The ‘Neural Space’: a Physiologically Inspired Noise Reduction Strategy Based on Fractional Derivatives

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Abstract – **We present a novel noise reduction strategy that is inspired by the physiology of the auditory brainstem. Following the hypothesis that neurons code sound based on fractional derivatives, we develop a model in which sound is transformed into a ‘neural space’. In this space sound is represented by various fractional derivatives of the envelopes in a 22 channel filter bank. We demonstrate that noise reduction schemes can work in the neural space and that the sound can be resynthesized. A supervised sparse coding strategy reduces noise while keeping the sound quality intact. This was confirmed in preliminary subjective listening tests. We conclude that new signal processing schemes, inspired by neuronal processing, offer exciting opportunities to implement novel noise reduction and speech enhancement algorithms.**

Keywords-neural coding; sparse coding; fractional derivation; bio-inspired

I. INTRODUCTION

Speech enhancement and noise reduction strategies have been developed on the basis of various mathematical principles. Common strategies that are used today in acoustical signal processing are spectral subtraction, Wiener filtering and subspace algorithms [1-6]. These methods, although based on fundamentally different strategies share commonalities: first, they are based on the signal amplitude and are therefore sensitive to the signal energy, and second, although they can improve the speech quality, they generally do not improve the speech intelligibility. A system to improve speech intelligibility in noisy situation, possibly one that performs as good as a human listener, has become the holy grail of the speech processing community. Among the many applications that such a system would have are hearing aids, mobile phones, and automatic speech recognition and telecommunication systems. Despite the demand for better solutions recent improvements in research have been incremental. It is generally assumed that the required breakthrough in development requires novel ideas that consider information of a higher statistical order and contextual information.

Here we suggest a novel noise reduction strategy which is inspired by a modern interpretation of the physiology of the auditory brainstem. We try to emulate the workings of the mammalian brainstem by distilling a mathematical concept of neuronal speech processing. Specifically, we present a method that is inspired by our knowledge of how neurons in the ventral cochlea nucleus (VCN) code sound. Recordings in our lab allow the hypothesis that neurons code sound based on fractional derivatives of the sound envelope and from this basis we develop a noise reduction algorithm. We utilize a modified well-known algorithm for noise reduction: sparse code shrinkage. However, instead of selecting basis vector components only on the basis of amplitude, we also take the temporal derivative into account. On top of the traditional well known first derivative, we also introduce here the concept of fractional derivatives of order $0 \le k \le 1$. We expect that by doing this, more powerful basis functions can be found and that the signal can be better reconstructed than in traditional methods.

The structure of the paper is as follows: Section II presents the study of physiology of sound coding in the ventral cochlear nucleus. Section III describes the principle of sparse coding and implementation of a sparse coding method. Section IV describes the implementation framework of sparse coding based on fractional derivatives of spectral envelopes. Section V presents preliminary experimental evaluation results.

II. PHYSIOLOGICAL MOTIVATION

A. Sound coding in the ventral cochlear nucleus

The cochlear nucleus (CN) is the first processing station of sound in the mammalian auditory brainstem. It provides the first opportunity in the ascending auditory pathway to recode the input from the cochlea. It is a mandatory processing station for all auditory nerve fibres. Anatomically it has been described as containing more than fifty discrete cell types [7] in two sub-divisions; the Dorsal Cochlear Nucleus (not considered here) and Ventral Cochlear Nucleus (VCN). Sound is coded neurally by short electrical bursts (pulses) which are termed ‘action potentials’ (or spikes). Traditionally it is thought that the rate by which a neuron fires per time is a good measure of the information that it is coding, but today it is clear that information is coded by rate and higher order statistics in single neurons as well as in the temporal correlation of population of neurons.

We have investigated in recent years, how neurons’ response and interval statistics are correlated with the sound stimulus. An important visual tool to investigate neuronal rate responses over time are ‘post stimulus time histograms’.

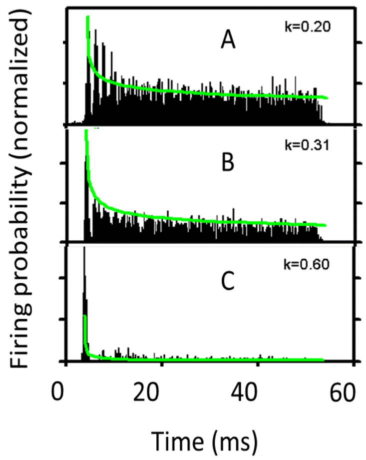

Figure 1. Rresponse probability of 3 different neurons in the ventral cochlea nucleus to a short sound burst of 50 ms. The green line indicates best fits with a power function t^{-k}. The fitted k-values are shown in the top right corners.

Post stimulus time histograms (PSTHs) represent the averaged neurons' response to repetitions of the same sound. Properly normalized these represent the probability density function of the neurons in response to a specific stimulus.

In contrast to its input from the auditory nerve, the temporal discharge characteristics of VCN neurons in response to sound are heterogeneous. Using the shape of the PSTH in response to tone bursts at the unit's best-frequency (the frequency at which the neuron responds at lowest threshold), single unit responses from the VCN have traditionally been classified as belonging to one of several main types [8]. Three examples of PSTHs in response to pure tones of 50 ms at 20 dB above the neurons threshold are shown in Figure 1. The responses vary in a number of aspects. Important here is the 'adaptation', the gradual decline of response after the onset. In the three examples, adaptation increases from top to bottom, and we observe that a power function, t^{-k} with increasing power models this behaviour well.

The observed variability in temporal adaptation leads to a particularly simple interpretation of the neuron's role in a coding scheme. Neurons with less adaptation (low *k*-value) are better suited for measuring energy (Blackburn, Sachs, 1990): they respond continuously during a stimulus; the response of an ideal neuron with $k=0$ would be proportional to the stimulus itself (Fig. 2A, top panel). Neurons with high *k*-value adapt strongly and have no sustained rate and responds only to changes in the stimulus (Rhode, 1994; Winter, Palmer, 1995); an ideal neuron with $k=1$ would make the response proportional to the temporal derivative of the stimulus with respect to time (Fig. 2A bottom panel). Neurons between these extremes perform a mixture of these two processes, their response shows some adaptation and some sustained response.

Accordingly, the mathematical differentiation operator is defined for non-integers orders, this branch of mathematics is known as fractional calculus. Differentiation generalizes to d^k/dt^k where *k* is not restricted to integer values [9].

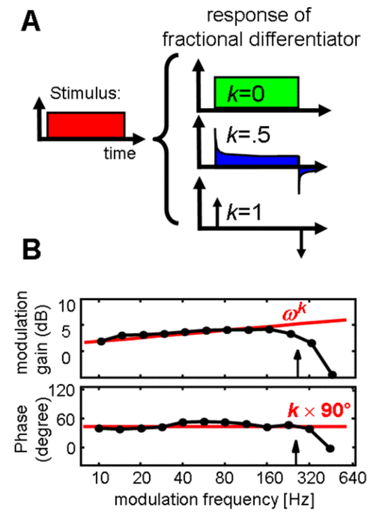

Figure 2. Neurons as fractional differentiators

A) Sketch of differentiation of orders 0, 0.5 and 1 (top to bottom) of a rectangular envelope. Arrows in the bottom sketch indicate Dirac delta functions. The behaviour at $k = 0$ corresponds to that of "sustained chopper" neurons that respond almost constantly during the whole stimulus. The behaviour at $k = 1$ corresponds to that of "Onsets" that respond best to the stimulus onset. "Primary-likes" (and essentially all other neurons) are somewhere between these extremes. The resulting adaptation pattern follows a power function.

B) Response of a neuron with a k-value of 0.47 to an amplitude modulated sinusoids. The top panel shows that modulation gain as function of modulation frequency is well described by ω^k; the bottom panel shows that the phase is almost constant at $k \times 90°$. The red lines in B are not fitted to the data, they show instead the prediction of the fractional model. To calculate the phase response a constant delay was subtracted from each period (indicated by the arrow in both panels as a cut-off modulation frequency).

The result of applying such an operation to a step input is a power-law decay t^{-k} as observed in the PSTHs. The middle panel of Fig. 2A shows the effect of this operation on a step input for $k=0.5$. For non-integer *k* the behaviour is non-local so that the output depends on the history of the input as well as its current value. It therefore appears that all neurons are differentiators, each neuron having a particular (non-integer) order. This kind of fractional behaviour, characterised by power function decay, has been observed and modelled in various sensory systems [10-12] e.g. tactile [13], baro [14], joint [14], atrial stretch [14] visual [15], electric [16, 17] and vestibulo-ocular reflex [18, 19].

To test the hypothesis that fractional differentiation is occurring in VCN neurons we tested the following hypothesis in a limited preliminary study with 11 units: fractional calculus predicts that the modulation transfer function will be proportional to $\omega^k \sin(\omega t + k\pi/2)$ [9] i.e. a gain of $(2\pi f)^k$ and a phase lead of $k \times 90°$. Neurons were stimulated by a sinusoidally modulated input sin(ωt) and their *k*-value was identified from the PSTH. The results from one neuron are shown in Fig. 2B. Out of the 11 neurons, 9 could be described well by the given equation. However, the sample size in this study was too small to allow a conclusive answer if this model is ultimately correct and more work is needed to underpin the physiological basis.

The observation that projecting neurons produce PSTHs that are well-modelled by power laws raises two immediate questions: how does their physiology cause them to do this, and why have they evolved so as to do it? For the 'how' Anastasio [18] has suggested that power law behaviour may result from parallel activation of many dendritic segments with differing time constants. For the 'why' one possibility is that sensory systems evolved as control systems and it has been shown that fractional differentiation is an efficient element in a control system for biological processes [10]. The impulse response of a fractional calculus systems can be approximated by many exponential decay times, thus having many time constants or in other words a decaying "memory strength". It is not possible to predict the system response at any time from one measurement without knowledge about the stimulus. With numerous exponential decays the memory of a signal gets forgotten the longer the signal is gone. Therefore fractional order controllers memorize the latest inputs more strongly and also forget the old inputs more completely than conventional integer order controllers. Drew and Abbot [10] describe power-law adaptation as temporal "generalist", an intermediate case between perfect adaptation (integration in which the signal history is never forgotten) and exponential adaptation (in which the signal history is forgotten after a few time constants). This behaviour seems to be advantageous for an organism that has to deal with stimuli that vary on multiple time scales. This behaviour is not strange or even rare, it is apparent in most natural systems [20], but not widely discussed probably because of the apparent complexity of the theory of fractional derivatives.

We use these experimental results as a template to develop a hypothesis how a description of the observed neuronal response could be used to develop the basis of a novel noise reduction method. In our model we assume that the neurons indeed perform a fractional derivation of the envelope waveform. Encapsulating this behaviour in a mathematical formulation, we are going to utilize fractional derivatives as the pool of potential functions for the basis transformation in a traditional noise reduction scheme.

B. The 'neural space'

We aim to develop a novel signal processing algorithms that comprise a physiological motivated model for noise reduction and speech enhancement applications. In each stage the model will attempt to simulate the physiological function. We are not trying to build or mimic neuronal networks, but to emulate their function on the basis of abstract mathematical principles. The first stages of the model are similar to the popular 'auditory image mode' (AIM) [21]. The frequency selectivity of the hearing process is modelled by a gammatone filter bank with 22 channels, the approximate number of human critical bands. In their temporal response, neurons cannot follow frequencies higher than 1-2 kHz, and therefore we apply a low pass filter of 1200 Hz to the envelope of the signal in each channel. The amplitude of this signal represents the probability that auditory nerve fibres fires; in AIM this stage is therefore called the 'neuronal activity pattern'. Instead of statistical analysis that follows in AIM, we take a different approach, and simulate the hypothetical fractional response of neurons in the brain stem. Real neurons would form a continuum of *k*-numbers between 0 and 1 with millions of neurons in each frequency channel. To simplify the processing, we assume here only three different *k*-values (0, 0.5 and 1) in each of the 22 frequency channels and only aim to demonstrate the principle.

At this stage in the model the amplitude of the signal in each channel is a representation of the sound in a new space: the $k=0$ channels represent signal energy, the $k=1$ channels its entropy [22], and the $k=0.5$ channel are a mixture and contain a memory. We call this representation the 'neural space' of the signal and we argue that advanced and novel signal processing that is inspired by physiology can take place in this space. After signal processing in the neural space, the signal is resynthesized back to the time domain, allowing applications where the user needs to hear a cleaned-up signal.

Although the focus in this paper is on the novel representation, we show that a simple noise reduction scheme – supervised sparse coding – can work in the neural space.

III. Sparse Coding Method

A. Principle of Sparse Coding

Sparse coding (SC) strategies offer a promising method to identify the most essential information in a speech sound. Recently, there has been significant development in SC algorithms, exploring sparse representations in the context of denoising and classification [23-27]. Sparse applications exploit the fact that most signals of interest are sparsely represented in an appropriate dictionary or base. However, most previous research utilize 'off-the-shelf' wavelet- and cosine-transform dictionaries but recent research has demonstrated the significant advantages of dictionary learning matched to the signals of interest [28].

We consider a signal $\mathbf{x} \in \mathbb{R}^D$ and a dictionary consisting of L basis vectors, $\mathbf{D} = [\mathbf{d}_{(1)} \cdots \mathbf{d}_{(L)}] \in \mathbb{R}^{D \times L}$, $\|\mathbf{d}_{(l)}\|_2 = 1, l = 1, \cdots, L$. A sparse coding $\mathbf{c} \in \mathbb{R}^L$ of signal x in dictionary D defines a sparse linear combination of $K \le L$ atoms, such that the approximation error $\|\mathbf{x} - \mathbf{Dc}\|_2$ is sufficiently small.

B. Dictionary Learning

Dictionary learning adapts an initial dictionary to a specific signal class (e.g. speech spectral envelopes). The signal in this specific class is represented by a linear combination of dictionary atoms. Learning the dictionary is critical in noise reduction, where signal of a specific class must have a sparse representation in its dictionary, while noise could not. Constructive dictionaries that are not signal class specific don not satisfy this criterion.

Dictionary learning is decomposition of a data matrix $\mathbf{X} = [\mathbf{x}_{(1)} \cdots \mathbf{x}_{(N)}] \in \mathbb{R}^{D \times N}$ into a dictionary $\mathbf{D}$ and a coding $\mathbf{C} = [\mathbf{c}_{(1)} \cdots \mathbf{c}_{(N)}] \in \mathbb{R}^{L \times N}$, given by

$$\arg\min_{\mathbf{D},\mathbf{C}} \left\| \mathbf{X} - \mathbf{D} \cdot \mathbf{C} \right\|_F^2 \quad (1)$$

subject to a sparsity constraint on **C.** The unit norm constraint on $\|\cdot\|_F$ denotes the Frobenius norm. *k*SVD (*k*-means singular value decomposition) algorithm of [23] implemented in Matlab by kSVD-Box[1] is used to realize dictionary learning. This method iteratively solves locally optimal solutions, by alternating between optimizing the coding and the dictionary.

Coding update: For a given dictionary, an orthogonal matching pursuit (OMP) regression was used to compute **C** by approximating the solution of

$$\arg\min \left\| \mathbf{c}_{(n)} \right\|_0 \quad (2)$$

$$s.t. \left\| \mathbf{x}_{(n)} - \mathbf{D}\mathbf{c}_{(n)} \right\|_2 \le \sigma \quad (3)$$

Dictionary update: For each column $\mathbf{d}_{(l)}$, *l*=1,…L, the contribution of basis vector $\mathbf{d}_{(l)}$ to the residual norm was separated and the residual norm was minimized using SVD. Specific details are described in [23].

C. Noise Reduction

We assume the observed noisy signal is the linear additive mixture of clean signal and noise. Given the dictionary and noise variance, the clean signal could be estimated by orthogonal matching pursuit (OMP) regression algorithm.

The signals here are fractional derivative representations of spectral envelopes from speech materials. BKB (Bamford-Kowal-Bench) sentences were chosen as both training and test speech materials [29]. BKB sentence lists are standard British speech materials containing 21 lists, each containing 50 keywords in 16 sentences. Six lists of BKB sentences were randomly chosen as training data and the other lists were used as test data. The training sentences were concatenated to derive spectral envelopes to get dictionary.

IV. Supervised Sparse Coding Strategy on Fractional Derivatives of Spectral Envelopes

A. Implementation of Sparse Coding Strategy in Speech Enhancement

Figure 3 shows the implementation of the supervised SCS strategy for the spectral envelopes. As most of the information is contained in the envelopes of each filter, processing in spectral envelopes is expected to extract the most critical information. In our strategy, the input signals are split into 22 channels by a gammatone filter bank, from which envelopes are extracted in each channel by Hilbert transform. In each frequency channel fractional derivatives are calculated for three *k*-domains: *k=0* (only coding Energy), *k=1* (only coding amplitude change or entropy) and *k=0.5*. In future application, we expect that these *k*-values fill the continuum between 0 and 1. In traditional noise reduction schemes, only *k=0* is taken into consideration.

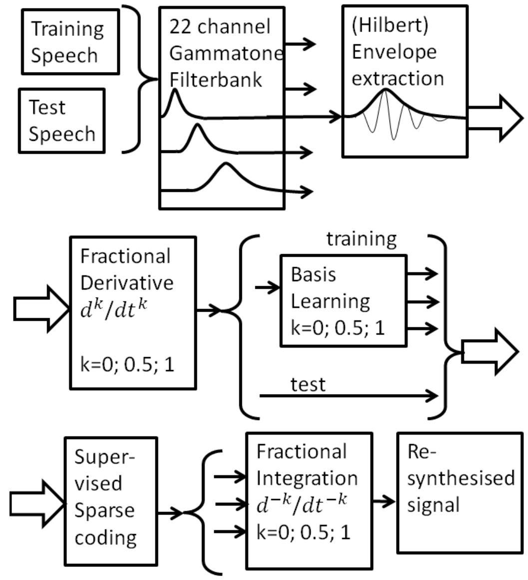

Figure 3. Flowchart of the spectra envelope sparse coding strategy. Supervised SCS is applied to the spectral envelopes of the fractional derivatives in each frequency channel.

Supervised SCS is implemented on the matrices of spectral envelopes where each column represents a short temporal frame and each row represents the envelopes in one channel. The training speech is processed in the same way and basis learning is applied in the spectral envelopes from the training data. The training data is about 10 seconds long. After supervised SCS, the basis vectors with highest Eigenvalues in each *k*-domain are selected for further processing and the de-noised signal is calculated by resynthesis on the basis of the selected basis vectors. The resynthesis works by averaging the three individual reconstructions according to *k=0, .5* and *1* with identical weight.

B. Implementation of Fractional Derivation and Integration

Fractional derivation and integration are implemented by using a matrix approach that enables convenient discretization of partial differential equations with derivatives of arbitrary real order [30]. We used software from the MATLAB file exchange[2] to realize fractional derivation and integration. Examples of fractional derivatives of spectral envelopes of a short segment word from high, middle and low frequency bands are presented in Figure 4. Columns show the fractional derivatives of the spectral envelopes from the same frequency band (centre frequencies: 3191, 1416, 212 Hz respectively) rows shows the fractional derivatives of the spectral envelops from different frequency bands with the same *k* value (0, 0.5, 1). The spectral envelopes are derived by Hilbert transform. The figures illustrate that, as expected, the fractional derivative fluctuates more at larger *k* and at higher frequency channels.

[1] http://www.cs.technion.ac.il/~ronrubin/software/html

[2] http://www.mathworks.com/matlabcentral/fileexchange/22071

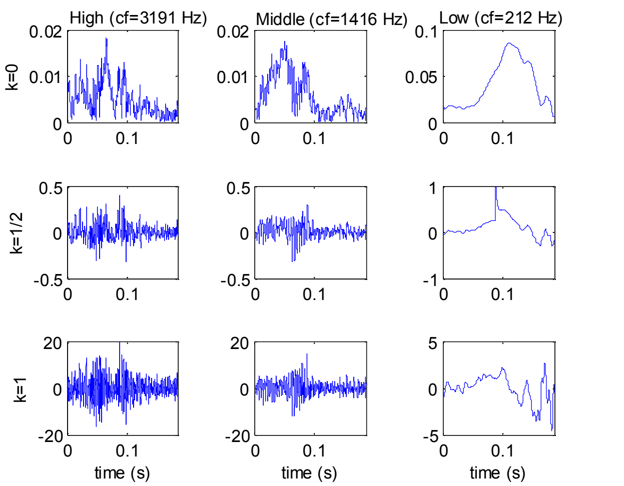

Figure 4. Fractional derivatives of the spectral envelopes of the word "pears". Different columns show: (left) High Frequency band (middle) middle frequency band (right) low frequency band. Different rows show different k-values. Cf indicates the centre frequency in a gamm tone filter bank.

The fractional derivatives emphasize the different temporal characters of the spectral envelopes. For example, for the zero-derivatives the sparse coding algorithm will emphasize the low frequency components which have higher energy; for the first derivatives the algorithm will pick out more high frequency components which show a higher rate of change. The fractional derivatives are in between these extremes (Figure 4 and Figure 5). The method is described in detail in [30].

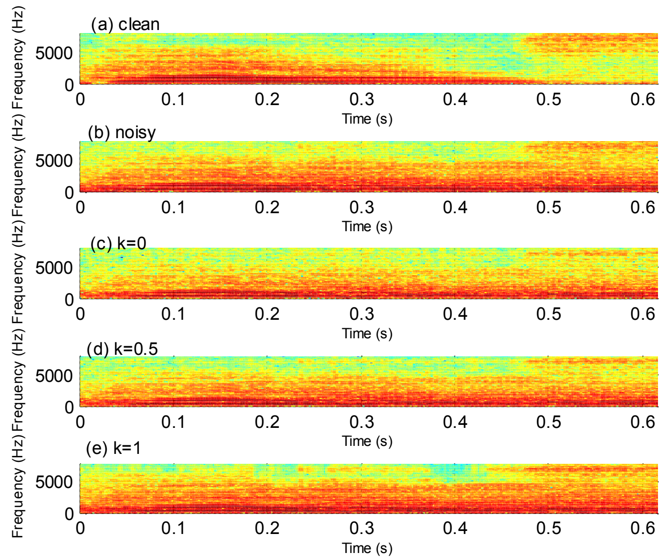

Figure 5. Spectrograms of outputs from sparse coding strategies on different fractional derivatives of spectral envelopes of word "pears" in babble noise (0 dB SNR). (a) clean speech; (b) noisy speech; (c) sparse coding on fractional derivatives of spectral envelopes (*k*=0); (d) sparse coding on fractional derivatives of spectral envelopes (*k*=0.5); (e) sparse coding on fractional derivatives of spectral envelopes (*k*=1).

Future work will choose appropriate k value to implement sparse coding so that noise is more efficiently reduced. Combination of strategies with different *k*-values is also practical as it will combine different advantages of different neural space (different k values) to enhance speech perception.

V. Experimental Results

Figure 5 shows spectrograms of the example sound *"pears"* clean (figure (a)) and in babble noise (0 dB SNR) (figure (b)). Also shown for comparison are the spectrograms of the resynthesized outputs after the sparse coding noise reduction strategies on different fractional derivatives (Figure 5 (c-e)). It can be seen that sparse coding on entropy (Figure 5 (e), *k=1*) emphasizes the high frequency bands and the noise, in the middle frequency bands is reduced; sparse coding on original envelopes (Figure 5 (c), *k=0*) keeps the energy in low frequency bands while losing some information in the higher frequency bands.

Results of noise reduction in the neural space were also evaluated by informal subjective listening tests. Three experienced listeners (first, second and last author), independently judged the resynthesized speech sounds. While the background noise was reduced slightly, the sound quality was as good as the original sounds. These preliminary results are encouraging.

This experiment shows that processing on fractional derivatives is practical to realize noise reduction and keep intelligibility. Standard subjective speech intelligibility tests with BKB sentences [31] will be used to test the performance of sparse coding in different fractional derivative of envelopes.

VI. Conclusions

This research presents several novel ideas for speech enhancement and noise reduction algorithms by mimicking mechanism of signal processing in auditory neurons. Novel aspects of our model are the physiologically motivated separation into frequency bands, the concentration on envelopes and finally the fractional derivatives of the envelope. We hypothesize that auditory neurons utilize fractional derivatives to code incoming sound in an optimal space in order to preserve information efficiently and reduce noise. This hypothesis needs further experimental exploration, but we think that this should not stop us from investigating the exciting mathematical possibilities that fractional derivatives offer for noise reduction algorithms.

Most state of art noise reduction strategies can reduce noise and increase speech quality, but they cannot improve speech intelligibility. Processing in the neural space aims to improve intelligibility while reducing noise and keeping the quality intact. Crucially have shown here that fractional derivatives allow transformation into the neural space and sounds can be resynthesized without loss of quality.

Sparse coding strategies form a family of different algorithms, of which the one used is a promising, but probably not the ultimate candidate. We hope that the demonstrated novel sound representations in the neural space can help to inspire improved sparse coding strategies that utilize the additional information that is provided by the fractional derivatives. Although we have not yet collected enough data to quantify the advantage conclusively, we have demonstrated the potential of this new representation. This also could further explore potential of sparse coding strategies in spectral envelopes for cochlear implant users [28, 32-34]. Furthermore, the transformation into the neural space allows further noise reduction schemes in future that are also inspired by physiology like lateral and forward inhibition, as well as more complex strategies e.g. feed-back inhibition and excitation. In future work we will investigate the effects of the supervised sparse coding strategy under different noise types. We will also develop and evaluate the performance of unsupervised sparse coding strategy for hearing aid users. We will implement methods that have been developed for the auditory image model to further reduce noise. This can be done by implementing local waveform averaging schemes as well as predictive or adaptive filtering both based on the signals' F0 or repetition rate.

In an interdisciplinary field like this with multi-disciplinary approaches to highly complex problems, it takes a group of researchers with different backgrounds to further investigate and develop physiologically inspired ideas towards the ultimate goal, a speech enhancer which is as good as a normal hearing human. We expect that this work will impact the speech perception, physiology and machine learning community in future.

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