Iterative Amplitude/Phase Multiple-Symbol Differential Sphere Detection for DAPSK Modulated Transmissions

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Abstract—Differentially encoded and non-coherently detected transceivers exhibit a low complexity, since they dispense with complex channel estimation. Albeit this is achieved at the cost of requiring an increased transmit power, they are particularly beneficial, for example in cooperative communication scenarios, where the employment of channel estimation for all the mobile-to-mobile links may become unrealistic. In pursuit of high bandwidth efficiency, differential amplitude and phase shift keying (DAPSK) was devised using constellations of multiple concentric rings. In order to increase resilience against the typical high-Doppler-induced performance degradation of DAPSK and/or enhance the maximum achievable error-free transmission rate for DAPSK modulated systems, multiple-symbol differential detection (MSDD) may be invoked. However, the complexity of the maximum-a-posteriori (MAP) MSDD increases exponentially with the detection window size and hence may become excessive upon increasing the window size, especially in the context of iterative detection aided channel coded system. In order to circumvent this excessive complexity, we conceive a decomposed two-stage iterative amplitude and phase (A/P) detection framework, where the challenge of having a non-constant-window size and hence may become excessive upon increasing the window size, especially in the context of iterative detection aided channel coded system. Thus, allowing the incorporation of reduced-complexity sphere detection (SD). Consequently, a near-MAP-MSDD performance can be achieved at a significantly reduced complexity, which may be five orders of magnitude lower than that imposed by the traditional MAP-MSDD in the 16-DAPSK scenario considered.

I. INTRODUCTION

DIFFERENTIAL phase shift keying (DPSK) relying on low-complexity non-coherent detection constitutes an attractive solution for wireless communications, especially in scenarios, such as for example, cooperative communications, since it is robust against the phase ambiguities induced by rapid fading, while dispensing with complex timing recovery and channel estimation for mobile-to-mobile links. For the sake of transmitting an increased number of bits/symbol, differential amplitude and phase shift keying (DAPSK) [1−3] was proposed, which expands the single-ring constellation of traditional DPSK to multiple rings. Essentially, the information is encoded both in the amplitude- and phase-differences between successively transmitted symbols.

In order to enhance the maximum achievable error-free transmission rate for a given DAPSK modulation as well as to eliminate the typical emergence of an error-floor at high Doppler-frequencies, the powerful multiple-symbol differential detector (MSDD) has been applied to uncoded DAPSK-modulated systems in [4], which relies on the joint detection of multiple consecutively received symbols. However, when employed in an iterative detection aided channel coded DAPSK-aided system, the maximum-a-posteriori (MAP) soft-decision MSDD [5,6] employing even a moderate observation window size may still exhibit an excessive complexity, since it has to generate soft information based on the brute-force search for every transmitted bit. As a potential complexity reduction technique, the well-known tree-search-based sphere detection (SD) mechanism has been proposed for MSDD of a conventional DPSK modulated system [7]. This solution was termed as multiple-symbol differential sphere detection (MSDSD). Unfortunately, the non-constant-modulus constellation DAPSK precludes the direct application of the MSDSD scheme of [7]. Thus, until now the conception of an efficient MSDD for DAPSK-aided systems remained an open problem.

Against this background, firstly, we close this open problem by proposing an iterative A/P detection framework for MSDSD-aided DAPSK systems; Secondly, the iterative information exchange between the above-mentioned A/P detection stages is specifically tailored for mitigating any potential performance penalty imposed by the separate A/P detection; Thirdly, we incorporate the SD mechanism in this new MSDD for the sake of further complexity reduction. Our simulation results demonstrate a near-MAP-MSDD performance can be achieved at a significantly reduced complexity, which may be five orders of magnitude lower than that imposed by the traditional MAP-MSDD in the 16-DAPSK scenario considered.

II. SYSTEM ARCHITECTURE & CHANNEL MODEL

The simplified overall system model of bit-interleaved coded differential modulation is depicted in Fig. 1, where 16-DAPSK is assumed to be employed without loss of generality. At the transmitter of Fig. 1, a block of $L$ information bits $u$ is first encoded by the channel encoder in order to generate the coded bits $v$, which are then interleaved by the interleaver $\pi$. The resultant permuted bits $b$ are then fed through the DAPSK modulator. The $2^q$-DAPSK, also known as the Star Quadrature Amplitude Modulation (Star-QAM) scheme [1], employs multiple concentric rings by combining the $2^q$-DASK and $2^{(q-p)}$-DPSK modulation schemes. Specifically, as illustrated in Fig. 1, the first $q$ bits, $b^q_1 = [b^q_1, \ldots, b^q_{p-q}]$, of the $n$th $p$-bit encoded DAPSK symbol $d[n] = \gamma[n]v[n]$ are mapped to one of the legitimate radii $R = \{\alpha^n\}$, where $\alpha = 0, \ldots, 2^{(q-1)}$ in order to generate the component ASK symbol $\gamma[n]$, for example, according to the mapping schemes of Table I. Meanwhile, the remaining $(p-q)$ bits, $b^{p-q} = [b^{p-q}_1, \ldots, b^{p-q}_{p-q}]$, are mapped to the component PSK symbol $v[n] = e^{i\phi[n]} \in \mathbb{V} = \{e^{(2\pi p/2^{q-p-1})i|\pi|} = 0, \ldots, 2^{(q-p-1)} - 1\}$. Based on the above ASK and PSK modulation, differential encoding of the resultant APSK symbol $d[n]$ may be performed similarly to the conventional DPSK process in order to generate the DAPSK symbol $x[n] = a[n]s[n]$ as follows:

\[
\begin{align*}
x[n] &= d[n] \oplus x[n-1] = \gamma[n]v[n] \oplus a[n]s[n-1], \\
&= a^{i_{\lambda}(\gamma[n]) + i_{\lambda}(a[n-1])) \mod 2^q} \cdot \exp\left\{\frac{2\pi i|\pi|}{2}{v[n]} + i\{s[n-1]\}/2^{(q-p)}\right\},
\end{align*}
\]

where $i_{\lambda}\{\cdot\}$ and $iv\{\cdot\}$ are the indices of the radius- and phase-arguments, respectively. We note that with the aid of the modulo-$2^q$ operation, the transmitted component DASK symbol $a[n]i$ is restricted to be taken from the same signal set as the ASK symbol $\gamma[n]$, i.e., $a[n] \in A = R$, as usual for DPSK, where we have $s[n] \in S = \mathbb{V}$ due to the inherent periodicity of the phase. For example, the signal constellation set $\mathcal{X}$ of 16-DAPSK ($p = 1, q = 4$) is constituted of two concentric rings of 8-PSK symbols.

For simplicity we consider narrow-band time-selective Rayleigh fading channels, where the fading coefficients have an autocorrelation function of $\varphi[n] \triangleq \mathbb{E}\{h[n] + e^{i\theta[n]}\} = J_0(2\pi f_d\nu)$, according to the widely-used Clarke model [8], with $J_0(\cdot)$ and $f_d$ denoting the zero-order Bessel function of first kind and the normalized Doppler frequency, respectively. Thus, we have the transmission model of $y[n] = h[n]x[n] + w[n]$, where the fading coefficient $h[n]$ and
The AWGN noise $w[n]$ obey a complex Gaussian distribution of $CN(0, \sigma_w^2)$ and of $CN(0, 2\sigma_w^2)$, respectively. Then, the received symbol $y[n]$ is processed by the turbo receiver of Fig. 1 constructed by serially concatenating the differential detector and the channel decoder, and then exchanging extrinsic information between them.

### III. Iterative Amplitude/Phase Multiple-Symbol Differential Detection

Conventional differential detection (CDD) techniques [5] proposed for DAPSK rely on the direct calculation of the amplitude and phase differences, namely on $\Delta[n] = |y[n]|/|y[n-1]|$ and $\Delta\phi[n] = \angle y[n] - \angle y[n-1]$, respectively, between two consecutively transmitted symbols. However, in pursuit of an improved maximum achievable error-free transmission rate and/or an increased resilience against the formation of a high-Doppler-induced error-floor, one has to exploit the correlation between the amplitude and phase distortions experienced by the consecutively transmitted symbols with the aid of multiple-symbol-based detection, i.e., by using multiple-symbol differential detection (MSDD) [4]. As another benefit, it is worthwhile noting that the MSDD is also capable of increasing the iterative gain attained by the turbo receiver in the context of channel-coded systems. This is because the generation of soft-information by the MSDD for the bits within the same detection window benefits from exploiting each other’s improved-confidence reliability information provided by the channel decoder. As a result, the enhanced iterative gain attained by the MSDD-aided turbo receiver may be translated to an increased error-free transmission rate for DAPSK-modulated systems, as it will be demonstrated in the following sections.

#### A. MAP-Based Multiple-Symbol Differential Detection

1) Principle of the MSDD: Basically, the MSDD makes a decision about the $k$th block of the $(N - 1)$ consecutively transmitted DAPSK symbols $x[kN] = \{x[kN], \ldots, x[(k+1)N-1]\}^T$ on the basis of $N$ consecutively received symbols stored in $y[kN] = \{y[kN], \ldots, y[(k+1)N-1]\}^T$. Since each element of $x[kN]$ is the product of the component DASK and DAPSK symbols, we have $x[kN] = a[kN] \cdot s[kN]$ with the vectors $a[kN]$ and $s[kN]$ containing the corresponding $N$ consecutively transmitted constituent DASK and DAPSK symbols, respectively. Thus, a multiple-symbol-based transmission may be ready as mode as follows:

$$y[kN] = X_a[kN] h[kN] + w[kN],$$

where $X_a[kN] = \text{diag}\{x[kN]\}$, $A_a[kN] = \text{diag}\{a[kN]\}$ and $S_a[kN] = \text{diag}\{s[kN]\}$ are all diagonal matrices with their first upper-left element being the reference DAPSK symbol $x[kN(N-1)] \pm \alpha_{ref} \in A$ and the reference component DAPSK symbol $s[kN(N-1)] \pm s_{ref} \in S$, respectively. Additionally, $h[kN] = [h[kN(N-1)], \ldots, h[(k+1)N-1]]^T$ and $w[kN] = [w[kN(N-1)], \ldots, w[(k+1)N-1]]^T$.

2) Complexity of the MAP-MSDD: According to (5) and (9), the asymptotic complexity of the MAP-MSDD of a $2^p$-DAPSK scheme using $2^q$ concentric rings is $O(p \cdot 2^{pN})$. Therefore, employing the brute-force search carried out by the MAP-MSDD, might impose a potentially excessive computational complexity and hence may preclude its practical implementation, especially for high-order modulation schemes and/or for high observation window sizes. For example, under the assumption of an observation window size of $N = 6$ and the 16-DAPSK scheme ($p = 1, q = 4$), the number of evaluations of the PDF $p(y|\Gamma, \Theta)$ of (5) required for each 4-bit-coded symbol is as high as $2^{26} = 6.7109 \times 10^7$. Table I

**Table I**

<table>
<thead>
<tr>
<th>16-DAPSK ($q = 1$)</th>
<th>64-DAPSK ($q = 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a[n]$</td>
<td>$a[n]$</td>
</tr>
<tr>
<td>$b_{n,1}^{0}$</td>
<td>$b_{n,1}^{0}, b_{n,2}^{0}$</td>
</tr>
<tr>
<td>$\gamma[n]$</td>
<td>$\gamma[n]$</td>
</tr>
<tr>
<td>$a[n-1]$</td>
<td>$a[n-1]$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>$\alpha$</td>
</tr>
</tbody>
</table>

where the table values are represented by the sets of $a[n]$ and $b_{n,1}^{0}, b_{n,2}^{0}$, respectively.
where $\mathbb{B}_{n,i}^{0,\pm1}$ denotes the set of $2^q(2^{(p−q)(N−1)−1})$ legitimate phase-modulation-related bit vectors $b_n$ associated with $b_{n,i}^{0,\pm1} = \pm 1 \ (i \in \{1, \cdots, p−q\})$. From the second iteration of the MSDPD process, the phase information feedback switch is toggled to the ‘2’ position, since \( \hat{\Phi} \) in (10) can be computed based on the DPSK processing of the \textit{aposteriori} phase-modulation-related bit LLRs, \( L_D(b_0|y, \hat{\Phi}) \) of (13), delivered by the MSDPD, as observed in Fig. 2, in the interest of exploiting the improved-confidence phase information in the MSDAD detection. We note that both the amplitude ratio and phase difference estimates, i.e., $\Phi$ and $\Theta$, can be obtained in either hard-decision or soft-decision manner from the corresponding \textit{aposteriori} bit LLRs.

In our investigations we found that the conditional autocorrelation matrix $\Psi(X_{\text{ref}})$ is dependent on $x_{\text{ref}}$, but not on $s_{\text{ref}}$. Hence, a further complexity reduction may be achieved by averaging $p(y|X_{\text{ref}})$ over all possible $s_{\text{ref}}$ values instead of $x_{\text{ref}}$; when computing the $p(y|\Phi, \Theta)$ of (5). Thus, the burden of computing $p(y|\Phi, \Theta)$ in (12) and $p(y|\Phi, \Theta)$ in (13) can be reduced by a factor of $2^{(p−q)}$ using

\[
p(y|\Phi, \Theta) = E_{s_{\text{ref}}} \{ p(y|X_{\text{ref}}) \},
\]

instead of using (5). Consequently, the asymptotic complexity per iteration of the proposed IAP MSDD scheme for the 2\(^p\)-DAPSK scheme using $2^q$-concentric-ring constellation is $O(q:2^{p(N−1)}+ (p−q)\cdot 2^{(p−q)(N−1)+q})$. Hence, under the same assumption of $N = 6$ and that of 16-DAPSK, the number of evaluations of (5) required for each symbol in the IAP-MSDD process becomes $2^6 \times 3 \times 2^7 = 1.9667 \times 10^5$ per iteration (99.97% of the total complexity is contributed by the phase-modulation-related bit detection). This total complexity is two orders of magnitude lower than that of the traditional full-search-based MSDD.

IV. COMPLEXITY REDUCTION STRATEGIES FOR THE IAP-MSDD

Although a substantial complexity reduction can be attained, the complexity imposed by the IAP-MSDD of Fig. 2 proposed for the DAPSK scheme may still be deemed to be excessive, as illustrated in Section III-B, thus preventing its implementation in most practical scenarios. Hence, we continue our quest for more efficient complexity reduction techniques designed for the IAP-MSDD, in particular for its computationally more demanding MSDPD stage, which contributes the majority of the total complexity imposed.

A. Estimation of the Transmit-Domain Symbol Amplitude

According to (14), an immediate further complexity reduction by a factor of $2^q$ may be achieved at the MSDPD stage, if the amplitudes $\hat{A}_i$ of the transmitted symbols, instead of the amplitude ratios $\hat{\Phi}$, are estimated on the basis of the \textit{aposteriori} LLRs $L_D(b_0^i|y, \hat{\Phi})$ provided by the MSDAD detector. This is because in the presence of the amplitude estimates $\hat{A}_i$, the MSDPD detector may become capable of approximately computing the \textit{aposteriori} LLRs $L_D(b_0^i|y, \hat{\Phi})$ without averaging over all possible amplitudes of the reference symbol by using

\[
L_D(b_0^i|y, \hat{\Phi}) \approx L_D(b_0^i|y, \hat{A}_i),
\]

\[
= \ln \frac{\sum_b \mathbb{B}_{n,i}^{0,\pm1} p(y|\hat{A}_i, \Theta) \Pr(b_0)}{\sum_{b_0} \mathbb{B}_{n,i}^{0,\pm1} p(y|\hat{A}_i, \Theta) \Pr(b_0)},
\]

where we have

\[
p(y|\hat{A}_i, \Theta) = p(y|\hat{X}_i = \hat{A}_i \times S_d(\Theta)),
\]

which are next delivered to the serially concatenated MSDPD. Similarly, with the aid of the amplitude ratio estimates $\hat{\Phi}$, the \textit{aposteriori} phase-modulation-related bit LLRs $L_D(b_0|y, \hat{\Phi})$ may be computed by the MSDPD as follows:

\[
L_D(b_0|y, \hat{\Phi}) = \ln \frac{\sum_{b_0} \mathbb{B}_{n,i}^{0,\pm1} p(y|\hat{\Phi}, \Theta) \Pr(b_0)}{\sum_{b_0} \mathbb{B}_{n,i}^{0,\pm1} p(y|\hat{\Phi}, \Theta) \Pr(b_0)},
\]
with the diagonal matrix $S_d(\Theta)$ containing the $N$-component DPSK symbols along its diagonal associated with the phase difference information $\Theta$. Bearing in mind the benefit of acquiring the transmitting-domain symbol amplitude estimates, let us now further elaborate on the amplitude estimation procedure, which is constituted by the following two major steps:

Step 1: Estimate the amplitude, $a_{\text{ref}}$, of the reference symbol, namely, the first symbol $x_0$, of the block of $N$ successively transmitted symbols on the basis of the amplitude ratio $\Gamma$ and phase difference estimates $\Theta$, provided by the MSDAD and MSDPD detectors, respectively.

By exploiting the fact that at the transmitter the equi-probable amplitudes of the reference symbol of a specific multiple-symbol block is independent of both the information-carrying amplitude ratios as well as of the phase differences among the successively transmitted symbols, we have $\Pr(a_{\text{ref}} = \alpha^k|\Gamma, \Theta) = \Pr(a_{\text{ref}} = \alpha^k)$, $k \in \{0, \ldots, 2^q - 1\}$. Then, by exploiting Bayes’ theorem, the soft-decision-based amplitude of the reference symbol may be calculated as follows:

$$\hat{a}_{\text{ref}} = \sum_{k=0}^{2^q-1} \alpha^k \cdot \frac{p(y|\hat{a}_{\text{ref}} = \alpha^k, \Gamma, \Theta)}{p(y|\Gamma, \Theta)}$$

$$= \sum_{k=0}^{2^q-1} \alpha^k \cdot \frac{p(y|\hat{a}_{\text{ref}} = \alpha^k, \Gamma, \Theta) \Pr(a_{\text{ref}} = \alpha^k|\Gamma, \Theta)}{\sum_{i=0}^{2^q-1} p(y|\hat{a}_{\text{ref}} = \alpha^i, \Gamma, \Theta) \Pr(a_{\text{ref}} = \alpha^i)}$$

$$= \sum_{k=0}^{2^q-1} \alpha^k \cdot \frac{p(y|\hat{a}_{\text{ref}} = \alpha^k, \Gamma, \Theta)}{\sum_{i=0}^{2^q-1} p(y|\hat{a}_{\text{ref}} = \alpha^i, \Gamma, \Theta)} \hat{a}_{\text{ref}}$$

$$= \sum_{k=0}^{2^q-1} \alpha^k \cdot \frac{p(y|\hat{a}_{\text{ref}} = \alpha^k, \Gamma, \Theta)}{\sum_{i=0}^{2^q-1} p(y|\hat{a}_{\text{ref}} = \alpha^i, \Gamma, \Theta)} \hat{a}_{\text{ref}}$$

where the diagonal matrix $\hat{A}^k_{\text{ref}}$ contains the amplitude estimates of the transmitted symbols along its diagonal associated with its first diagonal element $a_0 = \hat{a}_{\text{ref}} = \alpha^k$. Then, the diagonal elements of $A^k_{\text{ref}}$ may be calculated recursively as:

$$\hat{a}_{n+1} = \sum_{\gamma_{n+1}, a_{n+1} \in \mathcal{A}} (\gamma_{n+1} + a_{n+1}) \Pr(\gamma_{n+1}) \Pr(\hat{a}_{n+1})$$

(19)

where the ASK symbol probability $\Pr(\gamma_{n+1})$ may be readily calculated based on the ASK likelihood ratio (LLR), i.e., on $L_D(\Gamma_{n+1}, y, \Theta)$ of (10) generated by the MSDAD, while the DASK symbol probability $\Pr(\hat{a}_{n+1})$ can be approximately evaluated as:

$$\Pr(\hat{a}_{n+1} = \alpha^k) \approx \frac{1}{\sum_{a \in \mathcal{A}} \Pr(\gamma_{n+1} = a)} \Pr(\gamma_{n+1} = a)$$

for $\hat{a}_{n+1} \leq \alpha^k, k = 0$;

$$\frac{\alpha^{k+1} - \alpha^{k-1}}{\alpha^{k+1} - \alpha^{k-1}} \cdot \frac{\alpha^{k+1} - \alpha^{k+1}}{\alpha^{k+1} - \alpha^{k+1}}$$

for $\alpha^{k-1} \leq \hat{a}_{n+1} \leq \alpha^k$;

$$\frac{\alpha^{k+1} - \alpha^{k-1}}{\alpha^{k+1} - \alpha^{k-1}}$$

for $\alpha^{k-1} < \hat{a}_{n+1} < \alpha^{k+1}$;

$$0$$

for all the other cases,

which essentially reduces the computational complexity imposed by (19), especially when the size of $\mathcal{A}$ is high.

Step 2: Upon obtaining the amplitude estimate for the reference symbol from (18), we estimate the amplitudes of the remaining $(N-1)$ transmitted symbols of the specific multiple-symbol block with the aid of the amplitude ratio estimates $\hat{F}$.

More specifically, in order to generate $\hat{A} \_d$ for the MSDPD detection of (15), the soft-decision-based amplitude calculation criterion of (19) - which was employed when obtaining $\hat{A}^k_{\text{ref}}$ of (18) - is also invoked for recursively computing the diagonal elements of the matrix $\hat{A} \_d$ commencing from the first element $\hat{a}_{\text{ref}}$ of (18).

B. Incorporating a Structured Tree Search in the MSDPD Stage

As another benefit of estimating the amplitudes of the transmitted symbols, an efficiently structured tree search employed by the well-known SD may be incorporated into the computationally demanding MSDPD stage, as detailed in this section. We will demonstrate that this technique is capable of achieving a further significant complexity reduction. Provided that the amplitude estimate matrix $\hat{A} \_d$ has been obtained, we now further elaborate on (17) by reformulating it as follows (the argument $\Theta$ in $S_d(\Theta)$ is omitted for notational simplicity):

$$p(y|\hat{X}_d = \hat{A} \_d S_d) = \exp\left\{-y^H [\Psi(\hat{X}_d)]^{-1} y\right\}$$

(21)

where according to (7) we have:

$$\Psi(\hat{X}_d) = \hat{A} \_d S_d \Sigma_d (\hat{A} \_d S_d)^H + 2\sigma^2_x I_N$$

(22)

$$= S_d (\hat{\Sigma}_d + 2\sigma^2_x I_N) S_d^H$$

(23)

with $\hat{\Sigma}_d \triangleq \hat{A} \_d S_d \hat{A} \_d^H$ being termed as the equivalent channel covariance matrix. Note that for a given $\hat{A} \_d$, the denominator of (21) is independent of $S_d$, since $S_d$ is unitary, i.e., we have $S_d^H = S_d$. Thus, with the aid of the Max-log approximation, the calculation of the a posteriori phase-modulation-related bit LLRs of (16) may be simplified as:

$$L_D(\hat{b}_{\text{ref}}) \approx \max_{b_{\text{ref}} \in \hat{A}^k_{\text{ref}}, \Gamma \in \mathcal{A}} \left\{-y^H [\Psi(\hat{X}_d)]^{-1} y + \ln \Pr(\Theta)\right\}$$

$$= \max_{b_{\text{ref}} \in \hat{A}^k_{\text{ref}}, \Gamma \in \mathcal{A}} \left\{-y^H [\Psi(\hat{X}_d)]^{-1} y + \ln \Pr(\Theta)\right\}$$

(24)

Furthermore, since $S_d$ is unitary and owing to the independence of the elements of $\Theta$, (24) can be reformulated as follows:

$$L_D(\hat{b}_{\text{ref}}) \approx \max_{b_{\text{ref}} \in \hat{A}^k_{\text{ref}}, \Gamma \in \mathcal{A}} \left\{-y^H [\Psi(\hat{X}_d)]^{-1} y + \ln \Pr(\Theta)\right\}$$

(25)

where we have $Y_d \triangleq \text{diag}(y)$ and the lower triangular matrix $L$ satisfying $L^H = \hat{\Sigma}_d + 2\sigma^2_x I_N$ can be obtained by the Cholesky factorization of the symmetric positive definite matrix $(\hat{\Sigma}_d + 2\sigma^2_x I_N)^{-1}$ of (23). By defining the upper triangular matrix $U = L^H Y_d$ and after a few straightforward manipulations, we finally arrive at:

$$L_D(\hat{b}_{\text{ref}}) \approx \max_{b_{\text{ref}} \in \hat{A}^k_{\text{ref}}, \Gamma \in \mathcal{A}} \left\{-y^H [\Psi(\hat{X}_d)]^{-1} y + \ln \Pr(\Theta)\right\}$$

(26)

V. Performance Evaluation and Discussions

In order to visualize the Extrinsic Information Transfer (EXIT) characteristics of the proposed IAP-MSDD scheme, in Fig. 3 we plot the EXIT curves associated with different observation window sizes of $N$ for the IAP-MSDD against those of the CDD and of the traditional MSDD. Under the assumption of the 16-DAPSK modulated system of Fig. 1 and a normalized Doppler frequency of $f_D = 0.01$, the resultant EXIT curves seen in Fig. 3 are obtained by evaluating the extrinsic mutual information (MI), $I_E$, at the output of the specific differential detector for a given input stream of bit
LLRs along with the a priori MI $I_A$ at SNRs of 10 and 14 dB. According to the area properties of the EXIT chart, the upward-shifted EXIT curve of the IAP-MSDSD in Fig. 3 suggests that a significantly higher maximum transmission rate may be achieved in comparison to the CDD assisted system using $N_{\text{wind}} = 2$. The throughput gain achieved by jointly detecting $N > 2$ data symbols using the IAP-MSDSD is also visualized in the 3D plot of Fig. 4, where the maximum achievable throughput of the IAP-MSDSD-aided 16-DAPSK modulated system is depicted versus both the SNR and the ring-ratio $\alpha$. Additionally, the dotted and dot-dashed EXIT curves of Fig. 3 suggest that a compromise may be struck between the maximum achievable rate and the complexity imposed with the aid of a hybrid detection mechanism, namely by the combined conventional differential amplitude detection (CDAD) and MSDPD as well as by the amalgamated MSDAD and conventional differential phase detection (CDPD). Moreover, as implied by the small gap between the EXIT curve of the IAP-MSDSD and that of the traditional MSDD seen in Fig. 3, both the MSDAD and MSDPD of the IAP-MSDSD of Fig. 2 has to be invoked only once, in order to approach the performance of the traditional MSDD. Hence, this observation allows us to set the number of iterations between the MSDAD and MSDPD stages to one in our simulations throughout the paper in order to avoid any unnecessary operations. Thus, remarkably, the complexity imposed by the IAP-MSDSD becomes about five orders of magnitude lower than that of the traditional MSDD in the context of the 16-DAPSK modulation-aided system across a wide range of SNRs, as seen in Fig. 5, where the complexity quantified in terms of the number of transmitted symbol vector candidate enumerations during the differential detection is portrayed versus both the SNR and the ring-ratio $\alpha$. Furthermore, observe from the IAP-MSDSD-related throughput and complexity surfaces plotted in Figs. 4 and 5, respectively, that the ring-ratio $\alpha$ employed by 16-DAPSK plays a crucial role in determining both the system's achievable transmission rate as well as its detection complexity. Specifically, the simulation results seen in Figs. 4 and 5 suggest that setting the ring-ratio to $\alpha \approx 2.0$ constitutes an appropriate choice for maximizing the achievable throughput, while minimizing the complexity imposed by the proposed IAP-MSDSD scheme.

In conclusion, we proposed an IAP-MSDSD scheme for DAPSK modulated systems in this paper, which was shown to be capable of achieving a near-MAP-MSDD performance at a substantially reduced complexity, that was about five orders of magnitude lower than that imposed by the traditional MAP-MSDD in the case of 16-DAPSK.

**REFERENCES**


