower than when = 0.

As in section 3.2 I assume that countries believe that in state *j* in period 2 each has the same probability, of being a signatory (the Random Assignment Rule), so that expected payoff to a country in state *j* is

$j = l,h$

Finally I take expectations across the two states of the world. Define:

where is expected period 2 membership and is the expected savings in damage costs in period 2 from the stable IEAs that arise in the two states. Then expected payoffs across the two states are:

The properties of these functions are given in:

Lemma 10

(i)

(ii)

(iii)

The rationale is the same as for Lemma 4.

<u>3.4.2 Period 1 of Variable Membership, Learning .</u>

The expected payoff to country *i* in period 1 is:

.

Then it is straightforward to show that:

Lemma 11 (Period 1 Emissions Game– Strategies) *In the period 1 emissions game of the variable membership model with learning with n signatories, the optimal strategy is for non-signatories always to pol*

where .

The rationale is the same as for Lemma 5. From Assumption 1 and Lemma 10 it is straightforward to see that . So there exists a unique integer, , which is the smallest integer such that . Then:

Lemma 12 (Period 1 Emissions Game – Payoffs). *In the period 1 emissions game of the variable membership model with learning with n signatories, the expected payoffs to signatories and non-signatories are:*

Then it is readily shown that:

Result 5 *The unique stable IEA of the variable membership model with learning has members in period 1 and members in period 2 state j, j=l,h, with expected payoffs to signatories and non-signatories:*

In Ulph (2002) I showed that for = 0, the first period stable IEA had membership , so that first period membership was unaffected by whether there was learning or no learning. Since , in general it is to be expected that , so first period membership with learning will not be the same as with no learning. I discuss this in the next section as part of a general comparison of memberships and payoffs between the four cases analysed in this section.

4. Comparison of Learning/No Learning, Fixed/Variable Membership

In this section I consider the questions: how does learning affect membership and expected payoffs and how does having membership fixed rather than variable affect membership and expected payoffs?

4.1 Comparison of Learning and No Learning.

4.1.1 Fixed Membership.

It will be useful to recap the properties of the different stable IEAs for Fixed Membership with No Learning and Learning, and the parameter values for which they arise, which I do with reference to Result 4' and Figure 2. With No Learning, there are two possible stable IEAs: FMNL(i): membership * with strategy (for signatories) (0,0,0), which occurs in Regions A(i) and A(ii); and FMNL(ii): membership with strategy (0,1,1), which occurs in Regions B(i) and B(ii). With Learning there are three possible stable IEAs: FML(i): membership with strategy (0,0,0), which occurs in Region A(i); FML(ii): membership with strategy (0,0,1), which occurs in Regions A(ii) and B(ii); and FML(iii): membership with strategy (0,1,1), which occurs in Region B(i). .

I make comparisons between No Learning and Learning for each region in terms of membership, the total amount of pollution generated in each period of time and each state of the world, and global expected welfare. It is readily shown that if one outcome results in at least as much total pollution in every period and state as another outcome, then the second outcome generates at least as great global expected payoff as the first.

Result 6 *The comparison between Learning and No Learning varies over the four regions of parameter space as follows:*

A(i): *FML(i) involves the same strategy and hence same amount of pollution by each signatory as FMNL(i), but FML(i) has at least as many signatories ( ) as FMNL(i) (n**). Hence global pollution is at least as great in each period and state of the world with No Learning than with Learning, so expected global payoff is at least as high with Learning than No Learning.*

A(ii): *FML(ii) has the same or fewer signatories (as FMNL(i) (n**). But FML (ii) has signatories polluting in the low state in period 2. So global pollution is strictly higher with Learning than No Learning in at least one state and period, and no lower in any other. So expected global payoff is strictly greater with No Learning than Learning.*

B(i): *FML(iii) and FMNL(ii) are identical in all respects, so Learning and No Learning are equivalent.*

B(ii): *FML(ii) has at least as many signatories ( ) than FMNL(ii) ( ) . Moreover with FMNL(ii), signatories pollute in both states of the world in period 2 while with FML(ii) they pollute only in the low-cost state in period 2. So global pollution is* *strictly lower with Learning than No Learning in at least one state and period and no higher in any other. So expected global payoff is strictly greater with Learning than No Learning.*

Thus in Region A(ii) No Learning is better than Learning, in Region B(ii) Learning is better than No Learning, in Region B(i) No Learning and Learning are equivalent, and in Region A(i) Learning is better than No Learning if , and equivalent otherwise.

To gain more insight, I have carried out some numerical calculations. Since, as I have already noted, key parameters are the degree to which costs depend on stocks (captured by the parameter μ) and the degree of uncertainty (captured by the parameter θ) in Table 2 I follow Table 1(b) and present results for N =100, low (0.1), medium (0.5) and high (0.9) values for each of μ and θ and 1000 values of γ. In rows 1 and 2 I present average values for membership (Mfmnl) and expected global payoff (VGfmnl – normalised to lie between 0 and 1) for the fixed membership model with no learning, where the average is taken over the 1000 outcomes for different values of γ. Rows 3 and 4 present the same averages for the fixed membership model with learning. For the nine combinations of values of μ and θ the fixed membership model with learning leads (on avergare) to lower membership and lower global expected payoff than fixed membership without learning.

In Rows 11-14 I show for what proportion of the 1000 cases membership with learning lies above or below membership with no learning (for the remaining cases they are equal) and similarly for expected global payoff. To interpret these results it is useful to refer back to Table 1(b) for the different proportions of parameter space lying in Regions A(i), A(ii), B(i) and B(ii). Taking membership first, Result 6 tells us that membership with learning should be at least as high as membership with no learning in Regions A(i) and B(ii), and at most as high as membership with no learning in Region A(ii). Row 11 shows that membership with learning is only higher than with no learning in a small number of cases, much smaller than Regions A(i) and B(ii) combined, while Row 12 shows that, apart from μ =0.9, θ =0.1, the proportion of cases for which membership with learning is less than with no learning is exactly equal to area A(ii). In terms of expected global welfare, Result 6 says that learning is strictly better than no learning in B(ii) and strictly worse than no learning in Region A(ii), and would be better than no learning Region A(i) if membership was higher. There are no such last cases, so Rows 13 and 14 in conjunction with Table 1(b) just confirm Result 6.

The overall message is that, except for high values of both μ and θ, in the majority of cases there is no difference between learning and no learning for the fixed membership model in terms of either membership or expected global payoff. Where they differ, it is more likely that learning leads to lower membership and lower expected global welfare.

4.1.2 Variable Membership.

In Ulph (2002) I showed that, for the case where $\kappa = 0$, , so that second period membership was higher on average with learning than no learning, for any period 1 level of pollution, and since period 1 membership was the same with learning as with no learning, and hence period 1 pollution was the same with learning as no learning, in the stable IEA second period membership would be higher (on average) with learning than with no learning. A key motivation for developing the model in this paper in which unit damage costs depend on the stock of pollution is to have a model in which, in principle, period 1 membership in the variable membership model could differ depending on whether there was the possibility of future learning. I showed in sections 3.2 and 3.4 that the functions for determining were indeed different. I now try to make more precise comparisons between the two memberships. This is complicated by the need to treat membership as an integer, and the following result indicates what can be said analytically if, as an approximation, one ignores the integral nature of membership.

Result 7 *Treating membership as a real number, and approximating* $\delta = \theta\gamma$, $\lambda = \theta\kappa$, *then:*
(i) *(ii)* *(iii)*

Thus with learning there are at least as many signatories in period 1 and, on average in period 2, as with no learning.

However, Result 7 depends on two approximation arguments, and it is important to check what can be said when integer values for membership are used and the uncertainty parameters are: . Moreover Result 7 only indicates what can be said about membership. It is also important to know what happens to expected global welfare. I cannot use the argument I applied for fixed membership, of going from what can be said about aggregate pollution to aggregate payoff, because with variable membership with learning, period membership will be higher in low damage costs states than with no learning and lower in high damage cost states than with no learning, so one cannot Pareto rank aggregate pollution between learning and no learning.

To make progress I have again used numerical simulations, and I report the results in Table 2. Rows 5-7 report average values for period 1 membership, expected period 2 membership and expected global payoff for variable membership with no learning and Rows 8-10 report the same for variable membership with learning. Expected period 2 membership with learning is usually greater then period 2 membership without learning, except for low values of θ. This is also true for period 1 membership, though the differences are much smaller. Average expected payoff with learning is always less than average expected payoff with no learning.

Rows 15-20 show for what proportion of the 1000 cases learning gives strictly higher or strictly lower values of the three variables than no learning The results show that for period 2 membership, except again for low values of θ, in a very great majority (95-100%) of cases membership is higher with learning than no learning. For period 1 membership, for more than 95% of cases, period 1 membership is the same with learning as with no learning, and the small proportion of cases where they are not equal usually divide fairly evenly between those where period 1 membership with learning is above period 1 membership with no learning and those where it is below. In terms of expected payoffs, although on average learning has a lower expected payoff than no learning, in a small proportion of cases (between 5% and 30%) expected payoff with learning is above expected payoff with no learning, but in a much bigger proportion of cases (35-85%) the opposite is true.

Thus, like the fixed membership model, learning is more likely to lead to lower expected global welfare than no learning. But there is a significant difference. Generally, the higher the proportion of cases where period 2 membership with learning is above period 2 membership with no learning, the greater the proportion of cases for which expected payoff with learning is below expected payoff with no learning. So whereas for fixed membership there is a positive relationship between lower membership and lower expected global welfare, with variable mebershiop this relationship is a negative one. As noted in Ulph (2002) this reflects the fact that the variable membership models are a sequence of one-period models and a well-known property of the one period model is that there is an inverse relationship between the number of countries who join an IEA and the benefits which an IEA brings.

4.2 Comparison of Fixed and Variable Membership

In this subsection I am interested in the question whether it is better to fix membership (force countries to commit to being a signatory or non-signatory for both periods[14]) or to allow countries to decide each period (and each state of the world if learning takes place) whether to join or not. Since I have not been able to derive a closed form solution for the variable membership cases, it is not possible to make analytical comparisons, so I rely on numerical calculations. I consider first the case of No Learning and then consider Learning.

4.2.1 No Learning

Average memberships and expected global welfare are given in Rows 1-2 and 5-7 of Table 2. For all nine parameter sets, on average membership in the fixed membership case is above first period variable membership but below second period variable membership, but expected global welfare is always higher with variable than fixed membership. The analysis of proportions of cases in Rows 20-25 shows that first period variable membership is almost never above fixed membership, and in a significant number of cases (rising from about 25% to 70% as μ moves from low to high) is below fixed membership. Second period variable membership is never below fixed membership and in a significant proportion of cases (falling from about 80% to 30% as μ moves from low to high) it is strictly greater. In the same proportion of cases expected global welfare with variable membership is above expected global welfare with fixed membership. So with No Learning, variable membership is better than fixed membership on average and for a significant majority of parameter values, except for high values of κ.

4.2.2 Learning

[14] I ignore the issue of how such commitment could be enforced.

The comparison between fixed and variable membership with learning is a more extreme version of the one just provided for no learning. For all nine parameter sets, on average fixed membership is higher than first period variable membership, lower than second period variable membership and leads to lower expected global welfare. The analysis of proportions in Rows 26-31 shows that, except for μ = 0.9, θ =0.1, for the great majority of cases fixed membership is the same as first period variable membership, (though where they are not the same fixed membership is more likely to be higher). But in the great majority of cases (up to 100%) second period variable membership is above fixed, and expected global welfare is higher with variable than fixed membership.

In summary, variable membership is normally better than fixed membership, because it leads to higher second period membership, without significantly affecting first-period membership.

5. Conclusions

In a companion paper, Ulph (2002) I explored the issue of how the possibility of getting better scientific information in the future about the damages caused by a stock pollutant might affect the incentives and timing of countries to join an International Environmental Agreement to manage the stock pollutant. I showed that if countries had to decide once-and-for-all whether to join an IEA, then if there were high damage costs and a high degree of uncertainty, learning would be worse than no learning in terms of membership and global welfare, both otherwise learning would be at least as good as no learning. However if countries could decide in each period and state of the world whether or not to join, than learning would not affect current membership but would affect future membership – with higher membership in the *low* damage state than the high damage state – but membership being higher on average with learning. But this was inversely related to global welfare – the more future membership with learning lay above future membership without learning the more expected global welfare with learning lay below expected global welfare with no learning. In general, variable membership was better than fixed membership, especially when there was learning.

However, the simplicity of the model employed in that paper – in particular the assumption that unit damage costs did not depend on the past stock of emissions – meant that in the variable membership model current membership could not be affected by whether learning could take place, and that was a limitation in seeking to explore how learning might affect current incentives to join an IEA. In this paper I have extended the analysis in Ulph (2002) to allow for unit damage costs to depend on the stock of pollution. This ensures that in the variable membership model, in principle current membership will differ between the cases of learning and no learning. It turns out, however, that the results of Ulph (2002) are largely robust to this more general assumption about damage costs. In particular, although in principle current membership in the variable membership model can differ between learning and no learning, for the vast majority of parameter values (over 95%) learning does not affect current membership, and where they differ it is as likely that current membership is higher with learning than no learning as it is that it lower. So, the possibility of learning better scientific information does not appear to have an important effect in causing countries to delay joining an IEA. The other findings in Ulph (2002) are also robust. With fixed membership, while it remains the case that for large values of damage costs and uncertainty, learning would be worse than no learning but otherwise learning is at least as good as no learning, as the importance of stock dependent cost increases, the regions of parameter space in which learning is better than no learning decrease while those in which learning is worse than no learning increase, so that for large values of both κ and θ in a majority of cases learning leads to lower membership and lower expected welfare. With variable membership, as already noted, current membership is almost unaffected by whether or not there is future learning, but second period membership on average is significantly greater with learning (though with the same 'perverse' effect that membership is lower in high damage cost states), but this inversely related to global welfare. However these effects are not strongly affected by the importance of stock-dependent damage costs. Finally fixed membership remains generally worse than variable membership.

Although the model used in this paper has been made more general than that in Ulph (2002) in one respect, it remains extremely simple in the other important respects discussed in the conclusions to Ulph (200). The model is limited to 2 periods. With variable membership, membership rises over time when there is No Learning and rises on average when there is Learning. This just reproduces the result in Rubio and Ulph (2002a) without uncertainty. But this is an artefact of the two-period horizon, which effectively makes pollution in the second period less damaging than in the first (because it does not last as long). Rubio and Ulph (2002b) consider the infinite-horizon model without uncertainty and show that membership declines over time. An obvious line for future research is to extend the analysis with uncertainty and learning to an infinite-horizon model.

Second, in both papers I have focussed on the case where countries are identical *ex ante* and *ex post*, uncertainty is solely about the extent of global net benefits, and learning is a very simple process. As noted in the introduction, Na and Shin (1998) focussed only on the case where there was uncertainty about the distribution of known total global benefits. It would clearly be desirable to have a model which combined both features and where there are asymmetries between countries both *ex post* and *ex ante.* Moreover in a model of more than two periods it would be desirable to model much richer processes of learning, in which there could be active and passive forms of Bayesian learning (along the lines of Karp and Zhang (2000), Kelly and Kolstad (1999)). However, unlike those models which had a single regulator, it would be desirable to model which countries engage in active or passive learning, how are costs shared, and how does this affect incentives to join an agreement.

Third, in both models countries are restricted to only two actions (pollute or abate). I conjecture that this is not a major restriction. Finally it would be desirable to consider other concepts of stable coalitions. Clearly, there is much to be done before we really understand how uncertainty and learning affect incentives to join international environmental agreements.

Appendix

Proof of Result 1.

(i) I give the proof for the case of learning; no learning is just a special case. Define: ; then ignoring terms: (1a) becomes:

(A1):

It is straightforward to see that a sufficient condition to ensure that, for any it is always optimal to set is or . Since this also ensures that is always optimal to set . Finally it is always optimal to set if:

Assumption 1 (i) and (ii) guarantee that these two sufficient conditions hold.

The expected payoff is got by setting in (1a). The fact that the expected payoff is the same under learning and no learning follows from the fact that the optimal strategy is to pollute no matter what the state of the world, so learning has no benefit.

(ii) Again I just give the proof for the case of learning. Suppose countries have collectively determined emissions for all countries other than *i* and are now deciding its emissions. They do so to maximise aggregate net benefits of all countries, which differs from (A1) in that all damage costs are now multiplied by *N*. Thus the payoff to all *N* countries is given by:

(A2):

It is straightforward to see that a sufficient condition to ensure that, for any it is always optimal to set is or . This also ensures that it is always optimal to set . It is always optimal to set if , but this is always satisfied if , so this is the single sufficient condition to ensure that it is always optimal for countries to collectively abate in all periods and all states of the world. Assumption 1 (iii) guarantees that this sufficient condition is satisfied.

The expected payoff is obtained by setting in (1a). Again that the fact that the expected payoff is the same under learning and no learning arises because the optimal strategy is to abate no matter what the state of the world is.

(iii) This follows directly from (i) and (ii). QED

Proof of Lemma 1.

(i) Follows from Result 1(i).

(ii) The n signatories take as given the emissions by non-signatories in each period: $N-\ n$. I model the choice of strategy by signatories as choosing a common strategy for each member so as to maximise the payoff of a typical member. Evaluating for the four possible strategies yields:

Clearly , so (1,0) will never be chosen (if they are going to pollute in only period it is better to do so in period 2). I define:

Now . It is clear to see that each have two positive roots, and it is straightforward to show that the upper roots both lie above N. The lower roots are, respectively:

It is readily checked that: . From the definitions of it is easy to see that

Then:

(iii) The expected payoffs are readily derived by just substituting the optimal emissions into the payoff function (1b).

QED

Proof of Result 2.

(i) Stability of n^*.

External Stability:

Since, by Assumption 1, $1 + \kappa > 2\gamma + 3\kappa N$ this last inequality must hold. So n^* is always externally stable (and hence any larger IEA would be internally unstable).

Internal Stability:

Now, when $\kappa = 0$, the LHS is the condition for stability of n^*, and as shown in Ulph (2002), will be non negative as long as $\gamma \geq 0.4$. When $\kappa > 0$, then, for $N \geq 5$, the RHS will be negative, so this reinforces the internal stability of n^* when $\gamma \geq 0.4$. Moreover, when $\gamma < 0.4$, although the LHS will now be negative, so will be the RHS, especially for large values of κ. Thus for $\gamma \geq 0.4$, n^* will be internally stable. For $\gamma < 0.4$ there will a large enough value of κ for which n^* will be internally stable, though this will have to be increasingly large the smaller is γ.

(ii) Stability of :

Internal Stability:

But this last inequality must hold by the definition of . So is always internally stable (and hence any smaller IEA would be externally unstable).

External Stability:

(a)

By Assumption 1, the term in parenthesis is non-negative and so this inequality is satisfied.

(b)

L

HS is condition is condition for stability of when $\kappa = 0$, and, as shown in

Ulph (2002), is negative when $\gamma \geq 0.4$. So this externality stability condition will certainly not be satisfied when $\gamma \geq 0.4$ and $\kappa > 0$.

When $\kappa = 0$, case (a) required small values of γ, specifically, < 0.4. Having $\kappa > 0$, reduces both and n^*, and so means that case (a) may not hold when $\gamma < 0.4$. Moreover, if case (b) arises, the stability condition is unlikely to be satisfied for large κ. So will not be externally stable when $\gamma \geq 0.4$, and may not be stable for $\gamma < 0.4$ and large enough values of γ.

Proof of Lemma 4.

In differentiating I will treat as the real number .

Differentiating: .

.

Proof of Lemma 5

From (x), it will pay a country acting non-cooperatively to pollute if:

(A3)

Since (A3) needs to hold for all possible, it is sufficient if it holds for

Using Lemma 4, the required sufficient condition is:

which is satisfied by Assumption 1.

For signatories, comparing the expected payoff each gets if they all abate and the expected payoff if they all pollute, then it will pay to abate if

QED

Proof of Result 3.

Internal Stability:

which is true by definition of

External Stability:

which holds by (A3). QED

Proof of Lemma 7

(i) Follows from Result 1(i).

(ii) Signatories take as given the emission by non-signatories, $N - n$, in each period and each state of the world. As in Lemma 1 I model the choice of strategy by signatories as that which would maximise the expected payoff of a signatory assuming that all signatories pursue the same strategy. Then, ignoring a common term and multiplying by 2, the expected payoffs to a signatory for the 8 possible strategies they could choose are:

It is straightforward to see that , so the corresponding strategies will never be selected. The rationale is that, in the first two cases, if the signatories are going to abate in only state in the second period, it is better to abate in the high cost state, while in the last case, if the signatories are going to abate in only period, it is better they abate in period 1 than period 2. I will show shortly that is dominated by the other payoffs. So the only undominated payoffs are: and . Then:

Recalling, from Lemma 1, that , it is straightforward to see that:

. (A4)

All these 4 quadratic expressions have 2 positive roots, and, as in Lemma 1, it is easily shown that all the upper roots are greater than N, so I am only interested in the lower roots. In Lemma 1, were defined as the lower roots of respectively. Define as the lower roots of respectively, and define . Then it follows from (A4) that . Hence:

so the optimal strategies for signatories in these four cases are (0,0,0), (0,0,1), (0,1,1), (1,1,1) respectively.

Finally I need to show that is dominated. Now it is easy to see that it is also straightforward to show that, given Assumption 1, and that . Finally it can be shown that the slope of is less than the slope of at , and below the slope of . So .

(iii) The expected payoffs follow by substituting the optimal strategies into the expected payoff function (1a). QED.

Proof of Result 4.

(i) Since payoffs for are same as for in the No Learning case the proof of Result 2 can be used to show that are externally stable and internally stable respectively.

(ii) Internal stability of . As in (i), if the proof of internal stability is the same as for n^{**} in Result 2. If then :

The term on the LHS is non-negative, and, all the terms on the RHS are non-negative (by Assumption 1). If is very close to , the root of , then the LHS will be very close to zero, and the condition is unlikely to be satisfied.

(iii) External Stability of . Again, as in (i), if then the proof of External stability is the same as for Result 2 for the case where . If then:

Again all the terms on both the LHS and RHS are non-negative. However all the terms on the LHS are less than 1, so there will be many parameter values for which this condition is not satisfied.

(iv) External Stability of There are two cases:

(a) :

All the terms on the LHS are non-negative, and the first 2 are positive (by Assumption 1), so the condition is satisfied.

(b) :

The terms on the LHS are non-negative and the first is positive (by Assumption 1). The term on the LHS will be negative if , in which case the condition is satisfied. If , then the RHS is also non-negative, so the condition may no be satisfied.

Internal Stability of : Again there are two cases:

(a)

Again the terms on both sides are non-negative, and the first term on the RHS is positive (by Assumption 1). However the term on the LHS is likely to be close to zero, so the condition may not be satisfied.

(b)

Since, by definition of , both terms on LHS are non-negative, the condition is satisfied. QED.

Proof of Result 6.

Suppose the aggregate levels of emissions in period 1, period 2 state *h*, period 2 state *l* are . Then, from (1), the expected global payoff, aggregated over signatories and non-signatories is:

By Assumption 1 the terms in square brackets are negative. So, if we compare two paths of aggregate emissions, and where: , and at least one of the inequalities is strict, then . QED.

Proof of Result 7.

(i) Using approximations, define:

(ii)

Assuming approximation in (i) yields:

∴Root of root of .

(iii) . QED

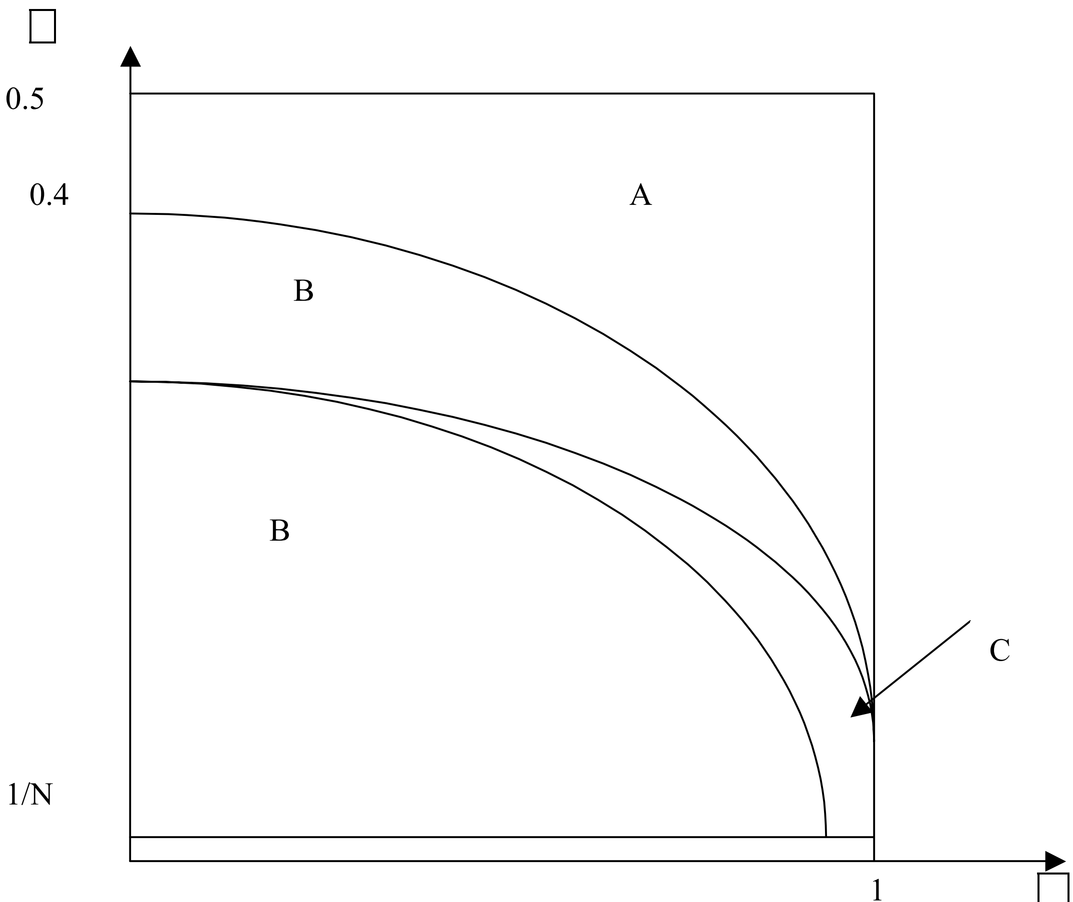

Figure 1(a): Fixed Membership – No Learning Regions for Different IEAs

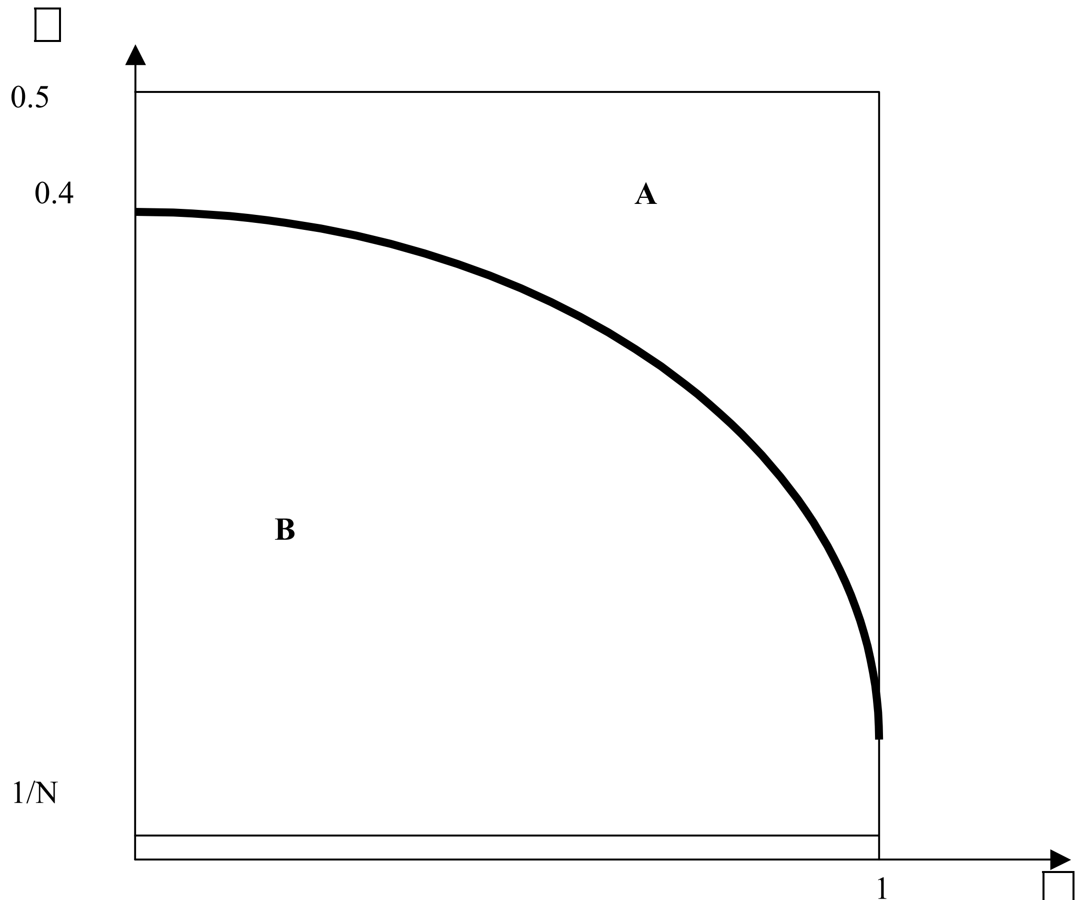

Figure 1(b): Fixed Membership – No Learning Regions for Different IEAs – Ignoring Multiple Stable IEAs

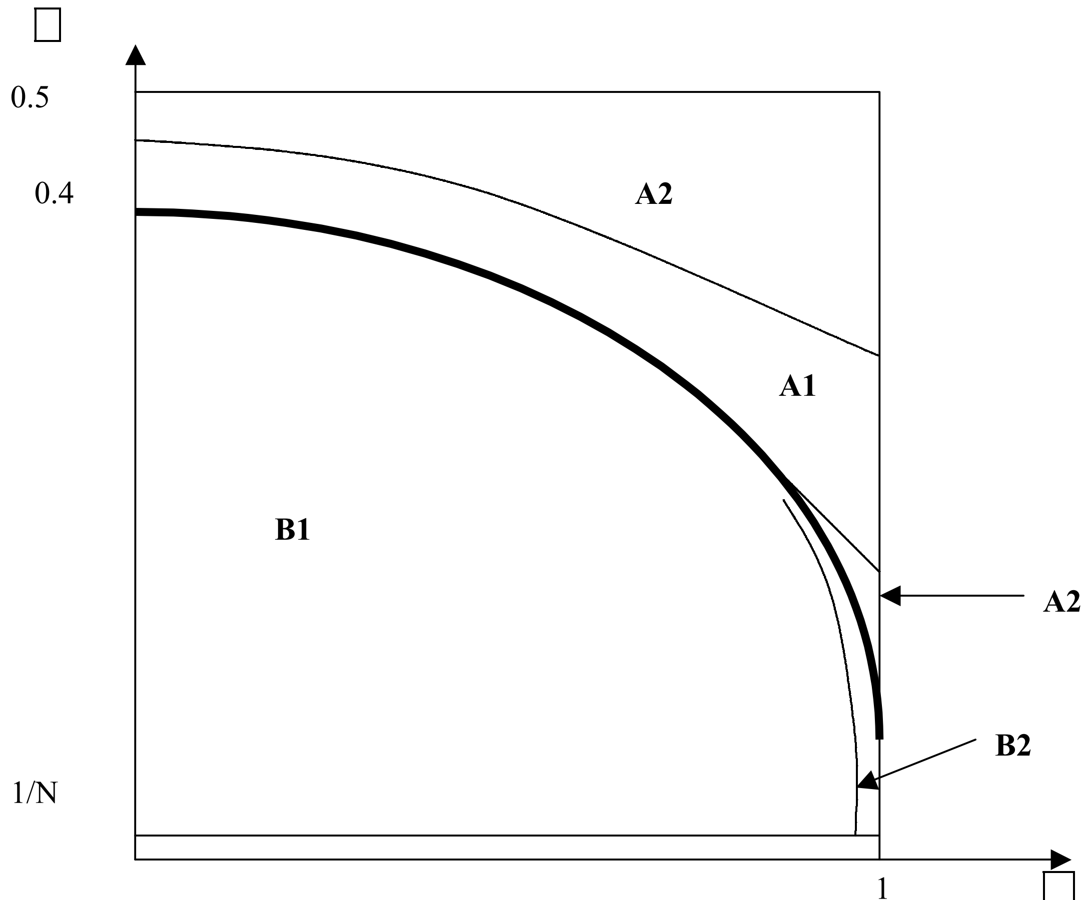

Figure 2(a): Fixed Membership – Learning Regions for Different IEAs – □ = 0.1

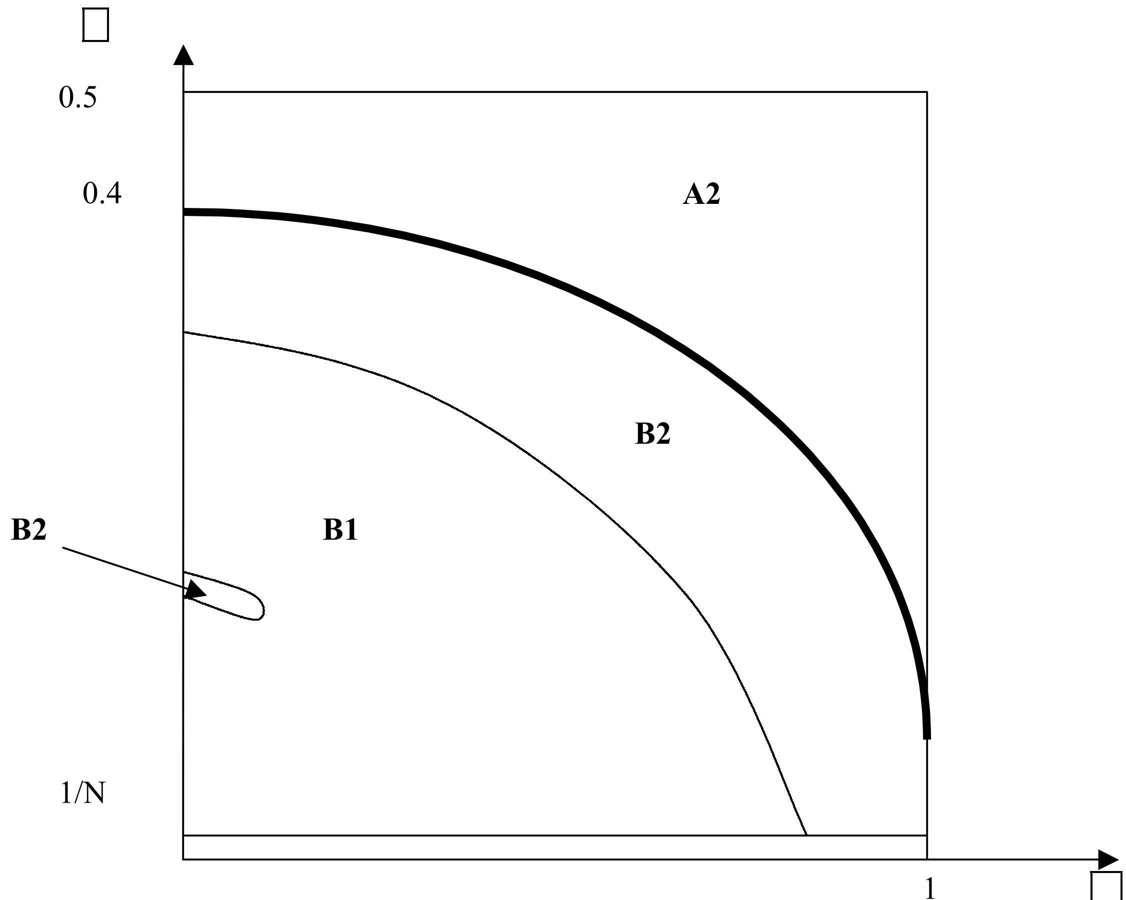

Figure 2(b): Fixed Membership – Learning Regions for Different IEAs - □ = 0.5

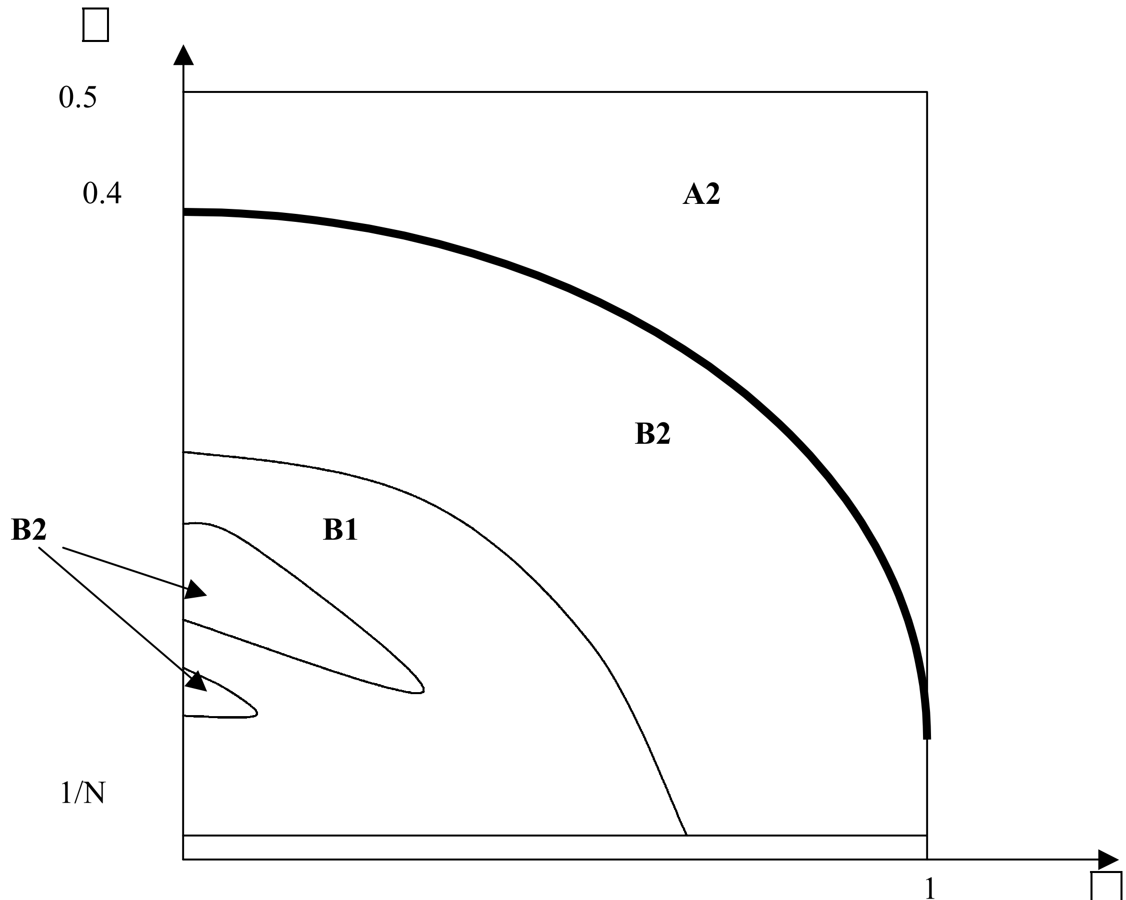

Figure 2(c): Fixed Membership – Learning Regions for Different IEAs - □ = 0.9

Table 1(a) Proportions of (γ, μ) Parameter Space For Regions A(i), A(ii), B(i), B(ii)

θ:
A(i)
A(ii)
B(i)
B(ii)

μ:
θ:
A(i)
A(ii)
B(i)
B(ii)

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