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**Discussion Papers in
Economics and Econometrics**

GROWTH WITH COMPETING TECHNOLOGIES AND BOUNDED RATIONALITY

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No. 0205

ISSN 0966-4246

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July 2002

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The model is investigated by means of local stability and computer numerical analysis. Two types of steady states obtain, each characterised by the complete specialization of production into one of the two technologies. Convergence towards the low-growth steady state, associated with the unskilled labour intensive technology, occurs under adverse structural conditions, such as marked initial skill shortage and high skill upgrade costs. This result of lock-in to the inferior steady state is interpreted as co-ordination failure, in that market forces do not always provide sufficient incentives to ensure a high-growth path.

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Growth with Competing Technologies and Bounded Rationality

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Abstract

I develop a model of growth based on three assumptions: first, a variety of technologies characterised by different degrees of labour skill intensity, where technological change is localized; second, agents are boundedly rational, and the aggregate rule of motion of their behaviour follows a replicator dynamics; third, markets do not clear instantaneously, with prices adjusting gradually. For simplicity, I study the case of two technologies and two labour markets, one for skilled and one for unskilled labour.

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JEL classification: O41, O33, E11, C62
Keywords: Unbalanced Growth, Localized Technical Change, Bounded Rationality

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1 Introduction

In the past few years, the search for new insights in growth theory has intensified under the pressure of the so-called convergence controversy, i.e. the empirical debate over the pattern of convergence, if any, of per capita income levels across countries. After common agreement has been reached over the rejection of the absolute convergence hypothesis (e.g. Barro (1991)), efforts have focussed on testing the validity of the conditional convergence hypothesis, which maintains that per capita incomes of countries sharing the same structural characteristics, e.g. preferences, technologies, population growth and government policies, etc., should converge to the same level regardless of their initial conditions. Evidence has been gathered that conditional convergence holds, but is slow, being about 2% per year (e.g. Mankiw, *et al.* (1992), Barro and Sala-i-Martin (1995)).

However, not only do some researchers question the validity of such evidence, calling for the use of different econometric techniques (e.g. Durlauf et al. (2001), Quah (1997)), but they also doubt that this is the really interesting issue to address (e.g. Azariadis and Drazen (1990), Quah (1996)). When we face such striking differences in per capita incomes across countries as we observe in the real world, the dramatic question is not whether or not poor countries converge to their own level of steady states, but what causes such steady state levels to be so low. Consequently, a sense of dissatisfaction with the neo-classical Solow model, in which steady states are essentially exogenously determined, led to various refinements of both the neo-classical and endogenous-growth approach. What all of these refinements have in common is the notion of multiple steady states, i.e. the simultaneous presence of two, or more, equilibria, one of which involves a long-run per capita income growth rate that is lower than the other(s). This equilibrium may be called a poverty trap[1], and its empirical counterpart is the so-called club-convergence hypothesis: per capita income of countries that are identical in their structural characteristics converge to the same level provided that their initial conditions belong to the basin of attraction of the same steady-state equilibrium (Galor (1996)).

There are many reasons why multiple steady states may occur. In a Solow-setting, it suffices to introduce heterogeneity across individuals, and different propensities to save out of interest and labour income in order to imply multiple steady states (Galor (1996)). In overlapping generation models, the role of human capital has received much attention. On the one hand, some authors have stressed the social increasing returns to scale from capital accumulation, either because of the positive externalities brought about by individual human capital (Lucas (1988)), or because of some threshold effects in technical progress (Azariadis and Drazen (1990)). Others have focussed on the constraints on individual capital formation stemming from capital market imperfections, especially in the presence of income inequality (e.g. Aghion *et al.* (1999), Aghion and Bolton (1997), Galor and Zeira (1993)), or local externalities (e.g. Durlauf (1996), Benabou (1996)). Closely related is the issue of the impact of financial institutions on growth (e.g. Banerjee and Newman (1998)). Another account hinges more directly on the distribution of income, especially through politico-economic channels: higher inequality lowers the median income position thus bringing about higher tax rates and lower growth (Persson and Tabellini (1994)); or, an initially low level of wealth may generate social conflict, thus hampering the chances of catching up (e.g. Benhabib and Rustichini (1996)). Other causes of multiple steady states include endogenous fertility (e.g. Galor and Weil (1996)).

As emphasised by Bernard and Jones (1996), however, very few of the explanations provided focus on technology, despite its undoubted importance for growth, but rather insist on capital accumulation

[1] There exists another notion of poverty trap in the literature, which is associated with the persistence of an individuals position in the income distribution rather than with aggregate variables. Under this definition, a poverty trap is a state in which dynasties starting out with income below a threshold, converge to a low-level of income, whereas others converge to a high-level (e.g., Moav, 2002, Durlauf (1996)). However, in this paper I will always refer to the concept of poverty trap given in the text.

as the privileged factor in accounting for income disparities: in the words of Romer (1993), 'object gaps' seem to count more than 'idea gaps' for students of development. However, some studies have emphasised how technology gaps amongst countries do seem to occur in reality (Bernard and Jones (1996)), and have put forward theoretical explanations for this fact, which hinge on a country's institutional and economic structure as possible barriers to the adoption of the 'leading-edge' technology (Parente and Prescott (1994)). The so-called "appropriate technology" approach has systematically taken this view, emphasising the necessity of a "good match" between the technology adopted and the specific structural characteristics of an economy (e.g. Basu and Weil (1998)). For instance, if the frontier technology is produced in advanced countries, and this is designed for the use by skilled workers, then developing countries, which can only rely on unskilled workers, cannot exploit technical advances and bridge the gap with the most advanced ones (Acemoglu and Zilibotti (2001)). Other research, rather than focussing on comparative issues, have modelled technology and industrial structure as key factors in orientating the growth path of an economy. In particular, the role of demand spillovers across the various sectors of an economy have been indicated as crucial in making it possible the 'take-off' from a state of low to one of high growth (Murphy, *et al.* (1989)). Analogously, focussing on the supply side, localized technological complementarities amongst industrial sectors can determine multiple steady states, and the presence of some leading sectors can again trigger a take-off process (Durlauf (1993)).

Another approach in which technical change is seen as the main dynamic engine of evolution for the economic system, is developed within so-called "evolutionary economics" (e.g. Nelson and Winter (1982), Hodgson (1999)). Its methodology is different from the approaches illustrated above, in that it takes on bounded rationality, and an off-equilibrium standpoint, rather than optimising agents and market equilibrium. Indeed, selection, rather than equilibrium, is the main conceptual tool of analysis employed (Dosi and Nelson (1993)). Multiple equilibria can occur here because of the path-dependency of economic systems, which arise from the cognitive and informational limits of agents and from the cumulative characteristics typical of many economic processes and technical change in particular (e.g. Dosi *et al* (1988)).

This discussion sets the framework of analysis of this paper. The main goal is to develop a model of growth where technical change and the structural conditions of an economy occupy the centre-stage of analysis as the main dynamic engines for the economic system. Moreover, I also adopt methodological assumptions rather similar to those employed in evolutionary economics, in that the process of adjustment towards market equilibrium and individual optimality is only gradual, and exchanges take place outside equilibrium. Therefore, in this setting multiple steady states and lock-in effects arise through channels different from those emphasised in 'mainstream' growth theory, specifically as a failure of the co-ordinating power of the market forces in driving the economic system towards an efficient outcome. The "neo-classical" case of instantaneous adjustment and full individual rationality is a limit case of the model presented here, and its implications are commented on.

As pointed out, the main characteristic of the model lies in the particular view of technical change that is adopted. In fact, both neo-classical and 'new' Growth Theory generally deals with technical change of the general purpose type, i.e. one capable of affecting the *whole* set of productive techniques. Indeed, it is this assumption that justifies the common representation of technical change as a uniform shift of the isoquants of the production function towards the origin. Instead, I take on the notion of localized technical change, which was originally put forward by Atkinson and Stiglitz (1969) and Salter (1969), and which has also been recently adopted by both the "appropriate technology" approach and the "evolutionary economics" approach. The basic idea behind localized technical change is that an innovation is generally capable of affecting only a limited subset of techniques, with general purpose technologies being a limiting case. This corresponds to the shift of some segments, or even single points, of the isoquant towards the origin, rather than taking for granted that the entire isoquant shifts.

Furthermore, I will focus on a structural factor sometimes overlooked in both evolutionary and mainstream approaches - namely the composition of the workforce in terms of its skill attainment. More precisely, labour is not considered *homogenous*, that is directly employable into all of the productive activities, rather the different technologies are constrained to employ only labour with a particular level of skill. In particular, two technologies are available, one of which is *skilled* whereas the other is *unskilled* labour intensive. Technical change is then made up of an exogenous and an endogenous component. The former advantages the skilled-intensive technique, whereas the latter makes individual productivity growth rates depend on the degree of concentration of economic activities in that technique. The reason is that technical knowledge is a public good at the individual technique level, but not at the economy-wide level, since knowledge spillovers can occur only amongst firms adopting the same technique.

This is the key characteristic causing the whole process of technical change to be cumulative, in the sense that a higher level of concentration within a sector makes the associated technique increasingly attractive to firms. Most importantly, this is also what possibly causes the economy to be stuck in a slow-growth trap: although the skilled labour intensive technique *ceteris paribus* brings about faster productivity growth rates, whenever the structural conditions of the economy are initially adverse to its development (e.g. because of skill shortage within the workforce) the economy will concentrate its activities into the other technique, thus precluding the possibility of catching-up. Therefore, in the model, low and high-growth steady states are identified as situations in which all of the productive factors, i.e. capital and labour, are allocated into the unskilled and the skilled intensive technology respectively. On the grounds of local stability analysis and of numerical computer analysis, it is shown that the economic system always converges towards one of these steady states; in particular, the steady state characterised by a balanced growth path between the two sectors can be proven to be unstable. The main contribution of the paper lies in the analysis of the economic conditions that determine the convergence towards one rather than the other steady state, of the path of convergence, and of the particular characteristics of the steady states thus obtained.

The resulting model has a flavour similar to evolutionary work dealing with the question of the contest between technologies, mainly that introduced by Arthur (1985), within a urn-scheme stochastic process, and also Durlauf (1993) and Corradi and Ianni (2000) in the context of a spatial-temporal analysis. These models prove the non-ergodicity of stochastic systems, thus presenting multiple equilibria and lock-in of inefficient outcomes as possible long-run steady states. It is also similar to other work that analyses the rise and extinction of different technologies and their impact on the macroeconomic performance of the economy, such as Nelson and Winter (1982), Verspagen (1993), Silverberg and Verspagen (1994). However, the present model is distinct from many of these contributions in that the final outcome is not merely due to random factors, but to underlying causes in labour markets dynamics and the pattern of technical change.

The model also relates to Baumol's analysis of the unbalanced development of the economy (1967), although in his approach the demand side of the economy is seen as the crucial determinant of the unevenness of the growth path, whereas in the present model I focus on the supply side. Furthermore, each of the sectors that composes the economy is modelled like Goodwin's single-sector model of growth (1967). Finally, some links with the "appropriate technology" approach are also evident, as the slow-growth steady state can be interpreted as a mismatch between the structure of the economy, and in particular its workforce composition, and the advanced technology.

The basic structure of the model is introduced in section 2: this is composed of two parts, one related to the sphere of production, where the notions of the pair of available technologies (section 2.1), of their productivity growth rates (section 2.2), and of the rules of capital accumulation (section 2.3) are outlined. The second part describes the dynamics of the labour market (section 2.4). Given the off-equilibrium nature of the model, it is also necessary to specify the behaviour of the economy in the recurrent situations of imbalances and rationing in the market (section 2.5).

The first version of the model, characterised by the impossibility of workers changing their initial type of skill, is presented in section 3. After an illustration of the economic forces driving the evolution of the model (section 3.1), an analysis of the local stability of its possible steady states is carried out (section 3.2). This turns out to be inconclusive, given the non-generic nature of the system of differential equations. Therefore, the results of a numerical analysis of the evolution of the system is presented. Not only does this allow us to discuss the final outcome of the evolution of the model, in terms of convergence towards one of the two technologies, but also it enables an analysis of the path actually followed by the economy along this process. Section 3.3.1 presents a scenario of lock-in towards the unskilled intensive technique when the skill shortage in the workforce is particularly marked, while section 3.3.2 shows how this result is reversed when the initial skill composition is even slightly modified in favour of skilled labour.

Section 4 studies a second and more general version of the model that allows the possibility of mobility of the workforce between the two sectors. In section 4.1 the presence of mobility costs for the workers is modelled, and section 4.3 shows how both outcomes of convergence towards the techniques are possible: in particular I focus on how reducing the mobility costs associated with skill upgrading determines the convergence to the low-growth steady state to the high-growth one. Section 5 draws out some conclusions, leaving a discussion of the mathematical details of the model to the Appendix.

2 The basic structure of the model

2.1 Production

Consider a market for a homogenous good Q, where *two* different *techniques*, differing upon their labour skill-intensity, are available to firms. For simplicity, I suppose that the first technique exclusively adopts *skilled* labour, while the second only employs *unskilled* labour. Each technique is uniquely determined by the pair of coefficients expressing the requirements of capital and labour per unit of output, so that a different pair of coefficients of production *per se* implies a different technique[2]. Nevertheless, as I shall elaborate in the next section, although the production coefficients of a technique are fixed in an instant of time, they may change over time as an effect of technical change. The two techniques of production can thus be represented by a fixed coefficient Leontief production function:

$$Q_i = \min\left\{a_i L_i, \frac{K_i}{c}\right\} \quad i = 1,2 \tag{1}$$

where L_1 and K_1 represents the employment of skilled labour and capital in the skill-intensive technique, and L_2 and K_2 the amount of unskilled labour and capital employed in the unskilled-intensive one. c is the fixed coefficient of the content of capital in one unit of output, assumed to be equal for the two technologies, and a_i is labour productivity. Obviously, total production is given by: $Q=Q_1+Q_2$.

This model can also be thought of as representing the whole of the economy. In this framework, Q is the aggregate bundle made up of two commodities produced in the two main industrial *sectors* of the economy, which employ techniques, or, rather, *technologies*[3], differing as to their labour skill-intensity. The sector employing the skilled labour intensive technology thereby represents an advanced hi-tech industry, whereas the other is a “traditional” low-tech sector. In order for Q to be a sound concept in this setting, one also needs to assume that the relative prices of the goods remain constant over time, for instance because of infinite elasticity of their demands. In a neo-classical framework, free entry into each sector and arbitrage conditions guarantee that the sectors grow at equal rates, thus making it possible to abstract away from individual outputs and to consider the aggregate of the two. In the present context, however, given the off-equilibrium approach, it may be possible that sectors grow at different rates even in a steady state, a characteristic which is usually referred to as unbalanced growth.

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[2] Dealing with only two techniques of production, rather than the continuum of techniques that would make up a typical “neoclassical” isoquant of production, precludes marginal substitutability of inputs, enabling firms to operate only a discrete choice between the two available techniques. This characterisation is consistent with indivisibilities in the production process (Antonelli, 1995).

[3] The term technology seems more suitable to dealing with industrial sectors rather than technique. However, I will consider the two as substantial synonymous.

Given the close correspondence between the two cases presented, in the following sections the terms technique and technology will be used interchangeably, and the term *sector* will represent at the same time the segment of a market employing one of the two techniques in the single market version of the model, or the industry adopting a certain technology in the aggregate version.

2.2 Technical change

As pointed out in section 1, the account of technical change that will be developed is of the localized type; that is, two independent laws of motion of technical advance will be set out for the two different techniques. Furthermore, technical change will depend upon an endogenous and an exogenous factor. Let us deal with the first factor.

Some notable economists, (e.g. Arrow (1962), Paul Romer (1994)), have emphasised the non-rival nature and the -at least partial- non-excludability of innovations. In fact, although there are many ways whereby firms can temporally appropriate the benefits deriving from an innovation, we can expect that in the long run a large amount, if not all, of the knowledge associated with a technical innovation will be spread over the rest of the economy. An opposite view is taken by economists of the evolutionary area (e.g. Dosi (1988)) who argue that the nature of technological knowledge is largely tacit, thus, at least to some extent, appropriable by firms. According to this view, even in the long run can first-innovators still keep their "technological lead" from the followers. Consequently, the scope of the phenomenon of technical spillovers throughout the whole of the economy would be limited.

In the present specification I take on an intermediate position between those two views, arguing that technical information spillovers take place at the level of the individual technique, but not at the economy-wide level. In other words, technical information is a public good only at the technique-specific level, in particular because innovations carried out by a firm adopting a certain technique can be imitated by firms employing the same technique, but not from those using the other. For simplicity, the imitation process is assumed to be instantaneous. An analogous interpretation of this idea would be that firms belonging to the same sector of the economy create a "net" through which technical information is transmitted amongst them (e.g. Antonelli (1995), Maurseth and Verspagen (1999)).

No explanation is given in the model about how and why such innovations are carried out, and the R&D variable is then omitted. However, it is assumed that the larger the share of firms active in a certain sector, the higher the probability that an innovation is carried out. That is, applying the large numbers law, the growth rate of technical change depends positively on the density of activity in a certain sector, which is measured by the share of capital invested therein. Finally, technical change is assumed to be of the labour-augmenting type.

The following equation summarises all these considerations. It describes the rule of motion of labour productivity for each technique:

$$\frac{\dot{a}_i}{a_i} = g_i k_i \qquad (2)$$

where k_i is the share of capital invested in technique i :

$$k_i = \frac{K_i}{K} \qquad (3)$$

g_1 and g_2 are fixed parameters satisfying the following condition:

$$g_1 > g_2 \qquad (4)$$

The fact that a_i depends on k_i captures the *endogenous* factor of technical change: the larger the concentration of economic activity in technique *i*, the larger the technique-specific innovation rate[4,5].

[4] In Grimalda (2001), some evidence has been collected confirming this relation: in particular, the manufacturing sectors have been classified in four groups on the grounds of their technological intensity, in accordance with OECD (1997). Hence, for a sample of OECD countries, the productivity within each group, normalised for a country's average productivity, can be shown to depend positively on their value added share, with respect to manufacturing value added.

The parameters g_i, instead, portray the *exogenous* factor of technical change, which is usually thought of as being linked with the rate of advance in scientific discoveries. In fact, many scholars of the subject emphasise the essential role of growth in general scientific knowledge to bring about technological innovations: this acts as the ultimate determinant and as a constraint to the technological knowledge present in an economic system (Rosenberg (1982)). The fact that science mainly evolves independently from economic activity makes it an exogenous factor of development[6]. Condition (4) takes account of the evidence that skilled labour intensive techniques have in the last decades been more efficient than unskilled intensive ones, particularly because of the complementarity between information technology and skilled labour (Berman *et al.* (1998)). In the present model, this is equivalent to saying that scientific discoveries are more easily applicable to technique 1. This implies that technique 1 has a higher *potential* for growth, in the sense that economic growth is higher when productive factors are entirely allocated in the skilled intensive technique rather than in the unskilled intensive one. Therefore, steady states characterised by entire allocation of factors within the skilled-intensive technique will carry out higher growth rates than alternative steady states.

2.3 Capital accumulation

2.3.1 Rule of motion of aggregate capital

Since two productive sectors are present, we need two different laws of motion of capital: one refers to the aggregate level of capital accumulation whereas the other shows how capital distributes between the two sectors. In what follows, we shall assume that there exists a *continuum* of firms, so that the *dimension* of each of them is negligible. Moreover, each firm possesses one (infinitesimal) unit of capital, which can be invested in either technique. The following are aggregate relations, which can nevertheless be accounted for in terms of sensible individual behaviours. Let us first introduce some variables:

$$K = K_1 + K_2 \qquad (5)$$

$$r_i \equiv \frac{p_i}{K} \equiv \frac{Q_i - w_i L_i}{K_i} = \frac{1 - \frac{w_i}{a_i}}{c} \qquad (6)$$

$$\bar{r} = k_1 r_1 + k_2 r_2 \qquad (7)$$

Equation (5) defines the aggregate level of capital as the sum of the amount of capital invested in each technique. r_i describes the individual profit rate, and $\bar{r}$ is the overall rate of profit obtained by weighing the individual profit rates with the respective capital shares. Let us call y_i the cost of labour per unit of output produced:

$$y_i \equiv \frac{w_i}{a_i} \qquad (8)$$

This supports the view that economic systems *specialising* in a particular technology, or industrial sector, in terms of their value added share, also experience higher productivity growth rates.

[5] It can be noticed that the specification of the endogenous component of technical progress comprises a *negative* externality between firms active in different sectors: not only does a firm leaving a technique for the other increase the productivity in the new sector, but also it decreases that of the previous sector. Indeed, a relation consistent with the theoretical considerations previously set out should make the productivity growth dependent on the *absolute* level of capital present in a sector, rather than on its *relative* value. However, the specification adopted can be justified on grounds of analytical tractability, and in any case it does not affect the results of the model, since, as we shall see in section 2.3, the choice of the firm as to their location in the technology scale is based on the *relative* profitability of the two techniques.

[6] I proved that other specifications of the productivity equations, e.g. a linear equation in g and k, lead to the same results as the one adopted; in general, though, I cannot provide a general proof of the robustness of the results.

Then, we can write a compact expression for the technique-specific rate of profit:

$$r_i = \frac{1 - y_i}{c} \tag{9}$$

The share of total labour income is thus given by the following expression:

$$\bar{y} \equiv \kappa_1 y_1 + \kappa_2 y_2 \tag{10}$$

As for consumption and investment behaviour, in order to avoid technical complexities I simply assume that at each instant of time workers spend a constant share of their income, for simplicity set equal to one, in consumption, and firms reinvest a constant proportion of their profits, assumed equal to one as well[7]. Therefore, the overall flow of investments coincide with the amount of profits in a period, and the proportional growth rate of aggregate capital is equal to the rate of profit:

$$\frac{\dot{K}}{K} = \bar{r} = \frac{1 - \kappa_1 y_1 - \kappa_2 y_2}{c} \tag{11}$$

2.3.2 Rule of motion of individual capital

I now specify the macroeconomic evolution of the system for what concerns the distribution of capital across technologies, based on the assumptions of boundedly rational behaviour at the individual level. The specification that will be adopted draws on a model that has been put forward in the evolutionary literature (Silverberg and Verspagen (1994)), which captures the idea that adjustments towards optimality can occur only slowly because of informational and/or cognitive constraints on the agents. Equation (12) describes the sectoral growth rate of capital:

$$\frac{\dot{K}_i}{K_i} = r_i + \alpha(r_i - \bar{r}) \qquad \alpha > 0 \tag{12}$$

This equation is similar to equation (11) in its first component, in that capital accumulation in sector *i* depends on the amount of profits made by firms active therein, which, in accordance with the behaviour assumed in the previous section, are immediately reinvested: this is the "normal" rate of accumulation r_i, that is. However, the second component takes into account the possibility that firms switch towards the technique that is currently more profitable, thus making its growth rate higher. Adjustment costs are assumed to be negligible. Such a second component is consistent with a bounded rational behaviour, in that only the extreme case of α equal to infinity does correspond with the situation of instantaneous adjustment to optimality. In all of the other cases, at each instant of time only a part of the firms switch to the more advantageous technique, at a rate that is proportional to the extent of the profitability difference. Therefore, α may be seen as an index of the "amount" of information available to agents. Instead, the term expressing the relative profitability between techniques is justified on the grounds of agents' cognitive limits: a higher difference in relative profitability implies a faster flow of firms, the idea being that firms can in this case spell out more easily the available information and thus move in the 'right' direction[8].

[7] Notice that nothing would be lost by setting the propensities to consume and invest at a level less than one. This assumption is common to models of the Kaldorian tradition, which, more generally, assumed that the saving propensity of firms was higher than that of consumers. In models of neo-classical growth, the same result of constant propensity to save obtains, but in that case the horizon span is infinite.

[8] To be sure, the two aspects of information and cognitive ability are interrelated between one another: agents with more sophisticated cognitive abilities will also have more incentive in collecting information, thus influencing the magnitude of the parameter α. In any case, a value of α equal to infinity will be taken to represent the limit situation of fully informed and perfectly optimising agents who immediately recognise in each period which is the best technique, and move towards it.

Under this general framework, various microeconomic accounts of firms' actual individual behaviour and of the extent of cognitive and informational constraints are possible[9]. An interesting characteristic of the rule of motion set out in equation (12) is revealed by expressing it in terms of the share of capital:

$$\frac{\dot{k}_i}{k_i} = \frac{(1+a)(1-k_i)(y_j - y_i)}{c} \tag{13}$$

It is now evident that a version of the *replicator dynamics* is implicit in the rule of motion of firms across techniques (Weibull, 1995). In fact, $(y_j\text{-}y_i)$ is nothing but the difference in the rates of profit[10]. We can thereby conclude that the dynamics of the motion of firms across sector is grounded on the idea that firms *imitate* the agent of greater success on the basis of a slow process of diffusion of the information and limited cognitive abilities.

2.4 Labour market

Let L^S define the overall level of the workforce present in the economy. Assuming that the population does not grow over time, we can with no loss of generality normalise this level to 1. The population is made up of two types of workers, skilled and unskilled, whose level is indicated by the variable *s* and *(1-s)* respectively. In this section I assume that workers cannot move across sectors, and this, together with the assumption that every worker supplies all of her endowment of labour, makes the level *s* fixed. It is helpful to introduce some notation for labour supply in a specific market:

$$L_i^S = \begin{cases} s & \text{if } \; i=1 \\ 1-s & \text{if } \; \mathrm{i}=2 \end{cases} \tag{14}$$

Due to the off-equilibrium nature of the approach, a pair of equations describing the motion of prices in response to an imbalance in the market is needed. Labour demand will be determined by the firms' level of capital, which yields the following linear relation:

$$x_i \equiv L_i^D = \frac{K_i}{a_i c} \tag{15}$$

In the first instance, we assume a simple linear relation between the proportional growth rate of wages and the imbalances in the labour market:

$$\frac{\dot{w}_i}{w_i} = g\left(x_i - L_i^S\right) \tag{16}$$

γ is a parameter expressing the speed with which imbalances on the labour market affect wages: in the hypothesis of instantaneous market clearing, this parameter would be equal to infinity.

However, I will add to this relation another term measuring the impact of redistribution policies that are carried out "externally" to the market:

[9] A second account that would be consistent with equation (12) would be one in which fully informed and perfectly rational agents, though unable to make forecasts over the future so that their horizon only spans the current period, face delays in the transfer of capital from one sector to the other. In this case, α would then measure the speed at which capital can be scrapped in one sector and reinvested in the other sector.

[10] Not surprisingly, the argument can be easily restated in the language of Game Theory: α, the parameter relative to the degree of information present in the economy, may be seen as depending on the probability of being chosen at random from the group of firms and matched with another firm. The difference $(y_j\text{-}y_i)$ is the differences in the payoffs currently earned by the two "players", thus representing the incentive to adopt the alternative technique in the circumstance it is performing better: in this case the alternative strategy best fits the environment, meaning a higher rate of "reproduction" of the technology.

$$\frac{\dot{w}_i}{w_i} = g\left(x_i - L_i^S\right) + b_i \tag{16'}$$

The parameters b_i may be thought of as the result of a bargaining between the relevant groups of agents present in the economy over income distribution. It is indeed clear that its introduction alters both the distribution of income between labour and profits and between the two wages with respect to that obtained through the market, allowing a sort of "guaranteed" increase in wages[11]. Therefore, it is perhaps more convenient to express it in terms of the increase in labour productivity, as it happens in "real" bargaining over income distribution:

$$b_i = h g_i \qquad h \geq 0 \tag{17}$$

Therefore, in this specification parameter b_i represents the share of productivity gains accrued to labour income independently from market interactions. As we shall see, the value assumed by parameter h will be crucial in order to determine whether the evolution of the system reaches a situation of structural unemployment or not.

A further constraint regards profits: the following condition allows for the fact that firms would temporarily shut down their activities when experiencing negative profits:

$$0 \leq y_i \leq 1 \tag{18}$$

Clearly, when y_i, hits the boundary level of 1, wage growth could not exceed productivity growth, since claims from workers to get wage increases above that level could not be accommodated by firms just breaking the even. This constraint, along with equations (2) and (8), yields the final expression for the law of motion of y_i:

$$\frac{\dot{y}_i}{y_i} = \begin{cases} g\left(x_i - L_i^S\right) + (h-1)k_i g_i & if\, y_i < 1 \\ \min\left\{0, g\left(x_i - L_i^S\right) + (h-1)k_i g_i\right\} & if\, y_i = 1 \end{cases} \tag{19}$$

The actual level of employment will obviously be the minimum between labour demand and supply. Indicating such variable with L_i, we have:

$$L_i = \min\left\{L_i^D, L_i^S\right\} \tag{20}$$

2.5 The behaviour of the system when labour demand is rationed

The explicit introduction of labour market requires an amendment of what outlined above regarding the rules of capital accumulation. In fact, the possibility of having an excess of labour demand over supply implies that a fraction of capital is actually left idle, and of course this will affect both the level of profits and of investments.

More precisely, suppose labour demand is in excess, so that only the fraction of capital that can be matched with labour supply is employed in production. The rest is unproductive, because of the perfect complementarity between labour and capital associated with the Leontief technology. Let K^E_i be the amount of capital *effectively* being employed, while K_i is the overall amount of capital *present* in a sector, but not necessarily employed. K^E_i will be given by the following expression:

[11] The reader may have noticed that in such a specification the wages law of motion is formally equivalent with a Phillips curve relationship, where the NAIRU is given by the expression $L_i^S - \frac{b_i}{g}$.

$$K_i^E = \begin{cases} c a_i L_i^S & when \quad L_i^D > L_i^S \\ K_i & when \quad L_i^D \le L_i^S \end{cases} \tag{21}$$

More generally, we can define the ratio of capital actually employed over the total as:

$$u_i = \begin{cases} 1 & when \quad L_i^S \ge L_i^D \\ \dfrac{c a_i L_i^S}{K_i} & when \quad L_i^S > L_i^D \end{cases} \tag{22}$$

The variable u_i will be called the *capacity utilisation of capital* in sector *i*. When labour supply is in excess there will be no rationing, leading to *full* utilisation of capital, represented by a value of 1 for such a variable. When labour demand is in excess u_i will take on values less than 1. Therefore, in situations of rationing, a percentage (1- u_i) of firms *present* in sector *i* is unable to undertake any productive activity. These firms will offer higher wages than the current one, thus hoping to attract workers currently working for other firms in order to enter the market. This has the effect of raising the level of wages in accordance with equation (20).

The eventuality of rationing affects both the levels of profits and investments, as these quantities are determined by the capital that is actually being employed. Therefore, the foregoing expressions for the growth of capital must be corrected to take into account its possible rationing. One finds that the overall rate of profit, which is equal to the overall rate of growth of capital, is now given by:

$$\frac{\dot{K}}{K} = \bar{r} = k_1 u_1 r_1 + (1 - k_1) u_2 r_2 \tag{23}$$

where, as stressed above, the rates of profit in each sector are computed in terms of effective capital:

$$r_i = \frac{Q_i - w_i L_i}{K_i^E} \tag{24}$$

This is the quantity that firms actually use when comparing their current level of profit and that made possible by the alternative technique. Analogously, the growth rate of capital in each sector will be given by:

$$\frac{\dot{K}_i}{K_i} = u_i r_i + a\left(u_i r_i - \bar{r}\right) = \frac{u_i\left[1 + a(1 - k_i)\right](1 - y_i) - a u_j (1 - k_{i1})(1 - y_j)}{c} \tag{12'}$$

which of course boils down to equation (12) when both u_1 and u_2 are equal to 1. Equation (13) is subject to an analogous change:

$$\frac{\dot{k}_i}{k} = \frac{(1 + a)(1 - k_i)\left[u_i (1 - y_i) - u_j (1 - y_j)\right]}{c} \tag{13'}$$

3 The model with no mobility of labour

3.1 An economic insight into the model

The dynamics of the model set out above can be represented by a non-autonomous system of differential equations in the six variables representing the dynamics of each sector, i.e. productivity, capital, unit cost of labour. However, letting x_1 and x_2 substituting for K_1 and K_2, and introducing k_1, whose value is indeed determined by x_1 and x_2, we can keep out productivity and reduce the system to an autonomous 5-dimension system. This is what obtains in the interior of the space:

$$\dot{k}_1 = \frac{(1+a)(1-k_1)[u_1(1-y_1)-u_2(1-y_2)]}{c}k_1 \tag{26}$$

$$\dot{y}_1 = [g(x_1 - s) + (h-1)k_1 g_1]y_1 \tag{27}$$

$$\dot{y}_2 = [g(x_2 - (1-s)) + (h-1)(1-k_1)g_2]y_2 \tag{28}$$

$$\dot{x}_1 = \left(\frac{u_1[1+a(1-k_1)](1-y_1) - au_2(1-k_1)\ (1-y_2) - ck_1 g_1}{c}\right)x_1 \tag{29}$$

$$\dot{x}_2 = \left(\frac{u_2[1+ak_1](1-y_2) - au_1 k_1(1-y_1) - c(1-k_1)g_2}{c}\right)x_2 \tag{30}$$

The dynamics forces driving the model can perhaps be better appreciated if they are considered each in turn:

A) *Productivity*: Productivity levels are one of the determinants of the relative profitability of the two techniques. It is affected by the concentration of firms in a sector because of the positive externalities gained through knowledge spillovers. This process determines a peculiar phenomenon of cumulativeness in technical change, since once an economy "specialises" in a technique, that is to say allocates a large share of capital in a sector, it becomes increasingly difficult for the other sector to bridge the productivity gap.

B) *Wages*: The intensive use of a certain technique brings about an increase in the associated level of employment, which in turn increases wages and reduce profitability.

C) *Skill shortage*: This factor refers to a *structural* characteristic of the economy, given by the condition of relative abundance of the workforce in each market. As outlined above, if the labour supply associated with a particular technique is relatively scarce then the excess of labour demand will bid wages up.

These three factors can have a counterbalancing effect on each other; particularly, the wage effect and the possible presence of skill shortages of skilled labour might slow down, or even impede the economic system from converging towards the efficient technology.

In more detail, one can notice that the model is symmetric in the pairs of variables (x_1,y_1) and (x_2,y_2). Moreover, when capital is completely allocated in one sector (that is, κ_1=0 or κ_1=1) then the pair of equations that are now relevant "loses" every link with the other two equations. For instance, when κ_1=1, then equations (27) and (29) are "autonomous" from the other two variables and make up a "sub-system" of equations that is known as Lotka-Volterra, or predator-prey, model. This two-dimension system of equations has been extensively studied both in the mathematical literature (Hirsch and Smale, 1974) and in Economics (Goodwin, 1967). Its basic characteristic is to display a persisting cyclical behaviour in the two relevant variables (capital and cost of labour), because an excess of labour demand drives wages up, thus reducing the rate of profit and investment. In turn this decreases the level of production and employment, so that wages diminish and trigger a new phase of increase in investments. The virtue of this simple model is that it generates *endogenous* cyclical fluctuations around a trend within a model of growth. Hence, the system under exam looks like a generalisation of the predator-prey model, being a "combination" of two such models, and boiling down to one of them when converging to the boundary of the κ_1 axis.

3.2 Analysis of local stability

The steady states of the system can be divided into three categories: convergence towards the high-growth equilibrium, convergence towards the slow-growth equilibrium, and, finally, a balanced growth path between the two sectors of the economy. For *convergence* to a sector I mean the process which leads asymptotically to the state of complete allocation of capital – and, from section 4, of labour as well – within that sector. So, we shall observe one sector becoming the *leading* one of the economy, as its share in overall production continuously arises, and the other being confined to a *residual* role. The balanced growth path solution, instead, depicts a situation in which the two sectors grow at the same rate.

An assessment of local stability is not possible for the first two categories of steady states, because the presence of some purely imaginary eigenvalues makes the system locally non-hyperbolic (Guckhenheimer and Holmes, 1990). Still, the numerical analysis that I have conducted, part of which is reported in section 3.3, shows that all of such solutions look like attractors of the system under some values of the parameters. Instead, the solution associated with the balanced growth path can be immediately proved to be unstable by local stability analysis. In what follows the three types of steady states will be presented in more detail.

3.2.1 High-growth steady states

A1) $$\left\{ k_1 = 1 \quad y_1 = 1 - cg_1 \quad x_1 = s - \frac{(h-1)}{g} g_1 \quad y_2 = \frac{1 + a(1 - cg_1)}{1 + a} \quad x_2 = 1 - s \right\}$$

This solution is characterised by complete allocation of capital into the efficient technique. It holds under the condition that h be greater than 1, thus it implies a positive level of unemployment for skilled labour and full employment for unskilled labour. One can also notice that a greater speed of adjustment in labour market, as measured by coefficient g, helps to reduce the level of unemployment, which at the limit is then equal to zero. Moreover, when a also goes to infinity, which corresponds to the case of perfect information and rationality of the agents (see section 2.3.2), the profit rate in sector 1, $1 - y_1$, equates that of sector 2, $1 - y_2$, thus making firms indifferent between the two sectors. This state seems indeed to reflect a typical 'neo-classical' equilibrium, where labour markets clear and all sectors of the economy have the same level of profitability, though all the activities are concentrated in the first sector. Hence, the introduction of non-instantaneous market clearing and limited information within the model brings about structural unemployment and a persistent gap in the two sectors profit rates.

Another steady state characterised by convergence towards the first sector obtains:

(A2) $$\left\{ k_1 = 1,\ y_1 = 1 - cg_1 - c\left(\frac{1-h}{g\ s}\right) g_1^2,\ x_1 = s + \frac{(1-h)}{g} g_1,\ y_2 = \frac{1 + a(1 - cg_1)}{1 + a},\ x_2 = 1 - s \right\}$$

This was found under the limitation that $\eta<1$, thus it implies full employment of labour and rationing of capital in the first sector, and full employment of both capital and labour in the second technique (see eqs. 41-42).

3.2.2 Low-growth steady states

We also find a couple of steady states symmetrical to those just found, though they are characterised by convergence towards the second sector:

(B1) $$\left\{ k_1 = 0 \quad y_2 = 1 - cg_2 \quad x_2 = (1 - s) - \frac{(h-1)}{g} g_2 \quad y_1 = \frac{1 + a(1 - cg_2)}{1 + a} \quad x_1 = s \right\}$$

(B2) $$\left\{ k_1 = 0,\ y_2 = 1 - cg_2 - c\left(\frac{1-h}{g\ (1\text{-}s)}\right) g_2^2,\ x_2 = (1\text{-}s) + \frac{(1-h)}{g} g_2,\ y_1 = \frac{1 + a(1 - cg_2)}{1 + a},\ x_1 = s \right\}$$

Solution (B1) is an equilibrium with “structural unemployment” in the leading sector of the economy, that is sector 2, and full employment of both inputs in the residual one; again this solution holds under the restriction that h is greater than 1. Conversely, solution (B2) predicts an outcome with full employment of labour in both sectors, under the condition that h is less than 1. The properties of stability of these steady states are the same as those found out for the case of convergence towards the first sector.

3.2.3 Balanced growth path

This is the only steady state in which both technologies coexist:

$$\text{(C1)} \quad \left\{ \begin{array}{lll} k_1 = \dfrac{g_2}{g_1 + g_2} & y_1 = \dfrac{g_2 + g_1(1 - cg_2)}{g_1 + g_2} & x_1 = s - \dfrac{(h-1)}{g} g_1 g_2 \\ y_2 = \dfrac{g_2 + g_1(1 - cg_2)}{g_1 + g_2} & x_2 = 1 - s - \dfrac{(h-1)}{g} g_1 g_2 & \end{array} \right\}$$

Notice that such a steady state is not constrained by any limitation to parameter η: it can thus depict both a situation of full employment of labour or of structural unemployment, boiling down to no unemployment when γ=0. It is easy to show that for this value of k_1 the productivity remains constant across the two sectors; the other values ensure that labour markets clear, so that there is no tendency for the system to move away from such a configuration. Since both sectors evolve according to the same growth rate, the economy can be said to follow a balanced growth path. Nevertheless, an analysis of its local properties of stability shows that such an outcome is in fact unstable (see section 6.1). The economic reason is to be found in the property of cumulativeness of technical change: if this state is perturbed even slightly, then the sectoral productivity will differ, thus attracting some firms to move to the more profitable technique. As a consequence, the sector that by chance happens to be the more profitable will experience positive sectoral economies of scale that will suffice to break the balance between the two profit rates.

3.3 Analysis of global stability

Due to the complexity of the model, a thorough global investigation of the properties of the system is not possible. Therefore, using a specifically designed programme for Maple V, I have worked out a series of numerical analyses to test the behaviour of the system. The interested reader can find some notes in the last section of the Appendix. Overall, the main conclusion one can draw is that all of the above dubious cases of steady states turn out to be stable equilibria of the model for some values of the parameters. In what follows I shall highlight some of their features.

3.3.1 First scenario: convergence towards the inefficient technology in the presence of a marked skill shortage

First, I consider a situation where the condition of skill shortage is quite marked, as the economy starts off from a position where two thirds of the population are unskilled; that is to say, s=1/3. Besides, I also assume that the initial situation is one of initial perfect symmetry for all of the other variables, such that the two techniques are equitably profitable, and firms are equally distributed across them[12]. This should be appropriate for a situation of absolute ignorance over the properties of the techniques at

[12] The particular values for the parameters have been chosen consistently with works by Silverberg and Verspagen (1994) – for what concerns α, the degree of individual rationality and information - and Barro and Sala-i-Martin (1996) – for the capital output ratio c and sectoral growth rates g_1 and g_2:
α=1; g_1=0,04, g_2=0,02; c=3; γ=0.5; s=1/3; η=1.
The set of initial conditions is meant to depict a situation of even distribution of firms across the two sectors:
$\kappa_1(0)=0.5$; $y_1(0)=0.5$; $y_2(0)=0.5$; $x_1(0)=0.5$; $x_2(0)=0.5$; $a_1(0)=1$; $a_2(0)=1$

the beginning of the "story". Moreover, parameter η, the key to income distribution, is assumed to be greater than one. Therefore, the feasible steady state between those listed in section 3.2 would be solution (B1).

The main result is that the system converges asymptotically to the slow-growth steady state, as all capital becomes invested into technique 2 through a series of periodical oscillations that progressively dampen down (Figure 3.1). The reason can be investigated by looking at the behaviour of productivity growth rates. Technique 1 starts off with a higher productivity, as a result of the even distribution of firms across technologies at instant 0 (figure 3.2). However, firms become soon attracted by the possibility of hiring cheap labour in the unskilled labour market, thus boosting technique 2's productivity. Hence, after few periods, the productivity in the second sector leapfrogs the other, and this is sufficient in order to determine a form of technological lock-in towards the second technique.

It is worth noticing that the first sector does not completely disappear over time: as Figure 3.3 shows, production growth rate settles on a 0-growth path on average, in which the flow of firms moving to the more profitable sector is exactly compensated by the flow of new investments into this sector. This is the sense in which we can call this sector the *residual* one of the economy. The overall growth rate tends to stabilise over the same growth rate as that of the leading sector – sector 2, that is - but the individual growth of the individual sectors show much widest fluctuations, which partly offset each other because of their different periodicity.

The subsequent diagrams depict the dynamics within the labour market in the leading sector; that of the residual sector is similar thus it will not be commented. Employment and unit cost of labour settle on a cyclical path (Fig. 3.4). In the long-run, their trajectory converges towards a limit cycle, typical of Lotka-Volterra systems (Fig. 3.5). Notice that the coordinates of the centre of the cycle correspond with solution (B1), once the parameters have been assigned their numerical values[13]. While the first sector periodically experiences phases of full employment, a state of structural unemployment in the leading sector of the economy occurs.

One may question the reason why the two markets do not clear, even in the presence of flexible prices. In general terms, this is the result of the non-instantaneous adjustment in the labour market (see discussion in section 3.2). However, there is a further reason, related to income distribution. Provided that techniques have by construction fixed coefficients of production, firm labour demand will depend on the level of wages only indirectly, by means of the following dynamical mechanism: a low level of wages triggers high profits and then high investments, thus increasing the stock of capital and the demand for labour, and vice versa. However, if the "institutional" redistributive component is too high, which is the case when η is greater than 1, then capital accumulation will be too low to match labour supply. On the other hand, as we shall observe in the following section, for values of η less than 1 we obtain full employment of labour, but this time it is a portion of capital that is left idle. Therefore, too large a share of income devoted to labour implies too slow capital accumulation, thus creating unemployment. Finally, Figure 3.6 shows the evolution of income shares over time.

[13] Plugging in the value of the parameters, one then obtains that the trajectories should either orbit or converge towards the following set of values:

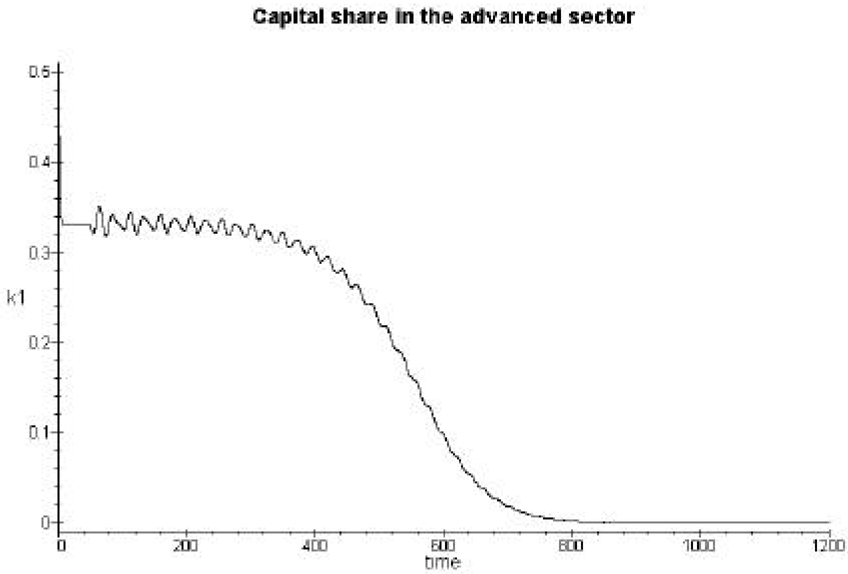


Figure 3.1

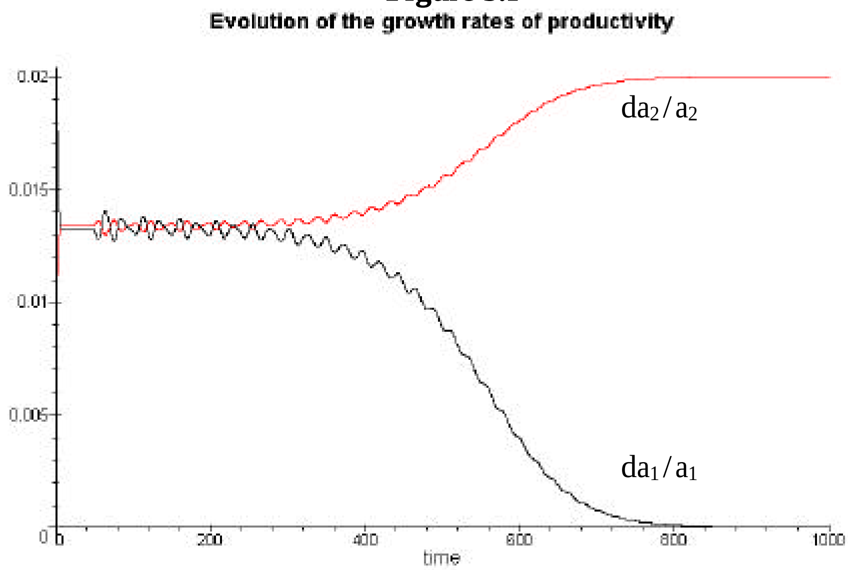


Figure 3.2

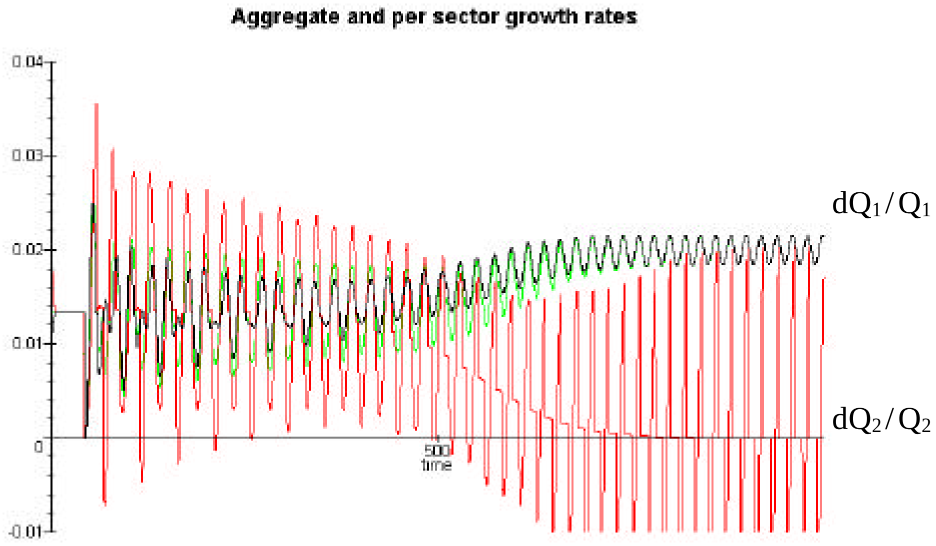


Figure 3.3

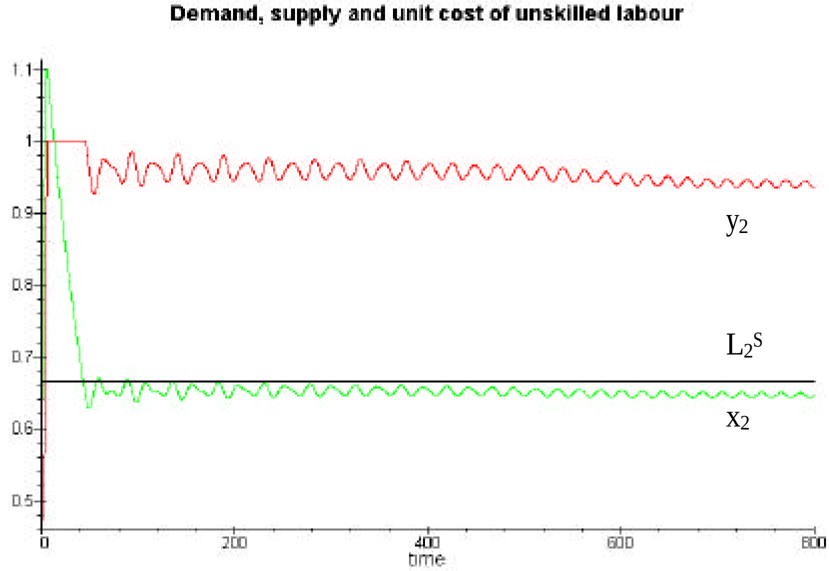


Figure 3.4

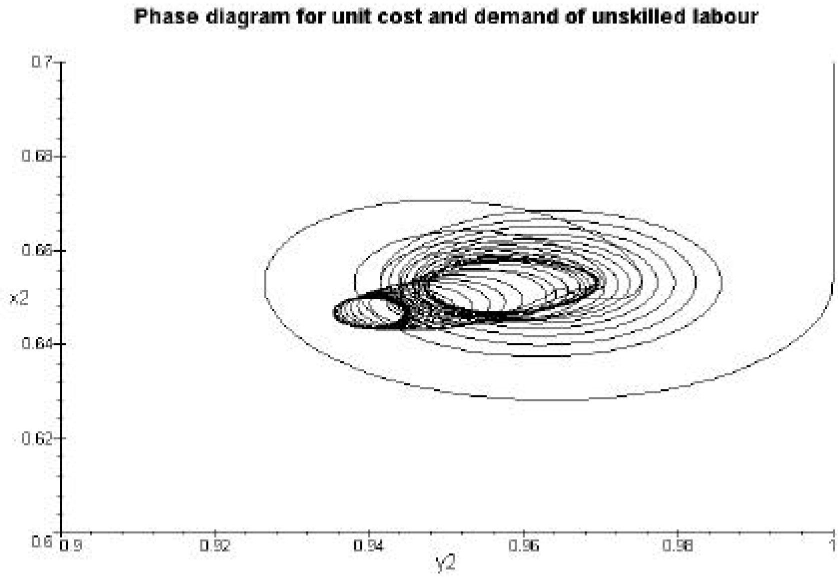


Figure 3.5

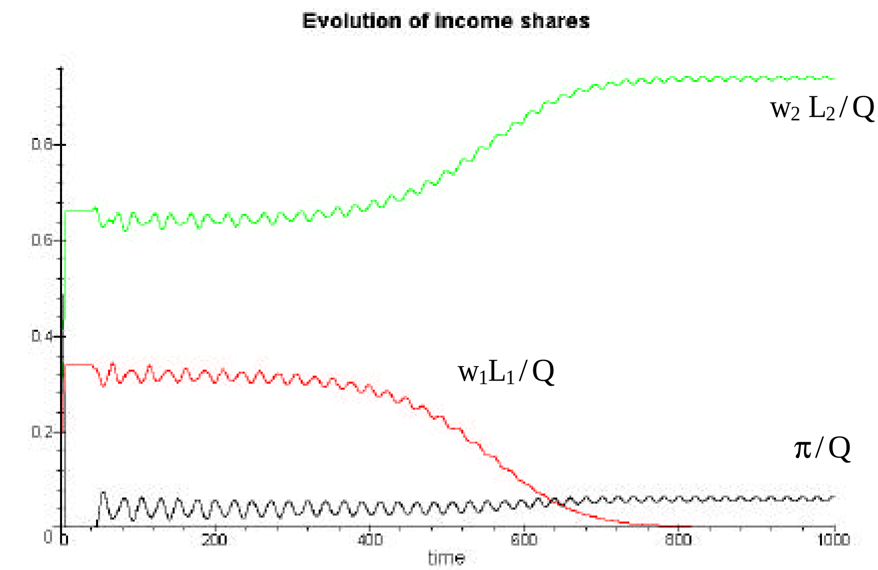


Figure 3.6

3.3.2 Second scenario: convergence towards the high-growth steady state with limited skill shortage

In this section I only slightly alter two of the parameters with respect to the previous case. This has the effect of completely upsettting the previous outcome. First, the percentage of skilled workers in the population now shifts from a third of the previous case to forty percent in the present one, thus implying a less marked skill shortage - namely s=0.4. Moreover, parameter ***h*** takes on a value less than 1, namely 0.5, indicating a less favourable distribution for labour income. All of the other values are those indicated in footnote 12.

As Figure 3.7 shows, convergence to the efficient technique now obtains. Accordingly, the analytical solution that is feasible for this setting is (A2)[14]. The individual and aggregate growth rates display the same behaviour as the previous case, but the roles are now inverted: it is the first sector that becomes the leading one, whereas the second sector dwindles to a zero-growth path (Figure 3.8). One of the distinguishing features of this case is that it is now capital that is rationed, while labour reaches full employment (Fig. 3.9 and 3.10). The presence of idle capital in this case as well as of unemployment in the previous scenario, is a typical characteristic of models of the Harrodian tradition, to which the present model is similar in that it precludes marginal substitution of factors of production. Finally, it can be noticed that all of the variables do not show the peculiar cyclical behaviour observed in the previous scenario, but tend instead to converge towards a point. This is due to the change in parameter h, which implies that the type of dynamics in the leading sector of the economy is that of a stable focus rather than a centre (see discussion in the Appendix, section 6.1). Figure 3.11 shows indeed that the variables in the leading sector display the spiralling behaviour typical of a focus.

Capital share in the advanced sector

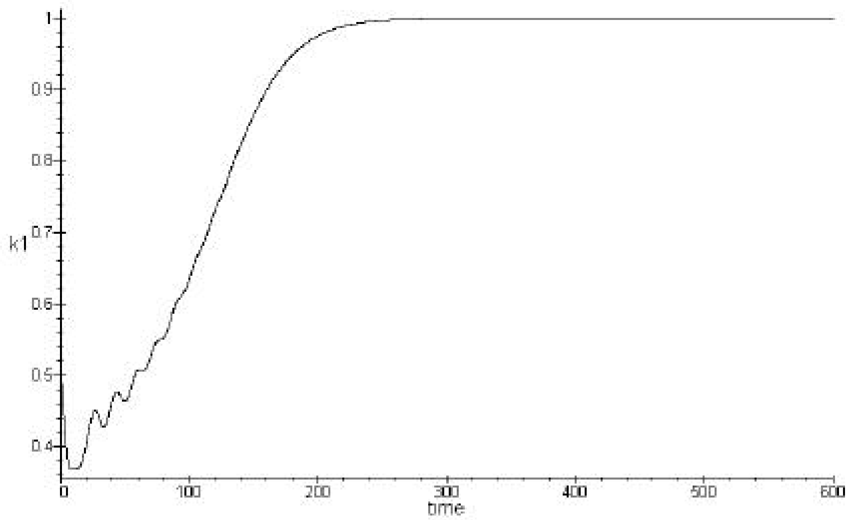


Figure 3.7

[14] This solution reads as follows after having substituted for the values of the parameters:

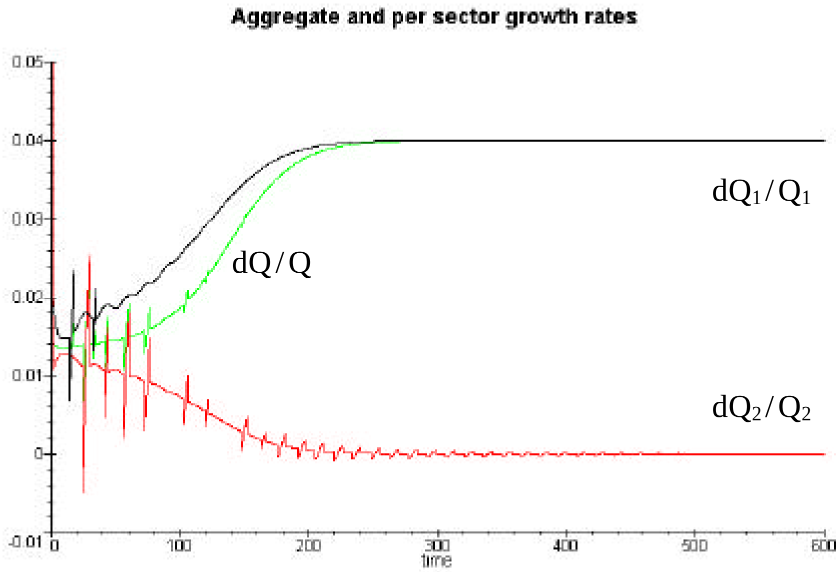


Figure 3.8

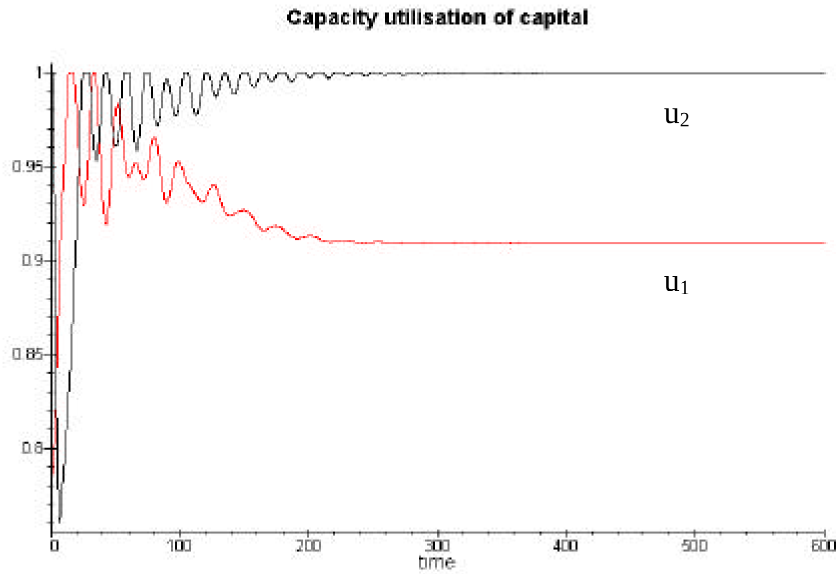


Figure 3.9

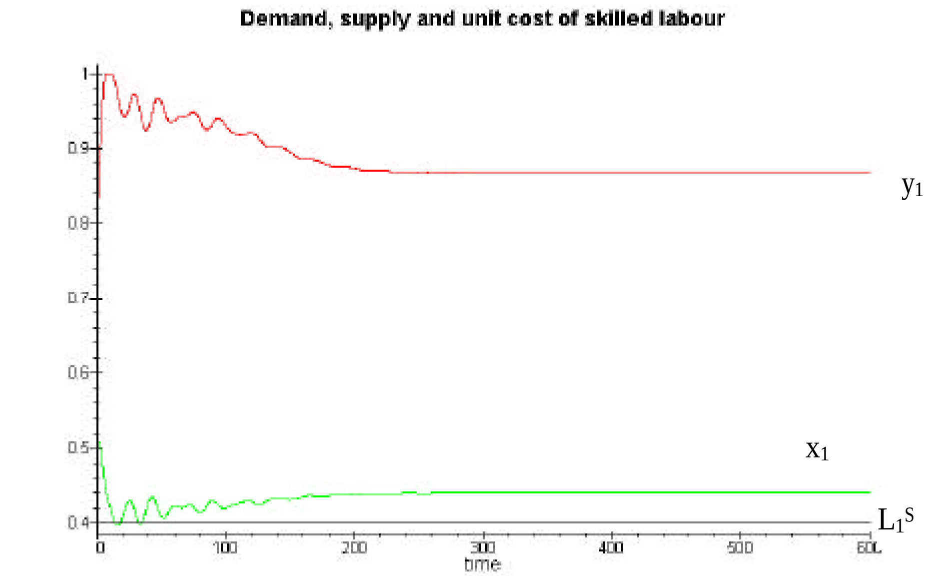


Figure 3.10

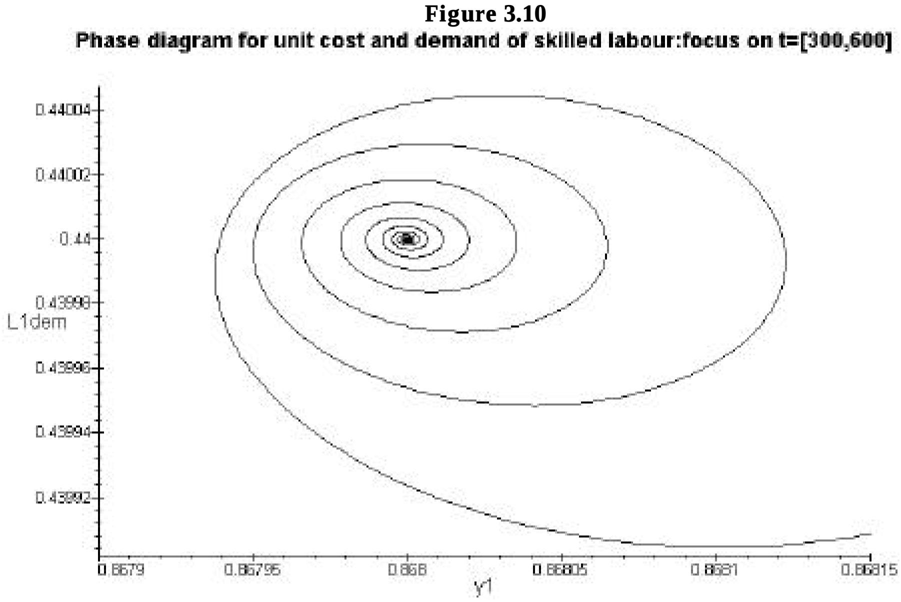
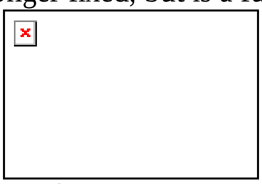


Figure 3.11

4 The case of labour mobility

4.1 Extension of the model

Let us now assume that workforce can be transferable from one sector of the economy to the other. Therefore, labour supply in each sector is no longer fixed, but is a function of time:



(31)

The normalisation to one of the whole of the workforce has again been adopted; furthermore, *s(t)* denotes the percentage of the population of workers supplying labour on the skilled market as a function of time *t*.

Generally, the skill upgrade for a worker demands some costs due to the necessity of increasing her stock of human capital. We shall assume that the distribution of "abilities" in the population is not homogeneous, but follows a uniform distribution on the interval [0,1], where the ranking goes from the ablest individual (s=0) to the least able (s=1). The cost of upgrading is determined accordingly by means of a function called $m_1(s)$, which is convex and monotonically increasing:

(32)

In this version, the cost of the upgrade is nil for the ablest worker and infinite for the least able, and the function is monotonic increasing. Notice that in this model the "ability" of a worker is not related with whether she has acquired the status of skilled, but with the ease with which such a change can be carried out.

Following an idea put forward by Blinder and Choi (1990), we can think that there also exists a "psychological" cost for the downgrading, since a worker who has started her career in the advanced sector is likely to experience a loss of social status. We can therefore assume a cost function for the downgrading symmetrical to the previous one:

(33)

An example of the two costs function is given in the following diagram, which assigns a higher relevance to the "material" cost of upgrading with respect to the psychological one of downgrading:

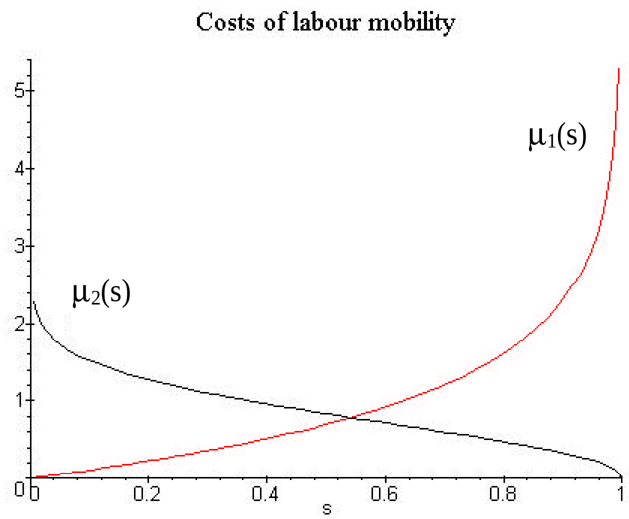


Figure 4.1

Let us assume that the utility function of the worker is linear in the wage and in the transfer cost. At each instant of time, she faces a binary decision as to whether stay in the current sector or move to the alternative one. She must then compare the expected utility gained in each sector, taking into account the mobility costs and the possibility of being unemployed. Let as assume that the probability of being left unemployed is proportional to the current unemployment rate, and that workers know the current levels of the unemployment rates in both sectors[15]. Therefore, given that the worker with ability labelled as $\underline{s}$ is currently working in sector *i*, her utility function will be given by:

(34)

where, recalling the notation already introduced, L_i/L^S_i and L_j/L^S_j stand for the unemployment rates in the two sectors. Hence, a worker will decide to move to the alternative sector if the associated expected utility, net of the costs associated with the change of skill, exceeds the expected utility earned by remaining in the current sector. Even in this setting, I shall assume bounded rationality, so that information diffuses slowly and follows a process of the replicator dynamics type. Accordingly, the rate of change in the composition of the workforce is proportional to the difference between the utility earned in the two sectors, where the constant of proportionality represents the speed with which information diffuses, the probability of being selected for a random matching that is (see section 2.3.2). In order to avoid unnecessary complications, I assume that the first workers to move across sectors, if convenient for them to do so, are those being at the "margin" of the markets, that is, the currently least able in the case of a shift from the skilled to the unskilled labour market, or the ablest in the case of a movement in the opposite direction. Therefore, the following rule of motion obtains:

(35)

As pointed out earlier, β measures the speed with which information is made available to agents, and their degree of rationality in processing this information is also taken into account by means of the difference in utilities.

Equation (35) forms a system of seven differential equations together with equations (27)-(30) and the two rules for productivity growth rates given by equation (2). As wages depend directly on productivity, it is no longer possible to make the system autonomous by means of the introduction of the auxiliary variable $\boldsymbol{k_1}$.

4.2 Analysis of local stability

Analysing the local stability of the steady state is now complicated by the presence of an additional equation and by the overall greater analytical complexity of the system. In order to make the derivation of some results possible at all, I first set the transfer costs $\grave{\imath}_1$ and $\grave{\imath}_2$ in equation 35 equal to 0, which permits a significant simplification of the expression to the following one:

(35')

[15] In this I am abstracting from the possibility that the probability of unemployment is proportional to the worker's level of ability. Also, the assumption that agents know the unemployment rate may seem to be at odds with the assumption of imperfect rationality and bounded rationality. The results we obtain, though, do not hinge upon this hypothesis.

In fact, on the grounds of the simulations that I have conducted, such costs only appear to have a key role in determining which of the steady states will be reached, but they do not seem to affect their nature.

Introducing some other minor simplifications, we can thus find the set of steady states for the system. It would be tedious to report all of the results of the analysis. The interested reader may find more information in the Appendix (section 6.3). I will limit here to summarise the main results that can be drawn. The most important conclusion is that the nature of the model does not seem to be affected by the inclusion of labour mobility. In fact, the steady states found for the previous model with no mobility of labour simply 'carry over' to the present setting. More precisely, for each steady state found in the previous model there exists a steady state in the present setting whose values coincide, apart from an exception, once the steady state value for the variable *s*, which was a parameter in the former model, has been substituted. For instance, solution (A1) of the previous model has a 'nearly-twin' solution here:

(D1)

In fact, all of the steady state values in (A1) are identical to those of (D1) once $s=1$ has been substituted into the expressions of (A1). The only difference is that in the present setting y_2 is undetermined. Furthermore, the steady state corresponding to the balanced growth path (C1) is also unstable. Moreover, the extra steady states in the latter version which do not match any of the previous all turn out to be unstable.

Even in this setting, therefore, steady states are found in which there is complete allocation of the resources of the economy, be it capital or labour, into one single sector, which is the *same* for both factors. The conclusion that one could draw is thereby that market forces are sufficient in order to drive both resources to the same sector, thus avoiding the possibility of skill mismatches between the technological requirements and the workforce qualifications. Nevertheless, even in this setting they do not suffice to co-ordinate the agents on the efficient technology. In other words, labour seems to behave in the same way as capital, despite the presence of switching costs that were not assumed for capital: the long-run outcome must be the complete shift of workers into the technique that becomes the leading one in the economy, the reason being that both workers and firms ultimately become attracted by the higher earnings that can be made in the leading sector of the economy. The following section reports some results of the simulations that have been conducted, focussing in particular on the role of adjustment costs on the determination of the long-run outcome.

4.3 Third and fourth scenarios: skill shortage and high vs. low costs of skill upgrading

First, we need to specify a functional form that can be used to represent the skill-upgrading costs. I shall employ a logarithmic form:

(36)

λ_1 is a parameter determining the magnitude of the upgrade costs: the higher the parameter, the higher the cost for every member of the population to improve their skill.

We shall then assume a function symmetric to the previous one for the downgrading cost:

(37)

The parameter values and initial conditions are such that the system starts with quite a marked skill shortage as in scenario 1, and with an upgrading cost relatively much higher than the downgrading cost.

All of the other conditions denote again a situation of initial symmetry between the two sectors[16]. The main upshot is that even in this case a result of convergence towards the slow-growth steady state obtains, which portrays a dynamics for the distribution of investments across sectors quite similar to that of the first scenario (Fig. 3.1). The underlying causes are the same as those stressed for the previous ones. In addition, productivity affects wages in such a way that workers are attracted to what soon becomes the leading sector of the economy, as shown by figure 4.2. The picture shows how the flow of labour is related to the wages differential. Therefore, productivity growth acts as the main factor to attract resources allocation both for capital and labour. According to the simulations conducted, this seems to be a general result, so that one could conclude that there is no risk of mismatch between labour demand and supply within this model. That is, a situation where capital and labour are allocated in the two different sector[17], is avoided. However, what cannot in general be impeded is, again, a lock-in effect to the inefficient technique.

An interesting feature of the model is its sensitiveness to the structure of the economy, and in particular to the magnitude of the skill upgrade costs. Indeed, it is sufficient to slightly shift the value of the related parameters to reverse the result of convergence towards the inefficient technique. In fact, after considering a set of parameters identical to the previous one but for the parameter l_1, which is now shifted to a lower value (0.975 instead of 1) denoting a lesser transfer cost for skill upgrading, we obtain a result of convergence towards the first sector, analogous to that obtained in the second scenario (Figure 3.7).

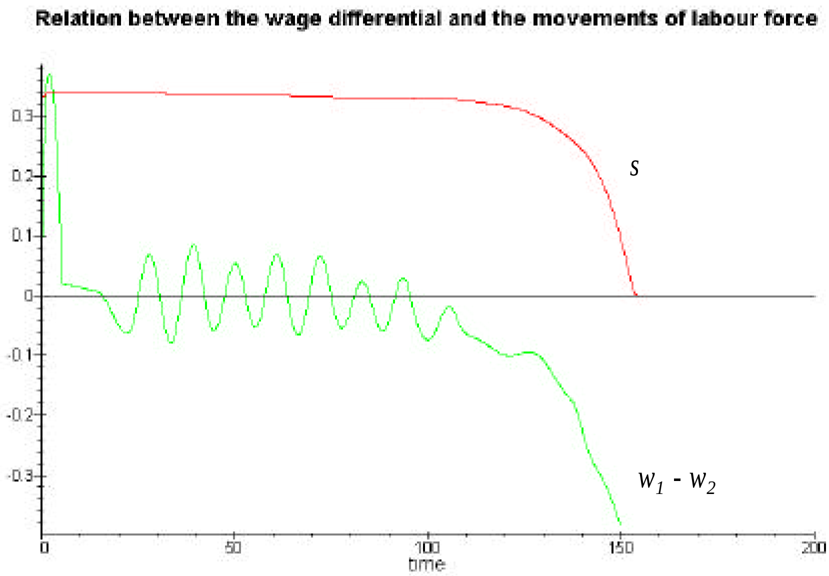


Figure 4.2

5 Conclusions

A model of growth with multiple steady states has been developed, which depicts the evolution of an economy characterised by localized technical change, i.e. spillovers taking place only at the technique-specific level, bounded rationality, which determine a replicator-type dynamics for aggregate behaviour of agents, and non-instantaneous market clearing. On the grounds of the local stability analysis and the

[16] In particular: $\{\alpha:=1; c:=3; \gamma_1:=1; \gamma_2=1; \beta=1; \eta:=1.5; g1:=0.04; g2:=0.02; \lambda_1=1, \lambda_2= 0.1\}$ $\{y_1(0)=0.5; y_2(0)=0.5; x_1(0)=.5; x_2(0)=.5; a_1(0)=1; a_2(0)=1; s(0)=1/3\}$.

[17] In fact, many of the additional steady states that can be found in the labour mobility version of the model display firms entirely concentrated in one sector of the economy and labour in the other. However, these steady states turn out to be unstable from a preliminary analysis.

simulations conducted, a high and a low-growth steady states, in which all factors are allocated to the skilled and unskilled-intensive technique respectively, obtain as stable equilibria of the model. A balanced growth path steady state, characterised by both sectors growing at the same rate, turns out to be unstable, because of the cumulative process of technical change. Some structural conditions determining the outcome of convergence have been highlighted, such as the degree of skill shortage in the case of non-transferability of labour, and the extent of skill upgrading costs in the case of labour mobility.

The low-growth steady state may be interpreted as the result of a co-ordination failure amongst the variety of agents making up the economy, i.e. workers and firms: both outcomes in which all agents converge towards the same sector are indeed 'equilibria' of the interaction, but the co-ordination on the high-growth equilibrium Pareto-dominates the other. In other words, in this world of gradual adjustment towards equilibrium and optimality and slow diffusion of information, market forces suffice to impede the mismatch of productive factors, but they do not always provide enough incentives to converge towards the efficient outcome. This in particular is the case when adverse initial structural conditions for the economy occur.

The analysis conducted has some straightforward, but significant, implications of political economy. In fact, a policy of training the unskilled work force, softening the initial skill shortage and lowering the skill upgrade costs, would make it possible to overcome the sub-optimal outcome. However, since the transition from the inefficient to the Pareto-efficient outcome requires some groups to give up part of their income shares in order to pay for the costs of the policy, in exchange of a benefit in the future, then some form of intertemporal agreement between the parties is necessary to guarantee the undertaking of the plan. As we all know, this is far from an easy requirement, though, especially in less developed countries where institutions are typically rather unstable. In more general terms, the paper stresses the complexity of a process of catching-up in presence of adverse structural conditions: even when a potentially more efficient technology is available in an economy, a lack of skill by the agents, concerning both workers as to their capacity to adapt to that technology, and firms as to their ability to exploit it, may thwart the economic incentives necessary to undertake the high-growth path.

6 Appendix

6.1 Analysis of the Steady States for the case of non-mobility of labour

A1)

The stability analysis is complicated by the fact that the variable x_2 is located just on the threshold level where the related equation (30) changes its expression. This implies that the Jacobian on the right neighbourhood of the point differs from that of the left neighbourhood. On the left neighbourhood of , where both u_1 and u_2 equals 1, we get the following set of eigenvalues:

(38)

Intuitively, the negative eigenvalue can be associated with the ${}_1$ axis, indicating the profitability of allocating capital in sector 1. Furthermore, two pairs of purely imaginary eigenvalues are obtained. Even though their sign is dubious, for the range of parameters economically meaningful we can be sure the argument of the square root is actually negative. In fact, g_i must have an order of magnitude at least 10 times smallest than the value of the other parameters, and this ensures in particular that

$$1 - cg_i < 1 \tag{39}$$

From linear stability theory we know that a two-equation linear system having a couple of purely imaginary eigenvalues is a centre. However, this conclusion does not necessarily carry over to non-linear systems. If this was the case, nevertheless, we may think that each couple of eigenvalues actually describes the dynamics of each of the two sectors, so that labour demand and wages should display the cyclical behaviour typical of a centre.

In the right neighbourhood of x_2, when $u_2<1$, we find the following set of eigenvalues:

$$\begin{Bmatrix} -g_1, & \frac{\sqrt{c\left[gL_1^S - g_1(h-1)\right](1-cg_1)}}{c}i, & -\frac{\sqrt{c\left[gL_1^S - g_1(h-1)\right](1-cg_1)}}{c}i, \\ -\frac{ag_1}{2} - \frac{\sqrt{c\left[4g\,L_2^S(1+a(1-cg_1)) - ca^2g_1^2\right]}}{c}i, & -\frac{ag_1}{2} + \frac{\sqrt{c\left[4g\,L_2^S(1+a(1-cg_1)) - ca^2g_1^2\right]}}{c}i, \end{Bmatrix} \tag{40}$$

It is notable that the negative eigenvalue and one of the two couples of imaginary eigenvalues coincide with what found for the left neighbourhood. Conversely, we now have a couple of complex conjugate eigenvalues with negative real part, whose dynamics would then be that of a stable focus generating trajectories that converge spiralling to a point.

We can indeed be sure, by means of analytical considerations, that the couple of purely imaginary eigenvalues that remain unchanged in the two neighbourhoods can be associated with the first sector. In fact, looking at the system of differential equations (26)-(30), one can notice that once $\boldsymbol{k_1}$ is equal to 0, as is the case asymptotically, then the second sector becomes "autonomous" from the variables of the first sector, thus assuming the form of a Lotka-Volterra two-equation system. Hence, the variables of the second sector must asymptotically behave like a centre, and converge towards a limit cycle. That this is the case can be derived from picture 3.5.

As for the first sector, the presence of different pairs of eigenvalues in the two neighbourhoods of the solution, which would generate different dynamical behaviour if taken singularly, makes it impossible to state their dynamical behaviour with certainty. The most likely conjecture, also supported by some further evidence derived from graphical analysis not reported here, is that variables of the first sector oscillate on a torus, although possible behaviours would also be those of a limit cycle with a slower process of convergence than the second sector, and a strange attractor, i.e. a behaviour characterised by chaotic evolution within a limited manifold.

(A2) $\left\{k_1 = 1,\ y_1 = 1 - cg_1 - c\left(\frac{1-h}{g\,L_1^S}\right)g_1^2,\ x_1 = L_1^S + \frac{(1-h)}{g}g_1,\ y_2 = \frac{1+a(1-cg_1)}{1+a},\ x_2 = L_2^S\right\}$

The properties of stability of the point must again be conducted considering two different sets of eigenvalues. In the left neighbourhood of $x_2 = L_2^S$ we shall observe:

$$\begin{Bmatrix} -g_1, & -\frac{g_1}{2} - \frac{\sqrt{c\left[4gL_1^S(1-cg_1) - cg_1^2(5-4h)\right]}}{c}i, & -\frac{g_1}{2} + \frac{\sqrt{c\left[4gL_1^S(1-cg_1) - cg_1^2(5-4h)\right]}}{c}i, \\ \frac{\sqrt{cgL_2^S(1+a(1-cg_1))}}{c}i, & \frac{\sqrt{cgL_2^S(1+a(1-cg_1))}}{c}i \end{Bmatrix} \tag{41}$$

In the right neighbourhood the eigenvalues are as follows:

$$\left\{ -g_1, \ -\frac{g_1}{2} - \frac{\sqrt{c\left[4gL_1^S(1-cg_1) - g_1^2 c(5-4h)\right]}}{c} i, \quad -\frac{g_1}{2} + \frac{\sqrt{c\left[4gL_1^S(1-cg_1) - g_1^2 c(5-4h)\right]}}{c} i, \right. \tag{42}$$

$$\left. -\frac{ag_1}{2} - \frac{\sqrt{c\left[4g\,L_2^S(1+a(1-cg_1)) - ca^2 g_1^2\right]}}{c} i, \quad -\frac{ag_1}{2} + \frac{\sqrt{c\left[4g\,L_2^S(1+a(1-cg_1)) - ca^2 g_1^2\right]}}{c} i, \right\}$$

Hence, even in this case we have three eigenvalues that remain the same in both neighbourhoods, which are possibly associated with the $\boldsymbol{k_1}$ axis and with the variables of the leading sector. However, instead of having two couples of imaginary values we find a pair of complex conjugates eigenvalues: this leads us to think that the variables in the first sector converge spiralling to a focus, as displayed in Fig. 3.11. As in the previous case, we have a couple of imaginary eigenvalues on one side and a pair of stable complex conjugates on the other for the residual sector: this is a dynamic behaviour not easily classifiable, alike that found for solution (A1).

I will not deal with the analysis of steady states (B1) and (B2) presented in section 3.2.2 since they are symmetric to solutions A1 and A2.

(C1) $$\left\{ \begin{array}{lll} \boldsymbol{k}_1 = \dfrac{g_2}{g_1+g_2} & y_1 = \dfrac{g_2 + g_1(1-cg_2)}{g_1+g_2} & x_1 = L_1^S - \dfrac{(\boldsymbol{h}-1)}{g} g_1 g_2 \\ y_2 = \dfrac{g_2 + g_1(1-cg_2)}{g_1+g_2} & x_2 = L_2^S - \dfrac{(\boldsymbol{h}-1)}{g} g_1 g_2 & \end{array} \right\}$$

The analytical expression of the set of eigenvalues for steady state (C1) relative to a balanced growth path is rather complicated and will not be reported. For an economically reasonable set of parameters, however, one obtains the following set of values, where I is the purely imaginary unit:

$$\{-.0165511798 + .5688615151\,I,\ .01356157422,\ -.0165511798 - .5688615151\,I,$$
$$-.0102296072 + .3609416065\,I,\ -.0102296072 - .3609416065\,I\}$$

Intuitively, we can associate the 4 complex eigenvalues that can be found to the variables related to each sector –labour demand and labour unit cost. The positive eigenvalue can instead be associated with the $\boldsymbol{k_1}$ co-ordinate, on the grounds of economic consideration set out in section 3.2.3.

6.2 Analysis of steady states for the case of labour mobility

As already pointed out, the analysis of local stability for the case of labour mobility reveals the close similarity from the economic standpoint between the steady states found in the two cases. However, the reader interested in the mathematical details may want to explore the peculiarities of this, more complex, case. In this section I thereby offer a brief summary of the results obtained.

D1) $$\left\{ k_1 = 1 \quad y_1 = 1 - cg_1 \quad x_1 = 1 - \frac{(h-1)}{g} g_1 \quad y_2 = \text{undetermined} \quad x_2 = 0 \quad s = 1 \right\}$$

I have already noticed how clearly does this solution correspond with solution (A1) in that, apart from y_2 being now undetermined, (A1) boils down to (D1) once the steady state value $s=1$ is substituted into the steady states values of the other variables. Since variable x_2 is located on the edge of its admissible values, it obviously suffices to find eigenvalues only on the relevant neighbourhood of the space. The following set obtains:

$$\left\{ \frac{(1+a)(1-y_2) - acg_1}{c}, \frac{(1+a)(1-y_2-cg_1)}{c}, \frac{\sqrt{c\left[g - g_1(h-1)\right](1-cg_1)}}{c} i, \quad -\frac{\sqrt{c\left[g1 - g_1(h-1)\right](1-cg_1)}}{c} i, \right.$$

$$\left. -\frac{a_1 b}{g}\left[g - g_1(h-1)\right](1-cg_1), \ 0 \right\}$$

No general conclusion can in general be drawn on the sign of the eigenvalues. However, some speculative considerations can be put forward. First, let us compare this with the set of eigenvalues found for (A1): two of the purely imaginary eigenvalues coincide, once the value of s has been substituted; presumably, they are associated with the Lotka-Volterra dynamics setting in within the leading sector of the economy. The first two eigenvalues of (D1) have now a dubious sign. However, for a significant set of the parameters, and for y_2 sufficiently close to 1, which on the grounds of the simulations conducted seems indeed to be the case, their values turn out to be negative. We finally have one negative eigenvalue, again for a realistic value of the parameters, and one equal to zero. Overall, therefore, this analysis cannot be conducive to any definite conclusion, because of the presence of eigenvalues with real part equal to nil. However, the simulations conducted prove indeed that this steady state turns out to be an attractor of the system, the two variables associated with the leading sector, x_1 and y_1 that is, moving along a close orbit whose centre is that indicated in (D1), y_2 converging to 1, and all of the other variables converging to the values prescribed in (D1).

(D2) $\left\{k_1 = 1,\ y_1 = 1 - cg_1 - c\left(\frac{1-h}{g}\right)g_1^2,\ x_1 = 1 + \frac{(1-h)}{g}g_1,\ y_2 = \text{undetermined},\ x_2 = 0,\ s = 0\right\}$

Alike (D1), solution (D2), which holds under the constraints that ç is less than 1, has a corresponding solution in (A2) in that the latter obtains if one substitutes $s=1$ for the parameter s in (A2). The only difference lies in that y_2 is undetermined in (D2), whereas it takes a definite expression in (A2). Assigning for simplicity $y_2=1$, which is the most likely value assumed by this variable in the residual sector of the economy, and the one actually observed in the simulations that have been conducted, the following set of eigenvalues obtains:

$$\left\{-\frac{g_1}{2} - \frac{\sqrt{c\left[4gL_1^S(1-cg_1) - cg_1^2(5-4h)\right]}}{c}i,\quad -\frac{g_1}{2} + \frac{\sqrt{c\left[4gL_1^S(1-cg_1) - cg_1^2(5-4h)\right]}}{c}i,\right.$$

$$\left. -\acute{a}g_1,\ -g_1(1+\acute{a}),\ 0,\ -ba_1\left[(1-cg_1) - cg_1^2\left(\frac{1-ç}{\tilde{a}}\right)\right] + \hat{a}a_2\right\}$$

Interesting analogies with solution (A2) can be found here, too. The first pair of complex conjugates eigenvalues is identical to that found for (A2): this is likely to be associated with the dynamics in the leading sector of the economy, and indeed in the simulations I have conducted variables x_1 and y_1 show the typical behaviour of a focus. The other eigenvalues are either negative – in particular the latter is certainly negative as a_1 in the long run outstrips a_2 by a large amount – or equal to 0, which is likely to be associated with y_2. That this solution is indeed an attractor for the system has been verified through numerical simulations.

Finally, symmetric solutions to the pair now illustrated, characterised by convergence to the second sector, can be found. These can be associated with solutions (B1) and (B2) for the same reasons set out above:

(E1) $\left\{k_1 = 0 \quad y_2 = 1 - cg_2 \quad x_2 = 1 - \frac{(h-1)}{g}g_2 \quad y_1 = \text{undetermined} \quad x_1 = 0 \quad s = 0\right\}$

(E2) $\left\{k_1 = 0,\ y_2 = 1 - cg_2 - c\left(\frac{1-h}{g}\right)g_2^2,\ x_2 = 1 + \frac{(1-h)}{g}g_2,\ y_1 = \text{undetermined},\ x_1 = 0, s = 0\right\}$

Not surprisingly, these steady states show symmetric properties of stability to those just examined.

Finally, an equivalent solution to he balanced growth path solution (C1) seems to obtain even in this case, even though a complete analytical solution is not possible. However, after having assigned the numerical values used in the previous simulations to the parameters, one can express all the variables as a function of the particular value taken on by s. This solution looks indeed the exact analogous of solution (C1):

$$\left\{\begin{array}{lll} k_1 = \dfrac{g_2}{g_1 + g_2} & y_1 = \dfrac{g_2 + g_1(1 - cg_2)}{g_1 + g_2} & x_1 = s - \dfrac{(h - 1)}{g} g_1 g_2 \\ y_2 = \dfrac{g_2 + g_1(1 - cg_2)}{g_1 + g_2} & x_2 = (1 - s) - \dfrac{(h - 1)}{g} g_1 g_2 & \mathrm{s} = \dfrac{1199999998}{2399999998} \approx 0.499 \end{array}\right\}$$

Not surprisingly, this turns out to be unstable.

6.3 Notes on numerical simulations

The simulations have been conducted with the Maple V package. Due to a certain inflexibility of the Maple's built-in function to embed boundary conditions on the variables, I have set up the set of equations composing the Runge-Kutta algorithm (e.g. Hildebrand, 1987) - the same adopted in Maple. I then applied it to the system under investigation. Furthermore, during the simulations I have replaced the workers' mobility costs functions specified in equations (36) and (37) with others having a constant, arbitrarily large, value in correspondence with their asymptotes.

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