Accepted Manuscript

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PII: S0165-1765(13)00331-5
DOI: http://dx.doi.org/10.1016/j.econlet.2013.07.007
Reference: ECOLET 6001

To appear in: *Economics Letters*

Received date: 15 April 2013
Revised date: 25 June 2013
Accepted date: 8 July 2013

Please cite this article as: Ioannou, C.A., Qi, S., Rustichini, A., A test of stability in a linear altruism model. *Economics Letters* (2013), http://dx.doi.org/10.1016/j.econlet.2013.07.007

A Test Of Stability In A Linear Altruism Model

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July 20, 2013

Abstract

Linear altruism predicts the estimated preferences to be independent of the subject's position in the game, if the role allocation is randomly determined, because subjects, in each role, have the same preferences *ex ante*. We test and reject this hypothesis.

JEL Classification: C51, C92, C72, D03

Keywords: Linear Altruism, Trust Game, QRE

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1 Introduction

Linear altruism is a functional form used extensively in outcome-based models of social preferences: the underlying assumption is that individuals have a utility over monetary outcome profiles that depends on their and other players' payments. Our objective in this exercise is to investigate the stability of the altruism parameters in the Trust game. We use the Quantal Response Equilibrium (QRE) of McKelvey and Palfrey (1995) to study first mover behavior.[1] As standard in this literature we assume that first mover beliefs are consistent with the observed probability distribution of the actions of second movers. We also study second mover behavior, which can be extrapolated without any rational expectations assumptions. Behavior in strategic interactions is explained as a Nash equilibrium of the game, where final payoffs are paid in utility units. Linear altruism and other theories of social preferences predict the estimated preferences to be independent of the subject's position in the game, if, in the experiment, the allocation to a role is randomly determined, because subjects, in each role, have the same preferences *ex ante*. Thus, a logical implication of the assumption of stability of preferences is that the estimated altruism parameters are statistically the same; that is, the weight assigned by the first mover on the second mover's payoff, estimated with the QRE approach, should be statistically indistinguishable from the weight assigned by the second mover to the first mover's payoff. Our results do not find support for this claim. In particular, we show that the representative first mover is less altruistic than the representative second mover in the second approach. We discuss plausible explanations for the discrepancy and caution researchers to accommodate for these possibilities before interpreting agents' behavior in strategic interactions.

2 Experimental Design

In the experimental session, the subjects had to play the Trust game for 15 rounds. The number of rounds to come was not communicated to the subjects. In each round, the subjects faced a different participant. With the conclusion of the experimental session, the subjects were privately paid their earnings in cash.

The Trust game is standard. One subject had the role of the first mover and the other subject had the role of the second mover. Let $m \in \{1, 2\}$ index the order of the mover, where $m = 1$

[1] QRE can be viewed as an extension of standard random utility models of discrete ("quantal") choice to strategic settings. Under this process best response functions become probabilistic. Much recent work has shown that QRE can rationalize behavior in a variety of experimental settings including: Alternating-Offer Bargaining (Goeree and Holt (2000)), Coordination games (Anderson, Goeree, and Holt (2001)), the Traveler's Dilemma (Capra, Goeree, Gomez, and Holt (1999), Goeree and Holt (2001)), All-Pay and First-Price Auctions (Anderson, Goeree, and Holt (1998), Goeree, Holt, and Palfrey (2002)).

denotes the first mover, and $m = 2$ denotes the second mover. The subjects' roles were determined by random assignment. The first mover was initially given an endowment of 4 quarters and was asked to specify an integer amount of quarters, between zero and 4 inclusive, to transfer to the second mover. Any quarters that were not transferred to the second mover were secured as profit for the first mover. Denote the amount of quarters transferred as $x \in \{0, 1, 2, 3, 4\}$. The amount transferred was multiplied by 4 before reaching the second mover; that is, the second mover received $4x$ quarters for a transfer x. If the first mover transferred 0 quarters, then the game ended. Otherwise, the second mover was asked to allocate $4x$ based on five options. Our experimental design secured that changes in the estimated parameters across movers were not affected by the cardinality of the choice set as both, first movers and second movers, had five choices to select from. Each option indicated the amount kept by the second mover, which was also the payoff of the second mover, and the corresponding profit of the first mover. The latter was provided in order to safeguard against calculation errors by subjects. The options were structured so as to provide variability in the allocation of $4x$. The first and fifth options were extreme in the sense that in the first option the second mover kept 0 quarters and the first mover got $4x$, whereas in the fifth option the allocation was flipped so that the second mover got $4x$ and the first mover got 0 quarters. The intermediate options were positioned across the two extremes. The second and fourth options distributed $4x$ unevenly, with the first mover getting the bigger portion in the second option, and the second mover getting the bigger portion in the fourth option. Finally, the third option split $4x$ more evenly across the first and second mover compared to the other options.[2] Let $y \in \{1, 2, 3, 4, 5\}$ denote the choice of the second mover. Furthermore, let π_m denote the payoff of mover m in quarters. Given a transfer x and choice y, the second mover's payoff is $\pi_2 = (y - 1) \times x$. On the other hand, the first mover's payoff is $\pi_1 = 4 + 3x - \pi_2$; that is, the first mover earns $4 - x$ from the first stage of the game and $4x - \pi_2$ from the second stage of the game. For example, assume the first mover transfers 3 quarters to the second mover, so that $x = 3$. The second mover receives $4x = 12$ quarters. Assume the second mover chooses the third option, so that $y = 3$. The second mover earns $\pi_2 = (y - 1) \times x = (3 - 1) \times 3 = 6$ quarters. The first mover gets the remaining 6 quarters; yet, this is not the payoff of the first mover who, also, secured 1 quarter in the first stage of the game. Therefore, the first mover's profit is $\pi_1 = 3x + 4 - \pi_2 = 3 \times 3 + 4 - 6 = 7$ quarters. Had the second mover chose the second option instead, so that $y = 2$, such a choice would correspond to an amount kept (payoff) by the second mover of $\pi_2 = (y - 1) \times x = (2 - 1) \times 3 = 3$ quarters, whereas the first mover would earn a payoff of $\pi_1 = 3x + 4 - \pi_2 = 3 \times 3 + 4 - 3 = 10$ quarters. The options were explicitly mentioned in the experimental instructions as well as indicated on the subjects' computer screens. The round

[2]For a transfer $x = 4$, the third option allocated 8 quarters to the first mover and 8 quarters to the second mover.

was completed with the earnings of the subject for the specific round indicated on the screen along with the cumulative earnings of the subject thus far in the game. The detailed instructions are reported in the Appendix.

The experimental sessions were conducted in the XSFS computer lab of the Florida State University in May of 2010. 16 subjects participated in each session; they were recruited from the undergraduate population of the Florida State University using an electronic recruitment system. Participants were allowed to participate in only one session. Each session lasted approximately 45 minutes. The experiments were programmed and conducted with the use of the experimental software z-Tree (Fischbacher (2007)).

Table 1 reports the descriptive statistics on the raw experimental data. Panel A presents the frequency of the transfer and the choice variables. 35% of first movers chose to transfer 0 quarters to second movers. Transfers of 1 quarter and 3 quarters were the least frequent choices of first movers. Furthermore, only 36.7% of the first movers transferred more than half of their endowment to second movers. On the other hand, 57.7% of second movers kept the entire allowable amount, whereas only 24.4% selected one of choices $y = 2, 3$. In Panel B, we show how the distribution of each choice y changes with the first mover's transfer. With the exception of 6 observations at choice $y = 2$ (for $x = 4$), all other observations for transfers greater than 1 quarter were allocated to choices $y = 3, 4, 5$. When first movers transferred only one quarter, then 100% of the second movers chose to keep the entire amount. The percentage of second movers keeping the entire allowable amount remained high at 42.9%, 60%, and 48.3% for transfers $x = 2, 3, 4$, respectively.

3 Structural Model

We assume a specific functional form of social preferences that has been used extensively in the literature to model linear altruism. We describe first the utility function of the first mover and then the utility function of the second mover. The first mover makes a choice of transfer $x \in \{0, 1, 2, 3, 4\}$. Payoffs that are incorporated into the utility function are based on the first mover's expectation over payoffs. More specifically, let the utility function of the first mover be

$$u_1^j(x) = v_1(x) + \epsilon^j(x) = (1 - w_1) \cdot E[\pi_1(x, y)|x] + w_1 \cdot E[\pi_2(x, y)|x] + \epsilon^j(x), \qquad (1)$$

where the utility of a first mover j is separated into a value that is common across all subjects $v_1(x)$ and an idiosyncratic preference shock ϵ^j. The altruism parameter w_1 is the weight a first mover j assigns to the payoff of the second mover. In addition, we assume that the idiosyncratic preference shocks are identically and independently drawn from a Type I extreme value distribution. Denote, next, the first mover's belief on the probability of the second mover choosing y given a transfer x,

Table 1: Descriptive Statistics of Transfers and Choices

Panel A	Transfer x	Freq.	Percent	Choice y	Freq.	Percent
	0	252	35.0	1	0	0.0
	1	78	10.8	2	6	1.3
	2	126	17.5	3	108	23.1
	3	90	12.5	4	84	18.0
	4	174	24.2	5	270	57.7
Total		720			468	

Panel B Choice y \ Transfer x	1		2		3		4	
	Freq.	Percent	Freq.	Percent	Freq.	Percent	Freq.	Percent
1	0	0.0	0	0.0	0	0.0	0	0.0
2	0	0.0	0	0.0	0	0.0	6	3.5
3	0	0.0	42	33.3	24	26.7	42	24.1
4	0	0.0	30	23.8	12	13.3	42	24.1
5	78	100.0	54	42.9	54	60.0	84	48.3

Notes: In Panel A, we provide the frequencies and percentages of each transfer x and choice y. If the first mover transfers 0 quarters, then the game ends; thus, the frequency of choice y is conditional on a transfer $x > 0$. In Panel B, we provide the frequencies and percentages of choices for each transfer amount x.

as $\rho(y|x)$. Thus, the expected payoff of the first and second movers for a given transfer x is

$$E[\pi_m(x,y)|x] = \sum_{y=1}^{5} \rho(y|x)\pi_m(x,y) \quad \text{for } m = 1, 2.$$

The choice probability of the first mover choosing transfer $x = 0, 1, 2, 3, 4$ is

$$\mathbb{P}_1(x) = \frac{\exp(v_1(x))}{\sum_{k=0}^{4} \exp(v_1(k))}. \tag{2}$$

On the other hand, the utility function of a second mover i has the functional form

$$u_2^i(x,y) = v_2(x,y) + \varepsilon^i(x,y) = (1 - w_2) \cdot \pi_2(x,y) + w_2 \cdot \pi_1(x,y) + \varepsilon^i(x,y). \tag{3}$$

Parallel to the first mover's utility specification, the second mover's utility function consists of a value that is common across all subjects $v_2(x,y)$ and an idiosyncratic preference shock ε^i. The common value $v_2(x,y)$ can be divided further into a subject i's own payoff π_2 and the paired first mover's payoff π_1. The altruism parameter w_2 is the weight a second mover i assigns to the payoff of the first mover. In addition, the idiosyncratic preference shocks are identically and independently drawn from a Type I extreme value distribution. The latter assumption yields the convenient logit choice specification

$$\mathbb{P}_2(y|x) = \frac{\exp(v_2(x,y))}{\sum_{k=1}^{5} \exp(v_2(x,k))} \quad \forall y \in \{1,2,3,4,5\}. \tag{4}$$

Estimation Technique

Our estimation techniques require the use of maximum likelihood to estimate the altruism parameters w_1 and w_2. Recall that the choice probability of first movers is given by (2); that is,

$$\mathbb{P}_1(x|w_1) = \frac{\exp((1-w_1)\cdot E[\pi_1(x,y)|x] + w_1 \cdot E[\pi_2(x,y)|x])}{\sum_{k=0}^{4} \exp((1-w_1)\cdot E[\pi_1(k,y)|k] + w_1 \cdot E[\pi_2(k,y)|k])}.$$

To calculate the expected payoffs, we use the QRE approach, which assumes that the beliefs of first movers are consistent with the observed probability distribution of the actions of second movers; that is, $\rho(y|x) = \frac{n_{y|x}}{n_x}$, where n_x is the observed number of occurrences of some transfer x, and $n_{y|x}$ is the number of observed occurrences of choice y given a transfer x.

The likelihood function is then

$$\mathbb{L}_1 = \prod_x \mathbb{P}_1(x|w_1)^{n_x},$$

and the log-likelihood function is

$$\widetilde{\mathbb{L}}_1 = \sum_x n_x \log \mathbb{P}_1(x|w_1).$$

Thus, we calculate w_{1*} so as to maximize the above likelihood function.

On the other hand, the choice probability of second movers is given by (4); that is,

$$\mathbb{P}_2(y|x,w_2) = \frac{\exp((1-w_2)\cdot \pi_2(x,y) + w_2 \cdot \pi_1(x,y))}{\sum_{k=1}^{5} \exp((1-w_2)\cdot \pi_2(x,k) + w_2 \cdot \pi_1(x,k))}.$$

Suppose we observe $n_{y|x}$ occurrences of choice y given transfer x; then, the likelihood function is

$$\mathbb{L}_2 = \prod_x \prod_y \mathbb{P}_2(y|x,w_2)^{n_{y|x}},$$

and the log-likelihood function is

$$\widetilde{\mathbb{L}}_2 = \sum_x \sum_y n_{y|x} \log \mathbb{P}_2(y|x, w_2).$$

We calculate w_{2*} so as to maximize the above likelihood function. Crucially, second mover behavior can be extrapolated without any *a priori* rational expectations assumptions in sharp contrast to first mover behavior as indicated above.

Testable Hypotheses

Recall that our experimental design imposes a random draw on the order of subjects in each period. Therefore, stability dictates that subjects approach the game with the same (that is, independent of the allocation to roles) *ex ante* preferences over monetary outcome profiles. Given this insight, we formulate next our testable hypotheses. The Null hypothesis is that the linear altruism model can explain in a compatible manner both, first and second mover behavior in the Trust game. In particular, the weight assigned by the first mover on the second mover's payoff w_1, estimated with the QRE approach, is statistically the same with the weight assigned by the second mover to the first mover's payoff w_2.

Null Hypothesis: $w^1 = w^2$

Alternative Hypothesis: $w^1 \neq w^2$

Accepting the Null hypothesis would provide evidence to support the specific functional form used to model linear altruism in the Trust game. Otherwise, rejecting the Null hypothesis would necessitate the need to look at alternative explanations to justify the discrepancy.

4 Results

In this section, we present the important results. The standard errors of the estimates are clustered at the individual level and are reported in parentheses. First, we estimate the first mover's weight w_1. We find that a first mover, on average, attaches a weight $w_{1*} = 0.018$ on the payoff of the respective second mover and $1 - w_{1*} = 0.982$ on his own payoff. The standard error is 0.0132. The weight w_{1*} is not significantly different from zero. In particular, the utility function of the first mover is

$$v_1(x) = \underset{(0.0132)}{0.018} \cdot E[\pi_2(x,y)|x] + \underset{(0.0132)}{0.982} \cdot E[\pi_1(x,y)|x].$$

On the other hand, for second movers, the estimated weight w_{2*} is 0.354 with a standard error of 0.053. Thus, the utility function of the second mover is

$$v_2(x,y) = \underset{(0.053)}{0.354} \cdot \pi_1(x,y) + \underset{(0.053)}{0.646} \cdot \pi_2(x,y),$$

which implies that second movers attach a strictly positive weight to a first mover's payoff. The estimated parameters are, at a 99% level, significantly different from zero. The estimation of w_{2*} allows us to approximate the second movers' conditional choice probabilities $\mathbb{P}_2(y|x,w_{2*})$. We compare this estimated choice probability with the actual observed probability $n_{y|x}/n_x$ in Table 2. We see from Table 2 that the model does a fairly good job in matching data except for the case of $x = 1$, where all subjects chose an amount kept of $y = 5$.

Table 2: SECOND MOVER CHOICE PROBABILITY $\mathbb{P}(y|x,w_{2*})$: MODEL PREDICTION VS DATA

	Transfer x							
	$x=1$		$x=2$		$x=3$		$x=4$	
Choice y	Model	Data	Model	Data	Model	Data	Model	Data
$y=1$	11.4%	0.0%	5.8%	0.0%	2.7%	0.0%	1.2%	0.0%
$y=2$	14.6%	0.0%	9.6%	0.0%	5.7%	0.0%	3.2%	3.4%
$y=3$	18.8%	0.0%	15.8%	33.3%	12.1%	26.7%	8.6%	24.1%
$y=4$	24.1%	0.0%	26.0%	23.8%	25.5%	13.3%	23.4%	24.1%
$y=5$	31.0%	100.0%	42.9%	42.9%	54.0%	60.0%	63.7%	48.3%

Notes: The table compares the estimated choice probability with the actual observed probability $n_{y|x}/n_x$. If the first mover transfers 0 quarters, then the game ends; thus, the statistics are conditional on a transfer $x > 0$.

In order to formally establish the difference between w_{1*} and w_{2*}, we present next the 95% confidence intervals of the estimates. It is clear from Table 3 that the estimated altruism parameters w_{1*} and w_{2*} have no overlap in the 95% confidence intervals. Therefore, the Null hypothesis is rejected: $w_1 < w_2$.

Table 3: CONFIDENCE INTERVALS OF THE ESTIMATED WEIGHTS

Estimated Weights	Coefficients	Confidence Intervals
w_{1*}	0.018	[-0.008, 0.044]
w_{2*}	0.354	[0.250, 0.457]

Notes: The table reports the confidence intervals of the altruism parameters at the 95% level.

5 Discussion

Preferences of subjects are unobserved but can be recovered via subjects' behavior. In our estimation, the recovered altruism parameters are different depending on the role of the subject in the game; that is, $w^1 < w^2$, which casts doubts on the stability of the model. A plausible explanation for this discrepancy can be attributed to inequity aversion (see Fehr and Schmidt (1999)) or risk aversion. For example, the CES utility function of Cox, Friedman, and Gjerstad (2007) would predict that the first mover discounts the amount sent back by the second mover, precisely, due to risk aversion. Such prediction is well in line with the results here since the estimates indicate that the first mover is less altruistic than the second mover. Yet, it is also possible that the inconsistency in the distribution of preferences is driven by intentions[3] or heterogeneous beliefs. For instance, instead of assuming that the first mover proposes a transfer to the second mover in the assumption that first and second movers have the same altruism weights, one can estimate the weight that a first mover believes a second mover to have (see for example, Rogers, Palfrey, and Camerer (2009)).

6 Concluding Remarks

Our objective in this exercise was to test the empirical validity of a specific class of utility functions that has been used extensively in the literature to model linear altruism. In its general form, an agent maximizes a weighted sum of his own payoff and the payoff of his match. We use an experimental Trust game to study the behavior of subjects and to estimate their altruism parameters. First mover behavior is studied under the Quantal Response Equilibrium (QRE), which maintains that first mover beliefs are consistent with the observed probability distribution. On the other hand, second mover behavior is extrapolated without any *a priori* rationality assumptions. A logical implication of stability is that the estimated altruism parameters are statistically the

[3]There exists an extensive literature on the role of intentions on experimental outcomes (see for example, Charness (2004)).

same, given that subjects' allocation to roles is randomly determined. We test this hypothesis on experimental data and reject it. In particular, our results indicate that the estimated weight placed on the payoff of a subject's partner depends significantly on the approach used. Under the QRE approach, a representative first mover is less altruistic than the representative second mover in the second approach. This discrepancy is alarming because researchers need to accommodate for possibilities such as inequity aversion, risk aversion, intentions, and heterogeneous beliefs prior to interpreting agents' behavior in strategic interactions.

Acknowledgments

The research was supported in part by research funds from the Florida State University, the Strategic Research Development Fund of the University of Southampton and the British Academy under grant number SG110905 to Christos A. Ioannou.

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Appendix

Experimental Instructions

This is an experiment on decision-making. The amount of money you earn will depend upon the decisions you make and the decisions other people make. Your cumulative earnings in quarters (1 quarter = 25 cents) will be the sum of your earnings from each round. At the end of the experimental session, you will be paid in cash your cumulative earnings. In addition, you will be paid a $10 participation fee. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Please do not communicate with each other during the experiment. If you have a question, feel free to raise your hand, and an experimenter will come to help you.

In the beginning of each period, you are matched with one other participant in the room. In every period you will be matched with different participants. You will never be matched with the same participant twice. Initially, your role as the first mover or as the second mover is decided by chance. In other words, you are equally likely to be selected as the first mover or as the second mover in a given period. The first mover will make a decision, and then the second mover will make a decision. In the beginning of a period, the first mover is endowed with 4 quarters. The first mover has to decide on how many quarters to transfer (0 or 1 or 2 or 3 or 4) to the second mover. The first mover keeps the quarters that he does not transfer to the second mover. The quarters that are transferred to the second mover are multiplied by a factor of 4; henceforth this is referred to as the new amount. Then, the second mover decides on how many quarters of the new amount to keep, while the remainder goes to the first mover. Depending on the transfer of the first mover, the second movers choices of Amount Kept are shown in the next pages. The appropriate screen can only be one of the following screens. This completes one period.

The profit of the second mover for the period is simply the quarters he kept from the new amount. The profit of the first mover is the quarters he chose to hold on to (and not transfer to the second mover), plus any quarters the second mover did not keep from the new amount. At the end of each period, your earnings for the specific period are indicated as well as your cumulative earnings so far in the game.

- We investigate the stability of the altruism parameters in an experimental Trust game.
- We use the Quantal Response Equilibrium (QRE) to study first mover behavior.
- We study second mover behavior, which can be extrapolated without rationality assumptions.
- Stability implies that altruism parameters are statistically the same, if a subject's allocation to a role is random.
- We test and reject this hypothesis. We also discuss plausible explanations.