Nonlinear output feedback and periodic disturbance attenuation for setpoint tracking of a combustion engine test bench

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Abstract

The quality of control actions depends strongly on the availability and the quality of signals to construct the controller. While most control design tools assume all states, hence signals, are measurable, this is often unrealistic. An observer is often necessary to provide signals to use in controller realization. This paper proposes the construction of a reduced order observer and an output feedback controller to solve a set point tracking problem of a combustion engine test bench modeled as an extended Hammerstein system. The asymptotic convergence of the observer is shown and separation principle is also proved. Because in real practice the measured signals are often affected by periodic disturbance due to the combustion oscillations, the controller is extended with an internal model based filter, to remove the effect of the periodic disturbance. Some simulation results are presented, comparing the performance of the proposed output feedback with the state feedback controller.

Key words: Automotive control; Combustion engine test bench; Internal model based filter; Nonlinear control; Observers; Output feedback; Setpoint tracking.

1 Introduction

Most feedback stabilization problems for nonlinear systems are theoretically solved using state feedback approach, assuming that all states are available from measurement. However this is often unrealistic in practice as very often only some of the states are measured. Thus the use of state observer and output feedback controller is inevitable for the controller realization (Abur and Exposito, 2004; Dunn et al., 2004). While there are various observer design techniques for linear systems, such as Kalman filter, Luenberger observer or Newton observer, the design of observer as well as output feedback controller for various classes of nonlinear systems is still a challenge. The separation principle as it works for linear systems does not always hold for nonlinear systems, which makes the design process more complicated (see e.g. Isidori and Astolfi (1992) where two Hamilton Jacobi equations have to be solved or Krstić et al. (1995) where observer backstepping is applied). Moreover, while there are ample tools to design a state feedback controller (see Khalil (1996)) and a state observer separately, proving the convergence of the combination, thus proving the nonlinear equivalence of the separation principle, is still an open challenge.

In this paper, a speed tracking problem for a combustion engine test bench is studied. To model the test bench dynamics, we exploit a structured class of nonlinear systems, namely the *extended* Hammerstein systems (Gruenbacher, 2005; Gruenbacher et al., 2008). This model is rather different from the commonly used models that mainly rely on employing engine maps to represent nonlinearities (Guzzella and Amstutz (1998); Kiencke and Nielsen (2005); Ohyama (2001)). A model built based on engine maps is in general not suitable for a nonlinear analytical feedback control design, whereas the extended Hammerstein model eases up this obstacle.

A standard Hammerstein system consists of a static nonlinearity followed by a dynamic linear system. In the extended Hammerstein structure, the linear part is replaced by a higher order polynomial function. This class of systems allow to describe the dynamical behavior of a combustion engine test bench, which has taken an increasingly important role in engine development (see

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Carlucci et al. (1984); Guzzella and Amstutz (1998); Outbib et al. (2006) and references therein). Such a test bench is mainly used for tracking given load patterns where the reference trajectory is often defined by a sequence of operating points. In this study, a diesel engine test bench is considered, taking the torque of the dynamometer and the accelerator pedal as the inputs, to control the speed of the engine and the torque of the shaft. We assume a very stiff shaft connection and trying to construct the torque at the engine flywheel.

The test bench model consists of four state variables; the angular velocity of the engine, the angular velocity of the dynamometer, the torsion angle and the engine torque. To solve the speed tracking problem, a Lyapunov based state feedback controller is first constructed to stabilize all the operating points in a given range. While the information of each state is necessary for the state feedback controller construction, of the four state variables, only the two angular velocities are measured directly by sensors. Therefore, to substitute the unmeasured states, a reduced order observer is constructed. We prove the convergence of the observer by showing the convergence of the observation error and also show that separation principle holds, to make sure that the state estimates produced by the observer can be employed to replace the state feedback controller with an output feedback.

Another problem in combustion engine control is the measurement noise. In practice, the batch behavior of the combustion that depends on the crankshaft angle (see Schmidt and Kessel (1999)) causes a combustion oscillation which is considered as a periodic noise to the engine speed. To suppress the combustion oscillation in order to eliminate its effects to the feedback loop, we apply a filter that involves an internal model of the combustion oscillations, which can be modeled as an exosystem (Gruenbacher and Marconi, 2009).

The contributions of this paper are two folds. First, we propose an observer design to be used in constructing an output feedback controller for the setpoint tracking. Second, we introduce a technique to attenuate the periodic disturbance due to combustion oscillation that affects the available measured signals. Simulations are carried out to test the performance of the observer and the filter in solving the setpoint tracking problem of the speed and the torque of the engine test bench.

2 Preliminaries

2.1 Notation and definitions

The set of real numbers is denoted by $\mathbb{R}$. A function $\alpha : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, strictly increasing and zero at zero. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. We often drop the arguments of a function whenever they are clear from the context.

Consider a general input affine nonlinear system

$$\dot{x} = f(x) + g(x)u \ , \quad y = h(x) \ , \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input and $y \in \mathbb{R}^p$ is the output. The functions f, g and h are smooth and $f(0) = 0$. If the input u is a state feedback controller, we write the closed-loop system of (1) as

$$\dot{x} = \tilde{f}(x) \ , \quad y = h(x) \ . \tag{2}$$

We use the following definitions throughout the paper.

Definition 2.1 *[Asymptotic stability] A continuous and differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ is called an asymptotic stability (AS) Lyapunov function for the system (2) if there exist class $\mathcal{K}_\infty$ functions $\alpha_1(\cdot)$, $\alpha_2(\cdot)$ and $\alpha_3(\cdot)$ such that the following holds*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \ , \tag{3}$$

$$\frac{\partial V}{\partial x}\tilde{f}(x) \leq -\alpha_3(|x|) \ , \tag{4}$$

for all $x \in \mathbb{R}^n$. ■

Definition 2.2 *[Asymptotic stabilizability] A nonlinear system (1) is asymptotically stabilizable by means of a state feedback if there exists a state feedback controller $u = u(x)$, such that the closed-loop system (2) with control u is asymptotically stable.* ■

Consider another dynamical system

$$\dot{z} = \Gamma(z, y, u) \ , \quad \hat{x} = \gamma(z, y, u) \ , \quad z \in \mathbb{R}^l \ . \tag{5}$$

Definition 2.3 *[Asymptotic observer] The system (5) is an asymptotic observer for (1) if for any $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $z \in \mathbb{R}^l$ the estimation state $\hat{x}$ asymptotically converges to the estimated state x. If $\hat{x} = z$, the system (5) is called an identity observer. Moreover, the system (1) is called asymptotically observable if it possesses an asymptotic observer.* ■

Definition 2.4 *[Uniform observability] A nonlinear system (1) is called uniformly observable if the observability of the system is independent of the input.* ■

2.2 Engine test bench model

A schematic diagram of the combustion engine test bench is illustrated in Figure 1. The test bench consists of two main power units, which are connected via a shaft. The main parts of such a dynamical engine test bench are the dynamometer, the connection shaft and the combustion engine itself. One of the design objectives for a dynamical engine test bench control is to stabilize the engine torque and the engine speed.

Considering the torque of the combustion engine, T_E, and the air gap torque of the dynamometer, T_{DSet}, as the inputs to the mechanical part of the engine test bench,

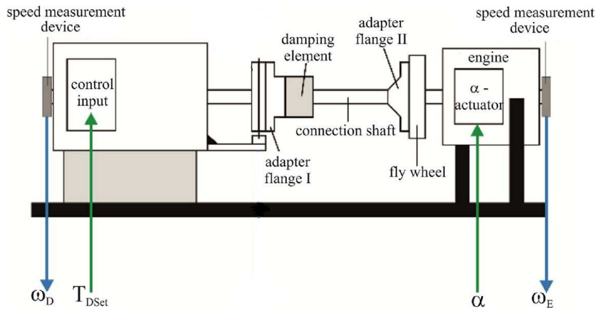

Fig. 1. The combustion engine test bench system

the model description can be reduced to a lumped engine connected to the dynamometer inertia by a damped torsional flexibility (see Kiencke and Nielsen (2005)). This mechanical part can then be modeled as a two mass oscillator

$$\dot{\varphi}_\Delta = \omega_E - \omega_D \tag{6}$$
$$\theta_E \dot{\omega}_E = T_E - c\varphi_\Delta - d(\omega_E - \omega_D) \tag{7}$$
$$\theta_D \dot{\omega}_D = c\varphi_\Delta + d(\omega_E - \omega_D) - T_{DSet} \ , \tag{8}$$

with φ_Δ the torsion angle, ω_E the engine angular velocity, ω_D the dynamometer angular velocity, θ_E and θ_D are the inertias of the engine and the dynamometer, respectively. The constant c characterizes the stiffness of the connection shaft and d represents the damping.

The dynamics of the combustion engine part is more complicated. The input to the combustion engine comes from the accelerator pedal α and the output that is important for the control purposes is the engine torque T_E. This engine torque can be divided into two parts: the cycle average value of the engine torque and the oscillating torque caused by the combustion oscillations. In the model we consider only the cycle average value, because the frequency of the oscillating torque is within the range that is sufficiently damped by the test bench system.

We consider a simplified model of the engine torque consisting of a static nonlinear map along with a nonlinear first order dynamical system represented as

$$\dot{T}_E = -\rho(T_{EStat}, \omega_E) T_E + \rho(T_{EStat}, \omega_E) T_{EStat}, \tag{9}$$

where T_{EStat} is the output of the static engine map ($T_{EStat} = S_{EM}(\alpha, \omega_E)$) and $\rho(T_{EStat}, \omega_E)$ is a nonlinear map that depends on the operating point. A polynomial approximation of ρ gives

$$\rho(T_{EStat}, \omega_E) \approx (c_0 + c_1\omega_E + c_2\omega_E^2) + \rho_\Delta(T_{EStat}, \omega_E),$$

with ρ_Δ containing all the terms of the polynomial that contain T_{EStat}. Defining a nonlinear static map m as

$$m(\omega_E, T_E, \alpha) := -\rho_\Delta(T_{EStat}, \omega_E) T_E + \rho(T_{EStat}, \omega_E) T_{EStat},$$

we can rewrite (9) as a class of extended Hammerstein model (see Gruenbacher (2005) for the detail modeling)

$$\dot{T}_E = -(c_0 + c_1\omega_E + c_2\omega_E^2) T_E + m(\omega_E, T_E, \alpha) \ . \tag{10}$$

To shorten the notation, we define $\tilde{\rho}(\omega_E^2) := (c_0 + c_1\omega_E + c_2\omega_E^2)$ in the rest of the paper. From the continuity of m in time and in its all arguments, without lose of generality, we assume that it is locally Lipschitz with respect to T_E. This assumption is valid as continuity is a necessary condition for Lipschitzity. The continuity of m is as a consequence of the continuity of $\rho(T_E, \omega)$ as shown in (Gruenbacher, 2005, Figure 8.12).

3 Observer design for the engine test bench

Consider the dynamical model of the test bench (6)-(10). Of the four states appearing in the model, only the two angular velocities ω_E and ω_D are measured. Therefore the output equations of the system are

$$y_1 = \omega_E \ , \qquad y_2 = \omega_D \ . \tag{11}$$

The control problem of an engine test bench usually involves torque control. Therefore it is very useful to include the torque signal T_E in the feedback loop. Because this quantity is not available from direct measurement, an observer is required to estimate the states T_E. It is the same case for the torsion angle φ_Δ. The following theorem proposes a reduced order observer construction, with the detail of the construction given in the proof.

Theorem 3.1 *Given the dynamical model of an engine test bench system (6)-(10) with the measured outputs (11). The following reduced order observer*

$$\begin{aligned} \dot{\hat{T}}_E &= -\tilde{\rho}(\omega_E^2)\hat{T}_E + m(\omega_E, \hat{T}_E, \alpha) + L_1 e_1 \\ \dot{\hat{\varphi}}_\Delta &= \omega_E - \omega_D + L_2 e_2 \ , \end{aligned} \tag{12}$$

where $L_2 > 0$, $L_1 > L_m - \tilde{\rho}(\omega_E^2)$, with $L_m > 0$ the Lipschitz constant of m and

$$e_1 = \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet} - \hat{T}_E \tag{13}$$
$$e_2 = \frac{1}{c}(\theta_D\dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet} - c\hat{\varphi}_\Delta) \ , \tag{14}$$

is an asymptotically stable observer for the system. ■

Proof of Theorem 3.1: Given the system (6)-(10) with outputs (11) and the reduced order observer (12). We define the estimation errors as

$$e_1 := T_E - \hat{T}_E \quad \text{and} \quad e_2 := \varphi_\Delta - \hat{\varphi}_\Delta. \tag{15}$$

First, we will show that the error terms satisfy (13),(14). By adding (7) and (8), we obtain T_E as:

$$T_E = \theta_E\dot{\omega}_E + \theta_D\dot{\omega}_D + T_{DSet}. \tag{16}$$

Moreover, φ_Δ can directly be obtained from (8), as:

$$\varphi_\Delta = \frac{1}{c}\Big(\theta_D\dot{\omega}_D - d(\omega_E - \omega_D) + T_{DSet}\Big) \ . \tag{17}$$

Substituting (16) and (17) into (15), we obtain (13) and (14), respectively. We can now write the error dynamics

$$\dot{e}_1 = \dot{T}_E - \dot{\hat{T}}_E = -\tilde{\rho}(\omega_E^2)e_1 + m(\omega_E, T_E, \alpha) - m(\omega_E, \hat{T}_E, \alpha) - L_1 e_1,$$
$$\dot{e}_2 = \dot{\varphi}_\Delta - \dot{\hat{\varphi}}_\Delta = -L_2 e_2.$$

To show the asymptotic stability of the error system, we use the Lyapunov function $V = \frac{1}{2}e^\top e$, with $e = [e_1 \; e_2]^\top$. Obviously,

$$\dot{V} = e_1\dot{e}_1 + e_2\dot{e}_2 \; . \tag{18}$$

From the local Lipschitzity of m with respect to T_E, there is a constant $L_m > 0$ such that

$$\left| m(\omega_E, T_E, \alpha) - m(\omega_E, \hat{T}_E, \alpha) \right| \le L_m \left| T_E - \hat{T}_E \right| = L_m \, |e_1| \, .$$

Hence, we have

$$\begin{aligned} \dot{V} &< -\tilde{\rho}(\omega_E^2)e_1^2 + L_m e_1 \, |e_1| - L_1 e_1^2 - L_2 e_2^2 \\ &< -\tilde{\rho}(\omega_E^2)e_1^2 + L_m e_1^2 - L_1 e_1^2 - L_2 e_2^2 \\ &= -\Big(\tilde{\rho}(\omega_E^2) + L_1 - L_m\Big)e_1^2 - L_2 e_2^2 \le -Le_1^2 - L_2 e_2^2. \end{aligned}$$

The existence of $L > 0$ is guaranteed by choosing L_1 such that $\tilde{\rho}(\omega_E^2) + L_1 > L_m$ for all ω_E. Therefore $\dot{V}$ is negative definite, thus it is proved that the observer (12) is an asymptotically stable observer for the system (6)-(10) with outputs (11). ■

Remark 3.1 *Note that the involvement of the derivative of the measured signals in the error equations (13), (14) is a standard feature of a reduced order observer, even in linear case (Franklin et al., 2010; Ogata, 2008). In practice, a high pass filter is commonly used to get this derivative while avoiding high frequency noises. Moreover, one may think that (16),(17) could be used directly instead of using the observer. While this is possible, as discussed in Laila and Grünbacher (2008), the performance of this so called "static observer" is quite poor.* ■

3.1 Separation Principle

Given a state feedback control u for the system (6)-(10), with $u(0) = 0$. To guarantee that the estimates $\hat{T}_E$ and $\hat{\varphi}_\Delta$ can replace the unmeasured states T_E and φ_Δ in the output feedback control construction, a separation principle must hold. For this, asymptotic stabilizability and uniform observability of the system with respect to the observer are required. Proposition 1 states the conditions for which the separation principle holds. The proof follows closely the proof of (Laila and Grünbacher, 2008, Proposition 3.2)

Proposition 1 *[Separation Principle] Consider the system (6)-(10). A continuous state feedback controller $u(t) = u(T_E, \varphi_\Delta, \omega_E, \omega_D)$ is an asymptotically stabilizing controller for the system. The asymptotic stabilization for the system using an output feedback $u = \hat{u}(\hat{T}_E, \hat{\varphi}_\Delta, \omega_E, \omega_D)$ with the observer (12) is solvable if the closed-loop system is uniformly observable.* ■

4 Set point tracking using output feedback

4.1 Output feedback controller design

We have now established that separation principle is valid for the state feedback controller and the observer. Hence, we can use the state estimates to construct an output feedback controller for the engine test bench. In Laila and Grünbacher (2007) we have designed a controller that guarantees asymptotic stability for a setpoint tracking problem of the test bench within a closed operating range. The construction follows the robust controller design proposed in Gruenbacher et al. (2008) that satisfies some robust optimal design criteria, via a model transformation approach, as briefly described next.

We define the state normalization as follows

$$\begin{aligned} x_1 &= \frac{T_E - T_{E0}}{\Delta T_E}, \quad x_2 = \frac{\varphi_\Delta - \varphi_{\Delta 0}}{\max(\varphi_\Delta)}, \\ x_3 &= \frac{\omega_E - \omega_{E0}}{\Delta \omega_E}, \quad x_4 = \frac{\omega_D - \omega_{D0}}{\Delta \omega_D}, \end{aligned} \tag{19}$$

with T_{E0}, $\varphi_{\Delta 0}$, ω_{E0} and ω_{D0} define the operating point and ΔT_E, $\max(\varphi_\Delta)$, $\Delta\omega_E$ and $\Delta\omega_D$ the maximum expected distance from the equilibrium point. With this scaling and taking $c\max(\varphi_\Delta) = \Delta T_E$ and $\Delta\omega_E = \Delta\omega_D$, the system (6)-(10) can now be represented in its normalized form as follows

$$\begin{aligned} \dot{x}_1 &= -(\tilde{c}_0 + \tilde{c}_1 x_3 + \tilde{c}_2 x_3^2)x_1 + u_1 \\ \dot{x}_2 &= b(x_3 - x_4) \\ \dot{x}_3 &= \frac{1}{\theta_E}\Big(\frac{c}{b}x_1 - \frac{c}{b}x_2 - d(x_3 - x_4)\Big) \\ \dot{x}_4 &= \frac{1}{\theta_D}\Big(\frac{c}{b}x_2 + d(x_3 - x_4)\Big) + u_2 \; , \end{aligned} \tag{20}$$

with $\tilde{c}_0 := c_0$, $\tilde{c}_1 := \Delta\omega_E(c_1 + 2c_2\omega_{E0})$, $\tilde{c}_2 := c_2\Delta\omega_E^2$, $b := \Delta\omega_E$. The inputs u_1 and u_2 are

$$\begin{aligned} u_1 &= \frac{m(\omega_E, T_E, \alpha) - m(\omega_{E0}, T_{E0}, \alpha_0)}{\Delta T_E}, \\ u_2 &= -\frac{T_{DSet} - T_{D0}}{\theta_D \Delta\omega_D}. \end{aligned}$$

The common control Lyapunov function used for designing the controller is

$$W(x_1, x_2, x_3, x_4) = k_1 x_1^2 + k_2 x_2^2 + k_3 x_3^2 + k_4 x_4^2 + k_5 x_2 x_4 \; ,$$

with $k_i \in \mathbb{R}^+$, $i = 1 \cdots 4$ and $k_5 \in \mathbb{R} - \{0\}$. The positive definiteness of $W(\cdot)$ is guaranteed within the considered operating range for some k_5 with $|k_5|$ sufficiently small. The controller takes the form

$$u = -[Rg(x)]^\top \left[\frac{\partial W(x)}{\partial x}\right]^\top , \tag{21}$$

where g is the input function matrix from (1) and R is a positive definite matrix. In Gruenbacher et al. (2008), this controller has been proved to asymptotically stabilize the system.

Note that the controller (21) is designed to asymptotically stabilize the normalized model (20) of the test bench, whereas our main objective is to apply the controller to the original system (6)-(10). For this, we need to transform back the normalized model and test the stability of the tracking for the original system. From the transformation (19), we have the following relations

$$m(\omega_E, T_E, \alpha) = u_1 \Delta T_E + T_{E0}(c_0 + c_1\omega_{E0} + c_2\omega_{E0}^2) \ ,$$
$$T_{DSet} = -u_2\theta_D\Delta\omega_D + T_{D0} \ ,$$

where we have chosen $\varphi_{\Delta 0} = \frac{T_{E0}}{c}$, $T_{D0} = T_{E0}$ and $\omega_{E0} = \omega_{D0}$. The setpoint tracking aims to follow the changing of the operating point (T_{E0}, ω_{E0}) of the engine.

Replacing the unmeasured states with their estimates, and applying the transformation (19), the output feedback controller takes the form

$$\begin{aligned} m(\omega_E, \hat{T}_E, \alpha) &= -2r_1k_1(\hat{T}_E - T_{E0}) \\ &\quad + T_{E0}(c_0 + c_1\omega_{E0} + c_2\omega_{E0}^2) \\ T_{DSet} &= k_5r_2\theta_D\Delta\omega_D\frac{c\hat{\varphi}_\Delta - T_{E0}}{\Delta T_E} \\ &\quad + 2k_4r_2\theta_D(\omega_D - \omega_{D0}) + T_{D0}. \end{aligned} \tag{22}$$

4.2 Simulation results I

In this subsection, we first show by simulation the convergence of the observer in estimating T_E and φ_Δ. Further, we will apply the output feedback controller (22) to control the test bench (6)-(10). The performance of the controller (22) is compared to the state feedback controller (21) for a setpoint tracking assignment.

In the simulation we have used the engine parameters $\theta_E = 0.32$ kgm^2, $\theta_D = 0.28$ kgm^2, $d = 3.5505$ Nms/rad and $c = 1.7441 \times 10^3$ Nm/rad, which are based on a dynamic test bench with a production BMW M47D diesel engine. The coefficients of the dynamic model of the combustion engine after the normalization are $\tilde{c}_0 = 6.3466$, $\tilde{c}_1 = 3.2096$, $\tilde{c}_2 = 2.7744$. For the controller we have chosen the parameters $k_1 = 1.5686$, $k_2 = 0.00174$, $k_3 = 0.88$, $k_4 = 1.05$, $k_5 = -0.0145$ and

$$R = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & * & 0 & r_4 \\ r_3 & 0 & * & 0 \\ 0 & 0 & 0 & r_2 \end{bmatrix}$$

with $r_i > 0,\ i = 1, \cdots 4$ and $*$ can be chosen freely such that R is positive definite (note that in this case because $*$ corresponds to the zero rows of g, it may be chosen zero, although this makes R only positive semidefinite). Hence the controller takes the form

$$u(t) = -\begin{bmatrix} 2r_1k_1x_1 + 2r_3k_3x_3 \\ r_2(2k_4x_4 + k_5x_2) + r_4(2k_2x_2 + k_5x_4) \end{bmatrix}, \tag{23}$$

and we have chosen $r_1 = 1, r_2 = 0.5, r_3 = 2$ and $r_4 = 0.5$. We apply the controller for a setpoint tracking when changing the operating point (T_E, ω_E) of the engine to follow a square wave reference signal. The initial condition of the engine test bench is $(50,\ 50/c,\ 300,\ 300)$ and of the observer is $(100,\ 100/c)$. We have chosen $L_1 = 1.5$ and $L_2 = 0.05$. Although $\dot{\omega}_E$ and $\dot{\omega}_D$ are not measured, because ω_E and ω_D are continuous signals, we take their derivative to use in the construction of the observer.

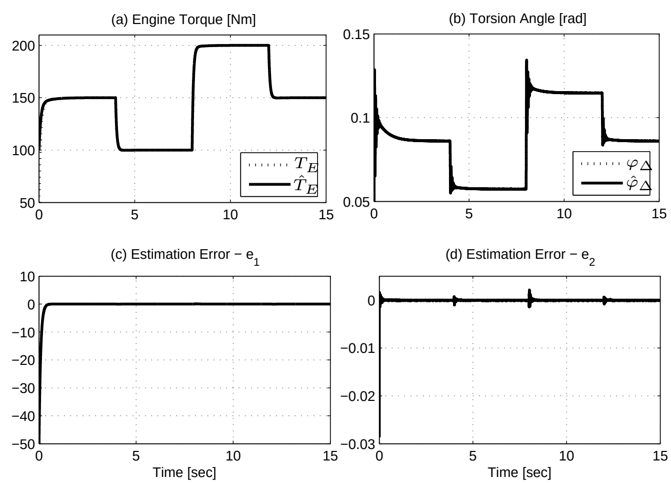

Fig. 2. Convergence test of the observer.

Figure 2 shows that the observer can estimate the unmeasured states T_E and φ_Δ very well as the responses of observer converge to the responses of the test bench very quickly, even when the initial condition of the two are very different. The response of the system with the output feedback is shown in Figure 3 which appears to almost overlap with the response with state feedback.

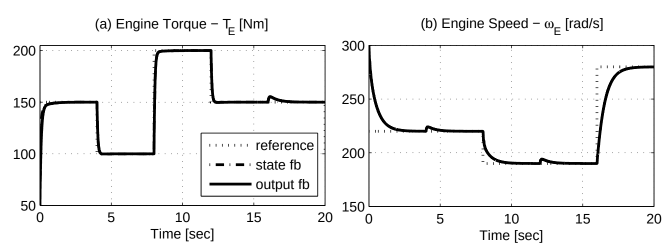

Fig. 3. Tracking using output feedback controller.

5 Filtering the periodic noise

In Section 4, the convergence of the observer and the applicability of its outputs to replace the unmeasured states in constructing the output feedback controller for the engine test bench have been demonstrated. However, this has not fully solved the issue that may arise in practical implementation. Because the model (6)-(10) is only an approximation of the highly nonlinear engine test bench dynamics, the performance limits of the actuators have to be considered. Furthermore, the measured signals ω_E and ω_D are affected by the batch behavior of the combustion that depends on the crankshaft angle (Schmidt and Kessel, 1999). Since each cylinder fires every 720° crankshaft angle (720°CA), it means for a four stroke engine a combustion occurs in every 180°CA. This creates the combustion oscillation which is considered as a periodic noise to the engine speed. While increasing the observer gains L_1 and L_2 to some extent yields a faster convergence of the observer, this unfortunately also increases the effect of the noisy speed measurement to the estimated signals, particularly as the error terms (13)-(14) depend on the derivatives of the measured speed signals. Neglecting all these sources of noise may deteriorate the quality of the generated output feedback, thus causes the closed-loop system to perform badly.

To minimize the effect of the periodic noise, a fast filter is used. The frequency of the fundamental oscillation of the noise is directly related to the engine speed and hence it is known. From the control point of view we are only interested in the cycle average value of the signals (T_E and φ_Δ), hence we need to separate the periodical part and the cycle average value part of each signal. A frequency varying internal model filter is then applied to reconstruct the estimated signals including the periodical parts. Using the states of the internal model it is then possible to calculate the cycle average value of the reconstructed signals. In the next subsections we will sketch the method and for further details we refer to Furtmüller and Grünbacher (2006) and Grünbacher et al. (2007).

5.1 Modeling the combustion oscillations via parameter varying exosystem

A combustion oscillation can be described by linear but frequency dependent harmonic oscillators

$$\dot{\omega}_i = S_i(\eta(t))\omega_i \ , \qquad d_{hi} = c_{S_i}\omega_i \ , \qquad (24)$$

with

$$S_i(\eta) = \begin{bmatrix} 0 & -i\eta(t) \\ i\eta(t) & 0 \end{bmatrix} \qquad \forall i = 1 \cdots 6 \ , \qquad (25)$$

where we consider up to the 6th harmonics, with $\eta(t)$ defines the frequency of the first harmonic of the combustion oscillations. The output maps are given by

$$c_{S_i} = [\alpha_i \ \ 0] \ \text{ or } \ c_{S_i} = [0 \ \ \alpha_i] \ . \qquad (26)$$

We assume a simple integrator

$$\dot{\omega}_0 = 0 \ , \qquad d_{h0} = \alpha_0 \omega_0 \ . \qquad (27)$$

Hence the full periodic signal with

$$\omega = [\omega_0 \ \omega_{11} \ \omega_{12} \ \cdots \ \omega_{61} \ \omega_{62}]^\top$$

can be represented as

$$\dot{\omega} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & S_1(\eta(t)) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & S_6(\eta(t)) \end{bmatrix} \omega \ , \qquad d_h = c_S \omega \ , \ (28)$$

and using the second output map in (26), we have

$$c_S = [\alpha_0 \ 0 \ \alpha_1 \ 0 \ \alpha_2 \ \cdots \ 0 \ \alpha_6] \ . \qquad (29)$$

Note that we have chosen to use the second output map for the same reason as in (Furtmüller and Grünbacher, 2006, proof of Lemma 1).

The internal model principle will be utilized to reconstruct the combustion oscillation. Usually the internal model principle is applied only for constant frequencies (Johnson, 1976). Thus, in this application the structure of the internal model description of the actual problem has to be rearranged slightly, by taking the exosystem to be parameter dependent and the internal model controller to be parameter varying. The structure of the applied internal model based filter is shown in Figure 4, in which the periodic part of the output of the observer is regarded as the output of the exosystem as explained in Remark 5.1.

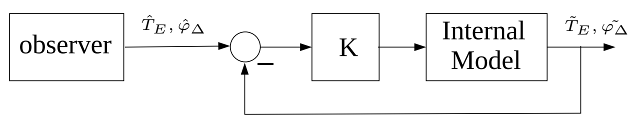

Fig. 4. Connection of the observer and the internal model filter

Remark 5.1 *Note that although the model (6)-(10) only represents the cycle average value, thus non-periodic, in reality the estimates $\hat{T}_E$ and $\hat{\varphi}_\Delta$ contain noises that include the periodical oscillation components d_h. Therefore, in the rest of the discussion we only consider this periodic combustion oscillation d_h. By slightly abusing the term, as d_h is actually part of the observer output, in this application the observer takes the place of the exosystem. Moreover, because $\hat{T}_E$ and $\hat{\varphi}_\Delta$ are treated in the same way, to avoid repetition, we only present the result for filtering $\hat{T}_E$.* ■

5.2 Design of the frequency dependent internal model

Denote the filtered engine torque by $\tilde{T}_E$. In the standard case when the oscillation frequency is constant and the exosystem is linear, the difference $(\hat{T}_E - \tilde{T}_E)$ tends to zero if the poles of the internal model are all equal to the eigenvalues of the exosystem and the internal model is controllable. In this application we extend the oscillation model (28) to get a controllable system that has the same eigenvalues (Internal Model Principle) as those of the exosystem.

The extension to the integrator subsystem comprises of adding a control input ν_0 so that

$$\dot{\xi}_0 = \nu_0 \ , \quad \tilde{d}_{h0} = \xi_0. \tag{30}$$

An input vector b_i=$[0 \ \ 1]^\top$ is also added to the oscillator subsystems so that the internal submodel takes the form

$$\begin{aligned} \dot{\xi}_i &= A_i(\eta)\xi_i + b_i\nu_i = \begin{bmatrix} 0 & -i\eta(t) \\ i\eta(t) & 0 \end{bmatrix}\xi_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\nu_i \\ \tilde{d}_{hi} &= c_{IM_i}\xi_i = [0 \ \ 1]\xi_i \ . \end{aligned} \tag{31}$$

Note that in (30)-(31) the gains α_0 and $\alpha_i, \ i = 1, \cdots, 6$, have been set equal to 1, because for the modeling purpose the magnitude of the oscillations may be assumed constant. The magnitude of the oscillation can also be defined by the initial states of the exosystem. Thus the composite internal model is

$$\begin{aligned} \dot{\xi} &= A(\eta)\xi + B\nu = A(\eta)\xi + B[\nu_0 \ \cdots \ \nu_6]^\top \\ \tilde{d}_h &= c_{IM}\xi \end{aligned} \tag{32}$$

with

$$A(\eta) = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & A_1(\eta) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & A_6(\eta) \end{bmatrix} , \quad B = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & b_1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & b_6 \end{bmatrix} ,$$

$$c_{IM} = [1 \ c_{IM_1} \ \cdots \ c_{IM_6}] \ .$$

5.3 Stabilizing parameter varying feedback controller

In this subsection we will design a converging, stabilizing controller for the internal model, aiming to get the steady state response of the internal model equal to the measured oscillation (Isidori, 1995). For a static but parameter varying feedback control law $\nu = K(\eta)e$, with $K = [\kappa_0(\eta) \ \ldots \ \kappa_6(\eta)]^\top$ and $e = \tilde{d}_h - d_h$, the closed-loop system becomes

$$\begin{aligned} \dot{\xi} &= A(\eta)\xi + BK(\eta)e \\ &= [A(\eta) - BK(\eta)c_{IM}]\,\xi + BK(\eta)d_h. \end{aligned} \tag{33}$$

According to Furtmüller and Grünbacher (2006), convergence is achieved if and only if the parameter varying closed-loop system (33) is asymptotically stable. With $c_{IM_i} = [0 \ 1]$ and $\kappa_i(\eta)$ a scalar function for all $i = 1, \ldots, 6$, the closed-loop of the oscillator subsystems becomes

$$\dot{\xi}_i = [A_i(\eta) - b_i\kappa_i(\eta)c_{IM_i}]\xi_i + b_i\kappa_i(\eta)c_{IM_i}d_{hi}. \tag{34}$$

For stability analysis we set $d_{hi} = 0$ and the system matrix of each closed-loop subsystem is

$$A_{i_{cl}}(\eta) = \begin{bmatrix} 0 & -i\eta \\ i\eta & -\kappa_i(\eta) \end{bmatrix} \quad \forall i = 1, \ldots, 6 \ . \tag{35}$$

Choosing the constant feedback gains $\kappa_i(\eta) = \tilde{\kappa}_i, \ \forall i = 1, \ldots, 6$, yields

$$A_{i_{cl}}(\eta) = \begin{bmatrix} 0 & -i\eta \\ i\eta & -\tilde{\kappa}_i \end{bmatrix} \quad \forall i = 1, \ldots, 6 \ . \tag{36}$$

Similarly for the integrator subsystem, choosing the feedback $\nu_0 = \tilde{\kappa}_0 e_0$ results in

$$\dot{\xi}_0 = -\tilde{\kappa}_0\xi_0 + \tilde{\kappa}_0 d_{h0} \ . \tag{37}$$

Hence the vector of the feedback gains of the controller that stabilizes the internal model is

$$K = [\tilde{\kappa}_0 \ \tilde{\kappa}_1 \ \cdots \ \tilde{\kappa}_6]^\top \tag{38}$$

where $\tilde{\kappa}_0$ to $\tilde{\kappa}_6$ are positive constants.

Remark 5.2 *Note that the feedback gains $\kappa_i(\eta)$ influence the convergence rate of the oscillator. Not only asymptotic stability, a fast convergence without overshoot is also importantly desirable. For each closed-loop subsystem (35), the characteristic polynomial with $\kappa_i(\eta)$ is $\Delta_i(s) = s^2 + \kappa_i(\eta)s + (i\eta)^2$. One possibility for fastest convergence without overshoot is at $\kappa_i(\eta) = 2i\eta$. However, as discussed in Furtmüller and Grünbacher (2006); Grünbacher et al. (2007), for a general application where the measured signal is always noisy and the output of the observer (and particularly the predicted output) is also noisy, if κ_i is too large, the convergence rate may become too fast (the observer tends to learn the noise). Therefore setting κ_i constant, e.g. $\kappa_i \leq 2i\eta_{min}$, is sufficient in the tracking problem considered in this paper.* ■

It is well known for linear parameter varying systems that fast changing parameters can deteriorate stability. Therefore, it is crucial to make sure that the internal model filter is stable in a given parameters range. The proof of stability of the filter follows exactly the same steps as in Grünbacher et al. (2007).

5.4 Simulation results II

In this section we will demonstrate by simulation, how the filter treats the noisy engine speed measurement. To do this, a sinusoidal periodic oscillation is introduced to the engine speed measurement. We show the effect of the internal model filter to the output feedback tracking performance. Figure 5 shows the engine torque (T_E) and the engine speed (ω_E), as well as the output feedback control signals α and T_{DSet}. In Figure 6 we show the comparison of the filtered signal using the internal model observer and a comparable Butterworth filter. It can be observed that with the internal model filter, even in a dynamic operation the estimation error of the cycle average value of the engine torque is quite small.

It is expected that the dynamics of the filter affect the dynamics of the closed-loop system. However, as the filter is designed such that stability is preserved, the effect

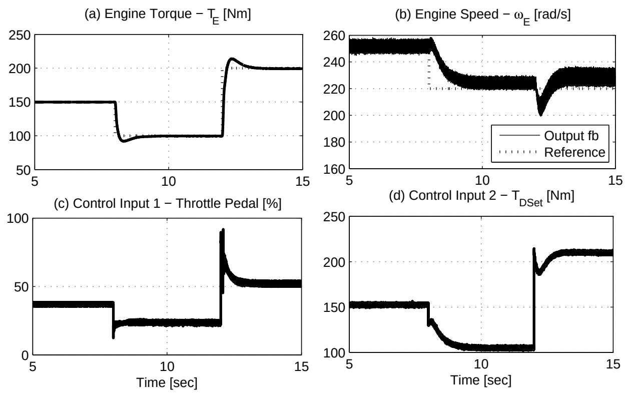

Fig. 5. Tracking using output feedback and noise filter.

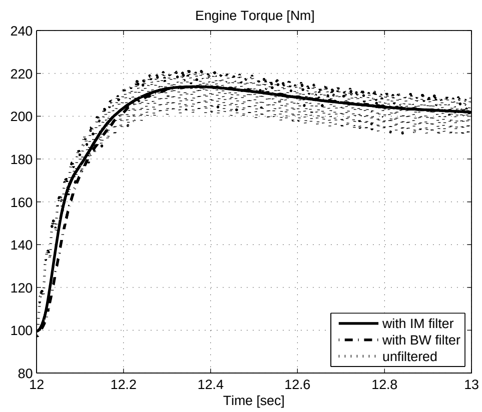

Fig. 6. Comparison of the measured and the filtered estimated torque.

occurs mainly only during the transient. In practice, to avoid a rough transient behavior, the filtered signals are therefore not applied from the start of the operation, but allowing few seconds delay until the transient is over before applying the filtered signals to the control loop.

6 Summary

In this paper we have presented a partial state observer design for a combustion engine test bench system. We have shown that the observer is asymptotically convergent to the system. We have also shown that separation principle is satisfied. We have demonstrated by simulation the performance of an output feedback controller constructed using the outputs of the proposed observer.

Moreover, as noise always involves in the real measurement, we have also discussed the use of an internal model filter to eliminate the effect of the periodic noise that is caused by the combustion oscillation. Some simulations results that illustrate a more realistic situation have also been provided.

As this current study is based only on simulation, the next challenge for this research is to implement and test the observer, controller and the filter design in an experiment, to solve the setpoint tracking problem of a real engine test bench.

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