Minimum Bit Error Rate Beamforming Receiver for Space-Division Multiple-Access Based Quadrature Amplitude Modulation Systems

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   - Motivations

2. Problem Formulation
   - System Description
   - MBER Beamforming

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Bit error rate is ultimate performance metric, but making bit decision is more complicated than making symbol decision.
Our Contributions

- Many previous works, including ours, have focused on **minimum symbol error rate** designs for QAM systems.
- It was generally believed that:
  1. A **minimum bit error rate** design is too complicated, and complexity may be much higher than MSER design.
  2. MSER design may be as good as MBER design.
- It would be nice at least intellectually to know the answers.
- In this work, we specifically look into **MBER design for QAM systems**, and our findings are:
  1. MBER design has similar complexity as MSER design, at least for 16QAM.
  2. MSER design indeed achieves the same performance of MBER design, in terms of BER.
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MIMO Model

- SDMA with $L$-element receive antenna array to support $M$ QAM users, where receive signal vector $\mathbf{x}(k) = [x_1(k) \; x_2(k) \cdots x_L(k)]^T$

$$\mathbf{x}(k) = \mathbf{H} \mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- Complex-valued AWGN vector $\mathbf{n}(k) = [n_1(k) \; n_2(k) \cdots n_L(k)]^T$ with covariance matrix $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}_L$

- Channel matrix $\mathbf{H} = [A_1 \mathbf{s}_1 \; A_2 \mathbf{s}_2 \cdots A_M \mathbf{s}_M] = [\mathbf{h}_1 \; \mathbf{h}_2 \cdots \mathbf{h}_M]$ with $i$th channel coefficient $A_i$ and steering vector for user $i$

$$\mathbf{s}_i = \begin{bmatrix} e^{j \omega_c t_1(\theta_i)} & e^{j \omega_c t_2(\theta_i)} & \cdots & e^{j \omega_c t_L(\theta_i)} \end{bmatrix}^T$$

$t_i(\theta_i)$: relative time delay at array element $l$ for user $i$, $\theta_i$: direction of arrival for user $i$, $\omega_c = 2\pi f_c$: angular carrier frequency

- Transmitted symbol vector of $M$ users $\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^T$
Beamforming Receiver

- Assume **user 1** is desired user, beamformer output

\[ y(k) = w^H x(k) = \bar{y}(k) + e(k) = c_1 b_1(k) + \sum_{i=2}^{M} c_i b_i(k) + e(k) \]

- \( c_1 b_1(k) \): desired signal, summation term: residual interfering signal, \( e(k) \): zero-mean Gaussian with \( E[|e(k)|^2] = 2\sigma_n^2 w^H w \)

- Weight vector \( w = [w_1 \ w_2 \cdots w_L]^T \), \( c_1 \) is made real and positive

- 16-QAM modulation, 4 bits per complex-valued symbol:

\[ b_i(k) = b_{R_i}(k) + j b_{I_i}(k) \in \{ \pm 1 \pm j, \pm 1 \pm 3j, \pm 3 \pm j, \pm 3 \pm 3j \} \]

- Two bits per in-phase / quadrature symbol mapping:

\[ 11, 10, 00, 01 \leftrightarrow -3, -1, +1, +3 \]

Notice the class 1 (C1) bit and the class 2 (C2) bit
Detection of Bits

- \( y(k) = y_R(k) + jy_I(k) \) used to detect four bits of \( b_1(k) \)

- Decision for in-phase C1 bit is given by
  \[
  \begin{cases}
    \text{C1 bit} = 0, & \text{if } y_R(k) > 0 \\
    \text{C1 bit} = 1, & \text{if } y_R(k) \leq 0
  \end{cases}
  \]

  and decision for in-phase C2 bit is given by
  \[
  \begin{cases}
    \text{C2 bit} = 0, & \text{if } -2c_1 < y_R(k) < 2c_1 \\
    \text{C2 bit} = 1, & \text{if } y_R(k) \leq -2c_1 \text{ or } y_R(k) \geq 2c_1
  \end{cases}
  \]

- Decisions for quadrature C1 and C2 bits are given similarly
  based on \( y_I(k) \)
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Bit Error Rate

- BER of 16-QAM beamformer with weight vector $w$ is defined by

$$P_E(w) = \frac{1}{4} \left( P_{E_{R},C_1}(w) + P_{E_{I},C_1}(w) + P_{E_{R},C_2}(w) + P_{E_{I},C_2}(w) \right)$$

- Let $b^{(q)}$, $1 \leq q \leq N_b = 16^M$, be legitimate sequences of $b(k)$:

$$\bar{x}(k) \in \mathbb{X} \triangleq \{\bar{x}^{(q)} = H b^{(q)}, 1 \leq q \leq N_b\}$$

- Set of beamformer scalar states

$$\bar{y}(k) \in \mathbb{Y} \triangleq \{\bar{y}^{(q)} = w^H \bar{x}^{(q)}, 1 \leq q \leq N_b\} = \mathbb{Y}_R + j \mathbb{Y}_I$$

- 16 subsets of beamformer scalar states

$$\mathbb{Y}^{(l,i)} \triangleq \{\bar{y}^{(q)} \in \mathbb{Y} : b_{R_1}(k) = l, b_{I_1}(k) = i\}$$

$$= \mathbb{Y}_R^{(l,i)} + j \mathbb{Y}_I^{(l,i)}, \ l, i = \pm 1, \pm 3$$
C1 Bit Error Rate

- In-phase C1 bit error probability

\[
P_{E_{R,C1}}(w) = \frac{1}{2N_{\text{sub}}} \sum_{\bar{y}_{R}^{(q)} \in Y_{R}^{(+1,+1)}} \left( Q(g_{R}^{(q)}(w)) + Q(g_{R}^{(q,a)}(w)) \right)
\]

\[N_{\text{sub}} = \frac{N_{b}}{16}, \quad Q(u) = \frac{1}{\sqrt{2\pi}} \int_{u}^{\infty} e^{-\frac{v^2}{2}} dv, \quad b_{1}^{(q)} = b_{R_{1}}^{(q)} + jb_{I_{1}}^{(q)} \quad \text{1st element of } b^{(q)},\]

\[g_{R}^{(q)}(w) = \frac{\text{sgn}(b_{R_{1}}^{(q)})\bar{y}_{R}^{(q)}}{\sigma_{n} \sqrt{w^{H}w}}, \quad g_{R}^{(q,a)}(w) = \frac{2c_{1} + \text{sgn}(b_{R_{1}}^{(q)})\bar{y}_{R}^{(q)}}{\sigma_{n} \sqrt{w^{H}w}},\]

- Quadrature C1 bit error probability

\[
P_{E_{I,C1}}(w) = \frac{1}{2N_{\text{sub}}} \sum_{\bar{y}_{I}^{(q)} \in Y_{I}^{(+1,+1)}} \left( Q(g_{I}^{(q)}(w)) + Q(g_{I}^{(q,a)}(w)) \right)
\]

with

\[g_{I}^{(q)}(w) = \frac{\text{sgn}(b_{I_{1}}^{(q)})\bar{y}_{I}^{(q)}}{\sigma_{n} \sqrt{w^{H}w}}, \quad g_{I}^{(q,a)}(w) = \frac{2c_{1} + \text{sgn}(b_{I_{1}}^{(q)})\bar{y}_{I}^{(q)}}{\sigma_{n} \sqrt{w^{H}w}}\]
**C2 Bit Error Rate**

- With some accurate approximation, in-phase C2 bit error probability

\[ P_{E_{R,C2}}(w) \approx \frac{1}{2N_{\text{sub}}} \sum_{\tilde{y}_R^{(q)} \in Y_R^{(+1,+1)}} \left( 2Q\left(g_R^{(q)}(w)\right) + Q\left(g_R^{(q,a)}(w)\right) \right) \]

- Quadrature C2 bit error probability

\[ P_{E_{I,C2}}(w) \approx \frac{1}{2N_{\text{sub}}} \sum_{\tilde{y}_I^{(q)} \in Y_I^{(+1,+1)}} \left( 2Q\left(g_I^{(q)}(w)\right) + Q\left(g_I^{(q,a)}(w)\right) \right) \]

1. **Class 2 error probability** approximately twice of **class 1 error probability**

2. For 16QAM, **complexity** of calculating bit error rate is **similar** to that of calculating symbol error rate
MBER Solution

- MBER beamformer solution is defined as
  \[ w_{MBER} = \arg\min_w P_E(w) \]

- MBER beamformer design may be obtained based on a gradient-descent numerical optimisation:
  1. Gradient of \( P_E(w) \) requires extensive computation
  2. Slow convergence and local minima problem

- Alternatively, evolutionary algorithms, such as **differential evolution** (DE) algorithm can be used:
  - DE is characterised by a) **initialisation**, b) **mutation**, c) **re-combination** and d) **selection** operations invoked for exploring the search space in an iterative procedure, until some termination criteria are met.
Simulation Systems

**Full-rank**: four-element antenna array supporting four users

- Minimum angular separation with desired user $\theta < 65^\circ$
- $E_b/N_0$: average bit energy over channel noise power
- All channel taps $A_i$ are identical

**Rank-deficient**: three-element array supporting four users
Benchmarks for Comparison

Two beamforming receiver designs are used as benchmarks

1 Conventional **minimum mean square error** (MMSE) solution that minimises MSE metric $E[|b_1(k) - y(k)|^2]$

$$w_{\text{MMSE}} = \left( HH^H + \frac{2\sigma_n^2}{\sigma_b^2} I_L \right)^{-1} h_1$$

$2\sigma_n^2$: channel noise power, $\sigma_b^2$: average symbol power

2 Our previous **minimum symbol error rate** (MSER) solution that minimises symbol error rate

$$\text{SER}(w) = \text{Prob}\{\hat{b}_1(k) \neq b_1(k)\}$$

$\hat{b}_1(k)$: detected symbol for $b_1(k)$

Same DE algorithm used to obtain MBER and MSER solutions
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Bit Error Rate (full-rank)

$P_s = 100, \gamma = 0.4, C_r = 0.4, G_{\text{max}} = 200$

*Four* receive antennas support *four* 16 QAM users

- MMSE-based beamforming
- MSER-based beamforming
- MBER-based beamforming

![Graph showing Bit Error Rate (BER) vs. $E_b/N_0$ in dB for different beamforming techniques.](image-url)

- $\theta = 20^\circ$
- $\theta = 30^\circ$
- $\theta = 40^\circ$

$\text{E}_b / N_0 \text{ in dB}$

$\text{BER}$
Bit Error Rate (rank-deficient)

\[ P_s = 100, \quad \gamma = 0.4, \quad C_r = 0.4, \quad G_{\text{max}} = 200 \]

Three receive antennas support four 16 QAM users

- MMSE-based beamforming
- MSER-based beamforming
- MBER-based beamforming

\[ E_b / N_0 \text{ in dB} \]

\[ \theta = 20^\circ, \quad \theta = 30^\circ, \quad \theta = 40^\circ \]
We have proposed a minimum bit error rate beamforming receiver for multi-user SDMA based QAM systems. More specifically, for 16QAM MIMO systems:

1. Derive explicitly bit error rate expression
2. Show MBER design has a similar complexity to that of MSER design
3. Confirm both MBER and MSER designs achieve same performance, in terms of BER

Future work will incorporate minimum bit error rate design in applications to unknown MIMO channel.
Adaptive Applications

- Previously we have developed a stochastic-gradient based adaptive MSER algorithm: least symbol error rate
  - Same approach can be adopted for adaptive MBER design
- More powerfully, previously we have applied MSER design in joint channel estimation and turbo detection
  http://eprints.ecs.soton.ac.uk/23148/
  - Same approach can be adopted by using MBER design
State-of-the-Art

- Existing schemes require an **extra iterative loop** between channel estimator and turbo detection/decoder.
- We have recently developed a new scheme where channel estimator is **embedded** in the original turbo iterative procedure.

![Diagram](image-url)