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EFFECTS OF DEMOGRAPHIC FACTORS ON TECHNOLOGICAL CHANGE AND ECONOMIC GROWTH

by

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EFFECTS OF DEMOGRAPHIC FACTORS ON TECHNOLOGICAL CHANGE AND ECONOMIC GROWTH

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The endogenous technology literature usually treats human capital as proportional to the population level. As a result, it finds a linear relationship between the population size or growth rate and technological improvement. In this paper I introduce human capital investment in an endogenous technology framework. It is shown that population affects technological improvement both directly and through the stream of human capital, with technology also having a feedback effect on the latter. These multiple effects of demographic factors on R&D, human capital and economic growth can explain certain facts, such as the growth patterns of the last two centuries.

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* The material in this paper draws on my PhD dissertation and is not to be identified with the position of the Office for National Statistics.

I. INTRODUCTION

The endogenous technological change literature implies a scale effect of the population size on the per capita growth rate of the economy. Yet this is not supported by the evidence: As Young [1998] argues, after the second world war not only the scale of the economy but also other growth promoting variables (such as trade liberalisation and increased education) were very favourable, yet without the growth rate increasing. The growth rate was indeed increasing during the industrial and pre-industrial ages, yet this increase was much more modest than the scale effect argument would imply.
Several authors have tried to fix this scale effect problem, while maintaining the endogeneity of technological progress. Two alternative approaches have been followed. The first (i.e., Kortum [1997] and Segerstrom [1998]) argues that the more advanced a technology the more difficult it is to improve it further, which implies that more and more resources are required for same amounts of improvement. This assumption is sufficient to eliminate the scale effect of population, albeit it yields the undesirable result that without population growth there is no output growth either.

The second approach (i.e., Young [1998], Dinopoulos and Tompson [1998] and Peretto [1998]) argues that R&D is 2-dimensional, that is both quality improving and variety expanding. According to these authors, as population increases the variety of products expands, which has a *dispersion effect* on the amount of resources that are allocated to quality-improving R&D. That is, although the total resources allocated to R&D increase, they are divided to an increasing number of products. The result is that it is the population growth rate rather than size that affects output growth.
Both approaches however maintained the "tradition" of the endogenous technological change literature, of ignoring the question of human capital formation: Although all

authors[1] recognise human capital as the engine of innovation and technological improvement, they take it as exogenous and proportional to the size of the population[2]. In this paper it is shown that when human capital investment is also taken into account, the relations between population, innovation and human capital become less straightforward: Although population growth has indeed a direct linear effect on innovation and growth, this faster productivity growth also encourages human capital investment, which reinforces the initial effect of population on R&D and investment. On the other hand though, population growth congests the economy's resources, which has a negative effect on human capital investment, which in turn reduces or even inverts the initial growth effect of population growth. It is also shown that when human capital is endogenous not only does the growth rate of the population matter, but also its age structure.

Finally, it is argued that these multiple effects of population on R&D and education can provide a better explanation of the increasing growth rates of the last two centuries as well as the more recent growth slowdown.

I assume an economy where the physical output is produced by means of a continuum of intermediate products which are not perfect substitutes for each other. The engine of growth is technological progress, which, as in the 2-dimensional R&D models, can be either variety expanding or quality improving. Both R&D activities use labour as the only input, measured in effective units, which takes human capital into account.
In this framework three types of economic agents are assumed: First, the firms that produce the intermediate products, which enjoy perpetual patent rights of their inventions and consequently maximise their profits under conditions of monopolistic competition. On the other hand, the firms producing final output operate in a perfectly competitive market. Extended families, which consist of members of all generations, are the third type

[1] Including the "founders" of the endogenous technology theory, i.e., Romer [1990], Aghion and Howitt [1992], etc.
[2] Zeng [1997] is among the few exceptions.

of agent. An extended family, which can be seen as all the descendants of an individual born far in the past, seeks to maximise an intertemporal utility function with respect to the members' average consumption. Decision variables of the family are, on the one hand, the allocation of its members' time between work and education and, on the other, the allocation of its total income between consumption and saving.

The main result is that population has a direct and, through human capital, an indirect effect on economic growth. Further, it is shown that it is not only the growth rate of population but its age structure as well that matters. A theoretical explanation is also offered for the finding of Bils and Klenow [2000] that "growth causes schooling rather than the other way round".

The structure of this paper is as follows: The model is presented in the next section and in section 3 the general equilibrium and steady state are derived. The comparative statics of demographic changes are studied in section 4, while section 5 summarises the main results.

II. ECONOMIC ENVIRONMENT

A closed economy is assumed, consisting of three different types of agent: families, final output firms, and firms that produce the intermediate products used as inputs in the final output sector.

A. FAMILIES AND POPULATION

By "family" an extended family is meant, which consists of all individuals with a common ancestor, whether this ancestor is still alive or not. It is assumed that the economy consists of many such extended families, identical in terms of preferences, real wealth, age structure and population dynamics. This assumption allows one to speak about a “representative family”. This representative family is assumed to maximise an intertemporal utility function with respect to the average consumption of its members, which is given by

(1) $U = \int_0^\infty e^{-\rho t} \ln c_t dt$

where c_t is per capita consumption. The fact that it is only the per capita consumption that matters implies perfect altruism among the family members, which in turn implies that the individual members of the family would take by themselves exactly the same decisions as a family planner.

The intertemporal budget constraint of the family is given by

(2) $\dot{q}_t = (r_t - n)q_t + P_t c_t - \ell_t$

where r_t is the interest rate at time *t*, *n* is the constant growth rate of population, q_t, c_t and ℓ_t are the per capita real assets, consumption, and effective labour supply, and P_t is the price of the single consumption good, with the wage rate as numeraire.

Real wealth consists of shares of the firms that produce the intermediate products. By “effective labour supply” the hours supplied to the labour market are meant, weighted by the human capital of the workers. The family therefore has two means of investment: shares of the “intermediate” firms, and human capital.

Regarding the population of the representative family, constant birth (ε) and death (λ) rates are assumed for simplicity. That is, at any point of time εN_t new members are born to the family and λN_t members die. It is also assumed that the probability of death (λ) is the same for all age groups. Thus the population growth rate (*n*) is given as $n = \varepsilon - \lambda$. The size of each generation as a portion of the total family population is given by

(3) $n_{st} = \varepsilon e^{\varepsilon(s-t)}$

where n_{st} is the “relative size” at time *t* of the generation born at time *s*.

B. UTILITY MAXIMISATION

The extended family maximises its utility (1) subject to the intertemporal budget constraint (2). However per capita effective labour ℓ_t is not exogenous to the family, but depends on its human capital investment. The two decisions can however be separated; the family can maximise the present value of the path of labour income, which is equal to its labour supply ℓ_t as the wage rate is used as numeraire, and then import the optimal solution into its utility maximisation problem.

The current value Hamiltonian of the second problem is given as

$$H = \ln c_t + \xi_t \left[(r_t - n) q_t + P_t c_t - \ell_t \right]$$

and the first order conditions are

(4) $c_t^{-1} = -\xi_t P_t$

(5) $\dot{\xi}_t = \xi_t (r_t - n - \rho)$

which give the optimal path for the per capita consumption (c_t) of the family:

(6) $\dot{c}_t = (r_t - n - \rho - \hat{P}_t) c_t$

C. HUMAN CAPITAL AND LABOUR SUPPLY

The effective labour supply of the family, mentioned previously, depends on the working hours supplied and on the human capital of the individuals that supply them. Following the literature[3] human capital can be built by investing time in education, which has to be taken out of current labour supply. In particular, it is assumed that each individual is endowed with one unit of non-leisure time, which is allocated between work and

[3] I.e., Ben-Porath [1967], Uzawa [1965], Mulligan and Sala-i-Martin [1993], Caballe and Santos [1993], etc. The accumulation function that is assumed in the paper is a simplified version of the functions they used.

education. The latter adds to the individuals' human capital[4], according to an accumulation function given by

(7) $\dot{h}_{st} = Bh_{st}u_{st}^{\delta} - \varphi h_{st}$

where h_{st} and u_{st} are the human capital and portion of time devoted to education for an individual born at time s. φ is a constant human capital depreciation rate which may be attributed to deterioration of skills, due to ageing. Finally, it is assumed that returns to education are diminishing, that is, $\delta < 1$.

As said, the family can separate the two decisions, that is, the decision on the optimal paths of its consumption and wealth, and the decision on its human capital investment and the resulting labour income. The first decision has already been treated; regarding the second, the family maximises the present value of its labour income by maximising the present value of the labour income of each of its members. The optimality condition requires that at any point in time the marginal returns to education (investment in future effective labour supply) and work (current effective labour supply) are equal. These returns are given by "what can labour buy", which depends on the price P_t of the final output, since the wage rate has been taken as numeraire.

The optimality condition is therefore given by

(8) $h_{st}P_t^{-1} = h_{st}\delta Bu_{st}^{\delta-1}\int_t^{\infty} e^{-\int_t^v (r_i+\varphi+\lambda)di} P_v^{-1} dv$.

The left hand side of (8) stands for the marginal returns to work, while the marginal returns to education are on the right hand side: The term $h_{st}\delta Bu_{st}^{\delta-1}$ corresponds to the human capital generated by the marginal unit of time that is invested in education, while the integral gives the present value of a unit of human capital. This is equal to the discounted stream of future wages in terms of the final good, which with the wage rate taken as numeraire are equal to the reciprocal of the price of the final good. The discount

[4] Providing of course that the individual will be alive in the next moment.

rate is equal to the interest rate, plus the depreciation rate φ of human capital and the probability λ that the individual will die in the next moment. The horizon is infinite, as a constant probability of death was assumed.

Assuming therefore that there are no corner solutions where the optimal education time exceeds unity- the individuals' time endowment- the optimal education choice is given by

(9) $u_{st}^{1-\delta} = \delta B \int_t^{\infty} e^{\int_v^t \left(r_i + \hat{P}_i + \varphi + \lambda\right) di} dv$.

It is noteworthy that the optimal education is the same for all age groups. This is due to the assumption of constant probability of death, and eliminates the life-cycle effect of the population age structure on human capital accumulation, discussed in Misoulis [2002]. It is further assumed that the new generations start at a human capital level that is equal to the average of the economy rather than a proportion of this average. These two assumptions together imply that all agents have the same stock of human capital and consequently the law of motion of the average human capital is the same as that of the human capital of the individual, that is,

(10) $g_t = Bu_t^{\delta} - \varphi$

where g_t is the growth rate of the average human capital and u_t is the equal among generations time invested in education, as given by (9).

D. THE FINAL OUTPUT SECTOR

A single good is produced in the economy, which is used entirely for consumption purposes. This good is assumed to be produced under conditions of perfect competition and with a C.E.S. production technology that demonstrates constant returns to scale. The inputs used are a variety of intermediate products which completely depreciate in the procedure. What is important for these intermediate products is that they are not perfect substitutes for each other.

The production function of the representative final output firm is therefore given by

(11) $Y = \left(\int_0^A x_i^a di \right)^{\frac{1}{a}}$

where x_i are the intermediate products used and A is the range of the available different types of intermediate products. It is also assumed that $a<1$. The firms of the physical output sector decide on the quantities of the inputs they use in order to maximise their profits, which are given by

(12) $\Pi = PY - \int_0^A p_i x_i di$

where p_i and P are the prices of the intermediate and final products respectively. In their maximisation problem the firms take the variety of the intermediate products A as given, and because of the assumption of perfect competition they do so for the prices p_i and P. Solving this maximisation problem yields the demand function for the intermediate products:

(13) $x_i = \left(\frac{P}{p_i} \right)^{\frac{1}{1-a}} Y$.

The next task is to derive an expression for the price of the final output. For that, equations (13) and (11) are substituted into (12) to yield

$$\Pi = P \left[\int_0^A \left(\frac{P}{p_i} \right)^{\frac{a}{1-a}} Y^a di \right]^{\frac{1}{a}} - \int_0^A p_i \left(\frac{P}{p_i} \right)^{\frac{1}{1-a}} Y di$$

which by arranging terms and using the property that under perfect competition the profits are zero, yields the following expression for the price of the final output:

(14) $P=\left(\int_0^A p_i^{\frac{a}{a-1}}di\right)^{\frac{a-1}{a}}$.

E. THE INTERMEDIATE PRODUCTS' FIRMS

The production of the intermediate products is assumed to be restricted by the perpetual patent rights of the firms that first introduced them[5]. This implies monopolistic competition in the intermediate products market. It is also assumed that no one but the initial patent holder can improve the quality of an intermediate product: Although it would be more realistic to allow for R&D races and business stealing, this would only complicate the analysis without adding anything to it.

The production of the intermediate products requires (effective) labour alone, and their "quality" is defined as the reciprocal of the labour input required for the production of one unit of the intermediate. In particular, it is assumed that the production function of the intermediates is given by

(15) $x_{it}=z_{it}^{\theta}\ell_{x_it}$

where θ is a constant, ℓ_{x_it} is labour input, and the labour productivity z_i evolves according to[6]

(16) $\dot{z}_{it}=\beta z_{it}\ell_{z_it}$

where ℓ_{z_it} is the labour input used for quality improving R&D. The intermediate firms maximise at any time *t* the present value of their expected profits, which is given by[7]

$$V_{it}=\int_t^{\infty}e^{\int_{\mu}^{t}r_v dv}\left[\left(p_{i\mu}-z_{i\mu}^{-\theta}\right)x_{i\mu}-\ell_{z_{i\mu}}\right]d\mu$$

[5] This subsection, as well as the next, draws from Peretto [1998].
[6] This quality improvement function is different from the one used in the literature, in the sense that in the literature it is the average rather than individual quality that matters.
[7] Recall that the wage rate is set as numeraire.

which by substitution of x_i from its demand function (13) yields

$$(17)\ V_{it} = \int_t^{\infty} e^{\int_{\mu}^{t} r_v dv} \left[\left(p_{i\mu} - z_{i\mu}^{-\theta}\right) p_{i\mu}^{\frac{1}{a-1}} P_{i\mu}^{\frac{1}{a-1}} Y_{\mu} - \ell_{z_{i\mu}} \right] d\mu .$$

The intermediate firms therefore maximise (17) under the constraint (16). The current value Hamiltonian is given as

$$H_i = \left(p_{it} - z_{it}^{-\theta}\right) p_{it}^{\frac{1}{a-1}} P_t^{\frac{1}{a-1}} Y_t - \ell_{z_i t} + \xi_t \beta z_{it} \ell_{z_i t}$$

and the first order conditions are

$$(18)\ p_{it} = \frac{1}{a} z_{it}^{-\theta}$$

$$(19)\ \xi_t = \frac{1}{\beta} z_{it}^{-1}$$

$$(20)\ \dot{\xi}_t = \xi_t \left(r_t - \beta \ell_{z_i t}\right) - \theta a^{\frac{1}{1-a}} P^{\frac{1}{1-a}} Y z_i^{-1-\theta+\frac{\theta}{1-a}} .$$

Taking next the time derivative of (19) and substituting into (20) we get after arranging terms the following expression:

$$(21)\ r_t = \beta \theta a^{\frac{1}{1-a}} P_t^{\frac{1}{1-a}} Y_t z_{it}^{\frac{a\theta}{1-a}} .$$

Equations (16), (18) and (21) give the paths of quality (z_{it}), quality improving R&D effort ($\ell_{z_{it}t}$), and output price (p_{it}), for the intermediate product industry i.

F. VARIETY EXPANSION

Although the intermediate products are protected with patent rights, there are no restrictions in inventing a new product. This implies perfect competition in the variety expanding R&D sector. The variety expanding technology is assumed of the type

$$(22)\ \dot{A}_t = \gamma L_{at}$$

where L_{at} stands for "effective" labour input in the "expansion R&D sector" and γ is a constant. For simplicity it is also assumed that the quality level of all new products is equal to the average quality of the existing ones. This is sufficient to achieve the same quality for all intermediate products.

In order to introduce a new variety, an R&D firm compares the cost of invention with the present value of the expected profits of this invention. From (22) the cost of invention is equal to $\frac{1}{\gamma}$. The present value of the expected profits on the other hand is given by (17). In other words, positive R&D in the expansion sector implies that

(23) $V_t = \frac{1}{\gamma}$.

III. GENERAL EQUILIBRIUM

Having described the model, the next task is to derive its general equilibrium. First though, the symmetry among the firms of the intermediate products' sector must be stressed: The assumption that the quality of new products is equal to the average quality makes all firms identical and they therefore make the same decisions. This allows one to talk about a "representative intermediate firm", which simplifies the notation and derivation of the general equilibrium.

DEFINITION 1

A general equilibrium is a set of variables c_t, r_t, q_t, u_t, ℓ_t, H_t, P_t, V_t, g_t, A_t, x_t, p_t, Y_t, z_t, ℓ_{xt}, ℓ_{zt} and L_{at} such as:

1. c_t and q_t are the per capita consumption and real assets of the family that maximise its intertemporal utility, given its expectations of future interest rates, price level, and its own effective labour supply.

2. u_t is the time spent in education by each individual, that maximises the present value of their intertemporal labour income, given their expectations for the future prices and interest rates.

3. H_t is the average human capital of the economy, which is also equal to the human capital of each individual agent and it is a function of their previous education decisions.

4. ℓ_t is the average effective labour supply of the representative family, and depends on the average human capital H_t and the time $1-u_t$ that is devoted to labour activities.

5. Y_t and x_t are the output produced and the inputs used by the final output firms, which maximise their profits given the prices P_t and p_t of the final output and intermediate products respectively.

6. V_t is the present value of the expected profits of an intermediate firm, and depends on the demand for their product, their current technology level z_t and the expected interest rates as well as the future decisions of the firm.

7. ℓ_{xt}, ℓ_{zt} and p_t are respectively the effective labour inputs in production and quality improving R&D of the intermediate firms and the price of their output, that maximise their value V_t just described.

8. L_{at} is the amount of effective labour employed in variety expanding R&D, given the value V_t of the intermediate firms.

9. z_t and A_t are the quality level and variety of the intermediate products respectively, and depend on the cumulative labour investment in quality improving (ℓ_{zt}) and variety expanding (L_{at}) R&D.

10. P_t is the price level of the final product, which clears its market.

11. r_t is the interest rate that achieves equilibrium between supply and demand for savings, the first given by the desired assets (q_t) of the families and the second by the investment plans of the R&D firms.

12. g_t is the growth rate of the average human capital.

The general equilibrium is described by the system of equations (2), (6), (9), (10), (11), (15), (16), (18), (21), (22), (23), as well as

(24) $P_t = A_t^{\frac{a-1}{a}} p_t$

(25) $V_t = \int_t^{\infty} e^{\int_{\mu}^{t} r_v dv} \left[\left(p_{\mu} - z_{\mu}^{-\theta} \right) x_{\mu} - \ell_{z\mu} \right]$

(26) $A_t \left(\ell_{xt} + \ell_{zt} \right) + L_{at} = N_t \ell_t$

(27) $\ell_t = (1 - u_t) H_t$

(28) $N_t q_t = A_t V_t$.

Equation (24) emerges by using the symmetry property of the intermediate firms in (14), and gives the price of the final output in terms of the price of the intermediate products. As can be seen, the price P_t is decreasing with respect to the variety A_t of the intermediates because higher variety allows higher production without reducing the marginal product of the intermediates used. (25) gives the value of intermediate firms, while (26) is the equilibrium condition in the labour market[8], (27) gives the aggregate labour supply in per capita terms, and (28) states the equilibrium between supply and demand for assets, that is, the value of all stocks of all intermediate firms must equal the real wealth of the representative extended family.

The next task is to reduce the number of equations and variables to those of interest, that is, r_t, u_t, g_t, ℓ_{xt}, ℓ_{zt}, L_{at}, ω_t and s_t. By ω_t is meant the growth rate of the per capita final output, while s_t stands for the aggregate labour supply per intermediate

[8] It is reminded that N_t is the population size while ℓ_t is the per capita effective labour supply.

firm and is defined by $s_t = \frac{N_t \ell_t}{A_t}$. After a considerable amount of algebra one ends up with the following simplified general equilibrium system:

(29) $\omega_t = r_t - \varepsilon + \lambda - \rho + \frac{1-a}{a}\hat{A}_t + \beta\theta\ell_{zt}$

(30) $\omega_t = \frac{1}{a}\hat{A}_t + \beta\theta\ell_{zt} + \hat{\ell}_{xt} - \varepsilon + \lambda$

(31) $u_{st}^{1-\delta} = \delta B \int_t^\infty e^{\int_v^t \left(r_i + \varphi + \lambda + \frac{a-1}{a}\hat{A}_i - \beta\theta\ell_{zi} \right) di} dv$

(32) $\ell_{xt} = \frac{1}{\beta\theta} r_t$

(33) $\ell_{zt} = \left(\frac{1-a}{a\beta\theta} - \frac{1}{\gamma} \right) r_t$

(34) $s_t = \ell_{xt} + \ell_{zt} + \frac{1}{\gamma}\hat{A}_t$,

along with equations (10), (22) and (26). For reasons that will become obvious the growth rate of varieties (A_t) is kept as it is. The steady state growth path is defined next:

A. STEADY STATE GROWTH

DEFINITION 2

Steady state is an equilibrium path that is characterised by the following properties:

1. The interest rate r_t, firm size s_t and the portion of time allocated to education u_t are all constant.

2. The total labour inputs in the production of intermediate products, quality improving R&D, and variety expanding R&D are proportional to the variety A_t of the intermediate products.

3. The per capita final output and human capital grow at the constant rates of ω and g respectively.

Property 2 is another way of saying that in the steady state growth path, ℓ_{xt}, ℓ_{zt} and $\frac{L_{at}}{A_t}$ are constant. This can only be the case if labour supply and product variety grow at the same rate. The first is given as $N_t\ell_t = N_t H_t(1-u_t)$, which by the steady state property of constant u_t implies that the growth rate of total labour supply is the sum of the growth rates of total population $(\varepsilon-\lambda)$ and average human capital (g). This in turn implies that the steady state growth rate of product variety is given as

(35) $\hat{A}_t = \varepsilon - \lambda + g$

Using next the properties of the steady state in the equations that describe the general equilibrium we end up with the following steady state system:

(36) $r - g = \varepsilon - \lambda + \rho$

(37) $Bu^{\delta} - g = \varphi$

(38) $(1-a)g + a\delta Bu^{\delta-1} + \left(1 - 2a - \frac{a\beta\theta}{\gamma}\right)r = a\varphi + (a-1)\varepsilon$

(39) $\ell_x = \frac{1}{\beta\theta}r$

(40) $\ell_z = \left(\frac{1-a}{a\beta\theta} - \frac{1}{\gamma}\right)r$

(41) $\omega = \frac{1-a}{a}(\varepsilon-\lambda) + \frac{1}{a}g + \beta\theta\ell_z$

(42) $s = \ell_x + \ell_z + \frac{1}{\gamma}(\varepsilon - \lambda + g)$.

To make things as simple as possible, the steady state system (36)-(42) was made block recursive, with the first three equations forming the first block and the remaining equations being one block each. Equation (36) is the steady state expression of (29), after subtraction of (30) and substitution of $\hat{A}_t$ from (35). (37) is the steady state expression of (10) while (38) emerges by substitution of (33) and (35) in (31), and solving the integral. (41) is the steady state expression of (30), after substitution of $\hat{A}_t$. Finally, (39), (40) and (42) are repetitions of (32), (33) and (34) without the time index, while (35) was also used in (42).

IV. STEADY STATE EFFECTS OF DEMOGRAPHIC CHANGES

This section studies the steady state effects of the two demographic parameters, fertility (ε) and mortality (λ) on the growth rates of per capita human capital (*g*) and output (ω), the interest rate (*r*), the time allocated to education (*u*), the labour allocation variables ℓ_x and ℓ_z, and the firm size *s*. Before proceeding, it is useful to recall that the demographic parameters give the population growth rate and age structure; in particular, it is $n = \varepsilon - \lambda$ and $n_k = \varepsilon e^{-\varepsilon k}$ where *n* is the population growth rate and n_k is the relative size of the generation of age k. Changes in λ reflect therefore opposite changes in the population growth rate while changes in ε reflect changes in both population growth rate and age structure. With this in mind the following proposition is next established:

PROPOSITION 1

Assuming that the C.E.S. parameter *a* is "high", an increase of the death rate λ ceteris paribus reduces all of *r*, *g*, *u*, ω, ℓ_x, ℓ_z, *s* and variety expansion $\hat{A}_t$, while an increase in the birth rate ε affects positively *r*, ℓ_x, ℓ_z, *s* and $\hat{A}_t$, negatively *u* and *g*, and has an ambiguous effect on the per capita growth rate ω.

The proposition is proved in the appendix. It is also shown that a value for $a \geq 2/3$ is sufficient for proposition 1 to hold, but not necessary; smaller values of a may still produce the same results. Noteworthy also is that the assumption of high a implies elastic substitution between intermediate inputs in the final output production function. Whether we interpret the intermediates as consumption goods[9] or as production inputs (i.e., Young [1998]), this assumption of elastic substitution is realistic; in modern economies there is a huge variety of both final products and skills, with often very minor differences between them.

Some of the results in proposition 1 were anticipated: Beginning with the interest rate, although it was shown in Misoulis [2002] that the population growth rate may affect it negatively, it was also shown that this can only happen if the elasticity of intertemporal substitution is strictly less than unity. However, in this paper this elasticity is exactly unity which means that the positive effect of ε and the negative effect of λ on *r* are in line with the previous findings.

As Peretto argued, higher population growth increases both the firm size *s* and the growth rate $\hat{A}_t$ of product variety. The reason is that higher population growth implies higher expected demand- and therefore profits- for the intermediate firms. As a consequence they increase both their production and quality improving R&D, or in other words, they increase their size. Yet this increases the firms' value, which results in more resources being allocated to variety expanding R&D. It is exactly for this reason that the

[9] This is the assumption of i.e., Peretto [1998], who studied the growth of utility rather than output, with an expanding variety of *consumption* goods. Both approaches give the same results, providing that one always remembers which of the two it is about.

birth rate ε is found to affect *s* and $\hat{A}_t$ positively, while the effects of the death rate λ are negative.

As the demographic variables affect ℓ_x and *r* in the same way, a positive correlation between the two is implied. This does not come as a surprise; A higher interest rate reduces the returns to future quality improvements, and the intermediate firms concentrate to production rather than R&D. However, the labour (ℓ_z) devoted to quality improving R&D is also positively related to the interest rate! The answer must be sought in the firm size: by affecting as said *s*, the birth and death rates affect (positively the first, negatively the second), the returns to R&D as well. This effect proves stronger than the one through the interest rate, with the result of a positive relation between *r* and ℓ_z.

On the other hand the effect of the birth rate on the amount *u* of time spent in education is negative. This is due to the positive effect of ε on *r*, whose effect on *u* is seen from (31) to be negative. This is due to the fact that a high interest rate reduces the expected returns to education. Yet equation (31) also reveals a second stream through which the birth rate affects education, that is, through variety expanding ($\hat{A}_t$) and quality improving (ℓ_{zt}) R&D. Both these factors are positively affected by ε while also positive is their own effect on education effort. As it is shown in the appendix, the growth rate of the price P_t of the final product is inversely affected by the amount of resources that are devoted to R&D, of either type. This is the same as saying that the growth rate of the purchasing power of effective labour is positively affected by R&D. Consequently, the higher the R&D effort, of either type, the higher the future returns to effective labour and the more it pays to invest in education.

This latter effect is reminiscent of the argument that not only schooling causes growth, but there is also an opposite causality between the two[10]. Under the assumption of unit elasticity of intertemporal substitution though this effect of population growth is dominated by that of the interest rate, with the result of lower education. Yet education does not fall now as much as it would in the absence of R&D.

However, education is negatively affected by the death rate λ as well. This may come as a surprise, as λ affects the interest rate and both types of R&D effort in a way opposite to that of ε. The answer is that λ also has a direct negative effect on education, as the probability of death raises the discount factor of future labour income. This effect dominates, resulting to a negative total effect of the death rate on education. Finally, from (37) it is straightforward that the way ε and λ affect education, the same way they affect the growth of the per capita human capital. That is, under the assumptions of the present model they both reduce g.

According to the 2-dimensional R&D literature, population growth boosts the per capita output growth as well, through the above mentioned stream of higher R&D effort in both quality and variety dimensions. The result of proposition 1 therefore, that λ reduces growth, is consistent with the literature. Puzzling however is the ambiguity of the sign of the effect of ε, which according to the literature should have been unambiguously positive. As can be seen from equation (41) the per capita output growth (ω) is a weighted sum of three factors: population growth, quality improvement, and per capita human capital growth. The first affects ω both directly (by increasing the number of shares the final output will be divided to) and indirectly, by increasing the growth rate of product variety. In the aggregate, though, the indirect effect dominates. Positive also is the effect of population growth on the quality improvement of the intermediate products, as shown previously. Yet the last factor, the per capita human capital, was found to be

[10] I.e., Bils and Klenow [2000].

affected negatively by both fertility ε and mortality λ. Although this leads to an unambiguously negative effect of λ on ω, it makes the total effect of ε ambiguous. This ambiguity is entirely attributable to human capital investment, which is exactly the factor the literature assumed away: Although what matters for per capita growth is the growth rate of the total human capital, by assuming its formation away the literature has regarded the growth rates of human capital and population as one and the same thing, which we have shown is not correct.

It is also interesting to study the effects of an equal increase of the birth and death rates. This will increase the portion of the young without altering the growth rate of the population. The results are summarised in the following proposition:

PROPOSITION 2

A demographic change that increases the proportion of the young but leaves the population growth rate unaffected has a negative effect on all of *r, g, u,* ω and *s*.

The proposition is proved in the appendix. The explanation of these results must be sought again in the area the 2-dimensional R&D literature assumed away, that is, human capital investment; by assuming it away, as long as the population growth rate does not change nothing else does. However, the population age structure is very important for human capital investment: A higher death rate λ increases the discount factor of future labour income, which reduces an individual's education and through it the growth rate of the average human capital. This effect is additional to the effect of λ through the channel of population growth and because of that it is not offset by the equal increase of the birth rate ε. In short, an equal increase of ε and λ, or in other words a younger but not faster growing population, implies less human capital investment, lower growth rate of the average human capital, and consequently lower growth of the total human capital.

In fact the effects of the population age structure on human capital investment and through it technological progress are much richer; in the present model very simplified assumptions were made with respect to the death rate and the human capital of the newly born. In particular, the assumption of age independent death rate results in the same education effort for all age groups which is not the case under more realistic assumptions (i.e., finite horizons). It is also more reasonable to assume that the new generations start at a human capital level that is proportional to the average of the economy, but less than that. The first implies a positive effect of the proportion of the young on human capital growth, while the second implies the opposite. What was however shown by proposition 2 is that even under very simplified assumptions the population age structure still affects innovation and technological progress.

The next task is to see how the above described effects of the demographic variables on economic growth explain the data. Similarly to Romer [1986], I study the annual per capita growth rate of the technology leader, which I compare to various demographic variables. This is done on table I.

Perhaps the very first thing one can see from the table is that the scale effect never existed, at least has not since the industrial revolution; although the per capita growth rate has been increasing, this increase was much more moderate than the one implied by the scale effect argument. Further, there is plenty of evidence of non-increasing growth in recent years. However, the per capita growth does not appear to keep pace with the growth rate of population either[11] as the 2-dimensional R&D literature implies, although authors like Dinopoulos and Thompson [1998] argued that the observed growth patterns may be due to a long adjustment period towards the steady state growth. Yet by stressing the multiplicity of the effects of the demographic factors on technological progress and

[11] A 20 years lag was used for the population growth, because it was assumed that it takes approximately that time for population to affect technological change and through it economic growth. The result however is exactly the same if contemporaneous population growth is used instead.

economic growth, this paper offers an alternative explanation for the growth patterns of the last two centuries: As can be seen from table I, the population growth rate has been steadily declining (with the exception of the decade of 1950-60), at least during the period of US leadership. On the other hand the median age of the population, which reflects its age structure, has been increasing. In addition, although it is not shown in the table, both fertility and mortality declined in the last two centuries. The fall in mortality had an unambiguously positive effect on growth, while the fall of fertility, although implying slower population growth, also resulted in more education and growth of per capita human capital. Overall, the combination of slower population growth and higher population age and life expectancy resulted in faster output growth. This is contrary to the recent literature that would expect the output growth rate to follow that of the population. However, it is doubtful that this growth pattern will not reverse if the demographic trends that generated it continue; as said above, the population age structure affects human capital investment in various ways and consequently if the birth rates in developed countries fall further we may well end up with an older population, less human capital investment, and slower economic growth.

V. CONCLUSIONS

In this paper I have studied the effects of population on technological progress and through it economic growth, when human capital investment is endogenous. In particular, I introduced human capital investment in a framework where technological progress can be either variety expanding or quality improving. This 2-dimensional approach for R&D has been shown in the literature to yield a linear effect from population growth to per capita output growth. I have shown that when human capital investment is also taken into account the relations between population, innovation, and growth become less straightforward. This is because what actually matters is not the growth rate of population but of human capital, the formation of which is also affected by other demographic variables.

In particular, the birth rate has a direct positive effect on innovation and technological improvement, precisely through the channel of the population growth rate. However a higher population growth rate also pushes the interest rate upwards, with adverse effects on education and human capital investment. Although the faster technological improvement encourages education, this cannot offset the negative effect of the higher interest rate, thus the overall effect of the birth rate on education attainment is negative. Consequently, the birth rate has a positive effect on innovation and economic growth through the channel of population growth and a negative effect through that of per capita human capital. It was shown that the total effect can go either way. This ambiguity contrasts with previous findings which would expect the birth rate to have an indisputable positive effect on economic growth, and it is exactly due to the effects of the birth rate on human capital investment.

The effects of the death rate on the other hand also include a direct negative effect on education effort, as a higher death rate also implies faster depreciation of human capital. This means that equal changes in the birth and death rates do not completely offset each other, and it was further found that a younger but not faster growing population implies slower human capital growth, slower technological improvement, and eventually slower economic growth. This finding implies more generally that with endogenous human capital investment it is not only the growth rate of population that matters for technological improvement, but also its age structure.

It was finally argued that these multiple effects of population on innovation and economic growth can also explain the observed growth patterns in the last two centuries: although the decline of population growth should according to the 2-dimensional R&D theory have reduced economic growth as well, the lower mortality and more balanced population age structure resulted in more education and through it increasing rather than decreasing growth rates. This however may be reversed if the current demographic trends in the developed countries continue, as education may not increase any more to make up for further reductions in the population growth rate.

APPENDIX: PROOF OF THE PROPOSITIONS

A. PROOF OF PROPOSITION 1

By total differentiation of equations (36) to (38) one gets

$\begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & \delta B u^{\delta-1} \\ K & 1-a & L \end{bmatrix} \begin{bmatrix} dr \\ dg \\ du \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ a-1 & 1 \end{bmatrix} \begin{bmatrix} d\varepsilon \\ d\lambda \end{bmatrix}$, where $K = 1 - 2a - \frac{a\beta\theta}{\gamma}$ and

$L = a\delta(\delta - 1)Bu^{\delta-2}$. The solution of this system is

(A1) $\begin{bmatrix} dr \\ dg \\ du \end{bmatrix} = \frac{1}{D} \begin{bmatrix} -L & L - a\delta B u^{\delta-1} \\ \delta B u^{\delta-1}(K - a + 1) & -\delta B u^{\delta-1}(K+1) \\ K - a + 1 & -(K+1) \end{bmatrix} \begin{bmatrix} d\varepsilon \\ d\lambda \end{bmatrix}$, where

(A2) $D = -L - \delta B u^{\delta-1} K - (1-a)\delta B u^{\delta-1} = -L - \delta B u^{\delta-1}(K + 1 - a)$.

It is assumed that $K + 1 - a < 0$. (A sufficient condition for this to hold is that $a \geq 2/3$).

Under this assumption it is *D>0*, since L is negative. It is also

(A3) $K + 1 = 2(1-a) - \frac{a\beta\theta}{\gamma} > (1-a) - \frac{a\beta\theta}{\gamma} > 0$.

The last inequality in (A3) stems from equations (39) and (40) and the assumption that ℓ_x and ℓ_z are both strictly positive.

Thus, *g* and *u* are negatively affected by both ε and λ, while the effects of ε and λ on *r* are positive and negative respectively.

Next we have that

(A4) $\frac{dg}{d\varepsilon} = \frac{\delta B u^{\delta-1}(K + 1 - a)}{-L - \delta B u^{\delta-1}(K + 1 - a)} \Rightarrow -1 < \frac{dg}{d\varepsilon} < 0$.

From (35) it is $\hat{A}_t = \varepsilon - \lambda + g$. From the above the negative effect of λ on $\hat{A}_t$ is obvious, while $\frac{d\hat{A}_t}{d\varepsilon} = 1 + \frac{dg}{d\varepsilon} \Rightarrow 0 < \frac{d\hat{A}_t}{d\varepsilon} < 1$.

Regarding next the firm size s, we use (35), (39) and (40) to rewrite equation (42) as

(A5) $s = \left(\frac{1}{a\beta\theta} - \frac{1}{\gamma}\right) r + \frac{1}{\gamma}\hat{A}_t$.

As already noted, ε has a positive effect on both *r* and $\hat{A}_t$ while the effect of λ is negative and the term in the parenthesis is from (A3) positive. Consequently the effect of ε on *s* is positive while the effect of λ is negative.

From equations (40) and (41) we have for the per capita growth rate

(A6) $\omega = \frac{1-a}{a}(\varepsilon - \lambda) + \frac{1}{a} g + \left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right) r$.

Since the effect of λ on both *g* and *r* is negative and $\frac{1-a}{a} - \frac{\beta\theta}{\gamma}$ is from (A3) positive, the negative total effect of λ on ω is straightforward. Regarding the effect of ε, we have $\frac{d\omega}{d\varepsilon} = \frac{1-a}{a} + \frac{1}{a}\frac{dg}{d\varepsilon} + \left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)\frac{dr}{d\varepsilon}$

Substituting $\frac{\partial g}{\partial \varepsilon}$ and $\frac{\partial r}{\partial \varepsilon}$ from (A1) we have

$\frac{d\omega}{d\varepsilon} = \frac{1}{D}\left[\frac{1-a}{a} D + \frac{\delta}{a} Bu^{\delta-1}(K+1-a) - L\left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)\right]$, where *D*, *K* and *L* were defined above. After a lot of work we get the final expression

(A7) $\frac{d\omega}{d\varepsilon} = \frac{1}{D}\delta B u^{\delta-2}\left[(K+1)(1-\delta) + u(K+1-a)\right]$

the sign of which can go either way and depends on the values of all parameters of the model.

Finally, it is obvious from (39) and (40) that the labour allocation variables ℓ_x and ℓ_z are affected by changes in the exogenous variables in the same way the interest rate is affected.

Q.E.D.

B. PROOF OF PROPOSITION 2

The effects on *r*, *g* and *u* of an equal change of ε and λ can be derived by adding together the two columns of the matrix in (A1), that is

$$dr = \frac{-a\delta Bu^{\delta-1}}{D} < 0$$

$$dg = \frac{-a\delta Bu^{\delta-1}}{D} < 0$$

$$du = \frac{-a}{D} < 0\,.$$

From (A5), (A6) and (35) it can be seen that the changes in s and ω are given by

$$ds = \frac{1}{\gamma}dg + \left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)dr$$

$$d\omega = \frac{1}{a}dg + \left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)dr$$

and they are negative because both *dg* and *dr* are.

Q.E.D.

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TABLE I

POPULATION AND GROWTH SINCE EARLY INDUSTRIALISATION

	Per capita growth (%)	Initial population (000)	Median age	Median age (start)	Population growth (%) (20 yr. lag)
UK 1785-20	0.5	7,434	25.8	26.5	1.24
UK 1820-90	1.4	16,736	25.6	24.9	1.39
US 1840-80	1.44	17,120	23.1	22.0	2.90
US 1880-20	1.78	50,262	26.1	24.5	2.23
US 1920-60	1.68	106,461	30.4	27.7	1.39
US 1960-90	1.97	179,979	33.0	31.6	1.46
US 1960-70	2.54	179,979	31.8	31.6	1.37
US 1970-80	1.61	203,810	32.7	32.0	1.76
US 1980-90	1.76	226,546	34.2	33.5	1.25

Notes:

UK population data refers to England only for 1785-1820 and to England and Wales only for 1820-1890.

Sources of the raw data:

(a) Romer [1986], Tables 1 and 2.
(b) Wrigley and Schofield [1981].
(c) Censuses of England & Wales.
(d) US Bureau of Census.
(e) OECD, National Accounts.