Dissemination of an optical frequency comb over fiber with 3×10^{-18} fractional accuracy

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Abstract: We demonstrate that the structure of an optical frequency comb transferred over several km of fiber can be preserved at a level compatible with the best optical frequency references currently available. Using an optical phase detection technique we measure the noise introduced by the fiber link and suppress it by stabilizing the optical path length. The measured fractional frequency stability of the transferred optical modes is 2×10^{-18} at a few thousand seconds and the mode spacing stability after optical-microwave conversion is better than 4×10^{-17} over the same time scale.

OCIS codes: (120.3940) Metrology; (120.3930) Metrological instrumentation; (140.4050) Mode-locked lasers ; (060.2360) Fiber optics links and subsystems.

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1. Introduction

A decade after the optical frequency comb technique transformed the field of frequency metrology, these devices have already found applications in other science areas as diverse as spectroscopy [1–3], attosecond physics [4] and astrophysics [5, 6]. When combined with atomic references, tests of fundamental physics that would have been unthinkable only a few years ago, and whose outcome could open a whole new era for physics, can now be performed [7, 8]. With optical frequency standards currently exhibiting a fractional accuracy better than 10^{-17} [9] and optical frequency combs making this accuracy available across a wide spectrum [10], new experiments could be devised in a wide range of research fields if ultra accurate optical frequencies were to be made available beyond the walls of metrology laboratories.

Much research has recently been undertaken on the transfer of ultra-stable frequencies over long lengths of optical fiber, driven by the need to compare state-of-the-art optical clocks in distantly located laboratories. Steps towards a world-wide optical fiber network linking these laboratories have already been undertaken, with a 900 km link now operative in Germany [11] and shorter links being progressively set up across Europe. Remarkable results have been demonstrated for transfer of a single microwave or optical frequency over long spans of dark fiber, using phase noise cancellation techniques to compensate for the noise introduced by environmental perturbations to the fiber [12–16]. Tests have also been performed on fiber carrying internet traffic [17], since international clock comparisons will inevitably need to use commercial optical networks. However in dedicated research networks a large optical bandwidth is available, making it possible to transfer a frequency comb rather than a single optical frequency [18]. It has been shown that microwave frequencies can be transferred over research networks with a stability and accuracy compatible with state-of-the-art frequency standards by using the mode spacing of an optical frequency comb [19, 20]. However, although preliminary experi-

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ments were carried out on a 1.5 km uncompensated fiber link [21], the issue of how accurately it is possible to preserve the optical comb structure over a long length of fiber has not been addressed until now. Here, we show for the first time that this is possible at a level exceeding that of the best frequency standards available today. We demonstrate that the mode frequencies of an optical comb can be transferred over a several km fiber link with a fractional accuracy better than 3×10^{-18}, a factor of three more accurate than the best optical clock reported to date [9]. We also test the stability of the mode spacing after optical-microwave conversion and find that it is preserved to better than 2×10^{-15} at 1 s and 4×10^{-17} for averaging times greater than 1000 s. This performance is achieved by using an optical phase detection technique to cancel the environmentally induced fiber noise, an approach that provides orders of magnitude higher sensitivity than techniques based on microwave detection [18].

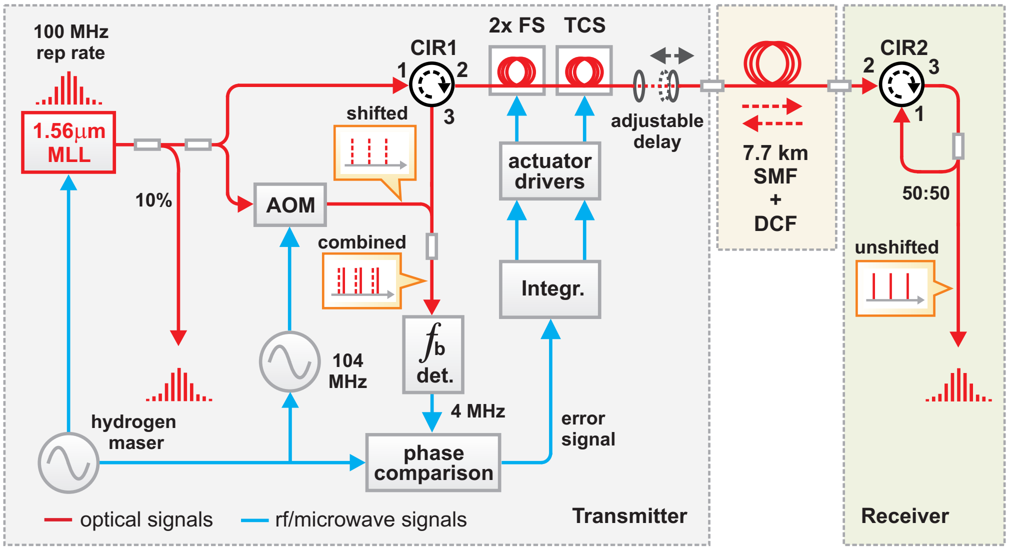

Fig. 1. Experimental setup. The detection stage (f_b det.) consists of a photodiode, a filter and cascaded amplifiers. MLL: mode-locked laser; CIR: circulator; FS: fiber stretchers; TCS: thermally controlled spool; Integr.: integrator; AOM: acousto-optic modulator; SMF: single-mode fiber; DCF: dispersion compensating fiber.

2. Fiber noise suppression principle

The fiber phase noise cancellation principle, which is related to one previously used for the transfer of ultra-short pulses over 60 m of fiber [22], is illustrated in Fig. 1. A commercial 1.56 μm amplified erbium-doped mode-locked fiber laser (MLL) generates sub-150 fs optical pulses at a repetition rate $f_\mathrm{r} = 100$ MHz. In the frequency domain this pulse train corresponds to a frequency comb with approximately 10^5 optical modes extending over an optical bandwidth of approximately 100 nm. The frequency of each optical mode can be described by $f_m = m f_\mathrm{r} + f_0$ where m is an integer, f_r is the repetition rate and f_0 is the carrier-envelope offset frequency. Both f_r and f_0 are stabilized to a 10 MHz signal from a hydrogen maser. Approximately 10 m of SMF-28 fiber are used to connect the experimental setup to the laser source, which is located in a different part of the laboratory, broadening the pulse duration to approximately 17 ps before it enters the first power splitter. The 90% output of this 90:10 splitter is used to propagate the comb over 7.7 km of spooled single mode fiber (SMF) to the receiver end where a portion is returned to the transmitter end via the same fiber. Both ends of the fiber are located in the same laboratory so that the accuracy and stability of the signal that has travelled 7.7 km can be compared to the signal injected at the input of the fiber. The forward and backward travelling pulse trains from the mode-locked laser are separated using optical circulators (CIR1, CIR2) and a dispersion compensating fiber module (DCF) recompresses the pulses to a duration of less than 100 ps. A free-space delay line is adjusted to achieve appropriate temporal overlap between the local (10% output of the power splitter) and the returned pulse trains. The returned frequency comb is combined with the original comb after the latter has been frequency shifted by f_{AOM} = 104 MHz using an acousto-optic modulator (AOM) and the beat notes between their optical modes are detected with a photodiode. The total length of out-of-loop fiber in the measurement setup was a few meters, with the lengths travelled by the shifted and unshifted combs being similar in order to improve the common-mode rejection of environmental effects that could increase the measurement noise floor.

The Doppler-induced frequency shift on the optical comb mode $f_p = pf_r + f_0$, due to environmental perturbations on the fiber, can be written as

$$\delta f_p = \frac{1}{c}\frac{d[n(f_p)l]}{dt} f_p \tag{1}$$

where $n(f_p)$ and $d[n(f_p)l]/dt$ are the refractive index of the fiber and the instantaneous rate of change of the optical path length at frequency f_p, and c is the speed of light in vacuum. This frequency shift can be measured by detecting any of the beat frequencies $f_{\mathrm{b}\,p,m}$ arising from interference between the optical modes f_m and f_p of the AOM-shifted and unshifted combs respectively:

$$f_{\mathrm{b}\,p,m} = f_{\mathrm{AOM}} - (f_p - f_m) - \frac{1}{c}\frac{d[n(f_p)l]}{dt} f_p \tag{2}$$

where we have assumed that the optical frequencies generated by the mode-locked laser are constant over the round trip time so that no noise is detected due to the self-heterodyne effect. This assumption is reasonable when the self-heterodyne noise is lower than the phase noise introduced by enviromental perturbations to the fiber link, which is the case in our experiment for frequencies within the feedback bandwidth of approximately 2 kHz.

These detected beats give rise to a current from the photodetector that can be described as

$$i(t) \propto \sum_{m=m_1}^{m_2}\sum_{p=p_1}^{p_2} \cos 2\pi \left[f_{\mathrm{AOM}} - (p-m) f_r - \frac{1}{c}\frac{d[n(f_p)l]}{dt} f_p \right] t \tag{3}$$

where m_1, m_2, p_1 and p_2 define the range of comb modes that contribute to the signal. Equation (3) illustrates the gain in sensitivity achieved using optical rather than microwave phase detection techniques; the frequency shift due to changes in the optical path length is greater by a factor $f_p/f_r \sim 10^6$. In our experiment f_{AOM} was 104 MHz, f_r was 100 MHz and we chose to detect the lowest frequency beat such that $p = m+1$, in which case

$$i(t) \propto \sum_{p=p_1}^{p_2} \cos 2\pi \left[f_{\mathrm{AOM}} - f_r - \frac{1}{c}\frac{d[n(f_p)l]}{dt} f_p \right] t. \tag{4}$$

For simplicity, Eqs. (3) and (4) assume that all comb modes have the same power and interfere with the same phase offset. In reality, the residual dispersion of the fiber link means that destructive interference between the beat frequencies will reduce $i(t)$, reducing the signal-to-noise ratio of the extracted beat signal. However, this effect will be partially counteracted by variations in mode power across the spectrum, and in practice a sufficiently high signal is detected.

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The lowest frequency beat, 4 MHz in our experiment, is amplified and phase compared with a maser-referenced synthesizer, generating an error signal which, after integration, is applied to two fiber stretchers and a thermally controlled fiber spool to compensate for fast (up to a few kHz) and slow phase fluctuations respectively.

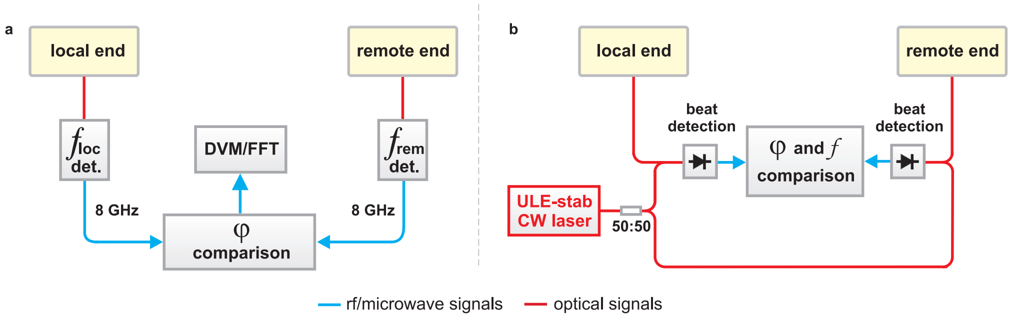

Fig. 2. Experimental setups for phase noise and frequency stability measurement of the mode spacing (a) and transferred optical modes (b). The detection stages $f_{\rm loc}$ and $f_{\rm rem}$ consist of a fast photodiode, a narrow bandpass filter and microwave amplifiers.

3. Results

To test the precision with which the full structure of the optical frequency comb can be preserved when transmitted over several-km-scale fiber links, two comb parameters must be measured: the repetition rate $f_{\rm r}$ (mode spacing) and the frequency of a selected optical mode.

The measurement of the mode spacing stability, accuracy and phase noise is performed by phase comparing the 80th harmonic (8 GHz) of $f_{\rm r}$ at the receiver end of the fiber with that detected directly at the output of the laser using a microwave mixer (Fig. 2a). The power spectral density of the phase noise fluctuations between 0.1 Hz and 100 kHz was measured with an FFT analyser. When the noise cancellation is activated, the measured phase noise is -91 dBc/Hz at 1 Hz offset from the carrier (Fig. 3a), very close to the noise measured when the 7.7 km SMF and the DCF are replaced by a 2 m fiber and an attenuator set to provide the same overall loss. We observe suppression of both the thermally induced fiber noise at low offset frequencies (by up to 20 dB) and the acoustic noise around a few hundred hertz. By integrating the phase noise between 1 Hz and 100 kHz we calculate the timing jitter to be less than 17.5 fs. The measurement of the frequency stability was achieved by converting the output voltage of the microwave mixer, logged every 0.5 s with a digital voltmeter (measurement bandwidth 7 Hz), into phase changes. The measured stability is shown in Fig. 3b. The Allan deviation of the data taken within the first two hours is 1.8×10^{-15} at 1 s and 7×10^{-17} at 100 s. Over longer periods, the signal-to-noise ratio (SNR) of the error signal degrades due to changes in the polarization of the returned optical signal with respect to that of the local signal. When the SNR falls below approximately 25 dB (in a 10 kHz bandwidth), we notice a degradation of the fractional frequency stability to 2.2×10^{-15} at 1 s and 1×10^{-16} at 100 s. This problem could be alleviated by using an optical amplifier to increase the power of the returned optical signal in order to maintain the SNR above this level. An alternative approach could be to insert an automatic polarization control system on the returned pulse train. However, even with the SNR degradation the fractional transfer stability is better than 4×10^{-17} at 1000 s, corresponding to a timing jitter of 40 fs.

To determine the stability and accuracy of the transferred optical modes a continuous-wave

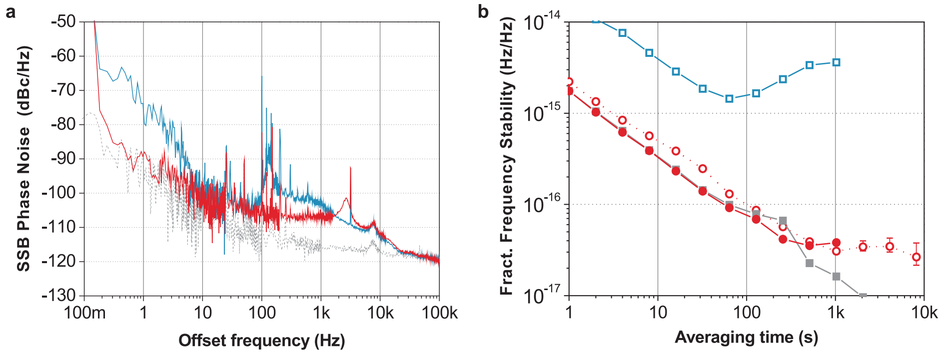

Fig. 3. Single sideband phase noise (a) and frequency stability (b) of the 80th harmonic (8 GHz) of the fundamental frequency spacing measured at the remote end. a) blue: fiber noise not suppressed; red: fiber noise suppressed; grey: measurement noise floor. b) Open blue squares: fiber noise not suppressed; open red circles: fiber noise suppressed; filled red circles: fiber noise suppressed, first two hours; filled grey squares: measurement noise floor.

(CW) 1542 nm laser stabilized to an ultra-low-expansion glass optical cavity is used as a common reference against which a selected optical comb mode frequency is measured before and after the fiber link (Fig. 2b). At each end two 35 MHz beats are generated between a selected comb line and the CW laser. At the remote end, in order to achieve a suitably high SNR of the beat, an erbium-doped optical amplifier is used at port 3 of CIR2. A frequency comb extending over 30 nm and with an average power of 2 mW is available at the output of the amplifier. The broadband noise of the 35 MHz beats is filtered using tracking oscillators with a bandwidth of approximately 200 kHz. Any difference observed between the two beat frequencies arises from the fiber noise since changes in the frequency of the CW laser are common mode. The lengths of the fibers linking the local end, the remote end and the CW laser to the photodetectors were approximately 1 m in each case. The power spectral density of the phase fluctuations between the two beat frequencies (and hence of the transferred optical mode) is measured with a digital phase detector followed by an FFT analyser. The digital phase detector has a linear range extending over 256π, sufficient to detect correctly the fast and slow phase fluctuations. As shown in Fig. 4a, the fiber-induced phase noise is reduced by up to 55 dB when the phase noise suppression loop is activated and is -40 dBc/Hz at 1 Hz offset from the carrier. Above 10 kHz, bumps in the phase noise are visible due to self-heterodyne detection of the mode-locked laser noise. The timing jitter measured from 1 Hz to 100 kHz is 5.2 fs corresponding to a phase change of approximately 2π. The transfer stability is measured in two ways: by phase comparing the two beats with the digital phase detector and by synchronously counting their frequencies. The difference between the two methods is the measurement bandwidth (7 Hz for phase comparison and approximately 200 kHz for frequency counting). The frequency stability calculated from the phase data is shown in Fig. 4b and is 4×10^{-17} at 1 s and approximately 2×10^{-18} for timescales of a few thousand seconds (corresponding to a timing jitter smaller than 10 fs). The mean frequency offset between the two beats corresponds to a transfer accuracy for the optical mode frequency of 2.6×10^{-18}. When a 200 kHz measurement bandwidth is used, the frequency stability is 5×10^{-15} at 1 s and reaches a few parts in 10^{18} at a few thousand seconds. The calculated accuracy in this case is 2.9×10^{-18}. We achieve these results despite the fact that we use only a single actuator to compensate for changes in both f_0 and f_r. Even better performance could potentially be achieved if a more sophisticated

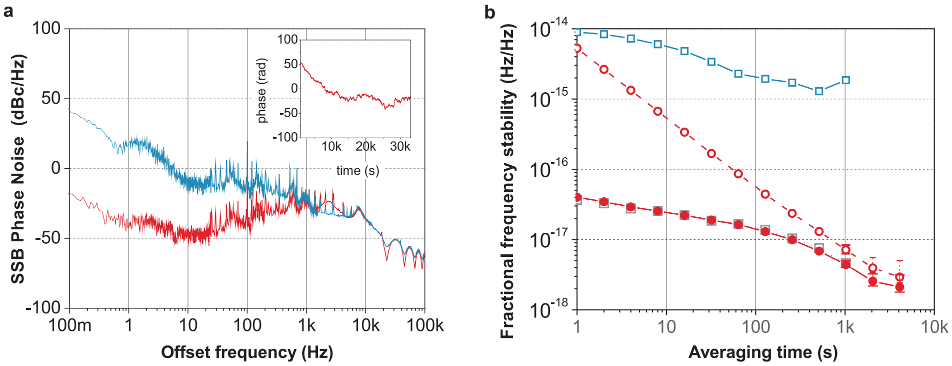

Fig. 4. Phase noise (a) and frequency stability (b) of the sample optical mode at the remote end. a) suppression inactive (blue); active (red). Inset: phase evolution over time of the optical mode when the suppression is active. b) Without (open blue squares) and with noise suppression, 7 Hz (filled red circles) and 200 kHz (open red circles) measurement bandwidth; open grey squares: measurement noise floor.

control system, such as that proposed in [18], were used to compensate for changes in group delay dispersion. Although the accuracy of the transferred repetition rate is worse than that of the optical modes because of noise introduced in optical-to-microwave conversion [23], it is still sufficient to demonstrate that the stability and accuracy measured for a single optical mode is preserved over the transferred optical bandwidth BW_{opt} (approximately 4 THz) since $(\delta f_r/f_r \times BW_{opt})1/f_{opt} \approx 8 \times 10^{-19}$. We also note that the actual stability delivered to a user at the remote end would actually be better than we measured with the frequency counter as no self-heterodyne noise would be present.

4. Conclusion

These results demonstrate that it is possible to preserve the overall structure of an optical frequency comb when it is transferred over several-km lengths of optical fiber, at a level better than the best optical frequency standards currently available. We measure the stability of the optical frequency of a sample mode of the comb to be comparable to that achieved when only a single optical frequency is transferred [13–17] rather than the tens of thousands transferred in our experiment. This frequency comb transfer technique could potentially be used over much longer distances (>100 km) by adding optical amplifiers, possibly bidirectional, to compensate for the extra loss of the fiber. We note that, employing a different phase detection technique, we have previously transferred a microwave frequency over 86 km of installed fiber using a frequency comb [20]. Microwave stability similar to that demonstrated in this paper was achieved in that earlier work, although the stability of optical modes was not measured. As for other frequency transfer techniques, the performance could be degraded by the longer round trip time which limits the fiber noise cancellation bandwidth. However, the degree of degradation will depend on the measurement bandwidth required by the user. For a similar measurement bandwidth to that used in this experiment (7 Hz), transfer over many hundreds of km would be possible.

The results make it possible to envisage the dissemination of highly stable and accurate optical frequency combs from metrology laboratories to other remotely located users. Simultaneous distribution to multiple users can be implemented easily and at low cost since only one AOM is required regardless of the number of fiber links. The availability of a wide comb of optical frequencies with stability and accuracy matching that of state-of-the-art optical clocks and, equivalently, an optical pulse train with ultra-low timing jitter could enable researchers from different scientific areas to devise new experiments and further extend the applications of optical frequency combs in the years to come.

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