Capacity Analysis of the Successive AF Relaying Aided Cooperative DS-CDMA Uplink

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Abstract—An amplify-and-forward (AF) based successive relaying aided network (SRAN) is designed for a multi-user spread-spectrum scenario conceived for noncoherent (NC) detection in order to convert the typical 50% half-duplex relaying-induced throughput loss to a potential user-load reduction of the CDMA system, where the NC detector allows us to avoid the extra power consumption imposed by channel estimation. With the same “green agenda” in mind, we limit the extra complexity imposed on the relay node (RN) by invoking the AF, rather than the more complex decode-and-forward protocol. We theoretically characterize the proposed AF based SRAN in the context of the DS-CDMA uplink by evaluating its noncoherent Discrete-input Continuous-output Memoryless Channel (DCMC) capacity. We demonstrate that the cooperative DS-CDMA uplink achieves a higher capacity gain with respect to its conventional direct transmission based counterpart, when the system has to support a high number of DS-CDMA users.

I. INTRODUCTION

The family of cooperation techniques heralded by van der Meulen in [1] is capable of achieving uplink transmit diversity by forming a virtual antenna array (VAA) in a distributed fashion. Hence, as demonstrated in [2], the outage probability of the conventional two-phase AF protocol based cooperative system significantly outperforms its direct transmission based counterpart. However, the conventional two-phase AF protocol halves the achievable throughput for the sake of gleaning a second-order diversity gain due to the half-duplex transmit/receive constraint of practical transceivers. Hence its capacity is lower than that of the classic direct transmission based system. A beneficial technique of mitigating the half-duplex relaying induced throughput-loss was advocated in [3] [4], where the successive relaying concept was proposed and analysed.

We extend the investigation of SRANs from the single-user scenario to a more realistic multi-user scenario of the DS-CDMA uplink (UL). The pathloss between the mobile station (MS) and base station (BS) will be mitigated with the aid of the proposed SRAN, hence the performance of cell-edge users may be remarkably improved [5]. In contrast to [6], the successive relaying induced interferences generated both at the relay node (RN) and at the destination node (DN) may be eliminated by directly exploiting the classic DS-CDMA principle upon assigning unique, link-specific spreading codes to the potentially interfering links. Naturally, this implies that the orthogonal time-slots used in

[4] are replaced by unique, link-specific CDMA spreading codes at the cost of imposing a soft-limit on the number of users that maybe supported, given the limited number of spreading codes.

The handsets of concentric users may be employed as the RN in the cooperation aided network, which however cannot afford a high system complexity and power consumption. Hence, in contrast to its more power-thirsty decode-and-forward (DF) counterpart, the AF scheme is preferred here, since it imposes a relatively low system complexity. Based on the idealized simplifying assumption that the channel state information (CSI) is perfectly known, the capacity of AF based cooperative systems was characterized in [2], [7]. However, it is unrealistic to expect that in addition to altruistic relaying, the RN could also afford the complexity and power-consumption of estimating the source-to-relay link. Consequently, instead of high-complexity coherent detection, low-complexity noncoherent detection is proposed for avoiding both the extra power consumption as well as the pilot overhead imposed by channel estimation.

Against this background, our novel contribution is:

- 1) We design a SRAN for a multi-user scenario, where a specific interference suppression regime is designed and the effects of both the successive relaying induced interference and of the multiple access interference (MAI) are evaluated.
- 2) We derive the noncoherent DCMC capacity of the AF based SRAN for multi-user scenarios.
- 3) We demonstrate that the proposed cooperative DS-CDMA uplink exhibits substantial advantages in high user-load situations.

The rest of this paper is organised as follows. Our system model and interference suppression regime is described in Section II. Section III derives the noncoherent DCMC capacity of the AF based SRAN. In Section IV, the simulation results are provided and the benefit of embedding DS-CDMA uplink in the AF based SRAN is analysed. Finally, we conclude in Section V.

II. SYSTEM OVERVIEW AND INTERFERENCE SUPPRESSION REGIME

A typical scenario of the cooperative DS-CDMA uplink is portrayed in Figure 1, where the MS s roaming close to the edge of the DS-CDMA cell activates the SRAN regime to improve its communication quality. The activated two relays r_0, r_1 and the BS d are specifically labelled in Figure 1. As detailed in [8], the average path-loss gain of the link from node a to node b with respect to the Source-to-Destination

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Fig. 1: The successive relaying aided cooperative DS-CDMA uplink topology.

(SD) link is given by $G_{ab} = \left(\frac{D_{sd}}{D_{ab}}\right)^\alpha$, $(a, b) \in \{s, r_0, r_1, d\}$, where the notation D_{ab} represents the distance between node a and node b . Throughout this paper, the path-loss exponent is fixed to $\alpha = 3$ for representing a typical urban area. To simplify our analysis, we assume that the SRAN has a symmetric topology, which implies that D_{sr_0} , D_{r_0d} , G_{sr_0} and G_{r_0d} are identical to D_{sr_1} , D_{r_1d} , G_{sr_1} and G_{r_1d} , respectively. Then, let P_a denote the transmit power of node a , and $\bar{i}$ is defined as the complement of i .

Furthermore, all the possible propagation paths between the s , r_i and d are assumed to be time-selective block-fading Rayleigh channels [9], where the fading coefficients $h_{ab}[k]$ of the channel between node a and node b fluctuate according to the normalized Doppler frequency in a block and change according to an independent and identical distribution (i.i.d) from one block to the next [10]. The correlated block-fading period of the channel is represented by T_b .

More details about the SRAN, especially the typical transmission processes of the different cooperative phases are depicted in Figure 2, where the source transmissions are segmented into identical-length transmission frames of L symbols. As detailed in [4] [11], an impediment of the SRAN is the interference generated among the relays, which is also referred as the inter-relay interference (IRI). The IRI is further aggravated by the co-channel interference (CCI) between the signals transmitted from the source node (SN) and RN contaminating the signal received at the DN, as seen in Figure 2. As a benefit of combining DS-CDMA with the SRAN philosophy, the inherent ability of CDMA to deal with the multipath effects assists the SRAN in overcoming its interference-limited behaviour.

Fig. 2: The successive relaying aided transmission process in its different communication phases.

A. Noise Accumulation Problem

A noise accumulation problem will occur owing to directly employing the classic AF protocol in the SRAN, where the RN simply amplifies its received signal before retransmitting it. When considering the classic DS-CDMA uplink embedded into the AF based SRAN, the transmit power of the RN r_i assigned to the k^{th} symbol of the $(l+1)^{st}$ frame consists of four components received during the most recent frame: the power $P_s^l[k]$ of the SN's broadcast signal, the power $P_{r_i}^l[k]$ of IRI, the power $P_{MAI}^l[k]$ of MAI and the power σ^2 of the AWGN noise, which may be simply formulated by omitting the index k , yielding

$$P_{r_i}^{l+1} = f_{AM_{r_i}}^2 \{G_{sd}P_s^l + G_{r_i r_i}P_{r_i}^l + P_{MAI}^l + \sigma^2\} \quad (1)$$

$$P_{r_i}^l = f_{AM_{r_i}}^2 \{G_{sd}P_s^{l-1} + G_{r_i r_i}P_{r_i}^{l-1} + P_{MAI}^{l-1} + \sigma^2\} \quad (2)$$

⋮

$$P_{r_1}^2 = f_{AM_{r_1}}^2 \{G_{sd}P_s^1 + G_{r_0 r_1}P_{r_0}^1 + P_{MAI}^1 + \sigma^2\}, \quad (3)$$

where the amplification factor $f_{AM_{r_i}}$ of RN r_i constrains the transmitted power of the RN r_i to P_{r_i} . Obviously, the noise component received by r_i in the $(l-1)^{st}$ frame will contaminate the transmitted signal of r_i in the $(l+1)^{st}$ frame, as seen by substituting (2) into (1). This process is then continued recursively. Based on the assumption that P_s^l , P_{MAI}^l and the Signal-to-Noise-Ratio (SNR) remain constant for the different frames, and proceeding backwards from the $(l+1)^{st}$ to the 2^{nd} frame, the variance of the total recursively accumulated noise component in $P_{r_i}^{l+1}$ may be approximated by

$$\begin{aligned} \text{Var}[N_{\text{total}}] &= \sigma^2 f^2 \left[1 + f^2 G + (f^2 G)^2 + \dots + (f^2 G)^{l-1} \right] \\ &\approx \sigma^2 f^2 \lim_{l \rightarrow \infty} \sum_{n=0}^{l-1} (f^2 G)^n = \frac{\sigma^2 f^2}{1 - f^2 G}, \end{aligned} \quad (4)$$

where f and G represent $f_{AM_{r_i}}$ and $G_{r_i r_i}$, respectively. This reveals that all the noise generated in different frames at the different relays will be consistently scaled and accumulated during the transmission process of the classic AF based SRAN. Quantitatively they impose an extra noise component having a variance of $\frac{\sigma^2 f^2}{1 - f^2 G}$ on the DN, which cannot be mitigated by the single despread operation at the DN.

B. The DS-CDMA Based Interference Suppression Regime

Let $\mathbf{C}_i = [c_i[1], c_i[2], \dots, c_i[Q]]$, $i = 0, 1$ denote a pair of PN sequences having a cross-correlation (CCL) of γ_{01} and a spreading factor of Q . Observe at RN r_0 of Figure 2 that the received AWGN vector $\mathbf{n}[k]$ of a symbol-duration consists of Q chip-related AWGN samples $n[k+q]$, which may be formulated as: $\mathbf{n}[k] = [n[k+1], n[k+2], \dots, n[k+Q]]$, where $n[k+q] \sim \mathcal{CN}(0, \sigma^2/Q)$. After despreading by $\mathbf{C}_0^T$, the corresponding term of $\mathbf{n}[k]$ is given by

$$\eta_1 = \sum_{q=1}^Q n[k+q]c_0[q] \sim \mathcal{CN}(0, \sigma^2), \quad (5)$$

where η_1 is a Gaussian variable. If η_1 is further spread by $\mathbf{C}_0$ and then despread by $\mathbf{C}_1$, we obtain

$$\eta_2 = \sum_{q=1}^Q \eta_1 c_0[q] c_1[q]. \quad (6)$$

Observe that η_1 of (5) obeys a Gaussian distribution in statistical terms, but each of its realization becomes a specific value in (6). Hence (6) may be rewritten as

$$\eta_2 = \eta_1 \sum_{q=1}^Q c_0[q] c_1[q] = \eta_1 \gamma_{01}. \quad (7)$$

This implies that the power of the AWGN vector $\mathbf{n}[k]$ cannot be reduced by a single combined spread-despread operation, but it can definitely be mitigated by multiple spread-despread operations.

Based on the above analysis, a specifically arranged DS-CDMA regime is designed here in order to circumvent the potential noise accumulation problem, and to mitigate both the successive relaying induced interferences and the MAI imposed on the cooperative DS-CDMA uplink. In more detail, at the SN, the modulated symbols $S^l[k]$ are alternately spread by $\mathbf{C}_0$ and $\mathbf{C}_1$ from frame to frame ($l = 0, 1, \dots$). The received signals are firstly despread by $\mathbf{C}_i^T$ in the listening mode of RN r_i , and then directly spread by $\mathbf{C}_i$ in the transmit mode of RN r_i before the amplification operation. Hence, when we employ an appropriate PN sequence to suppress the interfering signal transmitted by the RNs (e.g. the IRI) at the receiver (RN or DN), according to (7), the AF noise component inherent in the interference will be simultaneously suppressed. Our spreading scheme also guarantees the orthogonality of the two different components in the received signal at the DN, which correspond to the SN's transmitted signal and to the RN's forwarded signal, respectively. These operations are also illustrated in Figure 2.

C. System Model for AF Based SRAN

We assume that the channel's fading coefficients in the l^{th} frame $h_{ab}^l[k]$ maintain the same value in different chip-durations of the same symbol. Hence, the k^{th} received signal of the l^{th} frame at the DN may be formulated as

$$\mathbf{y}^l[k] = \sqrt{G_{sd}} h_{sd}^l[k] \mathbf{c}_s^l[k] + \sqrt{G_{r_i d}} h_{r_i d}^l[k] \mathbf{c}_{r_i}^l[k] + \mathcal{I}_{s^M d}^l[k] + \mathbf{n}_d^l[k], \quad (8)$$

where $i = \text{mod}[(l+1), 2]$, $l = (0, 1, \dots)$. The spread AWGN vector $\mathbf{n}_d^l[k]$ obeys $\mathcal{E}\{\mathbf{n}_d^l[k] \mathbf{n}_d^{lH}[k]\} = \frac{1}{Q} \sigma^2 \mathbf{I}_Q$, where $\mathbf{I}_Q$ denotes the identity matrix having a dimension of Q . We assume having $(M+1)$ users, namely the desired user plus M interfering users. Consequently, the MAI imposed by the M interfering users on the DN is given by

$$\mathcal{I}_{s^M d}^l[k] = \sum_{m=1}^M \sqrt{G_{s_m d}} h_{s_m d}^l[k] S_{s_m}^l[k] \cdot \mathbf{C}_{s_m}. \quad (9)$$

The SN's broadcast signal is given by $\mathbf{c}_s^l[k] = S^l[k] \cdot \mathbf{C}_i$ and the RN's forwarded signal of (8) can be detailed as

$$\mathbf{c}_{r_i}^l[k] = f_{AM_{r_i}} \times \left\{ \sqrt{G_{sr_i}} h_{sr_i}^{l-1}[k] \mathbf{c}_s^{l-1}[k] + \mathcal{I}_{r_i r_i}^{l-1}[k] + \mathcal{I}_{s^M r_i}^{l-1}[k] + \tilde{\mathbf{n}}_{r_i}^{l-1}[k] \right\}. \quad (10)$$

The component $\mathcal{I}_{r_i r_i}^{l-1}[k]$ in (10) captures the effect of IRI after the combined despread-spread operation of RN r_i , which can be detailed as

$$\mathcal{I}_{r_i r_i}^{l-1}[k] = \gamma_{ii} \sqrt{G_{r_i r_i}} h_{r_i r_i}^{l-1}[k] R_i^{l-1}[k] \cdot \mathbf{C}_i, \quad (11)$$

where the CCL between the PN sequence $\mathbf{C}_a$ and the PN sequence $\mathbf{C}_b$ is commonly represented by γ_{ab} . We assign a pair of PN sequences having $\gamma_{ii} = 1/Q$ as $\mathbf{C}_i^T$ and $\mathbf{C}_i$ in order to significantly reduce the IRI effects. Then, after the combined despread-spread operation, the MAI imposed on r_i during the $(l-1)^{st}$ frame becomes

$$\begin{aligned} \mathcal{I}_{s^M r_i}^{l-1}[k] &= \tilde{\mathcal{I}}_{s^M r_i}^{l-1}[k] \cdot \mathbf{C}_i \\ &= \sum_{m=1}^M \gamma_{mi} \sqrt{G_{s_m r_i}} h_{s_m r_i}^{l-1}[k] S_{s_m}^{l-1}[k] \cdot \mathbf{C}_i. \end{aligned} \quad (12)$$

From the central limit theorem [12], the summation of M independent random variables may be modelled by the Gaussian distribution. Hence, additionally based on the Gaussian approximation method [13], the variance of $\tilde{\mathcal{I}}_{s^M r_i}^{l-1}[k]$ may be approximated by

$$\text{Var}[\tilde{\mathcal{I}}_{s^M r_i}^{l-1}] = \sum_{m=1}^M \frac{1}{3Q} G_{s_m r_i} P_{s_m}. \quad (13)$$

where, $G_{s_m r_i} P_{s_m}$ - the power of the MAI imposed on the RN before the despread-spread operation can be approximated by the MAI imposed on the DN. If we further assume the DS-CDMA system has a perfect power control, we can readily derive the following relationships

$$\sum_{m=1}^M G_{s_m r_i} P_{s_m} \approx \sum_{m=1}^M G_{s_m d} P_{s_m} = \sum_{m=1}^M G_{sd} P_s. \quad (14)$$

By substituting (14) to (13), (13) is rewritten as

$$\text{Var}[\tilde{\mathcal{I}}_{s^M r_i}^{l-1}] = \frac{1}{3Q} M P_s. \quad (15)$$

In order to constrain the transmitted power of the RN r_i to P_{r_i} [14], and invoking (11) as well as (15), we can obtain

$$f_{AM_{r_i}} = \sqrt{\frac{P_{r_i}}{G_{sr_i} P_s + \frac{1}{Q^2} G_{r_i r_i} P_{r_i} + \frac{1}{3Q} M P_s + \sigma^2}}. \quad (16)$$

We also note that $\mathbf{c}_{r_i}^l[k]$ of (10) corresponds to the k^{th} modulated source symbol of the previous frame, namely to $S^{l-1}[k]$, instead of corresponding to $S^l[k]$ transmitted by the SN in the present frame.

At the DN, the different components of the received signal $\mathbf{y}^l[k]$ can be resolved by appropriately configuring the chip-waveform matched-filter for the different spreading codes. When the filter is matched to the waveform $\mathbf{C}_i$, the signal directly transmitted by the SN will contribute the main component of the despread signal, while the RN's

forwarded signal and the interfering users' signals become the interference. Hence, the associated output of the chip-waveform matched-filter is given by

$$z_s^l[k] = \sqrt{G_{sd}} h_{sd}^l[k] S^l[k] + I_{r_i d}^l[k] + I_{s M d}^l[k] + n_d^l[k], \quad (17)$$

where $I_{r_i d}^l[k]$ accounts for the relay's forwarded signal component after the despreading operation. Recalling the analysis of Section II-B, according to (7), the noise term $\tilde{n}_{r_i}^{l-1}[k]$ in (10) will be suppressed along with the other components of (10) by the CCL γ_{ii} . Then $I_{s M d}^l[k]$ is the despread version of $\mathcal{I}_{s M d}^l[k]$ in (8), whose variance may be calculated by a process similar to (12), (15). Hence we can derive that $\text{Var}[I_{r_i d}^l] = \frac{1}{Q^2} G_{r_i d} P_{r_i}$; $\text{Var}[I_{s M d}^l] = \frac{1}{3Q} M P_s$; $n_d^l[k] \sim \mathcal{CN}(0, \sigma^2)$. Similarly, the despread signal extracted from $\mathbf{y}^l[k]$ but dominated by the RN's forwarded signal $\mathbf{c}_{r_i}^l[k]$ can be expressed as

$$\begin{aligned} z_{r_i}^l[k] &= f_{AM_{r_i}} \sqrt{G_{sr_i}} \sqrt{G_{r_i d}} h_{sr_i}^{l-1}[k] h_{r_i d}^l[k] S^{l-1}[k] \\ &\quad + I_{r_i r_i}^{l-1}[k] + I_{s M r_i}^{l-1}[k] + I_s^l[k] + I_{s M d}^l[k] \\ &\quad + \tilde{n}_{r_i}^{l-1}[k] + n_d^l[k], \end{aligned} \quad (18)$$

where $I_{r_i r_i}^{l-1}[k]$, $I_{s M r_i}^{l-1}[k]$ and $\tilde{n}_{r_i}^{l-1}[k]$ are the amplified and faded, then despread versions of $\mathcal{I}_{r_i r_i}^{l-1}[k]$, $\mathcal{I}_{s M r_i}^{l-1}[k]$ and $\tilde{n}_{r_i}^{l-1}[k]$ in (10). $I_s^l[k]$ is the interference imposed by the SN's broadcast signal $\mathbf{c}_s^l[k]$. Their variances are summarized as $\text{Var}[I_{r_i r_i}^{l-1}] = (\frac{1}{Q^2} G_{r_i r_i} P_{r_i}) f_{AM_{r_i}}^2 G_{r_i d}$; $\text{Var}[I_{s M r_i}^{l-1}] = (\frac{1}{3Q} M P_s) f_{AM_{r_i}}^2 G_{r_i d}$; $\text{Var}[I_s^l] = \frac{1}{Q^2} G_{sd} P_s$; $\text{Var}[\tilde{n}_{r_i}^{l-1}] = (\sigma^2) f_{AM_{r_i}}^2 G_{r_i d}$.

III. NONCOHERENT DCMC CAPACITY OF THE AF BASED SRAN

The transmission arrangement of the twin-relay-aided successive relaying procedure may be viewed as the superimposed transmissions of two conventional single-relay aided two-phase cooperative links [11]. This is also illustrated in Figure 2, where the transmissions represented by the dashed lines in the even phase and odd phase constitute one of the conventional single-relay aided two-phase cooperative networks, namely Coop-I. Similarly, the transmissions represented by the solid lines in the even and odd phases constitute another one, namely Coop-II. According to this philosophy, the noncoherent DCMC capacity of the AF based SRAN is constituted by the sum of the capacities of the AF based sub-networks Coop-I and Coop-II, which is formulated as

$$C_{\text{Successive}}^{\text{AF}} = C_{\text{Coop-I}}^{\text{AF}} + C_{\text{Coop-II}}^{\text{AF}}. \quad (19)$$

Since the noncoherent DCMC capacity of the conventional AF based single-relay aided two-phase cooperative network e.g. $C_{\text{Coop-I}}^{\text{AF}}$ is unknown, it will be derived below.

In the sub-network Coop-I, during the broadcast interval of length T_s , a block of the modulated source symbols $\mathbf{S}^l = [S^l[k+1], S^l[k+2], \dots, S^l[k+T_b]]^T$ is broadcast by the SN. The transmissions via the SD link generate the signal vector $\mathbf{z}_s^l = [z_s^l[k+1], z_s^l[k+2], \dots, z_s^l[k+T_b]]^T$, which is actually constituted by the associated despread signals received at the DN, as detailed in (17). According to [12] again, by treating the interference components in (17) as a

Gaussian variable, the overall equivalent noise component imposed on the vector $\mathbf{z}_s^l$ may be expressed as

$$\mathbf{n}_s^l = \begin{bmatrix} I_{r_1 d}^l[k+1] + I_{s M d}^l[k+1] + n_d^l[k+1] \\ \vdots \\ I_{r_1 d}^l[k+T_b] + I_{s M d}^l[k+T_b] + n_d^l[k+T_b] \end{bmatrix}, \quad (20)$$

whose covariance matrix is given by $\Psi_{\mathbf{n}_s^l} = \{\frac{G_{r_1 d}}{Q^2} P_{r_1} + \frac{M}{3Q} P_s + \sigma^2\} \mathbf{I}_{T_b}$. Furthermore, the associated fading coefficients of the SD link are represented by $\mathbf{h}_{sd} = [h_{sd}^l[k+1], h_{sd}^l[k+2], \dots, h_{sd}^l[k+T_b]]^T$. Hence, given the definitions $\Phi_{sd} = P_s \mathcal{E}\{\mathbf{h}_{sd} \mathbf{h}_{sd}^H\}$ and $\bar{\mathbf{S}} = \text{diag}\{\mathbf{S}^l\}$, the covariance matrix Ψ_{sd} of the despread received signal vector $\mathbf{z}_s^l$ is given by

$$\Psi_{sd} = \mathcal{E}\{\mathbf{z}_s^l \mathbf{z}_s^{lH} | \mathbf{S}^l\} = \bar{\mathbf{S}} \Phi_{sd} \bar{\mathbf{S}}^H + \Psi_{\mathbf{n}_s^l}. \quad (21)$$

The detailed derivation of (21) is not provided here owing to space limitations, but similar procedures may be found in [15] and [16].

Similarly, the signal vector $\mathbf{z}_{r_0}^{l+1} = [z_{r_0}^{l+1}[k+1], z_{r_0}^{l+1}[k+2], \dots, z_{r_0}^{l+1}[k+T_b]]^T$ represents another replica of the same modulated source symbol block $\mathbf{S}^l$, which is generated by the consecutive transmissions from the SN to the RN during the broadcast phase of duration T_s , and then to the DN during the multiple-access phase of duration T_r . The equivalent fading coefficients of the associated channels are represented by $\mathbf{h}_{sr_0 d} = [h_{sr_0 d}^l[k+1], h_{r_0 d}^{l+1}[k+1], \dots, h_{sr_0 d}^l[k+T_b], h_{r_0 d}^{l+1}[k+T_b]]^T$. Then the overall equivalent noise component $\mathbf{n}_{r_0}^{l+1}$ imposed on the vector $\mathbf{z}_{r_0}^{l+1}$ is composed by all the interference and noise components in (18). The relevant covariance matrix is formulated as

$$\begin{aligned} \Psi_{\mathbf{n}_{r_0}^{l+1}} &= \left\{ \frac{G_{r_0 d} \left\{ \frac{G_{r_1 r_0}}{Q^2} P_{r_1} + \frac{M}{3Q} P_s + \sigma^2 \right\} P_{r_0}}{G_{sr_0} P_s + \frac{G_{r_1 r_0}}{Q^2} P_{r_1} + \frac{M}{3Q} P_s + \sigma^2} \right. \\ &\quad \left. + \frac{G_{sd}}{Q^2} P_s + \frac{M}{3Q} P_s + \sigma^2 \right\} \times \mathbf{I}_{T_b}. \end{aligned} \quad (22)$$

Similar to our previous analysis, let us define $\Phi_{sr_0 d} = P_s f_{AM_{r_0}}^2 G_{sr_0} G_{r_0 d} \mathcal{E}\{\mathbf{h}_{sr_0 d} \mathbf{h}_{sr_0 d}^H\}$, the covariance matrix $\Psi_{sr_0 d}$ of the signal vector $\mathbf{z}_{r_0}^{l+1}$ is formulated as

$$\Psi_{sr_0 d} = \bar{\mathbf{S}} \Phi_{sr_0 d} \bar{\mathbf{S}}^H + \Psi_{\mathbf{n}_{r_0}^{l+1}}. \quad (23)$$

In the capacity analysis, the sub-network Coop-I may be equivalently modelled by a single-input-multiple-output (SIMO) system having $N_T = 1$ transmit antenna, and $N_R = 2$ receive antennas, where the transmissions via the SD link during the broadcast phase can be viewed as the transmissions from the single transmit antenna to one of the two receive antennas in the equivalent SIMO system. Similarly, the consecutive transmissions from the SN to the RN ensuing during the broadcast phase, and then to the DN during the multiple-access phase in Coop-I can be treated as the transmissions from the single transmit antenna to another receive antenna, while experiencing the equivalent channel's fading effect and incurring the same interference-plus-noise impairment in the SIMO system. Hence, given

the definitions $\bar{\mathbf{Z}} = [\mathbf{z}_s^l \ \mathbf{z}_{r_0}^{l+1}]$, $\bar{\mathbf{H}} = [\mathbf{h}_{sd} \ \mathbf{h}_{sr_0d}]$ and $\bar{\mathbf{N}} = [\mathbf{n}_s^l \ \mathbf{n}_{r_0}^{l+1}]$, the entire input-output relation of the equivalent SIMO system may be formulated by

$$\bar{\mathbf{Z}} = \bar{\mathbf{S}} \cdot \bar{\mathbf{H}} + \bar{\mathbf{N}}. \quad (24)$$

It was pointed out by Ding *et al.* [7] that the equivalent channel's fading coefficients - the elements of $\mathbf{h}_{sr_0d}$, involve a product of Gaussian random variables, which is no longer Gaussian. Nevertheless, Ding *et al.* further demonstrated that the ergodic channel capacity of the AF aided single-relay system based on the Gaussian assumption is still achievable, provided that the covariance matrix of the system's input signal is diagonal, and the diagonal elements may be obtained by solving the resultant optimization problems invoking multidimensional integrals. In the spirit of [7], we may approximate the capacity of the equivalent SIMO system based on the Gaussian assumption. Hence, according to [17], the probability density function of $\bar{\mathbf{Z}}$ conditioned on $\bar{\mathbf{S}}$ is readily expressed as

$$\Pr(\bar{\mathbf{Z}}|\bar{\mathbf{S}}) = \frac{1}{[\pi^{T_b N_T} \det(\Psi)]^{N_R}} \cdot \exp \left[-\text{tr} \left(\bar{\mathbf{Z}}^H \Psi^{-1} \bar{\mathbf{Z}} \right) \right], \quad (25)$$

where the conditional covariance matrix Ψ is given by

$$\Psi = \bar{\mathbf{S}} (\Phi_{sd} + \Phi_{sr_0d}) \bar{\mathbf{S}}^H + \Psi_{\mathbf{n}_s^l} + \Psi_{\mathbf{n}_{r_0}^{l+1}}. \quad (26)$$

Let us define $\beta_1 = \frac{T_s}{T_s + T_r}$ and $\beta_2 = \frac{T_r}{T_s + T_r}$, which was introduced for evaluating the effect of the source- and relay- transmission duration ratio of a conventional single-relay aided two-phase cooperative network, as detailed in [18]. Apparently, the AF based sub-network Coop-I has the inherent relationship of $\beta_1 = \beta_2 = \beta = 0.5$. Then, according to [19] and [20], the capacity of Coop-I can be readily formulated as

$$C_{\text{Coop-I}}^{\text{AF}} = \frac{\beta}{T_b} I(\bar{\mathbf{S}}; \bar{\mathbf{Z}}) = \frac{\beta}{T_b} [H(\bar{\mathbf{Z}}) - H(\bar{\mathbf{Z}}|\bar{\mathbf{S}})], \quad (27)$$

where the conditional entropy $H(\bar{\mathbf{Z}}|\bar{\mathbf{S}})$ can be calculated in a closed form as

$$\begin{aligned} H(\bar{\mathbf{Z}}|\bar{\mathbf{S}}) &= - \int \Pr(\bar{\mathbf{Z}}, \bar{\mathbf{S}}) \ln [\Pr(\bar{\mathbf{Z}}|\bar{\mathbf{S}})] d\bar{\mathbf{Z}} d\bar{\mathbf{S}} \\ &= \log_2 \left\{ e^{T_b} \cdot [\det(\pi \Psi)]^{N_R} \right\}. \end{aligned} \quad (28)$$

However, the entropy $H(\bar{\mathbf{Z}})$ cannot be evaluated in a closed form. A practical approach to its numerical evaluation is constituted by the following Monte-Carlo integration

$$\begin{aligned} H(\bar{\mathbf{Z}}) &= -\mathcal{E} \{ \log_2 [\Pr(\bar{\mathbf{Z}})] \} \\ &= -\mathcal{E} \left\{ \log_2 \left[\frac{1}{M_c^{T_b}} \sum_{\bar{\mathbf{S}}' \in \mathcal{X}} \frac{\exp \left[-\text{tr} \left(\bar{\mathbf{Z}}^H \Psi^{-1} \bar{\mathbf{Z}} \right) \right]}{[\det(\pi \Psi)]^{N_R}} \right] \right\}. \end{aligned} \quad (29)$$

Since each element of the T_b -component differentially encoded signal vector $\mathbf{S}^l$ is chosen independently from an M_c -point constellation set $\mathcal{M}_c$ with equal probabilities, the number of all hypothetically transmitted symbol matrices $\bar{\mathbf{S}}'$ equals to $M_c^{T_b}$ and $\mathcal{X}$ in (29) represents the set of all hypothetical values of $\bar{\mathbf{S}}'$.

Fig. 3: The effect of the geographic position of the RN on $C_{\text{Successive}}^{\text{AF}}$, as evaluated from (19), (27), (28) and (29), where a normalized Doppler frequency of $f_d = 0.01$ is assumed.

Now, we are ready to calculate the value of $C_{\text{Coop-I}}^{\text{AF}}$. Then, we can simply attain the capacity of the sub-network Coop-II following a similar process. According to (19), the noncoherent DCMC capacity of the AF based SRAN becomes available.

IV. SIMULATION RESULTS AND ANALYSIS

Let us define $\theta = \frac{D_{sr_i}}{D_{r_i d}}$, and assume that $D_{r_0 r_1}$ equals to the relatively smaller value between D_{sr_i} and $D_{r_i d}$. Based on the symmetrical structure of the proposed SRAN assumed in Section II, the exact proportions of D_{sr_i} , $D_{r_i d}$, D_{sd} and $D_{r_0 r_1}$ associated with different θ values are summarized in Table I, where the shortest distance is always normalized to unity. Employing the normalized distance list

θ	D_{sr_i}	$D_{r_i d}$	D_{sd}	$D_{r_0 r_1}$
1/3	1	3	3.824	1
1/2	1	2	2.802	1
1.0	1	1	1.732	1
2.0	2	1	2.802	1
3.0	3	1	3.824	1

TABLE I: Distance proportions with respect to θ

of Table I, the path-loss gain associated with a certain θ may be obtained according to [8].

A cooperative-user-selection scheme is employed, where the effect of the RN's position is considered. Then, we do not employ any sophisticated power-allocation scheme. We simply equally share the total power between the SN and RN, which directly leads to $P_s = P_{r_0} = P_{r_1} = \frac{1}{2} P_{\text{total}}$. The corresponding simulation results of $C_{\text{Successive}}^{\text{AF}}$ as evaluated from (19), (27), (28) as well as (29) are displayed in Figure 3, where the following RN positions expressed in terms of $\theta \in \{3.0, 2.0, 1.0, \frac{1}{2}, \frac{1}{3}\}$ were considered and the number of interfering users was fixed to $M = 0$. Observe in Figure 3 that the capacity of the SN's uplink employing the AF based SRAN exceeds that of the conventional single-link (SL) direct transmission (DT) structure, when appointing RNs sufficiently close to the SN. By contrast, it results in a degraded capacity compared to the conventional direct transmission structure, when appointing RNs close to the DN. In our particular case, appointing the RNs at

Fig. 4: The noncoherent DCMC capacity $C_{\text{Successive}}^{\text{AF}}$ in zero, moderate and heavy MAI scenarios, as evaluated from (19), (27), (28) and (29), where a normalized Doppler frequency of $f_d = 0.01$ is assumed.

the position of $\theta = \frac{1}{2}$ is seen to be the best strategy in Figure 3, which slightly improves the capacity $C_{\text{Successive}}^{\text{AF}}$ compared to the scenarios of $\theta = \frac{1}{3}$ and $\theta = 1.0$. The capacity of the conventional single-relay aided two-phase AF based cooperative DS-CDMA uplink having $\theta = \frac{1}{2}$ is also portrayed in Figure 3, which is significantly exceeded by that of its AF based SRAN counterpart.

In our next investigation we fixed the position of RNs to $\theta = \frac{1}{2}$ and focussed our attention on the detrimental effects of having an increased number of interfering users M . Specifically, the values of $M \in \{0, 32, 64\}$ are considered for modelling the scenarios of zero, moderate and heavy MAI, respectively. The relevant simulation results displayed in Figure 4 demonstrate that the capacity of the AF based SRAN always exceeds that of the direct transmission system, regardless of the number of interfering users. It is more intriguing to find that the capacity advantage of the AF based SRAN increases with respect to the conventional direct transmission system, when the MAI becomes stronger. This implies that in contrast to the conventional DS-CDMA uplink, the cooperative DS-CDMA uplink will exhibit more substantial advantages in high-load situations.

Furthermore, it is expected that if we adjust the amplification factor $f_{AM_{r_i}}$ with the aid of accurate power-control instead of equally splitting the power between the SN and RN, the AF based capacity may be further improved, as also reported in [2], [7]. This benefit may indeed be anticipated, since having a reduced SN and RN transmit power results in reduced MAI in comparison to the direct transmission scenario.

V. CONCLUSIONS

In this contribution, we suppressed the successive relaying induced interference with the aid of a specifically designed DS-CDMA technique. Then, based on the capacity analysis of Section III and the simulation results of Section IV, we may reasonably conclude that the proposed cooperative DS-CDMA uplink embedded in the AF based SRAN outperforms the SL DT based DS-CDMA uplink in terms of the achievable capacity, especially in high-load situations.

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