THE EFFECT OF CUBIC DAMPING IN AN AUTOMOTIVE VEHICLE SUSPENSION MODEL

Pananda N[1], Ferguson N S and Waters T P
ISVR, University of Southampton
np2g09@soton.ac.uk[1]

ABSTRACT

A two degree of freedom quarter-car model comprising a linear suspension spring in parallel with a non-linear damper has been investigated. The tyre damping is assumed to be negligible compared to the suspension damper and the tyre stiffness is represented by a linear stiffness. A cubic damping characteristic is of interest as an alternative to the viscous linear damping normally assumed. The system analysed is assumed to be under the steady-state sinusoidal base input. The analytical solutions obtained using the Harmonic Balance Method (HBM) have been validated with direct numerical integration.

To facilitate a comparison with the linear system, the cubic damping coefficient is chosen such that the response level of the bounce mode resonance is approximately the same in each case. The effect of the cubic damping can then be easily distinguished for the frequencies above the bounce mode. The displacement transmissibility for base input at excitation frequencies above the bounce mode is much higher than the linear case.

To this end, one can conclude that the cubic damping is not preferable for a base excited isolation system including a vehicle suspension system. However, to reduce the transmitted force to the supporting ground for an excited isolated mass, then cubic damping in an isolation system appears to be better than linear damping.

1 INTRODUCTION

The suspension system is one of the crucial systems for automotive vehicles. One of the important roles of the suspension system is to attenuate the vibration transmitted from the wheel-road interface to the car body. Damping is required to limit the resonant response of the vehicle vibrating rigidly on the suspension springs. A linear viscous damping model is commonly assumed to represent an automotive fluid damper and this is adequate for the purpose, for example, of illustrating the detrimental effect that damping can have in the isolation frequency region. One consequence of the linear model is that motion and force transmissibility are identical. An automotive damper is inherently non-linear due to the fluid orifice damping mechanism, and further complicated by the design of variable orifice valves to tune damper behaviour. The transmissibility of force and motion can therefore be expected to differ with associated implications for the efficacy of non-linear damping in components such as road suspensions and engine mounts.

Fluid dampers are not only nonlinear but intentionally asymmetric, i.e. possess different characteristics in jounce and rebound directions. Surace et al [1], and Cui et al [2] report asymmetric curves with a discontinuity owing to the friction characteristics when the relative velocity is equal to zero.

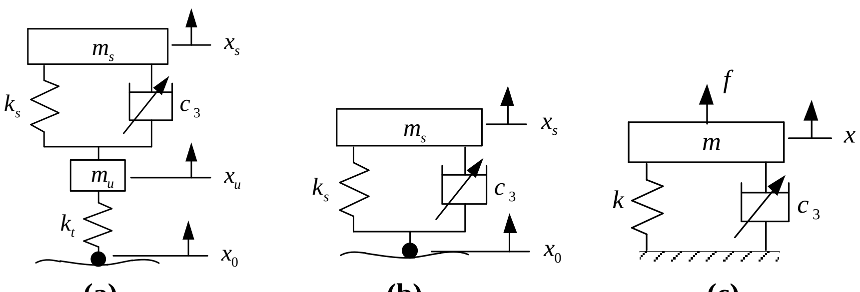
This paper investigates the physical effects of damping non-linearity by adopting a cubic damping model. There have been a number of similar previous studies. Milovanovic et al [3] and Kovacic et al [4] investigated the effect of cubic damping on the SDOF base excited isolation system using the method of averaging. The absolute displacement transmissibility for such a system was plotted in comparison to that for linear damping. The plots showed that the transmissibility of the absolute displacement tends towards unity as the excitation frequency tends to infinity. From these studies, one can conclude that using cubic damping is not preferable for the base excited isolation system. Shekhar et al [5] studied the effect of cubic damping in combination with linear damping for shock isolation for three different input shapes. The overall results reveal that cubic damping is detrimental to the system response.

However, the recent study of Peng et al [6] showed that the combination of linear viscous damping and anti-symmetric damping, including third and fifth powers of the velocity, can reduce the force transmissibility for the force excited isolation system. The comparison was made by keeping the value of linear damping ratio constant and varying the values of the non-linear damping terms, i.e. cubic and quintic terms. The results for the system with non-linear damping terms showed that the response around the resonance peak was significantly reduced with no change in the isolation region.

This paper presents the results obtained from closed form solutions using the Harmonic Balance method for both a single and two degree of freedom system with cubic damping. The reason for the stark difference between the performance of the base and force excited systems is discussed.

2 SYSTEM MODELS AND EQUATIONS OF MOTION

In this study, three models as shown in Fig. 1 are examined, i.e. a 2-DOF quarter-car model, a SDOF quarter-car model (SDOF base excited isolation model) and a SDOF force excited isolation model. The first model is used for investigation into the effect of cubic damping on the responses of the sprung and unsprung masses. By extension, the second model is subsequently examined to discover the influence of the cubic damping on the isolated mass (sprung mass) for which the response of unsprung mass is eliminated. In addition the influence of cubic damping on the response for the force excitation case is examined as the comparison to that of the base excitation.



(a) **(b)** **(c)**

Fig. 1: Lumped parameter models for (a) the 2-DOF quarter-car model configuration, (b) the SDOF quarter-car which is represented by a SDOF base excited isolation system and (c) the SDOF force excited isolation system.

2.1 A 2-DOF quarter-car model

A 2-DOF quarter-car model used in this study is shown in Fig. 1(a) for which the tyre damping is negligible compared to the suspension damper. Both suspension stiffness k_s and tyre stiffness k_t are represented by linear springs and are assumed to be constant. Parameters m_s, m_u and c_3 represent sprung and unsprung masses and cubic damping coefficient respectively. The equation of motion for the model shown in Fig. 1 (a) is given by

$$m_s\ddot{x}_s + c_3\left(\dot{x}_s - \dot{x}_u\right)^3 + k_s\left(x_s - x_u\right) = 0 \tag{1}$$

and

$$m_u\ddot{x}_u + c_3\left(\dot{x}_u - \dot{x}_s\right)^3 + \left(k_s + k_t\right)x_u - k_s x_s = k_t x_0 \tag{2}$$

Equations (1) and (2) can be written in non-dimensional form as

$$w_s'' + \zeta_3\left(w_s' - w_u'\right)^3 + \left(w_s - w_u\right) = 0 \tag{3}$$

and

$$w_u'' + M_r\zeta_3\left(w_u' - w_s'\right)^3 + \Omega_r^2 w_u - M_r w_s = M_r K_r w_0 \tag{4}$$

where the following substitutions have been made, i.e.

$$\Omega_r = \frac{\omega_u}{\omega_s},\ M_r = \frac{m_s}{m_u},\ K_r = \frac{k_t}{k_s},\ w_s = \frac{x_s}{X_0},\ w_u = \frac{x_u}{X_0},\ w_0 = \frac{x_0}{X_0}, \text{ and}$$

$(\ )'$ and $(\ )''$ are the first and second derivatives with respect to the non-dimensional time τ, which is given by $\tau = \omega_s t$

X_0 is the magnitude of base displacement input

ω_s and ω_u are the sprung and unsprung mass natural frequencies for the assumed two separate single degree of freedom systems and are given by

$$\omega_s = \sqrt{\frac{k_s}{m_s}} \text{ and } \omega_u = \sqrt{\frac{k_s + k_t}{m_u}}$$

The cubic damping term is given by

$$\zeta_3 = \frac{c_3}{\sqrt{\left(m_s k_s\right)^3}}\left(k_s X_0\right)^2 \tag{5}$$

Note that this depends upon both the coefficient for the cubic damping term and the base input amplitude.

2.2 A SDOF quarter-car model

To gain further insight into the effect of cubic damping, the tyre stiffness k_t is assumed to be infinitely stiff compared to suspension spring. Consequently the unsprung mass can be omitted. Then the model is simplified to be a SDOF quarter-car model or a SDOF base excited isolation system as shown in Fig. 1(b). The equation of motion for such a model is given by

$$m_s\ddot{x}_s + c_3\left(\dot{x}_s - \dot{x}_0\right)^3 + k_s\left(x_s - x_0\right) = 0 \tag{6}$$

However it is more convenient to consider the relative motion between the isolated mass and the base input, i.e. $z = x_s - x_0$. Then equation (6) becomes

$$m_s\ddot{z} + c_3\dot{z}^3 + k_s z = -m_s\ddot{x}_0 \tag{7}$$

Equation (7) can be written in non-dimensional form as

$$u'' + \zeta_3\left(u'\right)^3 + u = -w_0'' \tag{8}$$

where $u = \dfrac{z}{X_0}$.

The definition of cubic damping term for the SDOF base excited isolation is identical to that for the 2-DOF quarter-car model, i.e.

$$\zeta_3 = \frac{c_3}{\sqrt{\left(m_s k_s\right)^3}}\left(k_s X_0\right)^2 \tag{9}$$

2.3 A SDOF force excited isolation model

The response for the force excited isolation model is also examined, in order to compare the effect of cubic damping to that for the base excitation. The model is as shown in Fig. 1 (c), for which the equation of motion and equation of transmitted force are respectively given by

$$m\ddot{x} + c_3\dot{x}^3 + kx = f\left(t\right) \tag{10}$$

and

$$f_T\left(t\right) = c_3\dot{x}^3 + kx \tag{11}$$

The non-dimensional forms of these equations can be written as

$$w'' + \tilde{\zeta}_3\left(w'\right)^3 + w = p\left(\tau\right) \tag{12}$$

and

$$p_T\left(\tau\right) = \tilde{\zeta}_3\left(w'\right)^3 + w \tag{13}$$

where $w = \dfrac{x}{X_f}$, $p\left(\tau\right) = \dfrac{1}{m\omega_n^2 X_f} f\left(\frac{\tau}{\omega_n}\right)$ and $p_T\left(\tau\right) = \dfrac{1}{m\omega_n^2 X_f} f_T\left(\frac{\tau}{\omega_n}\right)$

with ω_n the natural frequency of the undamped system which is given by $\omega_n = \sqrt{k/m}$

X_f is the magnitude for the mass m which is given by $X_f = \left| f(t)/k \right|_{\max}$ where the assumed harmonic excitation is given by $f(t) = F\cos(\Omega\tau)$. Therefore for the case of force excited isolation, the cubic damping term is given by

$$\tilde{\zeta}_3 = \frac{c_3}{\sqrt{(km)^3}}\left(kX_f\right)^2 \tag{14}$$

3 RESULTS AND ANALYSIS

The equations of motion obtained in the previous section are integrated numerically using single sinusoidal frequency input with all zero initial conditions to obtain the steady-state numerical harmonic response. Fourier coefficients for the excitation frequency and its harmonics in the numerical predicted responses are determined and used to construct the frequency response function. For comparison, the HBM is applied to these equations to obtain the closed form solutions assuming just the response at the excitation frequency is to be determined. The solutions are plotted numerically in comparison to those from direct integration, which are in good agreement. Therefore the subsequent analysis and discussion in this study are based on the results obtained from HBM. Consequently the results and the effects of cubic damping are presented. Although the description of the procedures for solving these equations using the HBM are omitted, the reader is referred to previously published texts on the HBM method, e.g. Nayfeh [7] and Mickens [8].

To obtain a realistic set of results, the values for the cubic damping terms used are the values which provide a comparable amplitude of the displacement ratio at the bounce frequency in the case of a linear damping ratio. By doing this the responses above the bounce mode can be distinguished. The typical values of linear damping ratio regarding automotive suspension are around 0.25 to 0.75 [9]. In this study, the values of linear damping ratio are chosen to be 0.1, 0.2 and 0.3. The corresponding calculated values of cubic damping terms for the 2-DOF quarter-car model and the SDOF base excited isolation system, ζ_3, are 0.011, 0.101 and 0.407 respectively. For the case of force excitation, the same values of cubic damping are adopted for $\tilde{\zeta}_3$ even though the amplitudes of force transmissibility are slightly different.

It is important to note that using these cubic damping values is restricted for the specific value of the base input, since the cubic damping terms are input-dependent as given in equations (5) and (9). Whilst equation (14) is for the force input case which the values of cubic damping terms are restricted for the specific value of the force input. Hence the displacement ratios and the transmissibility for the cubic damping systems are affected by the amplitude of the input. In this paper, the values of cubic damping are calculated for an input amplitude of unity. However one can consider it the other way round, i.e. the higher value of cubic damping implies the larger amplitude of the input. The calculation procedure for getting the cubic term is not described in this paper.

3.1 The effect of cubic damping on the 2-DOF quarter-car model

In this section, the responses of the 2-DOF quarter-car model with cubic damping are plotted in comparison to the system with linear viscous damping as shown in Fig. 2(a)-(d). The displacement transmissibility for the non-linear system is defined by the approximate steady-state responses obtained from the HBM for which the responses at only the excitation frequencies are considered. The plots show the displacement ratio of the sprung mass to the base input for the system with linear damping and cubic damping respectively. The plots reveal that cubic damping causes an increase in the response magnitude for excitation frequencies above the bounce mode, especially for higher values of the cubic damping terms. Despite the levels of the displacement ratio around the wheel-hop mode for the cubic damping system being similar to those for the linear viscous damping, the isolation ability between the bounce mode and the wheel-hop mode is worse for the non-linear damping. This means the cubic damping might cause the sprung mass to have high level of response for the intermediate excitation frequencies.

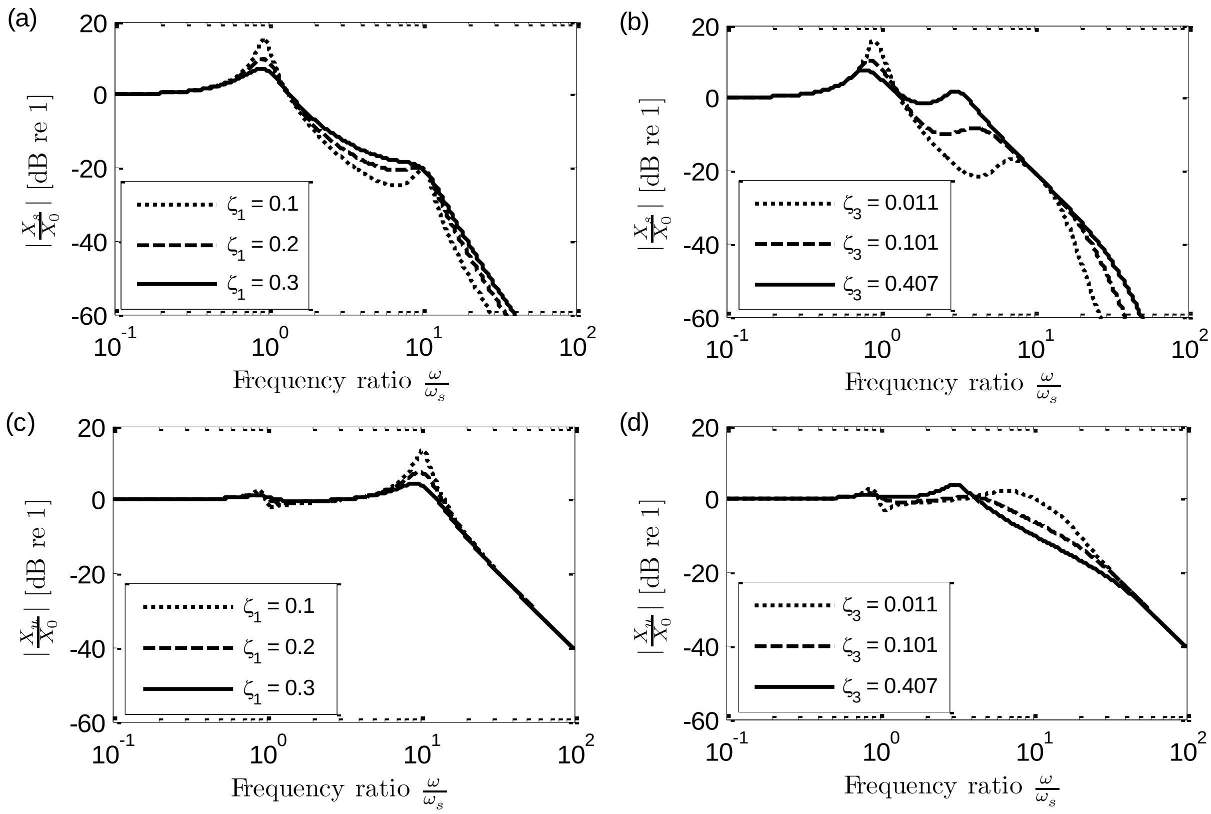


Fig 2. The transmissibility displacement ratios for a 2-DOF quarter-car model using HBM : (a) sprung mass for linear damping system, (b) sprung mass for the cubic damping system, (c) unsprung mass for linear damping system and (d) unsprung for cubic damping system.

The displacement ratio between unsprung mass and the base input for both linear and cubic damping systems are respectively shown in Fig. 2(c) and (d). One can describe the response at low excitation frequencies as a quasi-static system behaviour; the wheel is moving in unison with the base input. Theoretically, when the level of this ratio is around unity (0 dB) with no phase difference, the force between the wheel and the road is said to be the static load and the tyre deflection is constant [10]. One can say that with the higher force compressing the tyre, more grip will be obtained. The tyre is vibrating when either the ratio is not equal to unity or unity but with phase difference. If the tyre stiffness is too soft this will cause much vibration in the deflection of the tyre due to the undulating surface input. This can lead the wheel to vibrate against the stiffness of the tyre. This might result in poor quality of road holding, since to obtain good road holding the tyre deflection should be kept constant. However, the effect of cubic damping on the road holding is not discussed in this paper.

If the tyre stiffness is considered to be infinite, the motion of the unsprung mass and the base input are equal at all frequencies. This also means that there is no tyre deflection. Hence the forces arising from the wheel-road interface are transmitted directly to the sprung mass or isolated mass for the SDOF base excited isolation system. The effect of cubic damping for such a system is examined in the next section.

3.2 The effect of cubic damping on the SDOF base excited isolation model

By examining the SDOF base excited isolation system with cubic damping, the model for which illustrated in Fig. 1 (b), it is found that the displacement ratio for the isolated mass as shown in Fig. 3 (a) tends towards unity (0 dB) as the excitation frequency increases. Moreover, considering the phase information, Fig. 3 (b), reveals that in the high excitation frequency region the isolated mass is moving in-phase with the base input. As a result, one could hypothesise that when the excitation frequency is much higher than the undamped system natural frequency the damper starts behaving as a rigid link. This might be one culprit for the higher amplitude of the sprung mass for the 2-DOF quarter-car model above the bounce mode.

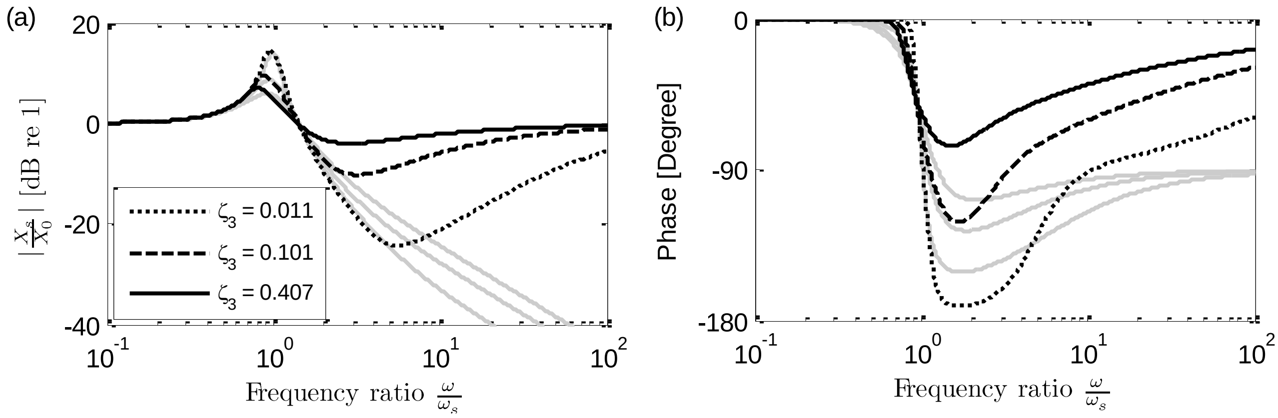
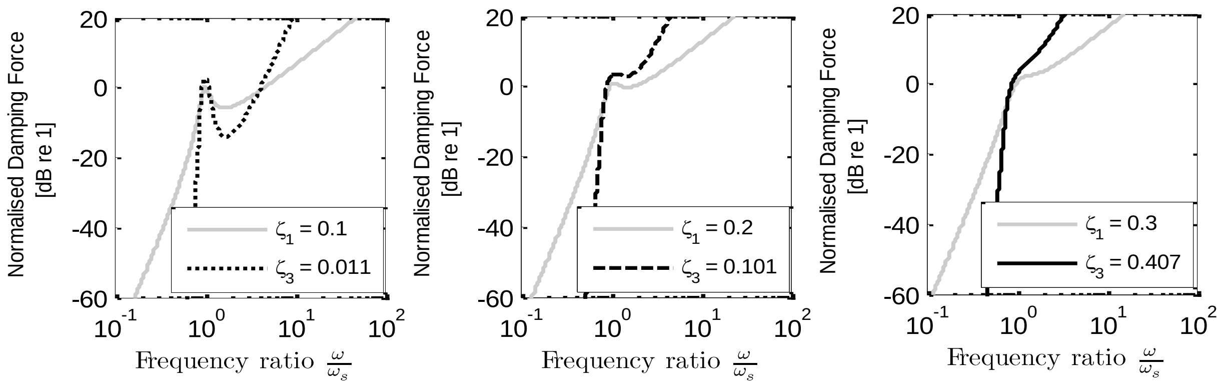


Fig 3. (a) The displacement transmissibility and (b) the phase lag for a SDOF base excited isolation system possessing cubic damping in comparison to those of linear damping (thick grey lines) where the damping ratios, ζ_1, are 0.1, 0.2 and 0.3.

In order to find the reason for the damper acting in such a way, the force contributions are of interest. Fig. 4 shows plots of the damping forces which contribute to the force acting on the isolated mass. One can see that the slopes of the cubic damping force with frequency above resonance are much steeper than those for the linear damper. This is because the damping force is resulting from the relative velocity of the isolated mass and the base input, which is proportional to the excitation frequency. Hence the higher excitation frequency causes higher damping forces which tend towards infinity. In the limit, motion across the damper is not possible and it acts as a rigid link.

However, one might get the benefit from the cubic damping if the normalised relative velocity across the damper is restricted to be lower than a specific value for which the damping forces from linear viscous damping and cubic damping are equal, i.e. $w_s' - w_0' = \sqrt{2\zeta_1 / \zeta_3}$. Fig. 5 shows the comparison of the restoring force characteristic diagram for linear damping and cubic damping. One can see that when the normalised relative velocity is lower than a certain value, the cubic damper



(a) **(b)** **(c)**

Fig 4. Force contributions in the damping element which contribute to the force acting on the isolated mass due to base excitation for both linear and cubic damping systems.

produces a lower damping force than that for the linear viscous damper. The force from cubic damping is much higher when the relative velocity exceeds a certain value as large relative velocity at high frequencies. This is a preferable characteristic around resonance since the relative velocity around resonance is very high. However, for the base excited isolation system for which the relative velocity is proportional to the excitation frequency, the damping force is higher for higher excitation frequencies. Thus there are some limitations to using the cubic damping for the base excited isolation system, including for the vehicle suspension system.

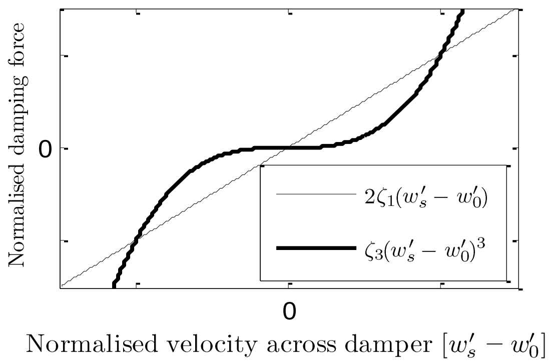


Fig 5. The characteristic restoring force diagram for cubic damping compared to linear viscous damping.

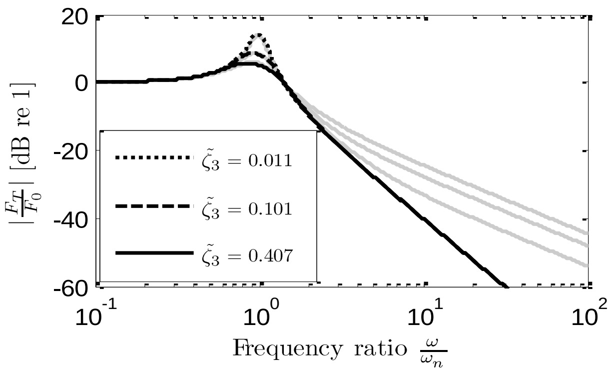


Fig 6. Force transmissibility for the force excited isolation systems with cubic damping in comparison to those with linear damping which the damping ratios, ζ_1, are 0.1, 0.2 and 0.3.

3.3 The effect of cubic damping on the SDOF force excited isolation model

In contrast to the case of base excitation, as one knows the highest relative velocity for the case of force excited isolation system occurs just around the resonance frequency and it is much lower for the excitation frequencies above resonance. As a result, higher damping forces are generated around resonance and much less for frequencies above resonance. In accordance with the restoring force characteristic diagram shown in Fig. 5, there is almost no damping force when the relative velocity is very low. This agrees with the principle idea for the isolation system, which needs to have less damping at high excitation frequencies. Thus the cubic damping is an alternative used for the force excited isolation system.

To support the preceding analysis, Fig. 6 shows the force transmissibility for the force excited isolation system for which the transmitted force around resonance is configured to have nearly the same level as those for the linear damping system. One can see that the isolation systems with cubic damping provide a better high frequency isolation. The decrease in the transmitted force is dominated by the mass line as for the undamped system, i.e. 40 dB per decade or inversely proportional to the excitation frequency squared. As one knows that the frequency response for the linear force excited isolation system at the isolation zone decreases by 40 dB per decade regardless the value of damping, so does the cubic damping system as shown in Fig. 7. Thus at high excitation frequencies, the motion of the vibrating mass appears to be very small. Hence the transmitted force is not influenced significantly from the stiffness component for this region whereas the linear damping still affects the force transmissibility. One can explain the advantage of cubic damping by examining the force contributed by the damper as shown in Fig. 8. The figures show the comparisons of the damping forces for the linear and cubic damping. One can see that the force contribution from the cubic damping is much lower than that from the linear damping. Then the transmitted force is totally influenced by the inertial force. This consequently results in the lower level of force transmissibility for the high frequency region.

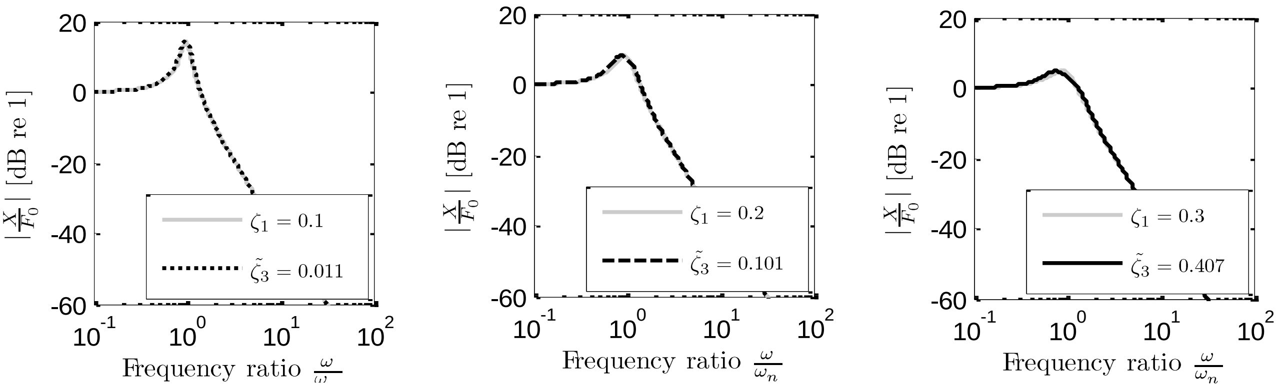


Fig 7. Frequency response for the force excited isolation systems with cubic damping in comparison to those with linear damping.

4 CONCLUSIONS

The damper element for the 2-DOF quarter-car model which is normally assumed as the linear viscous damping has been replaced by cubic damping as an alternative of the non-linear characteristic while the tyre damping is ignored. The responses for such a system were examined. The results show that cubic damping leads to worse response for the excitation frequencies between the bounce mode and wheel-hop mode. When the tyre stiffness is assumed to be infinitely stiff, the system was simplified to the SDOF base excited isolation system. The responses for the SDOF base excitation tend towards unity as the excitation frequency increases. By contrast, the effect of cubic damping for the case of the force excited isolation system is investigated. One can conclude that cubic damping produces a much higher damping force at high excitation frequencies for the SDOF base excited isolation system which is due to the higher relative velocity across the damper. As a result, the displacement transmissibility for the system with cubic damping is worse than that for systems with linear damping and tends towards unity as the excitation frequency increases. This includes using cubic damping as a damper element for a vehicle suspension system. The cubic damping causes a higher displacement and acceleration of the sprung mass for a broad frequency range above the bounce mode. In contrast, for the case of force excitation, there is less motion of the vibrating mass for higher excitation frequencies and hence less relative velocity across the damper and corresponding damping force. As a result cubic damping is beneficial in isolating the transmitted force from the vibrating mass to the supporting structure.

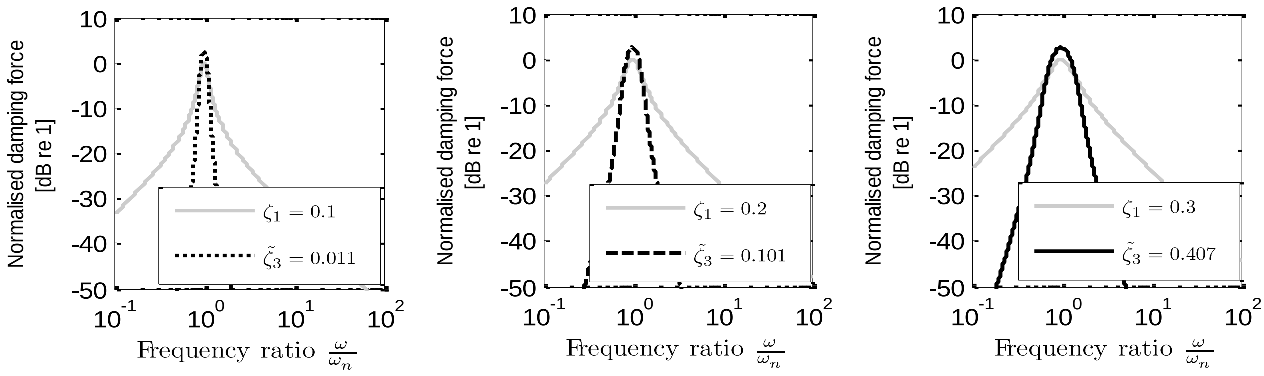


Fig 8. Force contributions in the damping element which contribute to the transmitted force acting on the supporting structure due to the excitation force for both linear and cubic damping systems.

REFERENCES

[1] C. Surace, K. Worden and G. R. Tomlinson, On the non-linear characteristics of automotive shock absorbers, *ARCHIVE: Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering 1989-1996 (vols 203-210)* 206 (1992) 3-16.

[2] Y. Cui, T. R. Kurfess and M. Messman, Testing and modeling of nonlinear properties of shock absorbers for vehicle dynamics studies, The World Congress on Engineering and Computer Science (2010)

[3] Z. Milovanovic, I. Kovacic and M. J. Brennan, On the displacement transmissibility of a base excited viscously damped nonlinear vibration isolator, *Journal of Vibration and Acoustics* 131 (2009) 054502-1 - 054502-7.

[4] I. Kovačić, Z. Milovanović and M. J. Brennan, On the relative and absolute transmissibility of a vibration isolation system subjected to base exctation, *FACTA UNIVERSITATIS Series: Working and Living Environmental Protection* 5 (2008) 39 - 48.

[5] C. N. Shekhar, H. Hatwal and A. K. Mallik, Performance of non-linear isolators and absorbers to shock excitations, *Journal of Sound and Vibration* 227 (1999) 293-307.

[6] Z. K. Peng, Z. Q. Lang, X. J. Jing, S. A. Billings, G. R. Tomlinson and L. Z. Guo, The Transmissibility of Vibration Isolators With a Nonlinear Antisymmetric Damping Characteristic, *Journal of Vibration and Acoustics* 132 (2010) 014501-1 - 014501-7.

[7] A. H. Nayfeh and D. T. Mook, *Nonlinear oscillations*, (1979) John Wiley & Sons, New York.

[8] R. E. Mickens, Comments on the method of harmonic balance, *Journal of Sound and Vibration* 94 (1984) 456 - 460.

[9] J. C. Dixon, *The shock absorber handbook*, 2nd Edition, (2007) John Wiley & Sons, Chichester, England.

[10] W. F. Milliken and D. L. Milliken, *Race car vehicle dynamics*, (1995) SAE International, Warrendale, PA, U.S.A.