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Anthony Kelly

University of Southampton, UK

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Measuring ‘equity’ and ‘equitability’ in school effectiveness research

Anthony Kelly*

University of Southampton, UK

This paper introduces a Gini-type index for measuring ‘attainment equity’ in schools; that is, how far a school (or group of schools) is from having a ‘fair’ proportion of its examination success attributable to a fair proportion of its student population. Using data from the National Pupil Database, the Index is applied to more than 20,000 students with matched attainment records at KS2 and KS4 in two ‘statistical-neighbour’ local authorities in England, capturing the extent to which they are meeting a public policy notion of equity. It is then combined with existing contextual value added measures to analyse school and local authority performance in terms of both attainment equity and context.

Introduction

Education policy makers and managers in the UK and elsewhere have struggled to marry their measures of effectiveness with the aspiration to provide equity in schooling outcomes; that is, how far a school (or group of schools) is from having a ‘fair’ proportion of its examination success attributable to a fair proportion of its student population. Whether using (now-preferred) raw threshold performance indicators such as the percentage of pupils achieving five or more A*-C grades at GCSE or (recently abandoned) contextual value added (CVA) measures that take account of pupil and school context, there is no metric for gauging the extent to which these outcomes meet expectations/aspirations/intentions of equity. Of course, defining equity, never mind measuring it, is problematic, especially in political terms as it links schooling and education policy to notions of freedom, ‘unfreedom’ (Sen, 1999; Wallace, 2004, p. 7) and the ‘negative liberty’ of avoiding arbitrary ‘regulation’ (Skinner, 2002). Yet many governmental and supra-governmental organisations...
have defined it non-problematically and explicitly in terms of outcomes. The European Union (EU) (2006) defines equity as:

...the extent to which individuals can take advantage of education in terms of opportunities...and outcomes. Equitable systems ensure that the outcomes of education are independent of [all] factors that lead to educational disadvantage...Inequity in relation to gender, ethnic minority status, disability and regional disparities etc. is not the prime focus, [except insofar] as it contributes to overall socio-economic disadvantage.’ (p. 2, emphasis added)

And the Organisation for Economic Co-operation and Development (OECD) (2005) says that:

The success of an education system depends upon an equal or fair distribution of inputs, processes and outcomes among participants in education with different characteristics. (p. 14, emphasis added)

In reality, wherever this ‘outcome view’ of equity is found in policy, it is actualised in practice through student attainment in public examinations. In Europe alone, it can be found in Denmark, Spain, Greece, France, Ireland, Italy, Cyprus, Latvia, Lithuania, Hungary, Netherlands, Austria, Poland, Portugal, Slovenia, Finland, UK, Norway and Romania and the situation is no different in the USA under the No Child Left Behind Act (Owens & Sunderman, 2006):

Accountability systems in the form of central exit examinations...exist in most European countries [and] there is some international evidence that [they] can improve student performance. However, accountability systems should be designed to ensure a full commitment to equity... (EU, 2006, p. 6)

For policy makers, then, the link between accountability, public examination success and equity is clear, but it is not clear what the targets should be, how they should be measured or how they should be spread across the range of prior attainment. For example, using unadjusted threshold measures, is it ‘fair’ to expect the bottom 40% (say) of pupils (as measured by their prior attainment at the end of KS2, say\(^3\)) to achieve 40% of the standard pass grades A\(^*\)–C\(^2\)? On the one hand, it could be argued that a school should expect a greater proportion of pass grades from its more able (and more privileged) pupils; on the other hand, that an equitable school might reasonably expect a greater proportion of higher grades (like A\(^*\) and A) to come from its more able pupils, but a more or less equal distribution of standard grades across the whole cohort. There is strong evidence in the official literature that the latter is the official aspiration (e.g. in the UK, DfEE [1997]; in the USA, No Child Left Behind [2001], c.f. Owens and Sunderman [2006]; in Europe, EU [2006]), which is consistent with national and trans-national policies in other social and economic areas (e.g. Chakravarty, 1990; Halfman & Leydesdorff, 2010) and with Rawlsian principles of justice (Rawls, 1971).\(^5\) So as a starting point for our analysis of state-funded comprehensive schools in the UK, we assume it too; that is to say, we assume that the intention of policy in terms of the attainment element of equity in non-selective schools is that a given proportion
of a school’s examination success be attributable to an equal proportion of its pupil population. Later, we will discuss departing from this *ab initio* assumption, but for now let us accept it as a reasonable ‘definition’ of what policy-makers aspire to in terms of attainment equity.

Of course, *any* single measure seeking to capture the effectiveness of schools across a range of outcomes masks a great deal of fine detail (e.g. Nuttall *et al*., 1989) and this is as true for a Gini-type coefficient of attainment equity as it is for school value-added measures. However, it would be a mistake to infer from this that all summative metrics are therefore useless. After all, they can identify weaknesses, set targets, focus effort and make comparisons over time easier between and within schools, the importance of which has been highlighted many times in research from different countries (e.g. Scheerens & Bosker, 1997; Goldstein *et al*., 2000; Teddlie *et al*., 2000; Gray *et al*., 2001; Kyriakides & Creemers, 2008) and, at their best, they can help us conceptualise and reconceptualise our understanding of effectiveness and its relationship to policy and the broader aims of schooling.

**Background: the Gini coefficient**

The Gini coefficient is a measure of statistical dispersion developed by the Italian statistician Corrado Gini in 1912 and is widely used today by organisations like the UN and the OECD to measure income and wealth distribution (Firebaugh, 1999; OECD, 2010; United Nations, 2010). In essence, it is a measure of the normalised inequality of a distribution, with ‘0’ representing full equality and ‘1’ representing full inequality. Figure 1 is a graphical representation. The straight line $y = x$ represents ‘perfect’ equality in the distribution of variable $y$ over the

![Figure 1. The concept of the Gini coefficient](image-url)
population \( x \); the curve represents an \textit{actual} distribution. A perfectly equitable distribution (the straight line) is one in which every \( x\% \) has the same \( y\% \), so a perfectly \textit{inequitable} distribution is one in which one person has all the \( y \) and everyone else has none, which is represented by the horizontal axis (\( y = 0 \)) for all \( x < 100 \) and the vertical line \( x = 100 \).

If \( B \) is the area under the curve and \( A \) is the area between the line and the curve, the Gini coefficient is defined as the ratio of \( A \) to the total area under the line (\( A+B \)):

\[ \text{Gini} = \frac{A}{A+B} \]

When \( A \) is zero (i.e., when there is no gap between the ‘ideal’ and the ‘actual’) the Gini is zero; and when \( A = A + B \) (i.e., when the gap fills the entire space), the Gini is maximal at unity.

If we normalise the axes, then \( A + B = 0.5 \) and:

\[ \text{Gini} = 2A \]
\[ = 1 - 2B \]

The curve that separates \( A \) and \( B \) is a ‘Lorenz’ curve, named after Max Lorenz who developed it in 1905 (Lorenz, 1905; Gastwirth, 1972). It plots the proportion of a variable \( y \) that is cumulatively attributable to the population \( x \) from the lowest to the highest and as such it is a cumulative distribution function where every point represents a Pareto-type statement (c.f. Appendix: Note 1). On normalised axes, Lorenz curves start at (0,0) and end at (1,1). They are continuous functions which, if the variable being measured cannot take negative values, cannot rise above the line of perfect equality and cannot sink below the \( x \) axis. They are always increasing and convex (c.f. Appendix: Note 2).

Since the Gini coefficient is defined as ‘1-2B’, if the Lorenz curve is represented by the function \( y = L(x) \), the value of \( B \), the area under the curve, can be found by simple integration, so the Gini can also be given by:

\[ 1 - 2 \int_0^1 L(x) \, dx \quad (\alpha) \]

**Developing an Attainment Equity (Æ) Index for schools**

While a Lorenz curve representing the distribution of pupil attainment in schools is likely always to be continuous, it will mostly (if not always) be the case that its full functional equation will be unknown; that is to say, only values at certain intervals, however dense or fine-grained the function, will be known. In that case, the Lorenz can be approximated on each interval as a continuous piecewise linear function and area \( B \) can be approximated by trapezoids.\(^6\) Then a Gini-type ‘Attainment Equity Index’ (Æ) will be given by:
\[ A = 1 - \sum_{k=1}^{n} (X_k - X_{k-1})(Y_k + Y_{k-1}) \]  \hspace{1cm} (\beta)

where \((X_k, Y_k)\) are the known points on the Lorenz, with \(X_0 = 0\) and \(X_n = 1\) and \(Y_0 = 0\) and \(Y_n = 1\).

Figure 2 shows a continuous piecewise linear function with known values at 20% intervals. Of course, this would be more fine grained if values were known every 5%, say, but it is not necessarily the case that this would result in any great improvement in accuracy (as a later example will demonstrate) because the Lorenz function is indexed in monotonic non-decreasing order (i.e., \(y_k \leq y_{k+1}\) for all \(k\)) so that mid-interval variability is dampened somewhat. However, in theory, the more fine-grained the better, especially if the calculations are done by a spreadsheet algorithm rather than manually, as here.

Again, we are assuming that it is a desirable benchmark in attainment equity terms, and a reasonable reflection of education policy in most western countries, for non-selective schools to have any given proportion of their examination grades (in the UK, GCSE A*-C, say) attributable to (or dispersed among) the same proportion of their student populations. This assumption, whose provenance lies in the derivation of the Gini coefficient and is supported by policy statements in the UK, the EU, the US and elsewhere, creates the ‘2’ in equation \((\alpha)\) above, but it need not be this way. It might be that a policy aspiration is instead piecewise and linear in terms of its notion of equity, like Figure 3; or concave and curvilinear.

Figure 2. The Lorenz curve replaced by a piecewise linear function whose values are known at 20% intervals.
Figure 3. The Attainment Equity Index for a ‘piecewise linear notion of equity’

Figure 4. The Attainment Equity Index for a ‘curvilinear notion of equity’
(e.g. quadratic, say $y^2 = x$) like Figure 4, in which case equations (α) and (β) would be replaced respectively by:

$$1 - 5/2 \int_0^1 L(X) \, dX \quad (\alpha 1)$$

$$\bar{E} = 1 - \sum_{k=1}^n (X_k - X_{k-1})(Y_k + Y_{k-1}) \quad (\beta 1)$$

and

$$1 - 3/2 \int_0^1 L(X) \, dX \quad (\alpha 2)$$

$$\bar{E} = 1 - 3/4 \sum_{k=1} (X_k - X_{k-1})(Y_k + Y_{k-1}) \quad (\beta 2)$$

However, there is a theoretical argument against assuming anything other than a linear representation of the policy benchmark/ideal and that is that the alternatives can reduce to near absurdities. If as in Figures 3 and 4 a concave (piecewise or curvilinear) notion of equity intention is assumed, it is possible for the Lorenz (representing the ‘actual’) to exceed it and thus for a school to operate with greater than intended/desired equity and were a convex notion assumed, it would imply that policy makers aspire to a greater proportion of grades coming from less able pupils. So if the $\bar{E}$ Index is to capture a reasonable view of attainment equity policy and if it is to define it in a reasonable way, then the assumption about linearity (made initially on the foot of policy statements) seems well-founded.

**Calculating the $\bar{E}$ Index for schools: some examples**

According to data from the National Pupil Database (NPD), School R has the following GCSE A*-C distribution for 2009 (using prior attainment at KS2 as benchmark):

- The bottom 20% (at KS2) obtained 8% GCSE grades A*-C
- The bottom 40% obtained 19% GCSE grades A*-C
- The bottom 60% obtained 40% GCSE grades A*-C
- The bottom 80% obtained 68% GCSE grades A*-C.

This is shown in Figure 5 as a piecewise linear function.
Using equation (β), with \( X_k - X_{k-1} = 0.2 \), the Attainment Equity Index for School R is:

\[
\mathcal{AE}_R = 1 - 0.2 \sum_{k=1}^{n} (Y_k + Y_{k-1})
\]

\[
= 1 - 0.2[(0.08 + 0) + (0.19 + 0.08) + \ldots + (1 + 0.68)]
\]

\[
= 0.260
\]

Is this \( \mathcal{AE} \) Index relatively small or large?

Well, a neighbouring school S had a similar but slightly different profile in the same year:

The bottom 20% (at KS2) obtained 7% GCSE grades A*-C
The bottom 40% obtained 20% GCSE grades A*-C
The bottom 60% obtained 38% GCSE grades A*-C
The bottom 80% obtained 71% GCSE grades A*-C

The \( \mathcal{AE} \) Index for School S (\( \mathcal{AE}_S \)) is 0.256, so the index is not disproportionately sensitive to small changes in the distribution.

Would it matter much if the cumulative intervals were calculated every 10% instead of every 20%? For School R, again from NPD data, the more fine-grained distribution was:
The bottom 10% (at KS2) obtained 3% GCSE grades A*–C
The bottom 20% obtained 8% GCSE grades A*–C
The bottom 30% obtained 12% GCSE grades A*–C
The bottom 40% obtained 19% GCSE grades A*–C
The bottom 50% obtained 33% GCSE grades A*–C
The bottom 60% obtained 40% GCSE grades A*–C
The bottom 70% obtained 50% GCSE grades A*–C
The bottom 80% obtained 68% GCSE grades A*–C
The bottom 90% obtained 86% GCSE grades A*–C

Using equation (β), with $X_k - X_{k-1} = 0.1$, the new $AE$ Index for School R is:

$$AE_R = 1 - 0.1 \sum_{k=1}^{n} (Y_k + Y_{k-1})$$

$$= 1 - 0.1[(0.03 + 0) + (0.08 + 0.03) + \ldots + (1 + 0.86)]$$

$$= 0.262$$

which is only a slight refinement on the original $AE_R$.

Like most measurements of this kind, the metric is lowered by lower ‘granularity’; that is to say, five 20% quantiles (say) will yield a slightly lower index than fifty 2% quantiles (say) taken from the same distribution.

**Calculating and comparing $AE$ indices for local authorities: an example**

It has been said that local authorities (LAs) in the UK today are no more than distributors and calculators of data, facilitating its flow between schools and central government (e.g. Ozga, 2009), but in many ways this means that they are well placed to demonstrate the value of indices like this because their data can be interrogated for within-LA and between-LA variation, while in the case of statistical neighbours, controlling for context.

Portsmouth (LA851) and Southampton (LA852) are ‘unitary’ city authorities on the south coast of England. They have much in common in terms of context and are very close statistical neighbours (Benton et al., 2007). NPD data show that for GCSE A*–C 2008/2009 with matched KS2 scores from 2003/2004:

- Portsmouth had 2215 pupils (1135 boys and 1080 girls) in 19 centres, which, excluding small (< 70 pupils) cohorts, reduced to 2007 pupils in 11 schools (see Table 1).
- Southampton had 2366 pupils (1142 boys and 1224 girls) in 18 centres, which, excluding small (< 70 pupils) cohorts, reduced to 2000 pupils in 11 schools (see Table 2).

Local authority-wide $AE$ indices were calculated for both LAs (across all centres) and then individually for each of the 22 schools. The results are shown in
Table 1. Attainment Equity Index for schools in Portsmouth LA in 2009

<table>
<thead>
<tr>
<th>School</th>
<th>( N_p )</th>
<th>% FSM</th>
<th>KS4 IDACI</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>( \beta ) Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>188</td>
<td>15</td>
<td>0.1974</td>
<td>0</td>
<td>5.90</td>
<td>17.26</td>
<td>37.37</td>
<td>63.82</td>
<td>100</td>
<td>0.303</td>
</tr>
<tr>
<td>B</td>
<td>237</td>
<td>18</td>
<td>0.2472</td>
<td>0</td>
<td>4.07</td>
<td>16.52</td>
<td>37.00</td>
<td>64.43</td>
<td>100</td>
<td>0.312</td>
</tr>
<tr>
<td>C(g)</td>
<td>198</td>
<td>22</td>
<td>0.3014</td>
<td>0</td>
<td>3.88</td>
<td>16.59</td>
<td>36.08</td>
<td>65.76</td>
<td>100</td>
<td>0.311</td>
</tr>
<tr>
<td>D</td>
<td>215</td>
<td>2</td>
<td>0.1063</td>
<td>0</td>
<td>8.96</td>
<td>24.20</td>
<td>44.05</td>
<td>69.65</td>
<td>100</td>
<td>0.213</td>
</tr>
<tr>
<td>E</td>
<td>186</td>
<td>28</td>
<td>0.3581</td>
<td>0</td>
<td>3.68</td>
<td>10.29</td>
<td>30.51</td>
<td>59.56</td>
<td>100</td>
<td>0.384</td>
</tr>
<tr>
<td>F</td>
<td>235</td>
<td>14</td>
<td>0.2292</td>
<td>0</td>
<td>4.66</td>
<td>10.52</td>
<td>22.59</td>
<td>50.69</td>
<td>100</td>
<td>0.446</td>
</tr>
<tr>
<td>G</td>
<td>198</td>
<td>9</td>
<td>0.1381</td>
<td>0</td>
<td>6.79</td>
<td>20.78</td>
<td>39.09</td>
<td>63.87</td>
<td>100</td>
<td>0.278</td>
</tr>
<tr>
<td>H</td>
<td>112</td>
<td>36</td>
<td>0.4543</td>
<td>0</td>
<td>6.40</td>
<td>15.20</td>
<td>30.40</td>
<td>56.00</td>
<td>100</td>
<td>0.368</td>
</tr>
<tr>
<td>I(b)</td>
<td>123</td>
<td>20</td>
<td>0.2780</td>
<td>0</td>
<td>5.36</td>
<td>15.82</td>
<td>34.44</td>
<td>59.44</td>
<td>100</td>
<td>0.340</td>
</tr>
<tr>
<td>J</td>
<td>168</td>
<td>14</td>
<td>0.2498</td>
<td>0</td>
<td>7.18</td>
<td>16.91</td>
<td>38.56</td>
<td>65.57</td>
<td>100</td>
<td>0.287</td>
</tr>
<tr>
<td>K</td>
<td>147</td>
<td>n/a</td>
<td>n/a</td>
<td>0</td>
<td>18.40</td>
<td>38.54</td>
<td>58.45</td>
<td>79.14</td>
<td>100</td>
<td>0.022</td>
</tr>
</tbody>
</table>

**Portsmouth LA**

851

<table>
<thead>
<tr>
<th>Overall% GCSE A*–C</th>
<th>0</th>
<th>5.30</th>
<th>16.70</th>
<th>35.97</th>
<th>64.19</th>
<th>100</th>
<th>0.311</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Overall% GCSE A*–G)</td>
<td>(0)</td>
<td>(13.6)</td>
<td>(31.5)</td>
<td>(51.9)</td>
<td>(75.1)</td>
<td>(100)</td>
<td>(0.112)</td>
</tr>
</tbody>
</table>

Notes: Eight schools with small GCSE cohorts have been removed; \( N_p \) = number of pupils with GCSE grades in 2008/9 and matched KS2 points in 2003/4; g/b = girls/boys only at time of census.
Table 2. Attainment Equity Index for schools in Southampton LA in 2009

<table>
<thead>
<tr>
<th>School</th>
<th>N&lt;sub&gt;p&lt;/sub&gt;</th>
<th>% FSM</th>
<th>KS4 IDACI</th>
<th>Cum.% by KS2 prior attainment</th>
<th>2008/9%GCSE A&lt;sup&gt;+&lt;/sup&gt;–C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>235</td>
<td>15</td>
<td>0.2291</td>
<td>0</td>
<td>1.57</td>
</tr>
<tr>
<td>B</td>
<td>193</td>
<td>25</td>
<td>0.3036</td>
<td>0</td>
<td>6.16</td>
</tr>
<tr>
<td>C</td>
<td>120</td>
<td>n/a</td>
<td>n/a</td>
<td>0</td>
<td>19.71</td>
</tr>
<tr>
<td>D(g)</td>
<td>180</td>
<td>13</td>
<td>0.1966</td>
<td>0</td>
<td>8.93</td>
</tr>
<tr>
<td>E</td>
<td>191</td>
<td>24</td>
<td>0.3062</td>
<td>0</td>
<td>2.63</td>
</tr>
<tr>
<td>F</td>
<td>153</td>
<td>37</td>
<td>0.3376</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>G</td>
<td>263</td>
<td>12</td>
<td>0.2095</td>
<td>0</td>
<td>5.31</td>
</tr>
<tr>
<td>H(b)</td>
<td>114</td>
<td>17</td>
<td>0.2688</td>
<td>0</td>
<td>4.31</td>
</tr>
<tr>
<td>I</td>
<td>162</td>
<td>26</td>
<td>0.3157</td>
<td>0</td>
<td>0.72</td>
</tr>
<tr>
<td>J(g)</td>
<td>201</td>
<td>15</td>
<td>0.2267</td>
<td>0</td>
<td>4.45</td>
</tr>
<tr>
<td>K(g)</td>
<td>188</td>
<td>20</td>
<td>0.2623</td>
<td>0</td>
<td>3.35</td>
</tr>
</tbody>
</table>

Southampton
LA 852

| Overall% GCSE A<sup>+</sup>–C | 0   | 2.86 | 11.08 | 29.01 | 59.45 | 100 | 0.390 |
| (Overall% GCSE A<sup>+</sup>–G) | (0) | (13.0) | (28.6) | (48.0) | (71.9) | (100) | (0.159) |

Notes: Seven schools with small GCSE cohorts have been removed; N<sub>p</sub> = number of pupils with GCSE grades in 2008/9 and matched KS2 points in 2003/2004; g/b = girls/boys only at time of census.
Table 1 and Table 2 alongside the raw percentage data and two measures of socioeconomic deprivation (KS4 IDACI\textsuperscript{13} [averaged for the school] and the percentage of pupils eligible for and claiming free school meals [FSM]), which are known to characterise low attainment (DfES, 2007).

In Portsmouth, one school was boys-only and one girls-only; in Southampton, one school was boys-only and three were girls-only. The $\AE$ indices were lower for girls-only schools than for boys-only schools in the same LA (see next section). In the other 16 schools, the proportion of boys to girls was fairly consistent, though not surprisingly the seven mixed schools in Southampton had a higher proportion of boys than the mixed schools in Portsmouth.\textsuperscript{14}

What is perhaps most significant is the difference between $\AE_{\text{Ports}}$ and $\AE_{\text{Soton}}$ (0.311 and 0.390, respectively). Given that the two authorities are so similar in intake characteristics, it suggests that GCSE attainment in Southampton is less equitable across the range of prior attainment at KS2 than Portsmouth. It is tempting to claim that the difference might have been even greater but for the presence of more girls-only schools in Southampton, but this would be to ignore the opposite effect of the higher proportion of boys in the remaining mixed schools.

Table 3 (which also shows the raw percentile data) and Figure 6 show the five-year trends for the two local authorities. They are very similar up to 2007/2008,\textsuperscript{15} after which time attainment equity in Portsmouth improved and attainment equity in Southampton worsened (c.f. Appendix: Note 3 for a back-story to secondary school reorganisation in Southampton at this time), but of course what is missing from this analysis is context and that is what we will now address.

**Taking account of context: teasing out the difficulties with adjustments**

At first glance it might seem a simple matter to develop a ‘contextual $\AE$ Index’ that itself takes account of school-level factors like gender balance, percentage ethnic mix, whether or not students speak English as an additional language (EAL), level of parental education, percentage Special Educational Need (%SEN) and various proxy variables at school level that reflect family income and social circumstance, like the percentage of pupils entitled to free school meals and IDACI. These are all factors for which ‘schools should to some extent be responsible’ (Ray, 2006, p. 21), though research findings in relation to the relative size of their impact is mixed (e.g. Milne et al., 1986; Cherlin et al., 1991; Kiernan, 1992; Downey, 1994; Thomas & Mortimore, 1996; Davis-Kean, 2005; Duckworth & Sabates, 2005; Chiu & Ho, 2006) and UK government policy has recently (November 2010) turned against including them because they are thought to lower expectations and encourage stereotyping:

We will put an end to the current ‘contextual value added’ (CVA) measure. This measure attempts to quantify how well a school does with its pupil population compared to pupils with similar characteristics nationally. However, the measure is difficult for the
### Table 3. Attainment Equity Index five-year trend for Portsmouth and Southampton in 2009

<table>
<thead>
<tr>
<th>Year</th>
<th>Np</th>
<th>0</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>( \mathcal{E} ) Index</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portsmouth LA 851</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2004–5</td>
<td>1932</td>
<td>0</td>
<td>8.74</td>
<td>27.96</td>
<td>40.87</td>
<td>67.90</td>
<td>100</td>
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<tr>
<td>2005–6</td>
<td>1990</td>
<td>0</td>
<td>2.43</td>
<td>10.37</td>
<td>32.07</td>
<td>58.92</td>
<td>100</td>
<td>0.385</td>
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<tr>
<td>2006–7</td>
<td>1853</td>
<td>0</td>
<td>8.03</td>
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<td>42.7</td>
<td>67.83</td>
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<td>2007–8</td>
<td>1856</td>
<td>0</td>
<td>3.53</td>
<td>8.62</td>
<td>34.66</td>
<td>63.19</td>
<td>100</td>
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<td>0</td>
<td>5.30</td>
<td>16.7</td>
<td>35.97</td>
<td>64.19</td>
<td>100</td>
<td>0.311</td>
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<tr>
<td><strong>Southampton LA 852</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2004–5</td>
<td>2319</td>
<td>0</td>
<td>6.64</td>
<td>23.31</td>
<td>37.97</td>
<td>65.35</td>
<td>100</td>
<td>0.267</td>
</tr>
<tr>
<td>2005–6</td>
<td>2243</td>
<td>0</td>
<td>3.48</td>
<td>11.94</td>
<td>31.66</td>
<td>60.71</td>
<td>100</td>
<td>0.369</td>
</tr>
<tr>
<td>2006–7</td>
<td>1848</td>
<td>0</td>
<td>7.35</td>
<td>21.25</td>
<td>40.16</td>
<td>66.34</td>
<td>100</td>
<td>0.260</td>
</tr>
<tr>
<td>2007–8</td>
<td>1946</td>
<td>0</td>
<td>5.11</td>
<td>14.25</td>
<td>34.97</td>
<td>62.56</td>
<td>100</td>
<td>0.332</td>
</tr>
<tr>
<td>2008–9</td>
<td>2366</td>
<td>0</td>
<td>2.86</td>
<td>11.08</td>
<td>29.01</td>
<td>59.45</td>
<td>100</td>
<td>0.390</td>
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</tbody>
</table>

Note: \( N_p \) = number of pupils (in all schools) with matching GCSE and KS2 data.
public to understand, and...has the effect of expecting different levels of progress from different groups of pupils on the basis of their ethnic background, or family circumstances, which we think is wrong in principle.’ (DfE, 2010, p. 68)

Notwithstanding their current unpopularity among policy makers, educational effectiveness research suggests that contextual measures are methodologically sound and practically important and whether or not they are used publicly by government in its accountability metrics, all the relevant data will remain available to schools and local authorities to reflect their own circumstances as they see fit, especially as the government is simultaneously committed to giving schools ‘greater confidence to set their own direction’ (DfE, 2010, pp. 31, 52) and local authorities the freedom ‘to provide whatever forms of support they choose’ (p. 14).

When making comparisons across similar contexts (as with statistical neighbours) or within the same context over time, it is relatively easy to avoid making adjustments, but for schools with disproportionate numbers of students from disadvantaged groups, the conceptual problem is how to incorporate factors like %FSM and %EAL into the Index post facto when academics are unable to quantify from research the precise percentage impact of these variables at school-level, though qualitatively they are known to have a negative effect, especially for pupils with low prior attainment (e.g. Edmonds, 1979; Thomas & Mortimore, 1996; Sammons & Smees, 1998; Fischer Family Trust, 2005; Palardy, 2008). One way forward might be to make an ‘elbow’ in the line of intended equity (like that shown in Figure 3, say) to reduce area $A$ at the low attainment end.
By way of example, consider adjusting the $AE$ Index for $\%\text{FSM}$. Let $F = \text{FSM}\% / 100$ and suppose that the ‘elbow’ reduces area $A$ to $A_F$ by the same percentage as $\%\text{FSM}$. Then:

$$A_F = (1 - F)A$$

and

$$AE_{\text{FSM}} = A_F/(A_F + B) = A(1 - F) \left[ A(1 - F) + B \right]$$

but $A = 0.5 - B$, so

$$AE_{\text{FSM}} = (1 - 2B)(1 - F)/[1 - (1 - 2B)F]$$

and $AE = 1 - 2B$, so

$$AE_{\text{FSM}} = (1 - F)AE/[1 - FAE] \quad (\gamma)$$

So if FSM\% = 10\% in a school and $AE = 0.26$, as was the case for School R, the $AE$ Index contextualised for 10\% FSM works out as 0.24, a reduction of about 8\% and, if $AE$ is adjusted for 30\% FSM, it is reduced by about 23\%.

It is worth noting that when $F = 1$, $AE = 0$ and there is no inequity. This means in effect that the area $A$ between the policy benchmark/ideal and the actual has been reduced to zero, not by the Lorenz lifting up to the line $y = x$, but by the line elbowing down to the curve. And this is not an unreasonable thing to happen because if every student were entitled to FSM there is no reason why a school should not aspire to attainment being spread just as equally over all its quantiles as would be the case were none of its pupils entitled to FSM.

Figure 7 shows a range of ‘adjustment curves’ for different percentages of a variable conceived as a percentage reduction in area $A$ (e.g. $\%\text{FSM}$, $\%\text{EAL}$, $\%\text{SEN}$, etc). Essentially, they are graphs of equation $(\gamma)$ for different values of the $AE$ Index. Such a set of curves could form a useful tool for school and LA managers as they estimate the likely impact on attainment equity of various targeted interventions or changing demographics.

Clearly, when $F = 0$, equation $(\gamma)$ reduces $AE_{\text{FSM}}$ to $AE$ as it should do. And it is right that the adjustment for $\%\text{FSM}$ lowers the original index to reflect the fact that the presumptive aspiration that attainment equity depends on equal attainment across all quantiles is relaxed a little; that it to say, it is (and should be) easier for a school ceteris paribus to do well in terms of the spread of achievement if it has a high $\%\text{FSM}$.

The same set of curves and the same equation $(\gamma)$ can also be adapted to gender balance. Since we know that girls outperform boys at GCSE, it is consistent with the approach used above to consider a reduction in area $A$ (and thus in the index) for schools with a higher proportion of boys (B\%) and an increase for schools with a higher proportion of girls (G\%).
If \( S = (B\% - G\%) / 100 \), the adjusted index becomes:

\[
AE_{\text{Gend}} = (1 - S)AE / [1 - SAE]
\]  
(γ1)

which properly reduces to \( AE \) when the boy/girl mix is equal. When \( B > G \) (\( S > 0 \)), \( AE_{\text{Gend}} < AE \) (i.e., area \( A \) has been decreased) and when \( G > B \) (\( S < 0 \)), \( AE_{\text{Gend}} > AE \) (i.e., area \( A \) has been increased) as it should do, which is consistent with the findings from the two local authorities in the previous section.

This all seems reasonable, but the contextual approach has several flaws. Firstly, equation (γ) produces the same reduction in \( AE \) for any given percentage FSM, EAL, SEN or gender difference, say, though it is unlikely that they all have the same impact. Secondly, there is no theoretical warrant for iterating two or more consecutive adjustments—for example, by adjusting the \( AE \) Index first to take account of FSM\%, say, (thus producing \( AE_{\text{FSM}} \)) and then subsequently adjusting \( AE_{\text{FSM}} \) to take account of gender, say (to produce \( AE_{\text{FSM-Gend}} \))—and the alternative, making a series of independent adjustments and then summing them all to get a single collective adjustment, runs the risk of reducing the index to zero for no good reason or of compounding errors where they exist.

There are some modifications that could be made to overcome (partially) these objections. Firstly, since socioeconomic status is known to be the most significant
determinant of academic attainment after ‘prior attainment’, which has already been catered for in the index via the quantile calculations, it could be argued than a one-off socioeconomic adjustment along the lines described above is justified and just leave it at that. Alternatively, we could avoid post facto adjustments to the index altogether, but restrict its use to comparisons between statistical neighbours (to control for the adjusting variables) and comparisons between the same schools or groups of schools over time where contexts have remained broadly the same. A third modification, which the author favours above the other two, is to combine the $\mathcal{AE}$ Index (which is output-focused) and existing CVA measures (which are process-focused) to categorise schools and local authorities in terms of both attainment equity and context (see Figure 8). In effect, this approach redefines ‘equity’ more comprehensively as ‘equitability’.$^{18}$ Since context includes measures of disadvantage and equitability reflects opportunity, this ‘$\mathcal{AE}$-CVA measure’ goes a long way towards providing a metric for capturing policy statements like those from the EU cited at the start of the paper (EU, 2006, p. 2). It is also arguably a more justified and acceptable metric for holding teachers accountable to central government for the delivery of education.

Schools with a low $\mathcal{AE}$ Index and high CVA are adding value and doing so across the range of pupil ability and background characteristics (‘High Equitability Schools’$^{19}$); schools with a high $\mathcal{AE}$ Index and high CVA are adding value but not across the range of ability and background and are by definition ‘Differentially Effective Schools’; schools with a high $\mathcal{AE}$ Index and low CVA are failing to add value (and therefore not raising pupils’ capability) and are inequitable in attainment terms, so can be termed ‘Low Equitability Schools’; and schools with a low

![Figure 8. Combining the Attainment Equity Index with CVA (KS2-4) measures to define equity in terms of outcomes and context](image-url)
AE Index and low CVA are ‘Uniformly Ineffective’. Schools close to the centre of Figure 8 are ‘average’ in terms of CVA and equitability, so we should expect some clustering in this region and apply the usual confidence restrictions to avoid over-differentiating schools on this basis.

Table 4 shows the 2009 AE Indices for Portsmouth and Southampton and the CVA measures for KS2–4 for the same year. The CVA is almost identical for the two local authorities (averaging the 10 schools for which CVA data were publicly available).

Figure 9 shows the ‘AE-CVA’ plot. Looking at it against the horizontal axis it is clear that the AE indices for Southampton schools are more strongly linked to deprivation (higher IDACI and higher %FSM) than is the case with Portsmouth schools, but Figure 9 tells us something that neither the CVA measures (which were almost identical) nor the respective AE Indices (which suggest that Southampton was less equitable in terms of attainment) tell us: namely, that Portsmouth schools are more attainment-equitable only because so many of its schools are ‘Uniformly Ineffective’. And it is also apparent that Southampton LA is more polarised in terms of the ‘equitability’ of its schools, having approximately half in the ‘high equitability’ category and half in the ‘low equitability’ category.

**Conclusion and suggestions for improving the Index**

Equity has been a fundamental concern for school effectiveness research (SER) since its inception, but in this respect the field has largely focused on the moral imperative of improving the process of education and the extent to which it can attenuate the impact of disadvantage (Sammons, 2007, p. 59; 2010). Attempts to define equity have conflated it with equality and notions of social justice, inclusion, access and participation (Sammons, 2007, p. 6), so in this sense SER has traditionally viewed equity in terms of the educative process and in terms of equality of opportunity for different groups, without necessarily trying to equate it, as policymakers have done, to the spread of attainment in schools and local authorities. The purpose of the AE Index therefore is not to substitute for the SER approach to differential effectiveness (Goldstein et al., 1993) or to replace existing measures, but rather to augment them (as with the ‘AE-CVA measure’ above) as part of a suite of situated metrics for the school or local authority.

The AE Index gauges the extent to which a school, family of schools, local authority or education system meets one aspect of the ‘fairness’ to which policymakers from many countries and organisations have alluded. It uses ‘raw’ examination data, albeit adjusted for prior attainment, which, in the UK at least, is certainly in line with government thinking. It is true that the Index on its own ignores the variables that effectiveness research seeks to control, but unlike other metrics, the Index as a single measure does not seek to isolate the ‘school effect’ beyond catering a priori for the most important factor: prior attainment. The Index has several advantages over other measures: it can be used in combination with complementary measures like CVA; it is based ab initio on prior attainment, which school effectiveness research tells us is the main factor in subsequent
Table 4. Attainment Equity Index Indices and CVA measures for Portsmouth and Southampton in 2009

<table>
<thead>
<tr>
<th>School</th>
<th>Æ Index</th>
<th>CVA (KS2–4)*</th>
<th>Æ Index</th>
<th>CVA (KS2–4)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.303</td>
<td>965.6</td>
<td>0.470</td>
<td>944.3</td>
</tr>
<tr>
<td>B</td>
<td>0.312</td>
<td>989.8</td>
<td>0.382</td>
<td>1001.6</td>
</tr>
<tr>
<td>C</td>
<td>0.311 (g)</td>
<td>995.4</td>
<td>0.011</td>
<td>not available</td>
</tr>
<tr>
<td>D</td>
<td>0.213</td>
<td>1021.4</td>
<td>0.157 (g)</td>
<td>1001.4</td>
</tr>
<tr>
<td>E</td>
<td>0.384</td>
<td>1007.9</td>
<td>0.394</td>
<td>986.7</td>
</tr>
<tr>
<td>F</td>
<td>0.446</td>
<td>992.7</td>
<td>0.501</td>
<td>987.1</td>
</tr>
<tr>
<td>G</td>
<td>0.278</td>
<td>980.7</td>
<td>0.296</td>
<td>1006.9</td>
</tr>
<tr>
<td>H</td>
<td>0.368</td>
<td>983.2</td>
<td>0.399 (b)</td>
<td>1011.7</td>
</tr>
<tr>
<td>I</td>
<td>0.340 (b)</td>
<td>992.0</td>
<td>0.454</td>
<td>975.4</td>
</tr>
<tr>
<td>J</td>
<td>0.287</td>
<td>975.5</td>
<td>0.300 (g)</td>
<td>1006.1</td>
</tr>
<tr>
<td>K</td>
<td>0.022</td>
<td>not available</td>
<td>0.349 (g)</td>
<td>1003.0</td>
</tr>
</tbody>
</table>

LA | 0.311** | Mean = 990.4 | 0.390** | Mean = 992.4 |

Notes: *mid-interval values; **this is not an average but is calculated separately; g/b = girls/boys only at time of census; CVA = contextual value added.
attainment; it is a ratio analysis that allows suggestive (rather than definitive) comparisons to be made (e.g. Goldstein & Spiegelhalter, 1996; Knapp et al., 2006; Shen & Cooley, 2008; Wohlstetter et al., 2008); a ‘transfer principle’ applies to the Index (that is to say, if equity is transferred from a pupil or group of pupils with a lot of it, to a pupil or group of pupils with a shortage of it, the resulting distribution is more equal), which is not the case with other school effectiveness metrics like CVA; it is easily interpreted, can track changes over time and can prompt school improvement; and it can give feedback directly to national governments, local authorities and schools, which has been a feature of good school effectiveness research over the years (e.g. Sammons et al. 1997; Van Damme et al. 2002; Thomas et al. 2007). This is not to suggest that the Index does not also have its disadvantages: it measures equity, but on its own does not measure opportunity, capability or wider aspects of social injustice; and like other school effectiveness measures, indices for different sub-populations cannot be averaged to obtain an index for the whole population.

Comparing schools is always difficult because they do more than just ‘attain’ in examinations: they encourage a range of intellectual, sporting and cultural interests among young people, enable friendships and develop the ability to interact socially. As with all school effectiveness measures, if these desirable commodities are not counted in the input (the Lorenz curve) they cannot be reflected in the output (the Index). The Index cannot speak for itself. Turning it into knowledge is what gives it value, but this depends on a series of prior decisions about what has been measured and how it has been measured, decisions that necessarily involve value judgements as well as technical ones. Schools benefit from having good quality easily-interpreted diagnostics (e.g. Gray et al., 1990; Schagen, 1991; Mortimore

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**Figure 9.** Distribution (around a two-LA average) of Portsmouth (P) and Southampton (S) schools in 2009 (from Table 4)
et al., 1994; Jesson, 1996) and indices like the one proposed here, alone or in conjunction with existing measures like school CVA, can act as a locus for conversations around the use of data (e.g. Demie, 2003; Timperley, 2005) and bring about a culture more supportive of system-wide improvement, as all policy makers intend.

Acknowledgements

I would like to thank Pam Sammons and Daniel Muijs for their helpful discussions.

Notes

1. The term is not uncontested and is explored in the paper.
2. The ‘General Certificate in Secondary Education’ is the examination taken by nearly all 16-year-old pupils in England and Wales after (typically) five years of secondary schooling.
3. Compulsory schooling in England is divided into ‘Key Stages’: KS1 (Years 1 and 2) for ages 5–7; KS2 (Years 3–6) for ages 7–11; KS3 (Years 7–9) for ages 11–14; KS4 (Years 10 and 11) for ages 14–16.
4. GCSE data in the UK is available in different ‘bundles: the percentage of grades A–C across all subjects; the percentage of grades A–C including English and mathematics; the (‘capped’) percentage of grades A–C in pupils’ best eight subjects; and so forth. In this paper, we are using the first of these (viz. the simple percentage of grades A–C obtained across all subjects) because it is the metric most frequently valued by UK policy-makers in measuring rates of progression to post-compulsory and higher education, themselves held to be measures of equity in the system (e.g. HEFCE, 2005, 2010; Access 2008; Thompson 2010).
5. In particular, Rawls’s Second Principle of Justice—that socioeconomic inequalities should be arranged so that they are of the greatest benefit to the least advantaged members of society—the basis of which is the view that ‘arbitrary factors’ like innate intellectual ability and the benefits that derive from it, should not determine life-chances or opportunities. Critiques of the Rawlsian view by Nozick (1974), Sandel (1982) and others would not negate this interpretation of Rawls’s theory in relation to educational attainment.
6. The trapezoids in question each have area \( \frac{1}{2}(X_k - X_{k-1})(Y_k + Y_{k-1}) \).
7. In the case of the quadratic function \( y^2 = x \), area \( A+B = \int_0^1 y \, dx = 2/3 \).
8. The problem is analogous to the system of scoring value-added in the UK, where a negative result would be possible but for the fact that 100 is added to KS1–KS2 scores and 1000 is added to KS2–KS4 scores.
9. A total of 9.9% of pupil entries had incomplete data with regard to either GCSE or KS2 attainment. These were removed from the dataset.
10. The Index could be adapted to incorporate shrinkage measures (e.g. Kreft, 1996; DfES/Ofsted, 2004; Thomson, 2007) for small cohorts, which would cause their data to be moved closer to the mean and make it less likely that extreme scores were recorded.
11. It would have been possible to calculate \( \delta \) indices for hundreds of schools and local authorities in the NPD, but at such an early stage of developing and trialling the concept, it was more important to test for ‘face validity’, to which end the concepts and findings were presented to managers and headteachers from the LA/schools in question, who confirmed very strongly their value. This would not have been possible with a large number of schools/authorities.
12. The indices are given correct to three significant figures, but there is no suggestion that the measure is representative of that degree of precision. Local authority-wide indices were also calculated using GCSE A*–G (shown on the Tables), which, as expected, were lower in both authorities since the spread of lower-grade attainment was proportionately more attributable to low-attaining pupils.

13. The Income Deprivation Affecting Children Index (IDACI) is based on student home postcodes and is essentially a measure of the proportion of children under 16 living in families in receipt of at least one of a specified range of social welfare measures, including income support, job seeker’s allowance, working families’ tax credit and disabled person’s tax credit. It is closely correlated to %FSM. The IDACI ranges in value from 0 to 1, with a national average of approximately 0.14 (National Statistics, 2006).

14. The IDACI indices were predictably lowest for the two schools with a selective element: school ‘C’ in Southampton and school ‘K’ in Portsmouth.

15. The peaks in both trend graphs in 2005–2006 may reflect changes in GCSE qualification ‘equivalency’ kicking in at that time, which is itself interesting as it shows a worsening of the situation in equity terms between high- and low-(prior) attaining pupils.

16. But what follows could apply just as easily to %EAL or %SEN or to any ‘percentagized’ impact factor.

17. It is possible to develop adjustments with ceilings, although this would mean departing from the linearity of the ‘ideal’ and from the notion of an index warranted by school effects research.

18. Equity’ (noun): fairness; ‘Equitable’ (adjective): having equity; ‘Equitability’ (noun): in this sense, the capability of individuals to benefit from the unbiased distribution of equity.

19. The phrase ‘high equity school’ used in a positive sense, as is usually the case, is surely misleading, as a uniformly ineffective school failing all its pupils equally is hardly worthy of praise. The mistake is to confuse ‘sharing’ with ‘sharing success’.

20. See reports from the London Challenge (DfES, 2003, 2006a, 2006b; Ofsted 2010), though it is striking that there are no references in any of these reports to equity, equitability or equality.

References
Daily Echo (2008a, 21, 23 October) Head of troubled school defends decisions made, Daily Echo, p. 2.
Daily Echo (2008b, 8 October) Ofsted inspectors called into Oasis Academy Mayfield, Daily Echo, p. 2.


Appendix

Note 1

The Pareto Principle (also known as ‘the 80-20 rule’) states that, for many natural situations, roughly 80% of effects come from 20% of causes. Joseph Juran originally suggested the principle and named it after the Italian economist Vilfredo Pareto, who had observed in 1906 that 80% of the land in Italy was owned by 20% of the population...and that 20% of the pea pods in his garden contained 80% of the peas! It is actually not as weird as it seems. Mathematically, when something is shared across a large population, there must be some number \(n\) between 50 and 100 such that \(n\)% is taken by \(100 - n\)% of the population. There is nothing special mathematically about the ‘80%’, but many ‘natural’ systems seem to have \(n\) somewhere in this region.

Note 2

For a cumulative distribution function with mean \(\mu\), the tangent to the Lorenz curve is parallel to the line \(y = x\) when \(x = \mu\) and this is the point where the equality gap, the vertical distance between the curve and the line, is greatest.

Note 3

At the start of that school year, five of Southampton’s 14 secondary schools were graded in the lowest ‘satisfactory’ category by Ofsted (the UK schools inspectorate), with two schools just coming out of Special Measures (i.e., having been assessed as failing). The previous year, the LA had carried out a review of its schooling provision and decided, with effect from August 2008, to close four secondary schools and establish two new independent Academies. Despite the consultation exercise, the restructuring was controversial (e.g. Daily Echo, 2008a,
Initially, the plan was simply to reopen 2 of the 4 closed schools to allow greater staffing flexibility across the surviving 12, but the LA underestimated the interest that an open competition to run the new Academies would generate so that it then had to decide whether to compete for the new schools or to withdraw and adjudicate on the competition. Eventually, in the midst of local political upheaval—there was a change in the overall control of the council from Liberal Democrat to Conservative—the LA decided not to bid and awarded both academies to ‘Oasis Community Learning’, a faith-based registered charity. The decision was called in by the government’s Scrutiny Panel, but was subsequently confirmed. Two of the four closed schools were among the five worst-performing schools in the city, but the other two were not. There was (and remains) considerable disgruntlement in the city as a result of the reorganisation, which has not been helped by the fact that the two new Academies have ‘poached’ some of the most successful staff (including heads) from the remaining ten schools. Of course, none of this proves causality with the higher $\mathcal{A}E$ Index from that time, but it provides important ‘context’ and suggests an area for further investigation, especially with regard to the attainment of pupils entitled to free school meals in the Academies compared to those in the other schools.